2. Integrali indefiniti immediati

$\int x^\alpha\,dx = \frac{x^{\alpha+1}}{\alpha+1} + c \quad (\alpha \neq -1)$ $\int \frac{1}{x}dx = \log|x| + c$

$\int sinxdx = -cosx + c$ $\int cosxdx = sinx + c$

$\int \frac{1}{cos^2x}dx = tgx + c$ $\int \frac{1}{sin^2x}dx = -ctgx + c$

$\int e^x dx = e^x + c$ $\int a^x dx = a^x \log_a e + c$

$\int \frac{1}{\sqrt{1-x^2}}dx = arcsinx + c$ $\int \frac{1}{1+x^2}dx = arctgx + c$

$\int sinhxdx = \int \frac{e^x - e^{-x}}{2}dx = \frac{e^x + e^{-x}}{2} + c = coshx + c$

$\int coshxdx = \int \frac{e^x + e^{-x}}{2}dx = \frac{e^x - e^{-x}}{2} + c = sinhx + c$

Esempi

1. $\int 5x^4\,dx = 5\int x^4\,dx = 5\frac{x^{4+1}}{4+1} + c = x^5 + c$

2. $\int \frac{1}{x^4}dx = \int x^{-4}\,dx = \frac{x^{-4+1}}{-4+1} + c = -\frac{1}{3x^3} + c$

3. $\int 3^{x+2}\,dx = \int 3^2 \cdot 3^x\,dx = 9 \cdot 3^x \log_3 e + c = 3^{x+2}\log_3 e + c$

4. $\int (1 - 2sinx)dx = \int dx - 2\int sinxdx = x + 2cosx + c$

Esercizi

(gli esercizi con asterisco sono avviati)

1. $\int (2x^3 - cosx)dx$ *2. $\int \frac{3+\sqrt{x}}{x}dx$

*3. $\int \frac{5x+4}{x}dx$ *4. $\int \frac{x^3+3x-4}{x-1}dx$

5. $\int \frac{x^3-8}{x-2}dx$ *6. $\int \frac{1-x}{1+\sqrt{x}}dx$

*7. $\int \frac{3x+4\sqrt[3]{x}}{\sqrt{x}}dx$ *8. $\int \frac{x^2+5x+4}{2\sqrt{x}}dx$

*9. $\int \left(e^{x+2} + \frac{1}{x^2}\right)dx$ *10. $\int \frac{x^4+2x^2+2}{1+x^2}dx$

*11. $\int \frac{2cos^3x+5}{cos^2x}dx$ *12. $\int \frac{3-2sin^3x}{sin^2x}dx$

*13. $\int sin^2\frac{x}{2}dx$ * 14. $\int (4x + 4 + tg^2x)\,dx$

15. $\int (2x + \frac{1}{\sqrt{x}} + sinhx)dx$

Come immediata generalizzazione della precedente, si ha la seguente tabella

$\int [f(x)]^{\alpha} \cdot f'(x)\, dx = \frac{[f(x)]^{\alpha+1}}{\alpha+1} + c \quad (\alpha \neq -1)$	$\int \frac{f'(x)}{f(x)} dx = \log\lvert f(x)\rvert + c$
$\int sinf(x) \cdot f'(x)dx = -cosf(x) + c$	$\int cosf(x) \cdot f'(x)dx = sinf(x) + c$
$\int \frac{f'(x)}{cos^2 f(x)} dx = tgf(x) + c$	$\int \frac{f'(x)}{sin^2 f(x)} dx = -ctgf(x) + c$
$\int e^{f(x)} \cdot f'(x)dx = e^{f(x)} + c$	$\int a^{f(x)} \cdot f'(x)dx = a^{f(x)} \log_a e + c$
$\int \frac{f'(x)}{\sqrt{1-[f(x)]^2}} dx = arcsinf(x) + c$	$\int \frac{f'(x)}{1+[f(x)]^2} dx = arctgf(x) + c$
$\int sinhf(x) \cdot f'(x)dx = coshf(x) + c$	$\int coshf(x) \cdot f'(x)dx = sinhf(x) + c$

$$\int f'(x)[f(x)]^{\alpha} dx = \frac{1}{\alpha+1}[f(x)]^{\alpha+1} + c \qquad (\alpha \neq -1)$$

Esempi

1. $\int (4x + 3)^5 dx$, poiché $D(4x + 3) = 4$, moltiplichiamo e dividiamo per 4, si ha

$\int (4x + 3)^5 dx = \frac{1}{4}\int (4x + 3)^5 \cdot 4\, dx = \frac{1}{4} \cdot \frac{(4x+3)^6}{6} + c = \frac{(4x+3)^6}{24} + c$

2. $\int \sin(2x) \cdot cos^3(2x)dx$, poiché $D\cos(2x) = -2\sin(2x)$, si ha

$\int \sin(2x) \cdot cos^3(2x)dx = -\frac{1}{2}\int -2\sin(2x) \cdot cos^3(2x)dx = -\frac{1}{2} \cdot \frac{cos^4(2x)}{4} + c = -\frac{cos^4(2x)}{8} + c$

Esercizi

*16. $\int (2x - 1)^3 dx$ *17. $\int x(3 - 4x^2)^2 dx$

*18. $\int x^2(2 + 3x^3)^4 dx$ *19. $\int \frac{x}{(3+4x^2)^6} dx$

*20. $\int \sqrt[3]{4 - x}dx$ *21. $\int \frac{1}{\sqrt{4x-3}} dx$

*22. $\int \sqrt[3]{x^2 + 2x + 1}dx$ *23. $\int cosx \cdot sin^4 x dx$

*24. $\int sinx \cdot \sqrt{cosx}dx$ *25. $\int \frac{cosx}{(2-sinx)^2} dx =$

*26. $\int \frac{sinx}{cos^3 x} dx$ *27. $\int \frac{1-sin^2 x}{sin^4 x} dx$

*28. $\int \frac{1-sinx}{cos^2x}dx$ 29. $\int sinx\sqrt{cosx+1}dx$

*30. $\int sin^3x\,dx$ *31. $\int cos^3x\,dx$

*32. $\int sin^3x\,cos^2xdx$ *33. $\int \frac{1+tg^2x}{cos^2x}dx$

34. $\int \frac{arcsinx}{\sqrt{1-x^2}}dx$ 35. $\int \frac{arcsin^2x}{\sqrt{1-x^2}}dx$

*36. $\int \frac{arcsin^2(x+1)}{\sqrt{-x^2-2x}}dx$ 37. $\int \frac{1}{xlog^2x}dx$

38. $\int \frac{(1+logx)^4}{x}dx$ *39. $\int \frac{logx}{x\sqrt{1-log^2x}}dx$

*40. $\int \frac{1}{x\sqrt[3]{1+logx}}dx$ 41. $\int \frac{log^2(x+2)}{x+2}dx$

*42. $\int tgx\sqrt{log(cosx)}dx$ *43. $\int \frac{arccosx+x}{\sqrt{1-x^2}}dx$

44. $\int sinhx\cdot coshx\,dx$

$$\int \frac{f'(x)}{f(x)}dx = log|f(x)| + c$$

Esempi

1. $\int tgx\,dx = \int \frac{sinx}{cosx}dx = -\int \frac{-sinx}{cosx}dx = -\log|cosx| + c$

2. $\int ctgx\,dx = \int \frac{cosx}{sinx}dx = log|sinx| + c$

3. $\int \frac{x}{1+5x^2}\,dx =$ (tenendo conto che $D(1+5x^2) = 10x$) $= \frac{1}{10}\int \frac{10x}{1+5x^2}dx = \frac{1}{10}\log(1+5x^2) + c$

Esercizi

*45. $\int \frac{2}{3-5x}dx$ *46. $\int \frac{1}{7x-2}dx$

*47. $\int \frac{x^2}{x^3-1}dx$ *48. $\int \frac{x^4}{x^5+1}dx$

*49. $\int \frac{x^3}{3x^4+1}dx$ *50. $\int \frac{x^2+4x}{x^3+6x^2+1}dx$

51. $\int \frac{x^2-1}{x^3-3x}dx$ 52. $\int \frac{-1+x}{3-2x+x^2}dx$

53. $\int \frac{x}{x^2-3}dx$ *54. $\int \frac{x^4}{x-1}dx$

*55. $\int \frac{x^3-8}{x+2}dx$ *56. $\int tg\left(2x+\frac{\pi}{4}\right)dx$

*57. $\int \frac{sin2x}{sin^2x+1}dx$ *58. $\int \frac{1}{tg(3x)}dx$

59. $\int \frac{sinx+cosx}{cosx-sinx}dx$ 60. $\int \frac{cosx}{1+sinx}dx$

61. $\int \frac{sinx}{2-cosx}dx$ 62. $\int \frac{sin2x}{4+sin^2x}dx$

63. $\int \frac{sin2x}{3+cos^2x}dx$ *64. $\int \frac{1}{sin2x}dx$

*65. $\int \frac{1-tgx}{1+tgx}dx$ 66. $\int \frac{1+tg^2x}{tgx}dx$

*67. $\int \frac{tgx}{log(cosx)}dx$ 68. $\int \frac{e^x}{e^x+3}dx$

69. $\int \frac{1}{xlogx}dx$ 70. $\int \frac{1}{\sqrt{x}(\sqrt{x}+1)}dx$

71. $\int \frac{1}{(x^2+1)arctgx}dx$ 72. $\int \frac{3}{x(1-logx)}dx$

73. $\int \frac{1}{\sqrt{1-x^2}arcsinx}dx$ *74. $\int \frac{logx}{x-2xlog^2x}dx$

$$\int f'(x)e^{f(x)}dx = e^{f(x)} + c \qquad \int f'(x)a^{f(x)}dx = a^{f(x)}\log_a e + c$$

Esempi

1. $\int e^{3-2x}dx =$ (tenendo conto che $D(3-2x) = -2) = -\frac{1}{2}\int e^{3-2x}\cdot(-2)dx = -\frac{1}{2}e^{3-2x} + c$

2. $\int \frac{3^{logsinx}}{tgx}dx =$ (tenendo conto che $D(logsinx) = \frac{cosx}{sinx} = \frac{1}{tgx}) = 3^{logsinx}\log_3 e$

Esercizi

75. $\int e^{2x}dx$ *76. $\int \frac{dx}{\sqrt[3]{e^x}}$

77. $\int xe^{-x^2}dx$ 78. $\int \frac{e^{\frac{1}{x}}}{x^2}dx$

79. $\int x^2\, e^{x^3}dx$ 80. $\int x^3e^{-x^4}\,dx$

81. $\int sin2x\, e^{sin^2x}dx$ 82. $\int x\, e^{2x^2-1}dx$

83. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}dx$ 84. $\int cosx\cdot e^{sinx}dx$

*85. $\int \frac{xe^{\sqrt{1+x^2}}}{\sqrt{1+x^2}}dx$ 86. $\int(1+tg^2x)e^{tgx}dx$

87. $\int 2^{2-x} + 3^{4+2x} dx$ 88. $\int \frac{e^{arcsinx}}{\sqrt{1-x^2}} dx$

89. $\int sin4x\, e^{1+cos^2 2x} dx$ **90.** $\int \frac{e^{\frac{x}{x+1}}}{(x+1)^2} dx$

$$\int f'(x) sinf(x) dx = -cosf(x) + c$$

$$\int f'(x) cosf(x) dx = sinf(x) + c$$

$$\int f'(x)[1 + tg^2 f(x)]dx = \int \frac{f'(x)}{cos^2 f(x)} dx = tgf(x) + c$$

$$\int f'(x)[1 + ctg^2 f(x)]dx = \int \frac{f'(x)}{sin^2 f(x)} dx = -ctgf(x) + c$$

Esempi

1. $\int \cos(2x + \frac{\pi}{4})\, dx =$ (poichè $D\left(2x + \frac{\pi}{4}\right) = 2$, moltiplicando e dividendo per 2)=

$= \frac{1}{2}\int 2 \cdot \cos(2x + \frac{\pi}{4})\, dx = \frac{1}{2} sin\left(2x + \frac{\pi}{4}\right) + c$

2. $\int \frac{1}{cos^2(4x+1)} dx =$ (poichè $D(4x + 1) = 4$, moltiplicando e dividendo per 4)=

$= \frac{1}{4}\int \frac{4}{cos^2(4x+1)} dx = \frac{1}{4} tg(4x + 1) + c$

Esercizi

*91. $\int \sin(3x + \frac{\pi}{6})\, dx$ 92. $\int sin(4x)\, dx$

93. $\int cos\left(3x - \frac{\pi}{3}\right) dx$ 94. $\int \cos(2x + \frac{\pi}{4})\, dx$

95. $\int \cos(2 - x)\, dx$ 96. $\int [\sin(4x) + \cos(4x)]\, dx$

97. $\int x \sin(x^2 + 1)\, dx$ *98. $\int sin^2 x dx$

*99. $\int cos^2 x dx$ *100. $\int sin^2 3x dx$

101. $\int cos^2 4x dx$ 102. $\int [1 + tg^2(4x)] dx$

103. $\int \frac{1}{sin^2(2x+5)} dx$ 104. $\int \frac{x}{sin^2(2x^2+1)} dx$

105. $\int \frac{x^2+x}{cos^2(2x^3+3x^2)} dx$ *106. $\int \left(2 + tg^2 \frac{x}{2}\right) dx$

$$\int \frac{f'(x)}{\sqrt{1-[f(x)]^2}}dx = arcsinf(x)+c \qquad \int \frac{f'(x)}{1+[f(x)]^2}dx = arctgf(x)+c$$

Esempi

1. $\int \frac{x^2}{\sqrt{1-4x^6}}dx = \int \frac{x^2}{\sqrt{1-(2x^3)^2}}dx$ (tenendo conto che $D(2x^3)=6x^2$) =

$=\frac{1}{6}\int \frac{6x^2}{\sqrt{1-(2x^3)^2}}dx = \frac{1}{6}arcsin(2x^3)+c$

2. . $\int \frac{1}{4+x^2}dx = \frac{1}{4}\int \frac{1}{1+\left(\frac{x}{2}\right)^2}dx$ = (tenendo conto che $D\left(\frac{x}{2}\right)=\frac{1}{2}$) $=\frac{1}{2}arctg\frac{x}{2}+c$

Esercizi

*107. $\int \frac{1}{\sqrt{2x-x^2}}dx$ 108. $\int \frac{1}{x\sqrt{1-log^2x}}dx$

*109. $\int \frac{x^3+x}{\sqrt{1-x^4}}dx$ *110. $\int \frac{x}{1+16x^4}dx$

111. $\int \frac{1}{25+x^2}dx$ *112. $\int \frac{1}{26+10x+x^2}dx$

113. $\int \frac{e^x}{\sqrt{1-e^{2x}}}dx$ 114. $\int \frac{1}{x\sqrt{1-log^2x}}dx$

*115. $\int \frac{sinx\cdot cosx}{\sqrt{2-sin^4x}}dx$ 116. $\int \frac{2e^x}{e^{2x}+1}dx$

117. $\int \frac{logx}{x(1+log^4x)}dx$

Soluzioni

Integrali indefiniti immediati

1. S. $\frac{x^4}{2}-senx+c$

***2. S.** $\int \frac{3+\sqrt{x}}{x}dx = 3\int \frac{1}{x}\,dx + \int x^{-\frac{1}{2}}dx = 3\log|x| + \frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}+c = 3\log|x| + 2\sqrt{x}+c$

***3. S.** $\int \frac{5x+4}{x}dx = 5\int dx + 4\int \frac{1}{x}dx = 5x+4\log|x|+c$

***4. S.** (tenendo conto che $x^3+3x-4=(x-1)(x^2+x+4)$)

$\int \frac{(x-1)(x^2+x+4)}{x-1}dx = \frac{1}{3}x^3+\frac{1}{2}x^2+4x+c$; ;

5. S. $\frac{1}{3}x^3+x^2+4x+c$;

***6. S.** $x-\frac{2}{3}x\sqrt{x}+c$; $\left(1-x=\left(1+\sqrt{x}\right)\left(1-\sqrt{x}\right)\right)$;

***7. S.** $\int \frac{3x+4\sqrt[3]{x}}{\sqrt{x}}dx = \int \left(3x^{1-\frac{1}{2}} + 4x^{\frac{1}{3}-\frac{1}{2}}\right)dx = \cdots 2x\sqrt{x} + \frac{24}{5}\sqrt[6]{x^5}$+c

***8.S.** $\int \frac{x^2+5x+4}{2\sqrt{x}}dx = \frac{1}{2}\int \left(x^{2-\frac{1}{2}} + 5x^{1-\frac{1}{2}} + 4x^{-\frac{1}{2}}\right)dx = \cdots \frac{\sqrt{x}(3x^2+25x+60)}{15} + c$

***9.S.** $\int \left(e^{x+2} + \frac{1}{x^2}\right)dx = \int (e^2e^x + x^{-2})dx = e^{x+2} - \frac{1}{x} + c$

***10.S.** $\int \frac{x^4+2x^2+2}{1+x^2}dx = \int \frac{(1+x^2)^2+1}{1+x^2}dx = \cdots x + \frac{x^3}{3} + arctgx + c$

***11.S.** $\int \frac{2cos^3x+5}{cos^2x}dx = \int (2cosx + 5\frac{1}{cos^2x})dx = 2sinx + 5tgx + c$

***12. S.** $\int \frac{3-2sin^3x}{sin^2x}dx = \int (-2sinx + 3\frac{1}{sin^2x})dx = 2cosx - 3ctgx + c$

***13. S.** $\frac{1}{2}(x - sinx) + c; \left(sin^2\frac{x}{2} = \frac{1-cosx}{2}\right)$

***14. S.** $\int (4x + 4 + tg^2x)\,dx = \int (4x + 3 + 1 + tg^2x)dx = 2x^2 + 3x + tgx + c$

15. S. $x^2 + 2\sqrt{x} + coshx + c;$

$$\int \boldsymbol{f'(x)[f(x)]^{\alpha}dx} = \boldsymbol{\frac{1}{\alpha+1}[f(x)]^{\alpha+1} + c} \qquad \boldsymbol{(\alpha \neq -1)}$$

***16 .S.** $\int (2x-1)^3dx =$ (tenendo conto che $D(2x-1) = 2$) $= \frac{1}{2}\int 2(2x-1)^3dx =$

$= \frac{1}{8}(2x-1)^4 + c;$

***17.S.** $\int x(3-4x^2)^2dx = -\frac{1}{8}\int -8x(3-4x^2)^2dx = -\frac{1}{24}(3-4x^2)^3 + c;$

***18 . S.** $\int x^2(2+3x^3)^4dx = \frac{1}{9}\int 9x^2(2+3x^3)^4dx = \frac{1}{9}\cdot\frac{(2+3x^3)^5}{5} + c = \frac{(2+3x^3)^5}{45} + c$

***19. S.** $\int \frac{x}{(3+4x^2)^6}dx = \frac{1}{8}\int 8x(3+4x^2)^{-6}dx = -\frac{1}{40}\frac{1}{(3+4x^2)^5} + c$

***20. S.** $\int \sqrt[3]{4-x}dx = -\int -(4-x)^{\frac{1}{3}}dx = \cdots -\frac{3}{4}(4-x)\sqrt[3]{4-x} + c;$

***21. S.** $\int \frac{1}{\sqrt{4x-3}}dx = \frac{1}{4}\int 4(4x-3)^{-\frac{1}{2}}dx = \cdots \frac{1}{2}\sqrt{4x-3} + c;$

***22. S.** $\int \sqrt[3]{x^2+2x+1}dx = \int (x+1)^{\frac{2}{3}}dx = \cdots \frac{3}{5}(x+1)\cdot\sqrt[3]{(x+1)^2} + c;$

***23. S.** $\int cosx\cdot sin^4xdx =$ (si osserva che $Dsinx = cosx$) $= \frac{1}{5}sin^5x + c;$

***24. S.** $\int sinx\cdot\sqrt{cosx}dx$ = (si osserva che $Dcosx = -sinx$) $= -\frac{2}{3}cosx\cdot\sqrt{cosx} + c;$

***25. S.** $\int \frac{cosx}{(2-sinx)^2}dx = -\int -cosx(2-sinx)^{-2}dx = \cdots = \frac{1}{2-sinx} + c;$

***26.S.** . $\int \frac{sinx}{cos^3x}dx = -\int -sinx\cdot cos^{-3}xdx = \frac{1}{2cos^2x} + c = \frac{sin^2x+cos^2x}{2cos^2x} + c = \frac{1}{2}tg^2x + \frac{1}{2} + c =$
$= \frac{1}{2}tg^2x + c';$

***27. S.** $\int \frac{1-sin^2x}{sin^4x}dx = \int \frac{cos^2x}{sin^2x}\cdot\frac{1}{sin^2x}dx = \cdots = -\frac{ctg^3x}{3} + c$

***28. S.** $\int \frac{1-sinx}{cos^2x}dx = \int(\frac{1}{cos^2x} - \frac{sinx}{cos^2x}dx) = \cdots = tgx - \frac{1}{cosx} + c$

29. S. $-\frac{2}{3}(cosx+1)\sqrt{cosx+1} + c;$

***30. S.** $\int sin^3x\,dx = \int sin^2x \cdot sinxdx = \int(1-cos^2x)\cdot sinxdx = -cosx + \frac{cos^3x}{3} + c$

***31. S.** $\int cos^3x\,dx = \int cos^2x \cdot cosxdx = \int(1-sin^2x)\cdot cosxdx = sinx - \frac{sin^3x}{3} + c$

***32. S.** $\int sin^3x\,cos^2xdx = \int(1-cos^2x)\cdot cos^2x \cdot sinxdx = \ldots = -\frac{cos^3x}{3} + \frac{cos^5x}{5} + c$

***33. S.** $\int \frac{1+tg^2x}{cos^2x}dx = \int \frac{1}{cos^2x} + \frac{tg^2x}{cos^2x}dx = tgx + \frac{tg^3x}{3} + c$

34. S. $\frac{1}{2}arcsin^2x + c;$ **35. S.** $\frac{1}{3}arcsin^3x + c;$

***36. S.** $\int \frac{arcsin^2(x+1)}{\sqrt{-x^2-2x}}dx = \int \frac{arcsin^2(x+1)}{\sqrt{1-(x^2+2x+1)}}dx = \int \frac{arcsin^2(x+1)}{\sqrt{1-(x+1)^2}}dx = \frac{arcsin^3(x+1)}{3} + c$

37. S. $-\frac{1}{logx} + c;$ **38. S.** $\frac{(1+logx)^5}{5} + c$

***39. S.** $\int \frac{logx}{x\sqrt{1-log^2x}}dx = -\frac{1}{2}\int(1-log^2x)^{\frac{-1}{2}}(-2logx)\frac{1}{x}dx = -\sqrt{1-log^2x} + c;$

***40. S.** $\int \frac{1}{x\sqrt[3]{1+logx}}dx = \int \frac{1}{x}(1+logx)^{-\frac{1}{3}}dx = \cdots \frac{3}{2}\sqrt[3]{(1+logx)^2}$ +c

41.S. $\frac{1}{3}log^3(x+2) + c;$

***42. S.** $\int tgx\sqrt{log(cosx)}dx =$ $\left(\text{poichè } Dlog(cosx) = -\frac{sinx}{cosx} = -tgx\right) =$

$= -\frac{2}{3}log(cosx)\sqrt{log(cosx)} + c;$

***43.S.** $\int \frac{arccosx+x}{\sqrt{1-x^2}}dx = -\int -\frac{arccosx}{\sqrt{1-x^2}}dx - \frac{1}{2}\int -2x(1-x^2)^{-\frac{1}{2}}dx = \cdots =$

$= -\frac{1}{2}arccos^2x - \sqrt{1-x^2} + c;$

44. S. $\frac{1}{2}cosh^2x + c;$

$$\int \frac{\boldsymbol{f'(x)}}{\boldsymbol{f(x)}}\boldsymbol{dx} = \boldsymbol{log|f(x)| + c}$$

***45.S.** $\int \frac{2}{3-5x}dx =$ (D$(3-5x) = -5$, moltiplicando e dividendo per -5) $= -\frac{2}{5}log|3-5x| + c;$

***46. S.** $\int \frac{1}{7x-2}dx = \frac{1}{7}\int \frac{7}{7x-2}dx = \frac{1}{7}\log|7x-2| + c$

***47. S.** $\int \frac{x^2}{x^3-1}dx =$ (moltiplicare e dividere per 3)$= \frac{1}{3}\int \frac{3x^2}{x^3-1}dx = \frac{1}{3}log|x^3-1| + c;$

***48. S.** $\int \frac{x^4}{x^5+1}dx = \frac{1}{5}\int \frac{5x^4}{x^5+1}dx = \frac{1}{5}\log|x^5+1| + c = log\sqrt[5]{|x^5+1|} + c$

***49. S.** $\int \frac{x^3}{3x^4+1}dx = \frac{1}{12}\int \frac{12x^3}{3x^4+1}dx = \frac{1}{12}\log(3x^4+1)+c$

***50. S.** $\int \frac{x^2+4x}{x^3+6x^2+1}dx = \frac{1}{3}\int \frac{3x^2+12x}{x^3+6x^2+1}dx = \frac{1}{3}\log|x^3+6x^2+1|+c$

51. S. $\frac{1}{3}log|x^3-3x|+c = log\sqrt[3]{|x^3-3x|}+c;$

52. S. $\frac{1}{2}log(x^2-2x+3)+c = log\sqrt{(x^2-2x+3)}+c$

53. S. $\frac{1}{2}log|x^2-3|+c = log\sqrt{|x^2-3|}+c$

***54.S.** $\int \frac{x^4-1+1}{x-1}dx = \int (x+1)(x^2+1)dx + \int \frac{1}{x-1}dx = \frac{1}{4}x^4+\frac{1}{3}x^3+\frac{1}{2}x^2+x+log|x-1|+c$

***55.S.** (dividendo il numeratore per il denominatore si ha : $\frac{x^3-8}{x+2} = x^2-2x+4-\frac{16}{x+2}$)

$\int \left(x^2-2x+4-\frac{16}{x+2}\right)dx = \frac{1}{3}x^3-x^2+4x-16log|x+2|+c;$

***56. S.** $\int tg\left(2x+\frac{\pi}{4}\right)dx = -\frac{1}{2}\int \frac{-2sin\left(2x+\frac{\pi}{4}\right)}{cos\left(2x+\frac{\pi}{4}\right)}dx = -\frac{1}{2}log\left|\cos\left(2x+\frac{\pi}{4}\right)\right|+c$

***57. S.** $\int \frac{sin2x}{sin^2x+1}dx = \int \frac{2sinxcosx}{sin^2x+1}dx = \log(sin^2x+1)+c$

***58. S.** $\int \frac{1}{tg(3x)}dx = \frac{1}{3}\int \frac{3cos3x}{sin3x}dx = \frac{1}{3}log|\sin(3x)|+c$

59. S. $-log|cosx-sinx|+c;$ **60. S.** $log(1+sinx)+c;$ **61. S.** $log(2-cosx)+c;$
62. S. $log(4+sin^2x)+c;$ **63. S.** $-log(3+cos^2x)+c$

***64. S.** $\left(\frac{1}{sin2x} = \frac{sin^2x+cos^2x}{2sinx\cdot cosx} = \frac{1}{2}\left(\frac{sinx}{cosx}+\frac{cosx}{sinx}\right)\right)$

$\int \frac{1}{sin2x}dx = \frac{1}{2}\int \left(\frac{sinx}{cosx}+\frac{cosx}{sinx}\right)dx = \frac{1}{2}(-log|cosx|+log|sinx|) = \frac{1}{2}log|tgx|+c;$

*** 65. S.** $\int \frac{1-tgx}{1+tgx}dx = \int \frac{cosx-sinx}{cosx+sinx}dx = log|sinx+cosx|+c;$

66.S. $log|tgx|+c;$

***67.S.** $\left(\text{poichè } Dlog(cosx) = -\frac{sinx}{cosx} = -tgx\right)\int \frac{tgx}{log(cosx)}dx = -log|log(cosx)|+c;$

68. S. $log(e^x+3)+c;$ **69. S.** $log|logx|+c;$ **70. S.** $2log\left(\sqrt{x}+1\right)+c;$

71. S. $log|arctgx|+c;$ **72. S.** $-3log|1-logx|+c;$ **73.S.** $log|arcsinx|+c;$

***74. S.** $\left(\text{tenendo conto che } D(1-2log^2x) = -4(logx)\frac{1}{x}\right)$

$\int \frac{logx}{x-2xlog^2x}dx = -\frac{1}{4}\int \frac{-4(logx)\frac{1}{x}}{1-2log^2x}dx = -\frac{1}{4}log|2log^2x-1|+c;$

$$\int \boldsymbol{f'(x)e^{f(x)}dx = e^{f(x)}+c} \qquad \int \boldsymbol{f'(x)a^{f(x)}dx = a^{f(x)}\log_a e+c}$$

75.S. $\frac{1}{2}e^{2x}+c$;

***76. S.** . $\int \frac{dx}{\sqrt[3]{e^x}} = \int e^{-\frac{x}{3}}dx = -3\int e^{-\frac{x}{3}}\cdot\left(-\frac{1}{3}\right)dx = -3e^{-\frac{x}{3}}+c;$

77. S. $-\frac{1}{2}e^{-x^2}+c;$ **78. S.** $-e^{\frac{1}{x}}+c;$ **79. S.** $\frac{1}{3}e^{x^3}+c$

80. S. $-\frac{1}{4}e^{-x^4}+c;$ **81.S.** $e^{sin^2x}+c$ **82.S.** $\frac{1}{4}e^{2x^2-1}+c;$

83. S. $2e^{\sqrt{x}}+c$ **84.S.** $e^{sinx}+c;$

* **85. S.**(poichè $D\sqrt{1+x^2}=\frac{x}{\sqrt{1+x^2}}$) $\int \frac{xe^{\sqrt{1+x^2}}}{\sqrt{1+x^2}}dx = e^{\sqrt{1+x^2}}+c$

86.S. $e^{tgx}+c;$ **87. S.** $-2^{2-x}\log_2 e+\frac{1}{2}3^{4+2x}\log_3 e+c$

88. S . $e^{arcsinx}+c$ **89. S.** $-\frac{1}{2}e^{1+cos^2 2x}+c;$ **90. S.** $e^{\frac{x}{x+1}}+c$

$$\int \boldsymbol{f'(x)sinf(x)dx = -cosf(x)+c}$$

$$\int \boldsymbol{f'(x)cosf(x)dx = sinf(x)+c}$$

$$\int \boldsymbol{f'(x)[1+tg^2f(x)]dx = \int \frac{f'(x)}{cos^2f(x)}dx = tgf(x)+c}$$

$$\int \boldsymbol{f'(x)[1+ctg^2f(x)]dx = \int \frac{f'(x)}{sin^2f(x)}dx = -ctgf(x)+c}$$

***91. S**,(poichè $D\left(3x+\frac{\pi}{6}\right)=3$, si ha $\int \sin(3x+\frac{\pi}{6})\,dx = \frac{1}{3}\int \sin(3x+\frac{\pi}{6})\cdot 3dx =$

$= -\frac{1}{3}cos\left(3x+\frac{\pi}{6}\right)+\mathrm{c}$

92. S. $-\frac{1}{4}cos(4x)+c;$ **93. S.** $\frac{1}{3}sin\left(3x-\frac{\pi}{3}\right)+c;$ **94. S.** $\frac{1}{2}sin\left(2x+\frac{\pi}{4}\right)+\mathrm{c}$

95. S. $-sin(2-x)+\mathrm{c};$ **96. S.** $-\frac{1}{4}[\cos(4x)-\sin(4x)]+\mathrm{c};$ **97. S.** $-\frac{1}{2}cos(x^2+1)+\mathrm{c}$

***98.S.** $\int sin^2 xdx = \int \frac{1-\cos(2x)}{2}dx = \frac{1}{2}\int dx - \frac{1}{4}.\int 2\cos(2x)\,dx = \frac{1}{2}x-\frac{1}{4}sin2x+c$

***99. S.** $\int cos^2 xdx = \int \frac{1+\cos(2x)}{2}dx = \cdots = \frac{1}{2}x+\frac{1}{4}sin2x+c$

***100 S.** $\int sin^2 3xdx = \int \frac{1-\cos(6x)}{2}dx$ =...= $\frac{1}{2}x-\frac{1}{12}sin6x+c$

101. S. $\frac{1}{2}x+\frac{1}{16}sin8x+c;$ **102. S.** $\frac{1}{4}tg(4x)+c;$ **103.S.** $-\frac{1}{2}ctg(2x+5)+c$

104. S. $-\frac{1}{4}ctg(2x^2+1)+c;$ **105. S.** $\frac{1}{6}tg(2x^3+3x^2)+c$

***106.S.** $\int\left(2+tg^2\frac{x}{2}\right)dx = \int\left(1+1+tg^2\frac{x}{2}\right)dx = \int dx + 2\int(1+tg^2\frac{x}{2})\frac{1}{2}\,dx =$

$=x+2tg\left(\frac{x}{2}\right)+c.$

$$\int \frac{\boldsymbol{f'(x)}}{\sqrt{\mathbf{1}-[\boldsymbol{f(x)}]^2}}\boldsymbol{dx}=\boldsymbol{arcsinf(x)}+\boldsymbol{c} \qquad \int \frac{\boldsymbol{f'(x)}}{\mathbf{1}+[\boldsymbol{f(x)}]^2}\boldsymbol{dx}=\boldsymbol{arctgf(x)}+\boldsymbol{c}$$

***107. S.** $\int \frac{1}{\sqrt{2x-x^2}}dx=\int \frac{1}{\sqrt{1-(1-2x+x^2)}}dx=\int \frac{1}{\sqrt{1-(x-1)^2}}dx=\arcsin(x-1)+c$

108. S. $\arcsin logx+c$

***109. S.** $\int \frac{x^3+x}{\sqrt{1-x^4}}dx=\int \frac{x^3}{\sqrt{1-x^4}}dx+\int \frac{x}{\sqrt{1-x^4}}dx=..=\ -\frac{1}{2}\sqrt{1-x^4}+\frac{1}{2}arcsinx^2+c$

***110.S.** $\int \frac{x}{1+16x^4}dx=\frac{1}{8}\int \frac{8x}{1+(4x^2)^2}dx=\frac{1}{8}arctg(4x^2)+c$

111. S. $\frac{1}{5}arctg\left(\frac{x}{5}\right)+c$

***112. S.** $\int \frac{1}{26+10x+x^2}dx=\int \frac{1}{1+25+10x+x^2}dx=\cdots=arctg(x+5)+c$

113.S. $arcsin(e^x)+c;$ **114.S.** $arcsin(logx)+c;$

***115.S.** $\int \frac{sinx\cdot cosx}{\sqrt{2-sin^4x}}dx=\frac{1}{\sqrt{2}}\int \frac{sinx\cdot cosx}{\sqrt{\left[1-\left(\frac{sin^2x}{\sqrt{2}}\right)^2\right]}}dx=\frac{1}{2}arcsin\left(\frac{\sqrt{2}sin^2x}{2}\right)+c;$

116. *S.* $2arctg(e^x)+c;$ **117. S.** $\frac{1}{2}arctg(log^2x)+c.$