ХАРКІВСЬКИЙ НАЦІОНАЛЬНИЙ

УНІВЕРСИТЕТ РАДІОЕЛЕКТРОНІКИ

Кафедра Математики

Звіт

з індивідуального домашнього завдання №1

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ЗАДАЧА 1

3.

1) $\lim\limits_{x\to 3} \dfrac{x^3-8x-3}{x^3-9x^2+27x-27} = \left\{\dfrac{0}{0}\right\} = \lim\limits_{x\to 3} \dfrac{x^3-3x^2+3x^2-9x+x-3}{(x-3)^3}$

$= \lim\limits_{x\to 3} \dfrac{x^2(x-3)+3x(x-3)+1(x-3)}{(x-3)^3} = \lim\limits_{x\to 3} \dfrac{(x-3)(x^2+3x+1)}{(x-3)^3}$

$= \lim\limits_{x\to 3} \dfrac{x^2+3x+1}{(x-3)^2} = \underline{+\infty}$

2) $\lim\limits_{x\to 2} \dfrac{x^2-4}{\sqrt[3]{4x}-2} = \left\{\dfrac{0}{0}\right\} = \lim\limits_{x\to 2} \dfrac{(x^2-4)(\sqrt[3]{16x^2}+2\sqrt[3]{4x}+4)}{4x-8} =$

$\lim\limits_{x\to 2} \dfrac{(x-2)(x+2)(\sqrt[3]{16x^2}+2\sqrt[3]{4x}+4)}{4(x-2)} = \lim\limits_{x\to 2} \dfrac{(x+2)(\sqrt[3]{16x^2}+2\sqrt[3]{4x}+4)}{4} =$

$\dfrac{(2+2)(\sqrt[3]{16\cdot 2^2}+2\sqrt[3]{4\cdot 2}+4)}{4} = \underline{12}$

3) $\lim\limits_{x\to\infty} \dfrac{x^2\sqrt{x}+11x^2}{7x-4\sqrt{x^5-100}} = \lim\limits_{x\to\infty} \dfrac{x^2(\sqrt{x}+11)}{x\left(7-4\frac{\sqrt{x^5-100}}{x}\right)} = \lim\limits_{x\to\infty} \dfrac{x(\sqrt{x}+11)}{7-4\frac{\sqrt{x^5-100}}{x}} =$

$\lim \dfrac{x\cdot\infty}{-\infty} = \underline{-\dfrac{1}{4}}$

4) $\lim\limits_{x\to 0} \dfrac{1+x\sin x-\cos 2x}{\sin^2 x} = \left\{\dfrac{0}{0}\right\} = \lim\limits_{x\to 0} \dfrac{2\times\frac{1-\cos 2x}{2}+x\sin x}{\sin^2 x} =$

$\lim\limits_{x\to 0} \dfrac{2\sin^2 x+x\cdot\sin x}{\sin^2 x} = \lim\limits_{x\to 0} \dfrac{\sin x\cdot(2\sin x+x)}{\sin^2 x} =$

$\lim\limits_{x\to 0} \dfrac{2\sin x+x}{\sin x} = \lim\limits_{x\to 0} 2+\dfrac{x}{\sin x} = 2+1^{-1} = \underline{3}$

5) $\lim\limits_{x \to 0} \left(3 - \frac{2}{\cos 2x}\right)^{ctg^2 2x} = \lim\limits_{x \to 0} \left(3 - \frac{2}{\cos 2x}\right)^{\frac{\cos^2 2x}{4x^2}} = \lim\limits_{x \to 0} \left(1 + \frac{2(\cos 2x - 1)}{\cos 2x}\right)^{\frac{\cos^2 2x}{4x^2}}$

$= \lim\limits_{x \to 0} \left(1 + \frac{2(\cos 2x - 1)}{\cos 2x}\right)^{\frac{(\cos 2x - 1)(\cos 2x)}{2\cos^2}} = \lim\limits_{x \to 0} \left(1 + \frac{2\cos 2x - 1}{\cos 2x}\right)^{\frac{\cos^2 2x}{4x^2}} = \underline{e^{-1}}$

6) $\lim\limits_{x \to 1} \frac{2^x - 2}{\ln x} = \lim\limits_{x \to 1} \frac{\frac{d}{dx}(2^x - 2)}{\frac{d}{dx}(\ln x)} = \lim\limits_{x \to 1} \frac{\ln 2 \cdot 2^x}{\frac{1}{x}} =$

$\lim\limits_{x \to 1} \ln 2 \cdot 2^x \cdot x = \ln 2 \cdot 2^1 \cdot 1 = 2\ln 2 = \underline{1{,}38}$

ЗАДАЧА 1

4) $f(x) = 3^{\frac{x}{1-x}} + 1$

$x = 1$ $D(x)$ невизначена

$f(x)$ неперервна на $(-\infty\ 1) \cdot (1\ +\infty)$

$f(1+0) = \lim\limits_{x \to 1+0} 3^{\frac{x}{1-x}} + 1 = 3 + \infty = +\infty$

$f(1-0) = \lim\limits_{x \to 1-0} 3^{\frac{x}{1-x}} + 1 = 3^{-\infty} + 1 = 1$

В $x = 1$ $f(x)$ має розрив II роду

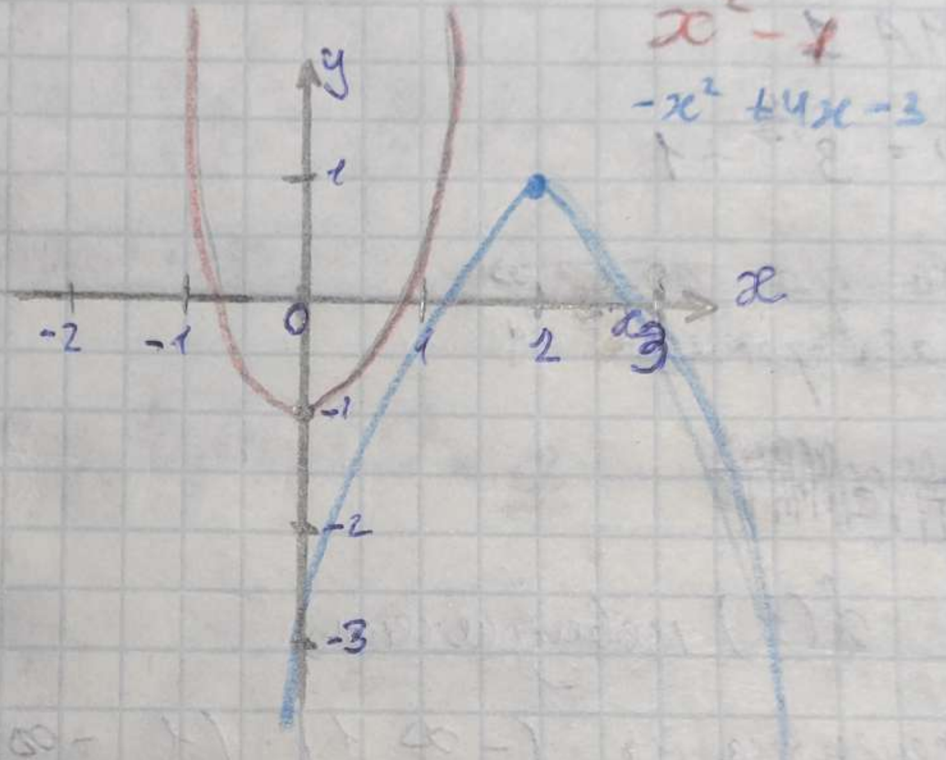
5 $f(x) = \begin{cases} -x^2 + 4x + A & x \leq 1 \\ x^2 - 1 & x > 1 \end{cases}$

$\lim\limits_{x \to 1} -x^2 + 4x + A = 0$

$x^2 - 1 = 0$

$-1 + 4 + A = 0$

$A = -3$



ЗАДАЧА 2

1) $y = x^6\sqrt{4-x^2} - \frac{8}{(4x^2-7)^5}$

$(x^6\sqrt{4-x^2})' = 6x^5 \cdot \sqrt{4-x^2} + x^6\left(-\frac{x}{\sqrt{4-x^2}}\right)$

$\left(-\frac{8}{4(x^2-7)^5}\right)' = 320\frac{x}{(4x^2-7)^6}$

$y' = -\frac{7^x}{\sqrt{4-x^2}} + 6x^5\sqrt{4-x^2} + 320\frac{x}{(4x^2-7)^6}$

2) $y = e^{\cos x^5} \sin^7 x + \operatorname{arctg}^6 7x$

$y' = -5x^4 \cdot e^{\cos x^5} \cdot \sin x^7 \cdot \sin x^5 + 7e^{\cos x^5} \cdot \sin x^6 \cdot \cos x +$
$+ 42\frac{\operatorname{arctg}^5 7x}{49x^2+1}$

3) $y = \ln^5\left(1+\frac{1}{\cos x^4}\right) + \ln^5 \operatorname{tg} x^4$

$y' = \frac{20x^3 \ln^4\left(1+\frac{1}{\cos x^4}\right)\sin x^4}{1+\frac{1}{\cos x^4}\cos x^{4^2}} + 20x^3\frac{\ln^4 \operatorname{tg} x^4}{\cos x^{4^2} \cdot \operatorname{tg} x^4} =$

4) $y = (\operatorname{tg} 7x^4)^{\sqrt{3x+1}}$

$\frac{y'}{y} = (\sqrt{3x+1} \cdot \ln(\operatorname{tg} 7x^4))'$

$y' = (\operatorname{tg} 7x^4)^{\sqrt{3x+1}} \cdot (\sqrt{3x+1} \cdot \ln(\operatorname{tg} 7x^4))'$

$(\sqrt{3x+1}\ln(\operatorname{tg} 7x^4))' = \frac{3}{2\sqrt{3x+1}} \cdot \ln(\operatorname{tg} 7x^4) + \sqrt{3x+1} \cdot 28 \cdot \frac{x^3}{(\cos^2 7x^4) \cdot \operatorname{tg} 7x^4}$

$y' = \frac{28x^3\sqrt{3x+1}}{\cos^2(7x^4) \cdot \operatorname{tg} 7x^4} + 3\frac{\ln(\operatorname{tg} 7x^4)}{2\sqrt{3x+1}} \cdot \operatorname{tg}(7x^4)^{\sqrt{3x+1}}$

5) $x - y + 7\cos y = 0$

$1 - y' - 7\sin y \cdot y' = 0$

$1 - y'(1 + 7\sin y) = 0$

$y' = \frac{1}{1+7\sin y}$

6) $y = e^{\arcsin x}$

$y' = e^{\arcsin x} \cdot \frac{1}{\sqrt{-x^2+1}}$

ЗАДАЧА 3

$t = 3x^2 - 6$

1) $\int \frac{x}{3x^2-6}dx = \int \frac{1}{6t}dt = \frac{1}{6}\int\frac{1}{t}dt = \frac{1}{6}\ln|t| = \frac{1}{6}\ln|3x^2-6| + C$

2) $\int e^{4-5x^2}x\,dx = -\frac{1}{10}\int e^{4-5x^2}dx = -\frac{e^{4-5x^2}}{10} + C$

3) $\int \frac{dx}{\sqrt{5+3x-x^2}} = \int \frac{1}{\sqrt{(x+\frac{3}{2})^2+\frac{11}{4}}}dx = \ln\left|x+\frac{3}{2}+\sqrt{(x+\frac{3}{2})^2+\sqrt{\frac{11}{4}}^2}\right| = \ln\left|x+\frac{3}{2}+\sqrt{x^2+3x+5}\right| + C$

4) $\int e^{2x}\cos x\,dx = \frac{e^{2x}\cos x}{2} - \int \frac{e^{2x}}{2}(-\sin x)\,dx =$

$\frac{\cos x - e^{2x}}{2} + \frac{1}{2}\cdot\left(\sin x\,\frac{e^{2x}}{2} - \frac{1}{2}\int \frac{e^{2x}\cos x}{2}\right)dx$

$\int e^{2x}\cos x\,dx = \frac{\cos x\, e^{2x}}{2} + \frac{\sin x\, e^{2x}}{4} - \frac{1}{4}\int e^{2x}\cos x\,dx$

$\frac{5}{4}\int e^{2x}\cos x\,dx = \frac{2\cos x\, e^{2x}}{2} + \frac{\sin x\, e^{2x}}{4}$

Відповідь: $\frac{2e^{2x}\cos x + \sin x\, e^{2x}}{5} + C$

5) $\int \frac{x^2+23}{(x+1)(x^2+6x+13)}dx = \int \frac{3}{x+1}dx + \int \frac{-2x-16}{x^2+6x+13}dx =$

$= 3\ln|x+1| - \ln||x^2+6x+13|| - 5\,\mathrm{arcctg}\left(\frac{x+3}{2}\right) + C$

6) $\int \frac{\sqrt{x}}{\sqrt[4]{x}+1}dx = \left| u = \sqrt[4]{x}+1 \to \frac{du}{dx} \right| = 4\int \frac{(u-1)^5}{u}du$

$= 4\left(\int u^4 du - 5\int u^3 du + 10\int u^2 du - 10\int u du\right) =$

$4\left(-\ln u + \frac{u^5}{5} - \frac{5u^4}{4} + \frac{10u^3}{3} - 5u^2 + 5u\right) = 20(\sqrt[4]{x}+1) +$

$\frac{4(\sqrt[4]{x}+1)^5}{5} - 5(\sqrt[4]{x}+1)^4 + \frac{40(\sqrt[4]{x}+1)^3}{3} - 20(\sqrt[4]{x}+1)^2 - 4\ln(\sqrt[4]{x}+1)$

8) $\int \frac{dx}{3\cancel{+}\cos x + \sin x} \; \frac{dx}{3 + \frac{1-tg(x/2)^2}{1+tg(x/2)^2} + \frac{2tg(x/2)}{1+tg(x/2)^2}} = \frac{1}{\sqrt{\frac{7}{4}}} \cdot arctg\left(\frac{tg\frac{x}{2} + \frac{1}{2}}{\sqrt{\frac{7}{4}}}\right)$

$= \frac{2\sqrt{7} \cdot arctg\frac{2\sqrt{7} \cdot tg(x/2) + \sqrt{7}}{7}}{7} + C$

9) $\int tg^3 x\, dx = \frac{1}{4}tg^4 x - \int tg^3 x\, dx = \frac{1}{4}tg^4 x - \left(\frac{1}{2}tg^2 x - \int \frac{\sin x}{\cos x}dx\right)$

$= \frac{1}{4}tg^4 x - \left(\frac{1}{2}tg^2 x + \ln|\cos x| d\cos x\right) = \frac{1}{4}tg^4 x - \frac{1}{2}tg^2 x + \ln|\cos x| =$

$= \frac{tg^4 x}{4} - \frac{tg^2 x}{2} - \ln|\cos x| + C$

ЗАДАЧА 3

2)

1) $\int_0^{13} \frac{x+1}{\sqrt{2x+1}}dx = \left| u = 2x+1 \to \frac{du}{dx} \right| = \frac{1}{4}\int \frac{u+1}{\sqrt{u}}du =$

$\frac{1}{4}\left(\int \sqrt{u}\, du + \int \frac{1}{\sqrt{u}}du\right) = \frac{1}{4}\left(\frac{2u^{\frac{3}{2}}}{3} + 2\sqrt{u}\right) =$

$\frac{(2x+1)^{\frac{3}{2}}}{6} + \frac{\sqrt{2x+1}}{2} = \frac{x+2\sqrt{2x+1}}{3} + C =$

$\sqrt{2x+1} - \frac{\sqrt{2x+1} - \frac{(2x+1)^{\frac{3}{2}}}{3}}{2}\Bigg|_0^{13} = 15\sqrt{3} - \frac{2}{3}$

$$2)\ \int_0^{\frac{\pi}{8}} \sin x \sin 3x\, dx = \int \sin x \sin 3x\, dx = \int \frac{1}{2}(\cos 2x - \cos 4x)\, dx$$

$$= \frac{1}{2}\left(\int \cos 2x\, dx - \int \cos 4x\, dx\right) = \frac{1}{2}\left(\frac{\sin 2x}{2} - \frac{\sin 4x}{4}\right) =$$

$$\frac{\sin 2x}{4} - \frac{\sin 4x}{8}\Big|_0^{\frac{\pi}{8}} = \frac{\sin(2 \times \frac{\pi}{8})}{4} - \frac{\sin(4\frac{\pi}{8})}{8} - \left(\frac{\sin(2\cdot 0)}{4} - \frac{\sin(4\cdot 0)}{8}\right) =$$

$$\frac{\sqrt{2}-1}{8}$$

ЗАДАЧА 3

3)

$$1)\ \int_{-\infty}^{-1} \frac{7}{(x^2-4x)\ln 5}\, dx = \lim_{a\to -\infty} \int_a^{-1} \frac{7}{(x^2-4x)\ln 5}\, dx =$$

$$\lim_{a\to -\infty}\left(\frac{7}{4} - \frac{-7\ln|a| + 7\ln|a-4|}{4\ln 5}\right) = \frac{7}{4}$$

$$2)\ \int_1^2 \frac{x}{\sqrt{(x^2-1)^3}\,\ln 2}\, dx = \lim_{a\to 1^+} \int_a^2 \frac{x}{(x^2-1)^{\frac{3}{2}}\ln 2}\, dx =$$

$$\left| \begin{array}{l} u = x^2-1 \to \frac{du}{dx} \\ = 2x \to dx = \frac{1}{2x}du \end{array}\right| = \frac{1}{2\ln 2}\int \frac{1}{u^{\frac{3}{2}}}\, du = -\frac{1}{\ln 2\sqrt{u}} =$$

$$= -\frac{1}{\ln 2\cdot\sqrt{x^2-1}} \quad \boxed{\int \frac{x}{\ln 2\,(x^2-1)^{\frac{3}{2}}}\, dx = -\frac{1}{\ln 2\sqrt{x^2-1}} + C}$$

интеграл расходится!