2. Regole di derivazione

Derivate delle funzioni elementari	
D k=0 k $\in\mathbb{R}$	
D x=1	
D x^{α}=α $x^{\alpha-1}$ $\alpha \in \mathbb{R}$	**D** $f(x)^{\alpha}$=α$[f(x)]^{\alpha-1}\cdot f'(x)$ $\alpha \in \mathbb{R}$
D $\frac{1}{x}=-\frac{1}{x^2}$	**D** $\frac{1}{f(x)}=-\frac{f'(x)}{[f(x)]^2}$
D$\sqrt{x}=\frac{1}{2\sqrt{x}}$	**D**$\sqrt{f(x)}=\frac{f'(x)}{2\sqrt{f(x)}}$
D$\sqrt[n]{x}=\frac{1}{n\sqrt[n]{x^{n-1}}}$	**D**$\sqrt[n]{f(x)}=\frac{f'(x)}{n\sqrt[n]{[f(x)]^{n-1}}}$
D $sinx = cosx$	**D**$sinf(x) = f'(x)\cdot cosf(x)$
D $cosx = -sinx$	**D**$cosf(x) = -f'(x)\cdot sinf(x)$
D $tgx = 1 + tg^2x = \frac{1}{cos^2x}$	**D**$tgf(x) = f'(x)\cdot[1 + tg^2f(x)] = \frac{f'(x)}{cos^2f(x)}$
D $ctgx = -1 - ctg^2x = -\frac{1}{sin^2x}$	**D** $ctgf(x) = f'(x)\cdot[-1 - ctg^2f(x)] = -\frac{f'(x)}{sin^2f(x)}$
D $log_a x = \frac{1}{x}log_a e$	**D**$log_a f(x) = \frac{f'(x)}{f(x)}log_a e$
D $logx = \frac{1}{x}$	**D**$\log f(x) = \frac{f'(x)}{f(x)}$
D $a^x = a^x loga$	**D** $a^{f(x)} = f'(x)\cdot a^{f(x)}loga$
D $e^x = e^x$	**D** $e^{f(x)} = f'(x)\cdot e^{f(x)}$
D $arcsinx = \frac{1}{\sqrt{1-x^2}}$	**D** $arcsinf(x) = \frac{f'(x)}{\sqrt{1-[f(x)]^2}}$
D $arccosx = -\frac{1}{\sqrt{1-x^2}}$	**D** $arccosf(x) = -\frac{f'(x)}{\sqrt{1-[f(x)]^2}}$
D $arctgx = \frac{1}{1+x^2}$	**D** $arctgf(x) = \frac{f'(x)}{1+[f(x)]^2}$
D $arcctgx = -\frac{1}{1+x^2}$	**D** $arcctgf(x) = -\frac{f'(x)}{1+[f(x)]^2}$

Regole di derivazione	
Prodotto per una costante	**D** $k \cdot f(x) = k \cdot f'(x)$
Somma	**D** $[\, f(x) + g(x)] = f'(x) + g'(x)$
Prodotto	**D** $[f(x) \cdot g(x)] = f'(x)g(x) + f(x) \cdot g'(x)$
Quoziente	**D** $\frac{f(x)}{g(x)} = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{[g(x)]^2}$
Funzione composta	**D** $f[g(x)] = f'[g(x)] \cdot g'(x)$
Funzione elevata a una funzione	**D**$[f(x)]^{g(x)} = [f(x)]^{g(x)} \cdot \left[g'(x) log f(x) + g(x) \cdot \frac{\boldsymbol{f'(x)}}{\boldsymbol{f(x)}}\right]$

Esercizi

(gli esercizi con asterisco sono avviati)

Somma

1) $f(x) = x^3 + cosx$

2) $f(x) = sinx - \sqrt{x}$

3) $f(x) = logx + 2x^4$

***** 4) $f(x) = x\sqrt[3]{x} - tgx$

***** 5) $f(x) = 2^x + \frac{x^2+x}{\sqrt{x}}$

6) $f(x) = e^x + ctgx$

7) $f(x) = \frac{1}{3}x^3 - \frac{x}{\sqrt{x}} + 2\frac{\sqrt[3]{x}}{x^4}$

8) $f(x) = arcsinx + \log_3 x$

9) $f(x) = arctgx - 3^x + sinx$

*10) $f(x) = \frac{x^4+2x^2-x+5}{x}$

11) $f(x) = \frac{3}{x} - \frac{1}{x^3} + 4$

12) $f(x) = arctgx - arcctgx$

Prodotto

Esempi

a) D$[x^4 sinx] = (x^4)' \cdot sinx + x^4 \cdot (sinx)' = 4x^3 sinx + x^4 cosx$

b) D$[x^2 logx] = (x^2)' \cdot logx + x^2 \cdot (logx)' = 2xlogx + x^2 . \frac{1}{x} = 2xlogx + x$

c) D$[e^x sinxcosx] = e^x sinxcosx + e^x cos^2 x - e^x sin^2 x =$

$$\left(\text{ricordando che} \quad sinxcosx = \frac{1}{2}\sin(2x) \quad \text{e} \quad cos^2 x - sin^2 x = \cos(2x) \right) =$$

$$= \frac{1}{2} e^x (sin2x + 2cos2x).$$

Esercizi

13) $f(x) = xe^x$

14) $f(x) = (2 + x^2)sinx$

15) $f(x) = \sqrt{x}tgx + \sqrt[3]{x}$

16) $f(x) = (2x^3 - x)(7x - x^2 + 1)$

17) $f(x) = (1 - x)e^x$

18) $f(x) = x2^x \log_2 x$

19) $f(x) = cosx \log x$

20) $f(x) = x^3 arcsinx$

21) $f(x) = \left(\sqrt{x} + 1\right)(x - x^2)ctgx$

22) $f(x) = \frac{4}{x} sinxlogx$

*23) $f(x) = x^3(\arcsin(x) + \arccos(x))$

24) $f(x) = (2 - 3x)^2(1 - x)^3$

25) $f(x) = (e^x + 2)^2(e^x - 1)$

26) $f(x) = log^2 x$

27) $f(x) = x^2 log^3 x$

28) $f(x) = \sqrt{1 - x^2} arcsinx$

29) $f(x) = (4 - x)\sqrt[3]{2 - x}$

30) $f(x) = (4x - 3x^2)^3$

31) $f(x) = (2 - e^x)^2(1 - e^x)$

32) $f(x) = sin^2 x \cdot cosx + sinx \cdot cos^2 x$

33) $f(x) = (e^\pi + 3)^3(2 - 3x)^2$

Quoziente

Esempi

a) $\mathrm{D}\frac{x-3}{1-x^2} = \frac{(x-3)' \cdot (1-x^2) - (x-3) \cdot (1-x^2)'}{(1-x^2)^2} = \frac{1-x^2-(-2x)(x-3)}{(1-x^2)^2} = \frac{x^2-6x+1}{(1-x^2)^2}$;

b) $\mathrm{D}\frac{logx}{x} = \frac{(logx)' \cdot x - logx \cdot (x)'}{x^2} = \frac{\frac{1}{x} \cdot x - logx}{x^2} = \frac{1-logx}{x^2}$;

c) $\mathrm{D}\frac{1}{(4-3x)^4} = \mathrm{D}(4 - 3x)^{-4} = -4(4 - 3x)^{-4-1}(-3) = \frac{12}{(4-3x)^5}$;

Esercizi

34) $f(x) = \frac{x^2+1}{3x-x^2}$

35) $f(x) = \frac{1}{x^2-x+1}$

36) $f(x) = \frac{sinx-cosx}{sinx+cosx}$

37) $f(x) = \frac{x}{\sqrt{x^2+1}}$

38) $f(x) = \frac{e^x-1}{e^x+1}$

39) $f(x) = \frac{1}{xlog^2 x}$

* 40) $f(x) = \sqrt{\frac{x+2}{x^2-1}}$

41) $f(x) = \frac{e^x(1-x^3)}{x+1}$

42) $f(x) = \frac{logx+1}{logx-1}$

43) $f(x) = \frac{(3x+1)^2}{(2-5x)^3}$

44) $f(x) = \frac{1+x^2}{arctgx}$

45) $f(x) = \frac{x^3}{log^2 x}$

Soluzioni

1.S. $f'(x) = 3x^2 - sinx$; **2. S.** $f'(x) = cosx - \frac{1}{2\sqrt{x}}$; **3. S.** $f'(x) = \frac{1}{x} + 8x^3$;

***4. S.** $f'(x) = \frac{4}{3}\sqrt[3]{x} - \frac{1}{cos^2 x}$; $(f(x) = x^{1+\frac{1}{3}} - tgx = x^{\frac{4}{3}} - tgx, f'(x) = \frac{4}{3}x^{\frac{4}{3}-1} - \frac{1}{cos^2 x} = \cdots)$;

***5. S.** $f'(x) = 2^x log2 + \frac{3x+1}{2\sqrt{x}}$;

$\left(f(x) = 2^x + \frac{x^2}{\sqrt{x}} + \frac{x}{\sqrt{x}} = 2^x + x^{2-\frac{1}{2}} + x^{1-\frac{1}{2}} = 2^x + x^{\frac{3}{2}} + x^{\frac{1}{2}}; f'(x) = 2^x log2 + \frac{3}{2}x^{\frac{1}{2}} + \frac{1}{2}x^{-\frac{1}{2}} = ..\right)$

6. S. $f'(x) = e^x - 1 - ctg^2 x$; **7. S.** $f'(x) = x^2 - \frac{1}{2\sqrt{x}} - \frac{22}{3x^4\sqrt[3]{x^2}}$; **8. S.** $f'(x) = \frac{1}{\sqrt{1-x^2}} + \frac{1}{x}\log_3 e$;

9. S. $f'(x) = \frac{1}{1+x^2} - 3^x log3 + cosx$;

***10. S.** $f'(x) = \frac{3x^4+2x^2-5}{x^2}$;

$\left(f(x) = \frac{x^4}{x} + \frac{2x^2}{x} - 1 + \frac{5}{x} = x^3 + 2x - 1 + \frac{5}{x}; f'(x) = 3x^2 + 2 - \frac{5}{x^2} = \cdots\right)$;

11. S. $f'(x) = \frac{3(1-x^2)}{x^4}$; **12. S.** $f'(x) = \frac{2}{1+x^2}$; **13. S.** $f'(x) = (1+x)e^x$;

14. S. $f'(x) = 2xsinx + (2+x^2)cosx$; **15. S.** $f'(x) = \frac{tgx}{2\sqrt{x}} + \frac{\sqrt{x}}{cos^2 x} + \frac{1}{3\sqrt[3]{x^2}}$;

16. S. $f'(x) = -10x^4 + 56x^3 + 9x^2 - 14x - 1$; **17. S.** $f'(x) = -xe^x$;

18. S. $f'(x) = 2^x \log_2 x + x2^x \log 2 \log_2 x + 2^x \log_2 e$; **19. S.** $f'(x) = -sinx \log x + \frac{cosx}{x}$;

20. S. $f'(x) = 3x^2 arcsinx + \frac{x^3}{\sqrt{1-x^2}}$;

21. S. $f'(x) = \frac{x-x^2}{2\sqrt{x}} ctgx + (1-2x)\left(\sqrt{x}+1\right)ctgx - \frac{(\sqrt{x}+1)(x-x^2)}{sin^2 x}$;

22. S. $f'(x) = \frac{4}{x^2}(-sinxlogx + xcosxlogx + sinx)$;

***23 S.** $f'(x) = \frac{3\pi}{2}x^2$; (ricordiamo che : $\arcsin(x) + \arccos(x) = \frac{\pi}{2}$) ;

24. S. $f'(x) = 3(1-x)^2(2-3x)(5x-4)$; **25. S.** $f'(x) = 3e^{2x}(e^x + 2)$;

26. S. $f'(x) = \frac{2}{x}logx$; **27. S.** $f'(x) = xlog^2 x(2logx + 3)$; **28. S.** $f'(x) = \frac{\sqrt{1-x^2} - x \cdot arcsinx}{\sqrt{1-x^2}}$;

29. S. $f'(x) = \frac{4x-10}{3\sqrt[3]{(2-x)^2}}$; **30. S.** $f'(x) = 6(2-3x)(4x-3x^2)^2$;

31.S. $f'(x) = e^x(2-e^x)(3e^x - 4)$;

32.S. $f'(x) = 2sinx \cdot cos^2 x - sin^3 x + cos^3 x - 2cosx \cdot sin^2 x$;

33. S. $f'(x) = -6(e^\pi + 3)^3(2-3x)$; **34. S.** $f'(x) = \frac{3x^2+2x-3}{(3x-x^2)^2}$; **35. S.** $f'(x) = \frac{1-2x}{(x^2-x+1)^2}$;

36. S. $f'(x) = \frac{2}{(sinx+cosx)^2}$; **37. S.** $f'(x) = \frac{1}{(x^2+1)\sqrt{x^2+1}}$; **38. S.** $f'(x) = \frac{2e^x}{(e^x+1)^2}$;

39. S. $f'(x) = -\frac{logx+2}{x^2 log^3 x}$;

***40. S.** $f'(x) = \frac{-x^2-4x-1}{2(x^2-1)\sqrt{(x+2)(x^2-1)}}$; $\left(f'(x) = \frac{D\left(\frac{x+2}{x^2-1}\right)}{2\sqrt{\frac{x+2}{x^2-1}}} = \frac{\frac{x^2-1-2x(x+2)}{(x^2-1)^2}}{2\sqrt{\frac{x+2}{x^2-1}}} = \cdots \right)$;

41. S. $f'(x) = \frac{-xe^x(x^3+3x^2+3x-1)}{(x+1)^2}$; **42. S.** $f'(x) = \frac{-2}{x(logx-1)^2}$; **43. S.** $f'(x) = \frac{3(3x+1)(5x+9)}{(2-5x)^4}$;

44. S. $f'(x) = \frac{2xarctgx-1}{arctg^2 x}$; **45. S.** $f'(x) = \frac{x^2(3logx-2)}{log^3 x}$;