CSCE 222-199 Discrete Structures for Computing – Summer 2020
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Problem Set 2

Due dates: Electronic submission of *yourLastName-yourFirstName-hw2.tex* and *yourLastName-yourFirstName-hw2.pdf* files of this homework is due on **Thursday, 6/4/2020 before 11:00 p.m.** on `http://ecampus.tamu.edu`. You will see two separate links to turn in the .tex file and the .pdf file separately. Please do not archive or compress the files. **If any of the two files are missing, you will receive zero points for this homework.**

Name: [redacted] **UIN:** [redacted]

Resources. For all parts of this question, I utilized the text from Perusall and any notes I took from the videos provided during the week. I also used some youtube tutorials to help me navigate through Latex.

On my honor, as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment. Furthermore, I have disclosed all resources (people, books, web sites, etc.) that have been used to prepare this homework.

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Total 100 points.

Problem 1. (10 points) Section 2.4, Exercise 2.20 (a)

Solution. Modus Tollens:
$A \to B$
$\neg B$
$\therefore \neg A$
$A \to B$ and $\neg B$ are the premises and $\neg A$ is what we can conclude.
Assuming that $A \to B$ and $\neg B$ are both true, and we know that $A \to B$ is equivalent to $A \vee \neg B$. We also know that if $\neg B$ is true then B should be false. Furthermore, when B is false, $A \to B$ is true, however A shouldn't be true. Therefore, A must be false and we can conclude $\neg A$ using Modus Tollens.

Problem 2. (15 points) Section 2.5, Exercise 2.21 (b) (without using a truth table)

Solution. 1) $\neg(A \to \neg B)$ - given
2) $\neg B \to A$ -implication of $A \to B$
3) $\neg B \to \neg A$ -by double negation
4)$(\neg B \to \neg A) \to (A \to B)$ -Axiom 3
5)$A \to B$ -by R1 (3,4)
6)$\neg(A \to B)$ -by negation
7) $\neg(\neg A \vee B)$ - negation of implication
8)$A \vee \neg B$ -distribution law

9) $\neg A \lor B$ -negation
10)$\neg A$ -disjunctive syllogism
11)$\neg A \lor \neg B$ -by R1 8, 10
12)$\neg(A \land B)$ -de Morgan's law
13)$\neg\neg(A \land B)$ -by negation
$=(A \land B)$

Problem 3. (15 points) Section 2.5, Exercise 2.24 (a) (without using a truth table)

Solution. There is no formal argument that can be made when it comes to proving axioms. However, we can note that tautologies lead to tautologies, and that axioms A1, A2, A3 are tautologies. Axiom A1 can be proved through the utilization of modus ponens. Modus ponens suggests that:
$A \to B$
A
$\therefore B$
Using the same logic for Axiom A1, $A \to (B \to A)$, we can break it down and apply modus ponens to the inside which is $(B \to A)$.
$B \to A$
B
$\therefore A$
Now, we can take that conclusion, and apply it to the outer half, which is $A \to A$. We know that $T \to T$ is True and $F \to F$ is also True, so whether A is true or false, $A \to A$ will always be True. Or we can apply modus ponens to $A \to A$, which is :
$A \to A$
A
$\therefore A$

Problem 4. (15 points) Section 2.6, Exercise 2.26 (a), (b), and (c)

Solution. a)The square of any real number is greater than or equal to 0.
b) For every real number x there exists are real number y such that x is less than y $(x < y)$.
c) For every real number x and y there exists a real number y such that if x is less than z, $(x < z\ )$, then x less than y and y less than z, $((x < y) \land (y < z))$.

Problem 5. (10 points) Section 2.6, Exercise 2.27 (a) and (e)

Solution. a) $\forall x$, $x \in Z$, $(x \in Q)$
e) $\forall x \exists y$, $x \in \mathbb{R}$, $(x = y^2)$

Problem 6. (10 points) Section 2.6, Exercise 2.28 (b) and (e)

Solution. b) $\forall a \exists b$, (a=b+2) is false because the statement doesn't apply to all a's. When a is equal to one there is not positive number b.
e) $\exists a \forall b$, (x $\leq$ b) is true because it is inclusive of when b=1.

Problem 7. (15 points) Section 2.8, Exercise 2.32

Solution. Original Statement: "If x is a real number, then x $\leq$ x^2".
a) $A \to B$
b) $(A \to B) \equiv \neg A \vee B$ so,
$\neg(A \to B) \equiv \neg(\neg A \vee B) \equiv A \wedge \neg B$, by de Morgan's law
c) Assuming that x is a real number is true, and x is not less than equal to x^2 is also True, then $A \wedge \neg B$ must be True.

Problem 8. (10 points) Section 2.9, Exercise 2.35

Solution. Given that m and n are two consecutive integers. We also know that n is even if and only if there exists an integer k such that n=2k, and n is odd if and only if there exists an integer k such that n=2k+1. So lets make n=2k and m=2k+1, and if k has the same value, then n and m will be consecutive. However, to prove that n+m is odd:
n+m = 2k +(2k+1)
=4k+1.
4k+1 will always be odd, hence proving the claim.
Test: If k=1
n=2(1)=2 and m=2(1)+1=3; 2 and 3 are consecutive integers.
m+n= 2+3= 5
and 4k+1=5
5=5

Checklist:
- ☐ Did you type in your name and UIN?
- ☐ Did you disclose all resources that you have used?
  (This includes all people, books, websites, etc. that you have consulted)
- ☐ Did you sign that you followed the Aggie honor code?
- ☐ Did you solve all problems?
- ☐ Did you submit the .tex and .pdf files of your homework to the correct link on eCampus?