Neural network for direct and inverse nonlinear Fourier transform

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Motivations

With the ever-increasing demand for the capacity of optical links, the possibility to constructively exploit nonlinear effects in fiber-optic systems using far-looking concepts, such as the nonlinear Fourier transform (NFT), has recently become the subject of research. One of the paramount challenges for the utilization of the NFT for optical signal processing and transmission is the sensitivity of NFT-based systems' performance to the deviations of the optical channels from the idealized model, with ASE noise be one of the main issues. In this work, we propose a neural network (NN) framework feasible to predict the continuous NFT spectrum of optical signals and to perform the inverse NFT for modulation of the signal.

Nonlinear Fourier Transform

The propagation of single light polarization in an optical fiber under some simplifying assumptions is well approximated by the NLSE:

$$i\frac{\partial q}{\partial z} + \frac{1}{2}\frac{\partial^2 q}{\partial t^2} + |q|^2 q = 0. \quad (1)$$

Here $q(z,t)$ is the optical field evolving down the fiber, z is the distance along with the fiber, and t is the time variable in the reference frame co-moving with the group velocity of the light envelope.

We address the direct transformation, which consists in solving the so-called Zakharov-Shabat spectral problem for the localized "potential" $q(z,t)$, i.e. for our signal. The nonlinear spectral parameter $\lambda = \xi + i\eta$, entering this problem, can be understood as the nonlinear analogue of frequency. We write down the direct NFT as

$$\begin{cases} -\partial_t \psi_1 + q(z,t)\psi_2 = i\lambda\psi_1, \\ \partial_t \psi_2 + q^*(z,t)\psi_1 = i\lambda\psi_2, \end{cases} \quad (2)$$

where ψ_i are some auxiliary functions.
The core part of NFT is the calculation of scattering coefficients $a(\xi)$ and $b(\xi)$, defined through $\Psi(t,\lambda)$ as follows:

$$a(\xi) = \lim_{t\to+\infty} \psi_1(t,\xi)e^{i\xi t},$$
$$b(\xi) = \lim_{t\to+\infty} \psi_2(t,\xi)e^{-i\xi t}.$$

The continuous spectrum $r(\xi) = b(\xi)/a(\xi)$ fills the real axis of the ξ-plane and corresponds to the dispersive wave component. The discrete part of the nonlinear spectrum is not addressed in our work here.

Quality metrics

To assess the quality of the NN prediction, we use the following formula defining the relative error for the continuous NFT spectrum:

$$\eta_r(\xi) = \frac{|r_{\text{predicted}}(\xi) - r_{\text{actual}}(\xi)|}{\langle |r_{\text{actual}}(\xi)| \rangle_\xi}, \quad (3)$$

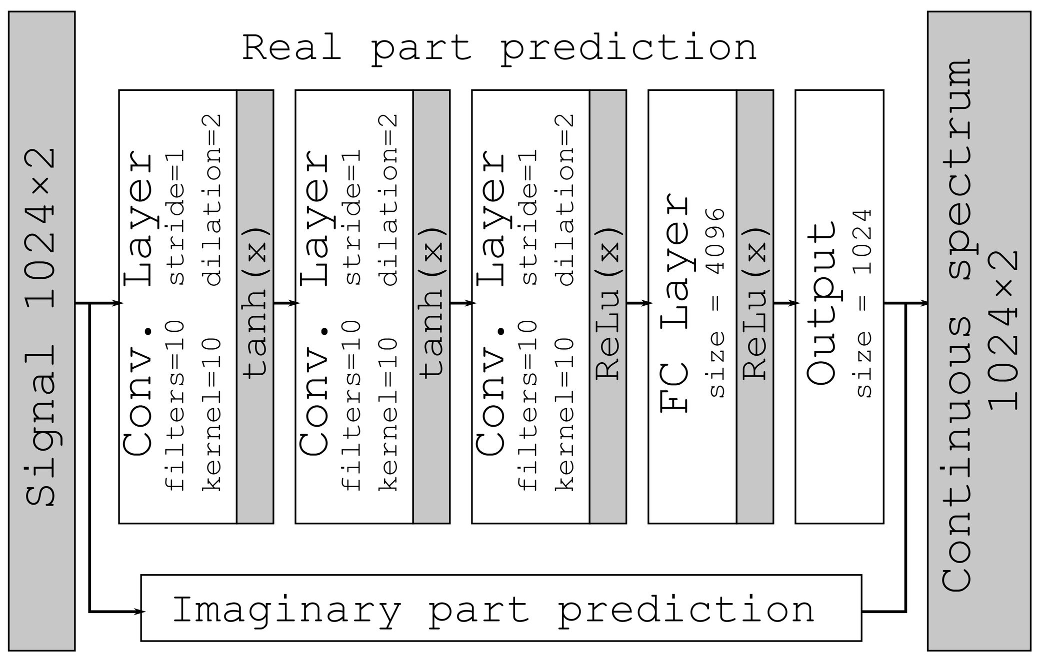
where $\langle\cdot\rangle_\xi$ denotes the mean over the spectral interval, the "predicted" and "actual" indices refer to the NN-predicted and precomputed values of the reflection coefficient $r(\xi)$, respectively. A similar formula for calculating the relative error for signal prediction:

$$\eta_q(t) = \frac{|q_{\text{predicted}}(t) - q_{\text{actual}}(t)|}{\langle |q_{\text{actual}}(t)| \rangle_t}, \quad (4)$$

where $q(t)$ is a signal and $\langle\cdot\rangle_t$ here is the mean over the time interval. The above relative error $\eta_r(\xi)$ and $\eta_q(t)$ is determined at the point ξ and t, respectively. We use $\langle\eta_r(\xi)\rangle_\xi$ and $\langle\eta_q(t)\rangle_t$ to estimate the overall mean of the error.

$$\eta_r = \frac{1}{S}\sum_{i=1}^{S}\langle\eta_{r,i}(\xi)\rangle_\xi, \quad \eta_q = \frac{1}{S}\sum_{i=1}^{S}\langle\eta_{q,i}(t)\rangle_t. \quad (5)$$

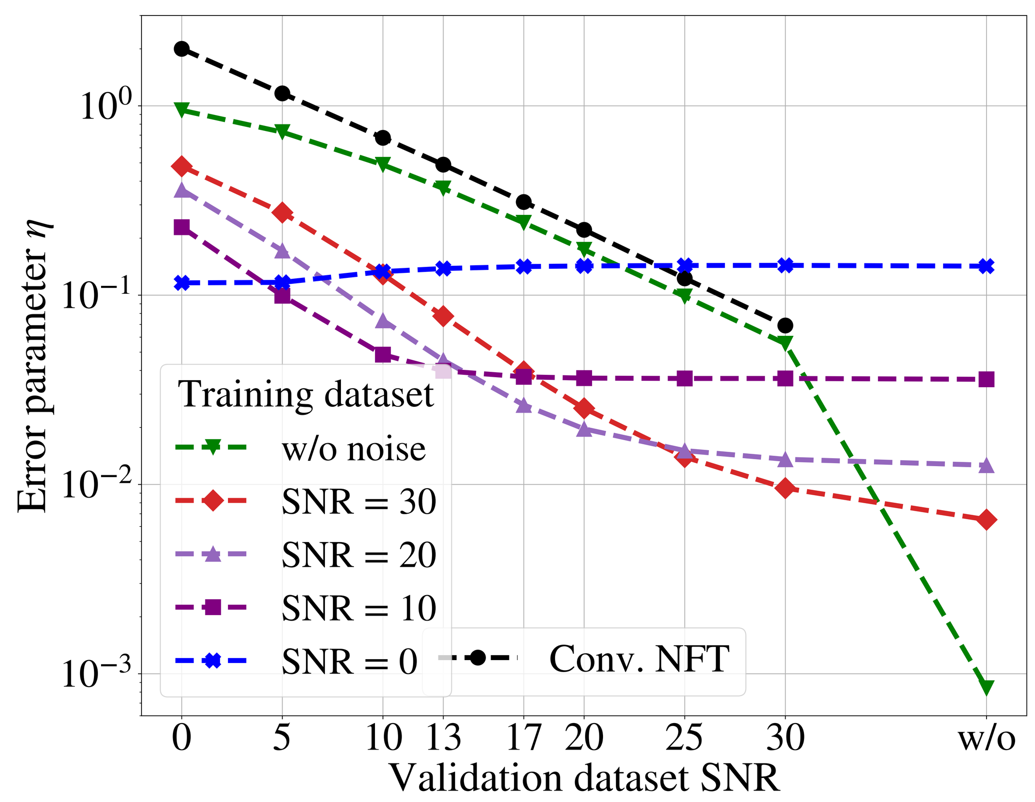
NN for direct NFT



Together with the set of deterministic signals, we generated the signal sets with the addition of uncorrelated Gaussian noise, adding the random value to each sample point. The signal-to-noise ratio (SNR) is a traditionally used characteristic for quantifying the level of a noisy corruption. In addition to the set without noise, which had 84632 signals, we used 8 sets of 423160 signals (5 different noise realisations). Each set corresponds to one of the following SNR values: $\{0, 5, 10, 13, 17, 20, 25, 30\}$ dB. 9 sets of 9403 signals with the corresponding noise levels were left to validate the network performance. Validation data sets were not used in the training process.

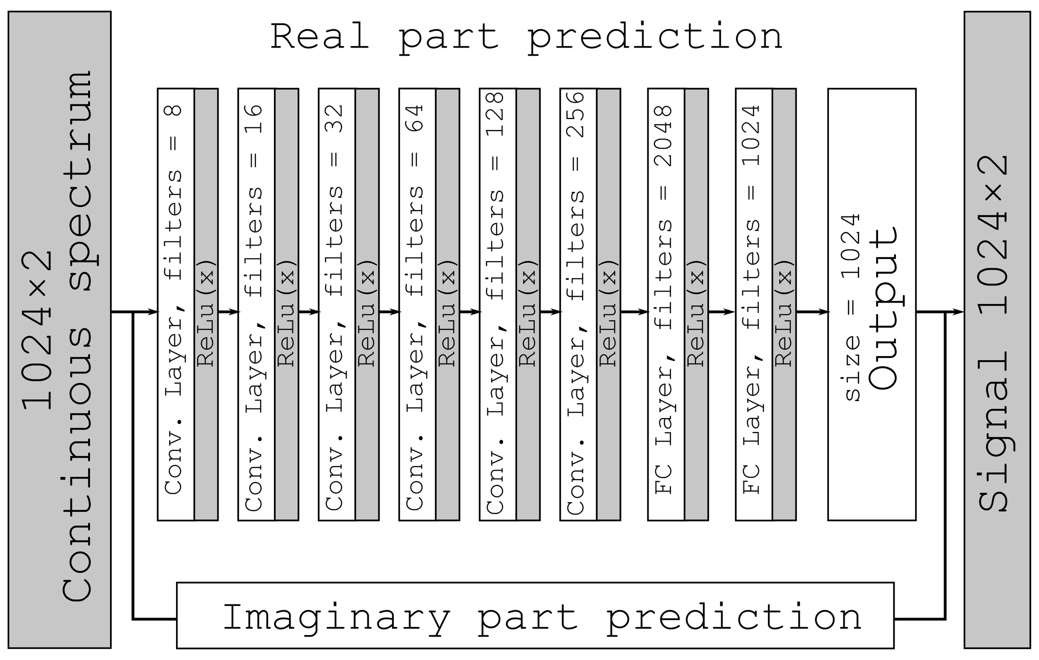
Results for Direct NFT

The values in the cells show the error value (5) for each specific pair of training and validation sets SNR. The gray cells correspond to the cases when the accuracy of the NFT-Net nonlinear spectrum restoration is lower than that of fast NFT, i.e. the NN does not denoise the signal well, while the white cells correspond to the cases when the accuracy of the continuous NF spectrum rendered by the NFT-Net is higher, i.e. the NN effectively denoises the result.



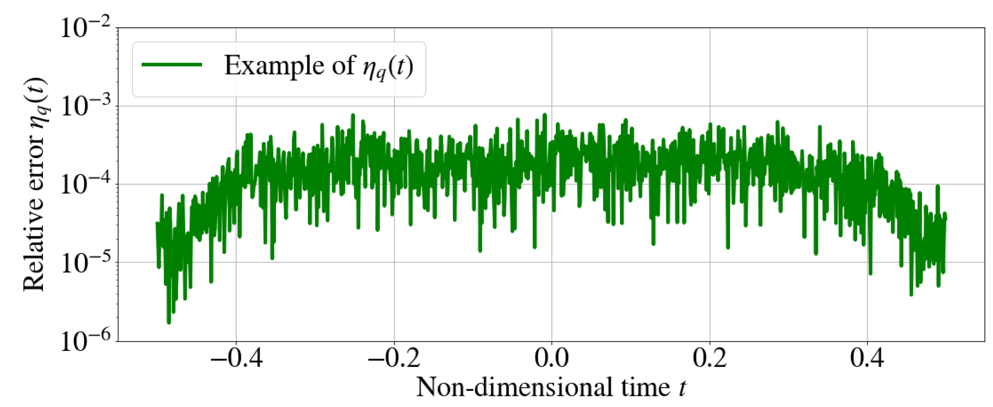
		Training SNR level, dB									
		Conv. NFT	w/o noise	30	25	20	17	13	10	5	0
	w/o noise	0	8.39e-4	6.52e-3	9.43e-3	1.26e-2	1.61e-2	2.38e-2	3.59e-2	7.43e-2	1.42e-1
Validation SNR level, dB	30	6.91e-2	5.54e-2	9.56e-3	1.11e-2	1.36e-2	1.68e-2	2.42e-2	3.63e-2	7.49e-2	1.44e-1
	25	1.23e-1	9.84e-2	1.40e-2	1.39e-2	1.51e-2	1.78e-2	2.45e-2	3.63e-2	7.47e-2	1.43e-1
	20	2.21e-1	1.74e-1	2.53e-2	2.18e-2	1.97e-2	2.08e-2	2.58e-2	3.65e-2	7.40e-2	1.43e-1
	17	3.10e-1	2.41e-1	3.96e-2	3.23e-2	2.63e-2	2.53e-2	2.78e-2	3.70e-2	7.31e-2	1.42e-1
	13	4.89e-1	3.66e-1	7.74e-2	6.12e-2	4.53e-2	3.97e-2	3.54e-2	3.98e-2	7.06e-2	1.38e-1
	10	6.78e-1	4.88e-1	1.29e-1	1.03e-1	7.36e-2	6.23e-2	5.12e-2	4.85e-2	6.87e-2	1.33e-1
	5	1.16e+0	7.26e-1	2.73e-1	2.31e-1	1.72e-1	1.43e-1	1.15e-1	9.93e-2	7.98e-2	1.17e-1
	0	2.00e+0	9.48e-1	4.79e-1	4.37e-1	3.60e-1	3.12e-1	2.59e-1	2.29e-1	1.74e-1	1.16e-1

NN for inverse NFT



Results for Inverse NFT

For the inverse transformation, the mean over the entire validation set of the mean relative signal prediction error $\langle\eta_q(t)\rangle_t = \mathbf{1.62 \cdot 10^{-4}}$.



Conclusion

The proposed NN architectures demonstrates the fundamental possibility of using the NNs to analyze and (de)modulate the complex optical signals used in communications. This opens up the prospects for improving existing systems without the need for a deep understanding of the internal nonlinear processes that affect the quality of signal transmission. We would like to stress that the method proposed in our current work is only the first step in the development of methods for machine processing of optical signals. It can be used to create smart receivers with digital back-propagation algorithms based on NFT and NN. Our findings demonstrate that the use of NN can allow studying not only the internal structure but also generating new signals using autoencoders. The fundamental possibility of using NN for NFT, in fact, can set up new areas for research related to the analysis of inherently nonlinear signal's structure and evolution characteristics. The next step in this direction can be the generalization of the results obtained in our work for the sake of our addressing the case of symbol sequences and simulating signal propagation in the NLSE-type channels [1].

References

[1] Egor V Sedov et al. "Neural networks for computing and denoising the continuous nonlinear Fourier spectrum in focusing nonlinear Schrödinger equation". In: *Scientific Reports* 11.1 (2021), pp. 1–15.

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