Stein's Lemma

March 14, 2018

Suppose X is a normally distributed random variable with expectation μ and variance σ^2. Further suppose g is a function for which the two expectations $E(g(X)(X-\mu))$ and $E(g^{'}(X))$ both exist (the existence of the expectation of any random variable is equivalent to the finiteness of the expectation of its absolute value). Then

$$E(g(X)(X-\mu)) = \sigma^2 E(g^{'}(X)) \tag{1}$$

In general, suppose X and Y are jointly normal distributed, Then

$$Cov(g(X), Y) = Cov(X, Y)E(g^{'}(X)) \tag{2}$$