CSCE 222-199 Discrete Structures for Computing – Summer 2020
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Problem Set 6

Due dates: Electronic submission of *yourLastName-yourFirstName-hw6.tex* and *yourLastName-yourFirstName-hw6.pdf* files of this homework is due on **Monday, 6/22/2020 before 11:00 p.m.** on `http://ecampus.tamu.edu`. You will see two separate links to turn in the .tex file and the .pdf file separately. Please do not archive or compress the files. **If any of the two files are missing, you will receive zero points for this homework.**

Name: [redacted]

Resources. For all parts of this question, I utilized the text from Perusall and any notes I took from the videos provided during the week. I also used some youtube tutorials to help me navigate through Latex, and a calculator.

On my honor, as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment. Furthermore, I have disclosed all resources (people, books, web sites, etc.) that have been used to prepare this homework.

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Total 100 points.

Problem 1. (15 points) Section 12.1, Exercise 12.3. Explain. Show your work.

Solution. We know that a password must be 6, 7, or 8 characters long, and the characters are restricted to lowercase letters and decimal digits. There are 26 lower case letters and 10 decimal digits. So not considering any other restrictions, there is a total of (26+10) possible characters so possible combinations would be, $36^6 + 36^7 + 36^8$ or $\sum_{k=6}^{8} 36^k$. However, we have to consider that the starting character of the password must be a lower case letter, hence subtract all the possible invalid passwords. Any invalid password would include if the first character is a decimal digit, hence $\sum_{l=6}^{8} 10^l$. So the possible passwords there could be is $\sum_{k=6}^{8} 36^k$ - $\sum_{l=6}^{8} 10^l$, which is 2.9015399e+12.

Problem 2. (15 points) Section 12.1, Exercise 12.5. Explain. Show your work.

Solution. Lets say the total length is n over the alphabet of k letters is, k^n. the length of a palindrome, k, means the first and last character must be the same. The middle letter can be any character, but the middle two characters must be the same, k^2. For n length word, the equation for the palindromes would be $k^{(n+1)/2}$. So the number of words that are not palindromes is $k^n - k^{(n+1)/2}$.

Problem 3. (10 points) Section 12.2, Exercise 12.7. Explain. Show your work.

Solution. For n number of people, there would be n (n-1)/2 clinks. For every 12 people they would clink glasses with another 11 people, hence (12 x 11) =132 clinks. Since we count twice when the first person clinks with the second person, and counted again when the second person clinked with the first, hence 132/2 =66. Hence there were 66 clinks at the party between a pair of people.

Problem 4. (15 points) Section 12.3, Exercise 12.11.

Solution. a) Using the definition of binomial coefficients:
$\binom{n}{k}k = \frac{n^{\underline{k}}}{k!}k = \frac{n^{\underline{k}}}{(k-1)!}$
$=\frac{n(n-1)^{\underline{k-1}}}{(k-1)!}$
$= n\binom{n-1}{k-1}$
b) Using a combinatorial proof:
Lets take the left side $\binom{n}{k}k$, which is determines the ways of picking a team of k members, form a set of n people. And choosing a captain with each section, k. This is the same as saying theres a number of ways to choose a captain from the set of n people, and another from the set of k-1, and the remaining people from n-1, for the other team members. This is makes the right hand side, $n\binom{n-1}{k-1}$. Thus this proved that $\binom{n}{k}k{=}n\binom{n-1}{k-1}$.

Problem 5. (5 points) Section 12.4, Exercise 12.21 (b). Show your work.

Solution. b) the equation of a permutation is ${}_nP_r = \frac{n!}{(n-r)!}$. n is the length of items and r is how many items are chosen. So in the word spoonfeed, has the length of 9. Hence, n = 9 and since we want all letters, r=9.
Hence, $\frac{9!}{(9-9)!}$
$= \frac{9!}{(0)!}$
= 9! is 362880.

Problem 6. (20 points) Section 12.6, Exercise 12.31. Explain. Show your work.

Solution. The inclusion-exclusion formula includes $|S\backslash S_1 \cup S_2| = |S| - |S_1| - |S_2| + |S_1 \cap S_2|$. In order to find the number of 1's in a page number within 1-500, is a see of 500 integers. First we have the numbers, where the 1's is in the 100's place. Then theres numbers with 1's in the 10's place and ones in the one's place. Then there are ones in both the hundred and tens place, ones that are in hundred and ones place, and ones in tens and ones place, and finally a one in all places.
So, P can be a place holder for all integers that is not a one. (1PP)=100, (P1P)=50, (PP1)=50,(11P)=10,(1P1)=10,(P11)=5, and (111)=1. Hence, using the inclusion-exclusion formula:
=(1PP) + (P1P) +(PP1) - (11P) + (1P1) +(P11) + (111)
= 100+50+50-10-10-5+1 =176
Thus, there are 176 number of pages that include a 1 in it.

Problem 7. (20 points) Section 12.7, Exercise 12.33. Explain. Show your work.

Solution. Given that the area is 400 ft^2, we know that each side is 20 ft. So divide each side to 10 equal pieces, which is 2ft each, so its a 10 x 10 grid. There are 101 dalmatians in the room, but only 100 smaller squares (the 10 x 10 grid), So using the pigeonhole principle there will be at least 2 in one of the same squares (or sharing a square). Within a square, the max distance is between points of the length on a diameter, and the length of the diameter of a square is $\sqrt{2^2+2^2} = \sqrt{8} < \sqrt{9}$ (or 3 ft), which proves the claim that two dalmatians in a small square will be less that 3ft apart.

Checklist:
- ☐ Did you type in your name and UIN?
- ☐ Did you disclose all resources that you have used?
  (This includes all people, books, websites, etc. that you have consulted)
- ☐ Did you sign that you followed the Aggie honor code?
- ☐ Did you solve all problems?
- ☐ Did you submit the .tex and .pdf files of your homework to the correct link on eCampus?