## 5. Limiti notevoli

I limiti notevoli, già considerati per le funzioni, valgono anche per le successioni.

1) Il numero ***e***

$$e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$$

2) $\lim\limits_{n\to\infty} n\sin\left(\frac{1}{n}\right) = 1$ ( porre $\frac{1}{n} = t$ e ricordare che $\lim\limits_{t\to 0}\frac{\sin t}{t} = 1$)

3) $\lim\limits_{n\to\infty} n^2\left(1-\cos\left(\frac{1}{n}\right)\right) = \frac{1}{2}$ ;

4) $\lim\limits_{n\to\infty} n\log\left(1+\frac{1}{n}\right) = 1$

5) $\lim\limits_{n\to\infty} n\left(e^{\frac{1}{n}}-1\right) = 1$

## Esercizi

**( gli esercizi con asterisco sono avviati )**

*Calcolare i seguenti limiti*:

1) $\lim\limits_{n\to\infty}(4n^3-n^2)$

2) $\lim\limits_{n\to\infty}(5+4n-2n^3)$

3) $\lim\limits_{n\to\infty}\frac{5n-2}{n-1}$

4) $\lim\limits_{n\to\infty}\frac{n^2-2}{2n-1}$

5) $\lim\limits_{n\to\infty}\frac{9n-2}{4-3n}$

6) $\lim\limits_{n\to\infty}\frac{n^2+2}{n^3-1}$

7) $\lim\limits_{n\to\infty}\frac{\sqrt{n}+5n^2}{(n-2)^2}$

8) $\lim\limits_{n\to\infty}\frac{\sqrt[3]{n}-3n^2+n^3-1}{(1-n)^3}$

9) $\lim\limits_{n\to\infty}\sqrt{\frac{4n}{9n+1}}$

*10) $\lim\limits_{n\to\infty}\frac{\cos n}{n}$

11) $\lim\limits_{n\to\infty}\frac{n}{\sin n}$

*12) $\lim\limits_{n\to\infty}\frac{\sqrt[3]{n^2}+\cos(n!)}{2+n}$

*13) $\lim\limits_{n\to\infty}\sqrt[n]{\frac{3n}{7n+1}}$

14) $\lim\limits_{n\to\infty}\frac{e^{-n}-n}{3+4n}$

*15) $\lim\limits_{n\to\infty}\left(\sqrt{n^2-2}-\sqrt{3+2n^2}\right)$

*16) $\lim\limits_{n\to\infty}\frac{1}{\sqrt{n+2}-\sqrt{n-1}}$

17) $\lim\limits_{n\to\infty}\frac{1}{\sqrt[3]{n^2-3n}+n}$

*18) $\lim\limits_{n\to\infty}\frac{n+1}{\sqrt{n^2+3n}}$

19) $\lim\limits_{n\to\infty}\frac{1}{\sqrt{3n-2}-\sqrt{n}}$

20) $\lim\limits_{n\to\infty}\frac{1}{\sqrt{n+2}-\sqrt{n}}$

21) $\lim\limits_{n\to\infty} \frac{1}{\sqrt[3]{n^2-3n}+n}$

22) $\lim\limits_{n\to\infty} \frac{\sqrt{n^2+1}}{3n+2}$

23) $\lim\limits_{n\to\infty} \frac{\sqrt{4n^2+1}}{\sqrt[3]{n^3-3n}+n}$

*24) $\lim\limits_{n\to\infty} nsin\left(\frac{-2}{n}\right)$

*25) $\lim\limits_{n\to\infty} \left[1-cos\left(\frac{1}{2n}\right)\right]\cdot n^2$

*26) $\lim\limits_{n\to\infty} \frac{sin\left(\frac{1}{n^2}\right)}{sin\left(-\frac{5}{n^2}\right)}$

*27) $\lim\limits_{n\to\infty} \frac{5n+1+\sin(2n)}{3n-1}$

28) $\lim\limits_{n\to\infty} \frac{cosn-2n}{sinn+3n}$

29) $\lim\limits_{n\to\infty} (4e^{\frac{2}{n-1}} - log\frac{en+1}{n})$

30) $\lim\limits_{n\to\infty} (2^{\frac{2n}{n-1}} - \log_{\frac{1}{3}}\frac{9n+1}{n})$

*31) $\lim\limits_{n\to\infty} \frac{n-arctgn}{n!}$

32) $\lim\limits_{n\to\infty} \log_{\frac{1}{3}}\left(\frac{1}{n}-\frac{1}{n^2}\right)$

*33) $\lim\limits_{n\to\infty} \frac{e^{n^2+n}}{e^{n^2-n}}$

*34) $\lim\limits_{n\to\infty}(arctg\frac{n+1}{n-1} - cos\pi\left(\frac{n+1}{n^2}\right))$

35) $\lim\limits_{n\to\infty} \left(log^2_{\frac{1}{2}}n - log_{\frac{1}{2}}n\right)$

36) $\lim\limits_{n\to\infty} (log(n^2+1) - 2logn)$

37) $\lim\limits_{n\to\infty} \left(arctg(n) - arctg(-n)\right)$

38) $\lim\limits_{n\to\infty} \left(arcsin\left(\frac{1}{n}\right) + arccos\left(\frac{1}{n}\right)\right)$

*39) $\lim\limits_{n\to\infty} \left(\frac{n+1}{n}\right)^{2n+3}$

40) $\lim\limits_{n\to\infty} \left(\frac{n+2}{n}\right)^{n+2}$

41) $\lim\limits_{n\to\infty} \left(1+\frac{1}{4+n}\right)^{1-n}$

42) $\lim\limits_{n\to\infty} \sqrt[n]{\frac{n}{n+1}}$

*43) $\lim\limits_{n\to\infty} \left(\frac{n+4}{n+2}\right)^{n}$

44) $\lim\limits_{n\to\infty} \left(1+\frac{1}{2n}\right)^{3n}$

45) $\lim\limits_{n\to\infty} \left(1-\frac{2}{n+1}\right)^{n}$

46) $\lim\limits_{n\to\infty} \left(1-\frac{7}{n^2}\right)^{n^2+1}$

*47) $\lim\limits_{n\to\infty} \left(\frac{n+3}{n-1}\right)^{2n-1}$

48) $\lim\limits_{n\to\infty} arctgn\cdot\left(1+\frac{1}{n}\right)^{n}$

49) $\lim\limits_{n\to\infty} (n+1)log\left(1+\frac{1}{n+2}\right)$

*50) $\lim\limits_{n\to\infty} n^2log\left(1+\frac{1}{n^2}\right)$

51) $\lim\limits_{n\to\infty} \left(1+\frac{1}{n^3+1}\right)^{\frac{n^3}{2}}$

*52) $\lim\limits_{n\to\infty} \left(nlog(n+4) - nlog(n+1)\right)$

*53) $\lim\limits_{n\to\infty} \frac{log(n+e^n)}{n}$

*54) $\lim\limits_{n\to\infty} \frac{log(4n^3+1)}{log(8n^2+1)}$

* 55) $\lim\limits_{n\to\infty} \sqrt[n]{3^n+5^n}$

## Soluzioni

**1. S.** $+\infty$ ; **2. S.** $-\infty$ ; **3. S.** $5$ ; **4. S.** $+\infty$;

**5. S.** -3; **6. S.** $0$; **7. S.** $5$ ; **8. S.** – $1$; **9. S.** $\frac{2}{3}$ ;

***10. S.** 0; ( $\forall n \in \mathbb{N}_0$ risulta $-1 < cosn < 1$ pertanto $\lim_{n\to\infty} \frac{cosn}{n} = 0$);

**11. S.** non esiste ;

***12. S.** $0$ ; ( $\forall n \in \mathbb{N}$ $-1 \leq cos(n!) < 1$ e il denominatore è un infinito per $n \to \infty$ di ordine 1 mentre il numeratore è un infinito di ordine $\frac{2}{3}$ ) ;

***13. S.** $1$ ; ( $\lim_{n\to\infty} \sqrt[n]{\frac{3n}{7n+1}} = \lim_{n\to\infty} \left(\frac{3n}{7n+1}\right)^{\frac{1}{n}}$ e poiché $\lim_{n\to} \frac{1}{n} = 0$ risulta $\lim_{n\to\infty} \sqrt[n]{\frac{3n}{7n+1}} = 1$ );

**14. S.** $-\frac{1}{4}$ ;

***15. S.** $-\infty$ ; ($\sqrt{n^2-2} - \sqrt{3+2n^2} = n\left(\sqrt{1-\frac{2}{n^2}} - \sqrt{\frac{3}{n^2}+2}\right) = \cdots$ ) ;

***16. S.** $+\infty$; ( razionalizzando il denominatore si ha $\frac{1}{\sqrt{n+2}-\sqrt{n-1}} = \frac{\sqrt{n+2}+\sqrt{n-1}}{3}$ ... ) ;

**17. S.** $0$;

***18. S.** $1$; ( $\lim_{n\to\infty} \frac{n+1}{\sqrt{n^2+3n}} = \lim_{n\to\infty} \frac{n\left(1+\frac{1}{n}\right)}{n\cdot\sqrt{1+\frac{3}{n}}} = \lim_{n\to\infty} \frac{n}{n} = 1$ );

**19. S.** $0$; **20. S.** $+\infty$; **21. S.** $0$; **22. S.** $\frac{1}{3}$ ; **23. S.** $1$;

***24. S.** $-2$; ( $\lim_{n\to\infty} nsin\left(\frac{-2}{n}\right) = \lim_{n\to\infty} -2\frac{sin\left(\frac{-2}{n}\right)}{\frac{-2}{n}} = -2$ tenendo presente che $\lim_{x\to 0} \frac{sinx}{x} = 1$ ) ;

***25. S.** $\frac{1}{8}$ ; ( $\lim_{n\to\infty} \left[1 - cos\left(\frac{1}{2n}\right)\right] \cdot n^2 = \lim_{n\to\infty} \frac{1}{4}\left[1 - cos\left(\frac{1}{2n}\right)\right] \cdot (2n)^2 = \frac{1}{4}\lim_{m\to\infty} \left[1 - cos\left(\frac{1}{m}\right)\right] m^2$ ... );

***26. S.** $-\frac{1}{5}$ ; (per $n \to \infty \Rightarrow sin\left(\frac{1}{n^2}\right) \sim \frac{1}{n^2}$ , $sin\left(-\frac{5}{n^2}\right) \sim -\frac{5}{n^2}$ );

***27. S.** $\frac{5}{3}$ ; ( $\lim_{n\to\infty} \frac{5n+1+\sin(2n)}{3n-1} = \lim_{n\to\infty} \frac{n\left(5+\frac{1}{n}+\frac{sin(2n)}{n}\right)}{n\left(3-\frac{1}{n}\right)} = \frac{5}{3}$ poiché $\lim_{n\to\infty} \frac{sin(2n)}{n} = 0$ ) ;

**28. S.** $-\frac{2}{3}$ ; **29.S.** $3$ ; **30.S.** $6$;

***31.S.** $0$; ( $\lim_{n\to\infty} \frac{n-arctgn}{n!} = \lim_{n\to\infty}\left(\frac{n}{n!} - \frac{arctgn}{n!}\right) = \lim_{n\to\infty}\left(\frac{1}{(n-1)!} - \frac{arctgn}{n!}\right) = 0$ essendo $0 \leq arctgn < \frac{\pi}{2}$;

***32. S.**$+\infty$;

***33. S.**$+\infty$; ( $\lim_{n\to\infty}\frac{e^{n^2+n}}{e^{n^2-n}} = \lim_{n\to\infty} e^{n^2+n-(n^2-n)} = \lim_{n\to\infty} e^{2n} = +\infty$ );

***34. S.** $\frac{\pi}{4}-1$ ; ( $\lim_{n\to\infty}(arctg\frac{n+1}{n-1} - cos\pi\left(\frac{n+1}{n^2}\right) = arctg(1) - \cos(0) = \frac{\pi}{4} - 1$ );

**35. S.** $+\infty$ ; **36. S.** $0$ ; **37. S.** $\pi$ ; **38.S.** $\frac{\pi}{2}$ ;

***39. S.** $e^2$ ; ( $\lim_{n\to\infty}\left(\frac{n+1}{n}\right)^{2n+3} = \lim_{n\to\infty}\left[\left(1+\frac{1}{n}\right)^n\right]^2 \cdot \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^3 = e^2$ ) ;

**40. S.** $e^2$; **41. S.** $e^{-1}$; **42. S.** $1$;

***43. S.** $e^2$; ( $\lim_{n\to\infty}\left(\frac{n+4}{n+2}\right)^n = \lim_{n\to\infty}\left(1+\frac{2}{n+2}\right)^n = \lim_{n\to\infty}\left(1+\frac{1}{\frac{n+2}{2}}\right)^{\frac{n+2}{2}\cdot\frac{2n}{n+2}} = e^2$ );

**44. S.** $e\sqrt{e}$; **45. S.** $e^{-2}$; **46. S.** $e^{-7}$;

***47. S.** $e^8$ ; ( $\frac{n+3}{n-1} = 1 + \frac{4}{n-1}$ , porre $\frac{4}{n-1} = \frac{1}{t}$ ...);

**48. S.** $\frac{\pi}{2}e$; **49.S.** $1$;

***50. S.** $1$ ; ( $\lim_{n\to\infty} n^2 log\left(1+\frac{1}{n^2}\right) = \lim_{n\to\infty} log\left(1+\frac{1}{n^2}\right)^{n^2} = 1$ );

**51. S.** $\sqrt{e}$ ;

***52. S.** $3$ ;

( $nlog(n+4) - nlog(n+1) = log\left(\frac{n+4}{n+1}\right)^n = log\left(1+\frac{3}{n+1}\right)^n = log\left(1+\frac{1}{\frac{n+1}{3}}\right)^{\frac{n+1}{3}\cdot\frac{3n}{n+1}}$ );

***53. S.** $1$ ;( $\lim_{n\to\infty}\frac{log(n+e^n)}{n} = \lim_{n\to\infty}\frac{log\left(e^n\left(\frac{n}{e^n}+1\right)\right)}{n} = \lim_{n\to\infty}\left(\frac{loge^n}{n} + \frac{log\left(\frac{n}{e^n}+1\right)}{n}\right) = 1 + \lim_{n\to\infty}\frac{log\left(\frac{n}{e^n}+1\right)}{n} = 1$

essendo $\lim_{n\to\infty}\frac{n}{e^n} = 0$ );

***54.S.** $\frac{3}{2}$ ; ( $log(4n^3+1) = log\left[4n^3\left(1+\frac{1}{4n^3}\right)\right] =$

$= log4n^3 + log\left(1+\frac{1}{4n^3}\right) \sim 3log4n$ per $n\to\infty$ ; $log(8n^2+1) \sim 2log8n$ ) ;

***55. S.** $5$ ; ( $\lim_{n\to\infty}\sqrt[n]{3^n+5^n} = 5\lim_{n\to\infty}\sqrt[n]{\left(\frac{3}{5}\right)^n+1} = 5$ poiché $\lim_{n\to\infty}\left(\frac{3}{5}\right)^n = 0$ );