Computing Continuous Nonlinear Fourier Spectrum of Optical Signal with Artificial Neural Networks

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Nonlinear Fourier transform (NFT) (also known in the mathematical and nonlinear science community as the inverse scattering transform [1]) has recently attracted a great deal of attention in the context of optical transmission in fiber channels [2], that can be approximated by the nonlinear Schrödinger equation. Within the NFT-based transmission approach, we modulate the parameters of the nonlinear spectrum (NS) and generate the respective information signal in time domain using inverse NFT. Both discrete and continuous parts of NS can be used, here we focus on the continuous spectrum only. Then, the signal is launched into the fiber, and at the receiver we apply direct NFT to the received signal's to retrieve the information encoded in NS. In this work we demonstrate that the *high-accuracy computation of the continuous NS can be performed by using artificial neural networks* (NN). The NS of a given localized signal $q(t)$ containing no solitonic (discrete) components is represented by the continuous complex-valued function $r(\xi)$ (the reflection coefficient) of the real spectral parameter ξ, where the latter plays the role of nonlinear frequency. The signals in our work have been specifically pre-selected to ensure that they contain no discrete spectrum. In the time domain, considered (normalized) symbol is given as the sum of independent sub-carriers [3]: $q(t) = \frac{1}{Q}\sum_{k=1}^{M} C_k e^{i\omega_k t} f(t)$ where M is a number of frequency channels, ω_k is a carrier frequency of the k-th channel, C_k corresponds to the digital data in k-th channel, and T defines the symbol interval; $f(t)$ is the return-to-zero carrier support waveform. To assess the quality of NN prediction, we use the following formula for the relative error: $\eta(\xi) = \frac{\langle |r_{predicted}(\xi) - r_{actual}(\xi)| \rangle}{\langle |r_{actual}(\xi)| \rangle}$, where $\langle \cdot \rangle$ denotes the mean over the spectral interval, the "predicted" and "actual" indices refer to the NN-predicted and precomputed values of the reflection coefficient $r(\xi)$, respectively.

Fig. 1(a) shows the architecture of the NN that performs the NFT operation, with the parameters given inside figure. The NN consists of sequential convolution layers and fully connected output layers. This NN predicts only one component of the continuous NF spectrum, such that two identical NNs have to be used to predict the real and imaginary $r(\xi)$ parts. Fig. 1(b) shows an example of the multi-carrier return-to-zero signal used for our analysis, while Fig. 1(c) depicts $\eta(\xi)$ - the difference between the predicted and actual (precomputed using conventional NFT method) continuous nonlinear spectrum for the example signal.

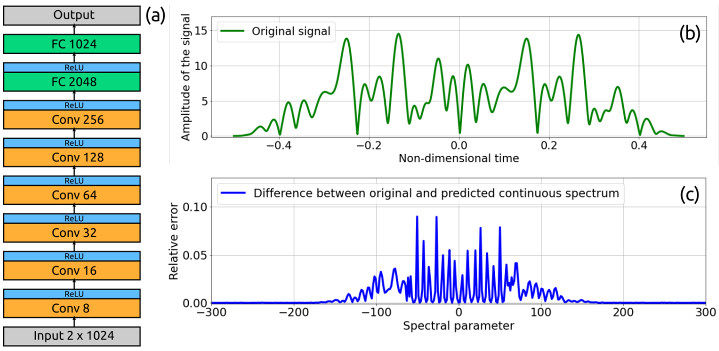

Fig. 1 (a) Neural network architecture. **(b)** Amplitude of the initial signal to decompose. **(c)** Absolute value of the relative error η between the precomputed and predicted continuous spectrum.

To train the NN, we precomputed 94035 signals, with C_k for each carrier drawn randomly from quadrature phase-shift keying (QPSK) constellations; the number of optical channels (carriers) M is 15. 9403 signals were used for validation and excluded from training. The network was trained for 20000 epochs, and the final value of the relative error for the entire validation dataset was less than 0.3%.

In conclusion, we demonstrated that the NNs can successfully perform the complicated nonlinear transformations, such as NFT operation, with good accuracy. Our findings highlight the fundamental possibility of using the NNs to analyze and process complex optical signals, when the conventional algorithms can fail to deliver an acceptable result. We anticipate that our results can be applied in various areas beyond optical communications.

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References

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