Lab 2: Second-Order Circuits

ECEN 325 - 511

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Due Date: September 28, 2021

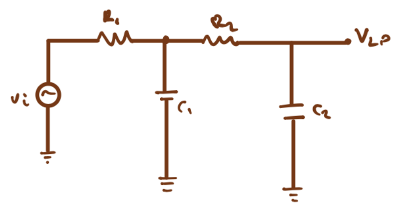
Purpose

The purpose of this lab was to be able to explore the frequency response for second-order circuits. This lab gave students experience with circuit design and analysis techniques, through simulations and measurements.

Calculations

part 1 → find transfer functions

Circuit A:



$$\frac{V-V_i}{R_1} + \frac{V}{Z_{C_1}} + \frac{V-V_{LP}}{R_2} = 0$$

$$V\left(\frac{1}{R_1} + C_1 S_1 + \frac{1}{R_2}\right) = \frac{V_i}{R_1} + \frac{V_{LP}}{R_2}$$

$$= V(R_1 + R_2 + C_1 S_1 R_1 R_2) = R_2 V_i + V_{LP} R$$

$$= \frac{R_2 V_i + V_{LP} R_1}{R_1 + R_2 + C_1 S_1 R_1 R_2}$$

and, $\frac{V_{LP}}{Z_{C_2}} + \frac{V_{LP}-V}{R_2} = 0$

$$sC_2 V_{LP} + \frac{V_{LP}}{R_2} = V \Rightarrow (sC_2R_2+1)V_{LP} = V$$

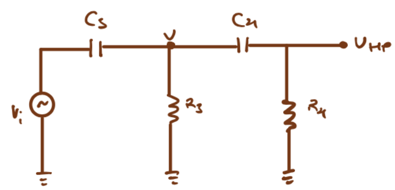
$$= V_{LP}(sC_2R_2+1) = \frac{R_2V_i + V_{LP}R_i}{R_1+R_2+C_1S_1R_1R_2}$$

$$= V_{LP}(R_1+R_2+C_1S_1R_1R_2+R_1R_2C_2 s + sR_2^2C_2 + R_1R_2^2C_1C_2s^2)$$
$$= R_2V_i + V_{LP}R_i$$

$$= V_{LP}(R_2 + C_1S_1R_1R_2 + R_1R_2C_2s + sR_2^2C_2 + R_1R_2^2C_1C_2s^2) = R_2V_i$$

$$\boxed{\frac{V_{LP}}{V_i} = \frac{1}{R_1 + C_1S_1R_1 + C_2R_1S + sR_2C_2 + R_1R_2C_1C_2S^2}}$$

circuit B:



$$\frac{V_0 - V_i}{2C_3} + \frac{V}{R_3} + \frac{V - V_{HP}}{R_4 C_4} = 0$$

$$v\left(\frac{1}{1/sC_3} + 1/R_3 + 1/1/sC_4\right) = sC_3 V_i + sC_4 V_{HP}$$

$$v\left(sC_3 + \frac{1}{R_3} + sC_4\right) = sC_3 V_i + sC_4 V_{HP}$$

$$v(sC_3R_3 + 1 + R_3 sC_4) = R_3 sC_3 V_i + R_3 sC_4 V_{HP}$$

$$v = \frac{R_3 sC_3 V_i + R_3 sC_4 V_{HP}}{(sC_3R_3 + 1 + R_3 sC_4)}$$

and ,

$$\frac{V_{HP}}{R_4} + \frac{v - V_0}{1/sC_4} = 0$$

$$\frac{V_{HP}}{R_4} \quad V_{HP}\left(\frac{1}{R_4} + sC_4\right) = sC_4 v$$

$$V_{HP}(1 + R_4 sC_4) = sC_4 R_4 v$$

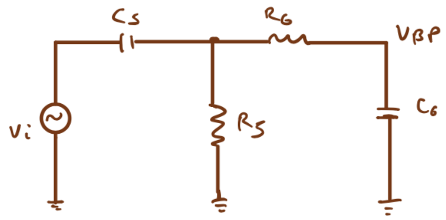
$$V_{HP}(1 + R_4 sC_4) = sR_4C_4\left(\frac{R_3 sC_3 V_i + sR_3 C_4 V_{HP}}{sC_3R_3 + 1 + R_3 sC_4}\right)$$

$$V_{HP}(1+R_4 s C_4)(sC_3R_3+1+R_3 s C_4) = s^2(R_3R_4C_3C_4 V_i + \cancel{R_3 R_4 C_4^2}\, V_{HP})$$

$$\hookrightarrow \quad sC_3R_3+1+R_3 sC_4 + s^2 R_3C_3R_4C_4 + C_4R_4 + s^2 \cancel{R_3R_4C_4^2})$$

$$\boxed{\frac{V_{HP}}{V_i} = \frac{s^2R_3R_4C_3C_4}{1+s(C_3R_4+C_3R_3+R_4C_4)+s^2R_3R_4C_3C_4}}$$

Circuit c:



$$\frac{V-V_i}{Z_{C_5}} + \frac{V}{R_5} + \frac{V-V_{BP}}{R_6} = 0$$

$$V\left(sC_5 + \frac{1}{R_5} + 1/R_6\right) = V_i sC_5 + \frac{V_{BP}}{R_6}$$

$$V\left(\frac{sC_5R_5R_6+R_6+R_5}{R_5R_6}\right) = \frac{R_6V_isC_5+V_{BP}}{R_6}$$

$$V(sC_5R_5R_6+R_6+R_5) = R_5R_6 sC_5V_i + V_{BP}R_5$$

$$V = \frac{sR_5R_6V_i + V_{BP}R_5}{sC_5R_5R_6+R_6+R_5}$$

and, $\dfrac{V_{BP}}{Z_{C_6}} + \dfrac{V_{BP}-V}{R_6} = 0$

$$= V_{BP}(sC_6 + 1/R_6) = V/R_6$$

$$= V_{BP}(sC_6R_6+1) = V$$

$$= V_{BP}(sC_6R_6+1) = \frac{sR_5R_6C_5V_i + V_{BP}R_5}{sC_5R_5R_6+R_6+R_5}$$

$$V_{BP}(1+sC_6R_6)(sC_5R_5R_6+R_6+R_5) = sR_5R_6C_5V_i + V_{BP}R_5$$

$$V_{BP}(1+sC_5R_5+s^2C_6C_5R_5R_6+sC_6R_6+sC_6R_5) = sR_5C_5V_i$$

$$\boxed{\frac{V_{BP}}{V_i} = \frac{sR_5C_5}{1+s(C_5R_5+C_6R_6+C_6R_5)+s^2C_6C_5R_5R_6}}$$

part 2 → find values for resistors and capacitors

circuit A: $H_{LP} = \frac{1}{1+\frac{s}{2\pi f_1}} \cdot \frac{1}{1+\frac{s}{2\pi f_2}} = \frac{1}{1+s\left(\frac{1}{2\pi f_1}+\frac{1}{2\pi f_2}\right)+s^2\left(\frac{1}{4\pi^2 f_1 f_2}\right)}$

lets say $C_1 = 47nF$ and $C_2 = 1nF$

then $\frac{1}{2\pi f_1} + \frac{1}{2\pi f_2} = C_1R_1 + C_2R_2 + R_1C_2$

$$\frac{f_1+f_2}{2\pi f_1 f_2} = R_1(C_1+C_2) + C_2R_2$$

$$= \frac{11000}{2\pi 10000000} = R_1[47\times10^{-9}] + 1\times10^{-9}R_2$$

$$R_2 = \frac{1.75\times10^{-4}}{1\times10^{-9}} - \frac{(47\times10^{-9})R_1}{1\times10^{-9}} \qquad R_2 = 1.75\times10^{-5} - 47R_1$$

$$\frac{1}{4\pi^2 f_1 f_2} = R_1R_2C_1C_2 \rightarrow 2.533\times10^{-9} = (1.75\times10^{5} - 47R_1)47\times10^{-9}R_1$$

$$5.39\times10^{-2} = 1.75\times10^{5}R_1 - 47R_1^2$$

$$47R_1^2 - 1.75\times10^{5}R_1 + 5.39\times10^{-2} = 0$$

$$\boxed{R_1 = 3723\,\Omega, \quad R_2 = 171277\,\Omega, \quad C_1 = 47\,nF, \quad C_2 = 1\,nF}$$

Circuit B: $H_{HP}(s) = \dfrac{s}{s+2\pi f_3} \cdot \dfrac{s}{s+2\pi f_4} = \dfrac{s^2 / 4\pi^2 f_3 f_4}{1 + \dfrac{2\pi(f_3+f_4)}{4\pi^2 f_3 f_4} + \dfrac{s^2}{4\pi^2 f_3 f_4}}$

$C_4 R_4 + C_4 R_3 + R_3 C_3 = \dfrac{f_3+f_4}{2\pi(f_3 f_4)}$ let $C_3 = 10\,\mu F$ and $C_4 = 1\,nF$

then, $1\times10^{-9} R_4 + 1\times10^{-9} R_3 + 10\times10^{-6} R_3 = 3.5\times10^{-4}$

$= 1\times10^{-9} R_4 + 10\times10^{-6} R_3 = 3.5\times10^{-4}$

$= 1\times10^{-3} R_4 + R_3 = 35$

so, $\dfrac{1}{4\pi^2 f_3 f_4} = R_3 R_4 C_3 C_4$

$= \dfrac{1}{4\pi^2 \cdot 10^7} = R_4 (35 - 1\times10^{-3} R_4) 1\times10^{-14}$

$$\boxed{R_3 = 24.77\,\Omega \quad R_4 = 10223.3\,\Omega \quad C_3 = 10\,\mu F \quad C_4 = 1\,nF}$$

circuit C: $\frac{s}{s+2\pi f_5} \cdot \frac{1}{1+\frac{s}{2\pi f_6}} = \frac{s/2\pi f_5}{\frac{s}{2\pi f_5}+1} \cdot \frac{1}{1+s/2\pi f_6} = \frac{s/2\pi f_5}{1+\left(\frac{1}{2\pi f_5}+\frac{1}{2\pi f_6}\right)s + s^2\left(\frac{1}{4\pi^2 f_5 f_6}\right)}$

$C_5R_5 + C_6R_6 + C_6R_5 = \frac{1}{2\pi}\left(\frac{1}{f_5}+\frac{1}{f_6}\right)$

$R_5(C_6+C_5) + C_6R_6 = \frac{1}{2\pi}\left(\frac{1}{1k}+\frac{1}{100k}\right)$

let $C_5 = 1\mu F$ and $C_6 = 1nF$

$C_5R_5 + C_6R_6 = \frac{1}{2\pi}\left(\frac{101000}{100000000}\right)$

$1\times10^{-6}R_5 + 1\times10^{-9}R_6 = 1.607\times10^{-5}$

$R_5 = 1607 + 1\times10^{-9}R_6$

so, $\frac{1}{4\pi^2 f_5 f_6} = C_6C_5R_5R_6$

$= \frac{2.535\times10^{-11}}{1\times10^{-15}} = (16.07 + 1\times10^{-9}R_6)$

$25330 = (16.07 + 1\times10^{-9}R_6)R_6$

$\boxed{R_5 = 16.07\Omega \quad R_6 = 1576\Omega \quad C_5 = 1\mu F \quad C_6 = 1nF}$

part 3 → plots

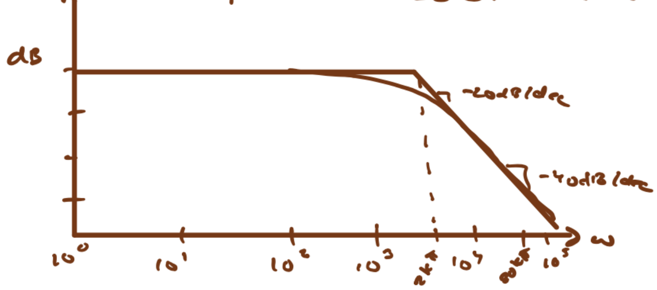
circuit A: $f_1 = 1kHz \quad f_2 = 10kHz$

$H_{2P} = \frac{1}{1+\frac{s}{2000\pi}} \cdot \frac{1}{1+\frac{s}{20000\pi}}$

$\lim_{s\to0}(H_{2P}) = 1$

$\lim_{s\to\infty} = 0$

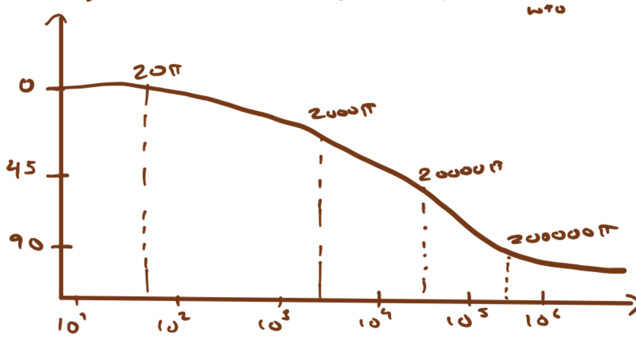
2 poles at 2000π and 10000π



magnitude response

$[20\pi, 20000\pi]$ & $[2000\pi, 200000\pi]$

$$-\lim_{\omega \to 0} \tan^{-1}\left(\frac{\omega}{2000\pi}\right) + \lim_{\omega \to 0} \tan^{-1}\left(\frac{\omega}{[illegible]}\right)$$



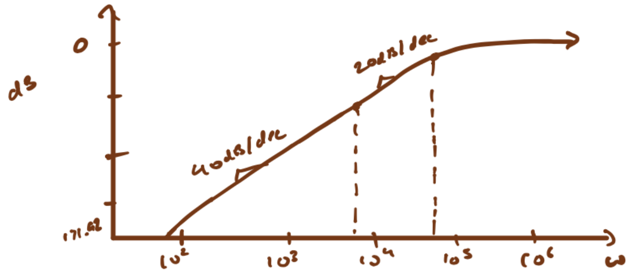
phase response

Circuit B: $H_{HP} = \frac{s}{s+2\pi f_3} \cdot \frac{s}{s+2\pi f_4}$ since $f_3 = 1kHz$, $f_4 = 10kHz$ $= \frac{s}{s+2000\pi} \cdot \frac{s}{s+20000\pi}$

there's two poles at 2000π, 20000π and 2 zeros at 0

$\lim_{s\to 0} |H_{HP}| = 10\log(s)^2 + 10\log_{10}(s)^2 - 10\log_{10}(2000\pi)^2 - 10\log_{10}(20000\pi)^2$

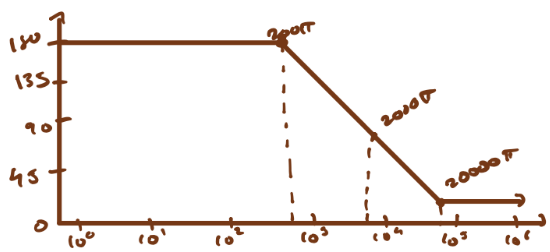
$= -75.96 - 95.46 = -171.92$



magnitude Response

$\lim_{\omega\to 0}$ phase $|H_{HP}| = \tan^{-1}(\frac{s}{0}) + \tan^{-1}(\frac{s}{0}) - \tan^{-1}(\frac{s}{2000\pi}) - \tan^{-1}(\frac{s}{20000\pi})$

$= 90 + 90 = 180°$

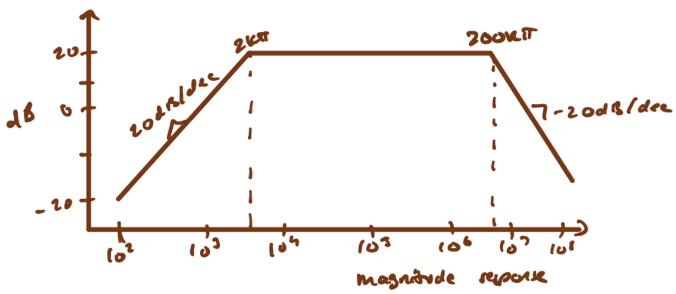


phase response

circuit 1: $\frac{s}{s + 2\pi f_s} \cdot \frac{1}{1 + s/2\pi f_b}$ $\quad f_s = 1\text{kHz}$, $f_b = 100\text{kHz}$ $\quad = \frac{s}{s + 2000\pi} + \frac{1}{1 + \frac{s}{200k\pi}}$

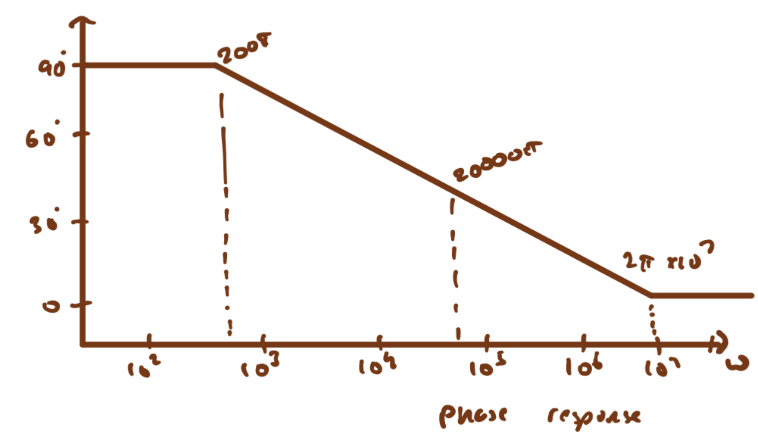
there's 0's on 0 and poles on 2000π and $200k\pi$

$$\lim_{\omega \to 0} |H_{BP}| = 10\log_{10}(s^2) - 10\log(2000\pi)^2 - 20\log_{10}(200k\pi)^2$$
$$= -24.42 \text{ dB}$$



$$\lim_{\omega \to 0} \text{phase} |H_{BP}| = \tan^{-1}\left(\frac{s}{0}\right) - \tan^{-1}\left(\frac{s}{2000\pi}\right) - \tan^{-1}\left(\frac{s}{200k\pi}\right) = 90°$$

$[200\pi, 20000\pi]$ $[20000\pi, 20000000\pi]$



part 4 → find V_o

circuit A: Here $V_i = 0.5 \sin(2\pi\, 10000t)$
$v_i = 0.5 \angle 0^\circ$

now $$H_{LP} = \frac{1}{1 + \frac{j\omega}{2000\pi}} \cdot \frac{1}{1 + \frac{j\omega}{20000\pi}}$$

$$H_{LP}(2\pi\, 10000) = \frac{1}{1 + j\frac{\cancel{20000\pi}}{\cancel{2000\pi}}} \cdot \frac{1}{1 + j\frac{\cancel{20000\pi}}{\cancel{20000\pi}}}$$

$$= \frac{1}{1+10j} \cdot \frac{1}{1+j} = -0.024 - 0.026i$$
$$= 0.035 \angle -132.376$$

so,
$$V_{LP} = H_{LP} \cdot 0.5 \angle 0$$
$$= 0.0176 \angle -132.376 = \boxed{V_{LP} = .0176 \sin(2\pi\, 10000 - 132.376)}$$

$f_3 = 1kH_z$
$f_4 = 10\,kH_z$

Circuit B: $$H_{HP} = \frac{s}{s + 2\pi f_3} \cdot \frac{s}{s + 2\pi f_4}$$

$$= \frac{s}{s + 2\pi\, 1000} \cdot \frac{s}{s + 2\pi\, 10000} = \frac{j\omega}{j\omega + 2000\pi} \cdot \frac{j\omega}{j\omega + 20000\pi}$$

$$H_{HP}(10000) = \frac{2\pi\, 10000\pi j}{20000\pi + 2000\pi} + \frac{20000\pi j}{20000\pi + 20000\pi}$$

$$= \frac{10j}{10+j} \cdot \frac{j}{1+j} = 0.70 \angle 129.28$$

$$V_{HP} = H_{HP} \cdot 0.5 \angle 0 = 0.70 \angle 129.28 \cdot 0.5 \angle 0 = .35 \angle 129.28$$

$$\boxed{V_{HP} = 0.35 \sin(2\pi\, 10000t + 129.28)}$$

$f_s = 1\text{kHz}$

$f_6 = 100\text{kHz}$

circuit c: $V_{BP} = \frac{s}{s+2\pi f_5} \cdot \frac{1}{1+\frac{s}{2\pi f_6}}$

$= \frac{j\omega}{j\omega + 2000\pi} \cdot \frac{1}{1 + \frac{j\omega}{200k\pi}}$

where $\omega = 2\pi \cdot 10000$

$$H_{BP}(10006) = \frac{j20000\pi}{j20000\pi + 2000\pi} \cdot \frac{1}{1 + \frac{2\cancel{0000}\pi j}{200\cancel{0000}\pi}} = \frac{10j}{10j+1} \cdot \frac{1}{1+j/100}$$

$$= \frac{10j}{10j+1} \cdot \frac{100}{100+j} = 0.995 \angle 5.138$$

$v_{BP} = H_{BP} v_i$

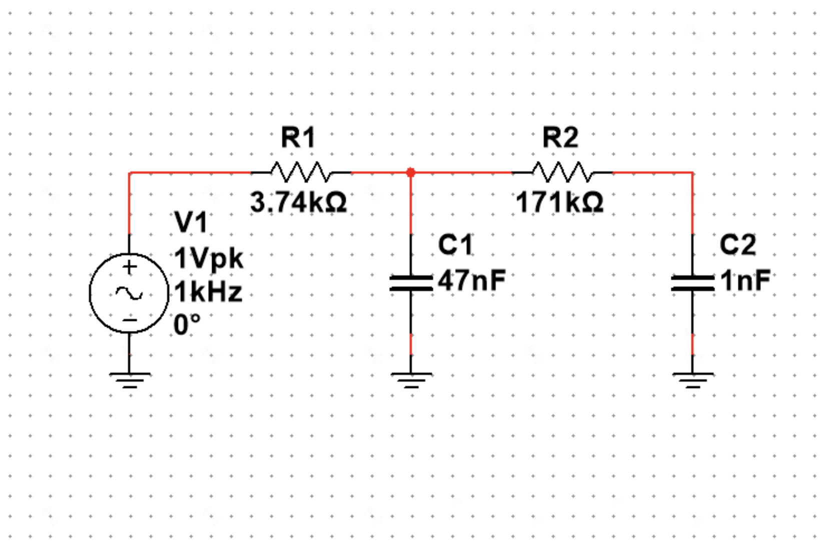
$= 0.995 \angle 5.138 \cdot 0.5 \angle 0 = 0.497 \angle 5.138$

$$\boxed{V_{BP} = 0.497 \sin(2\pi\, 10000t + 5.138)}$$

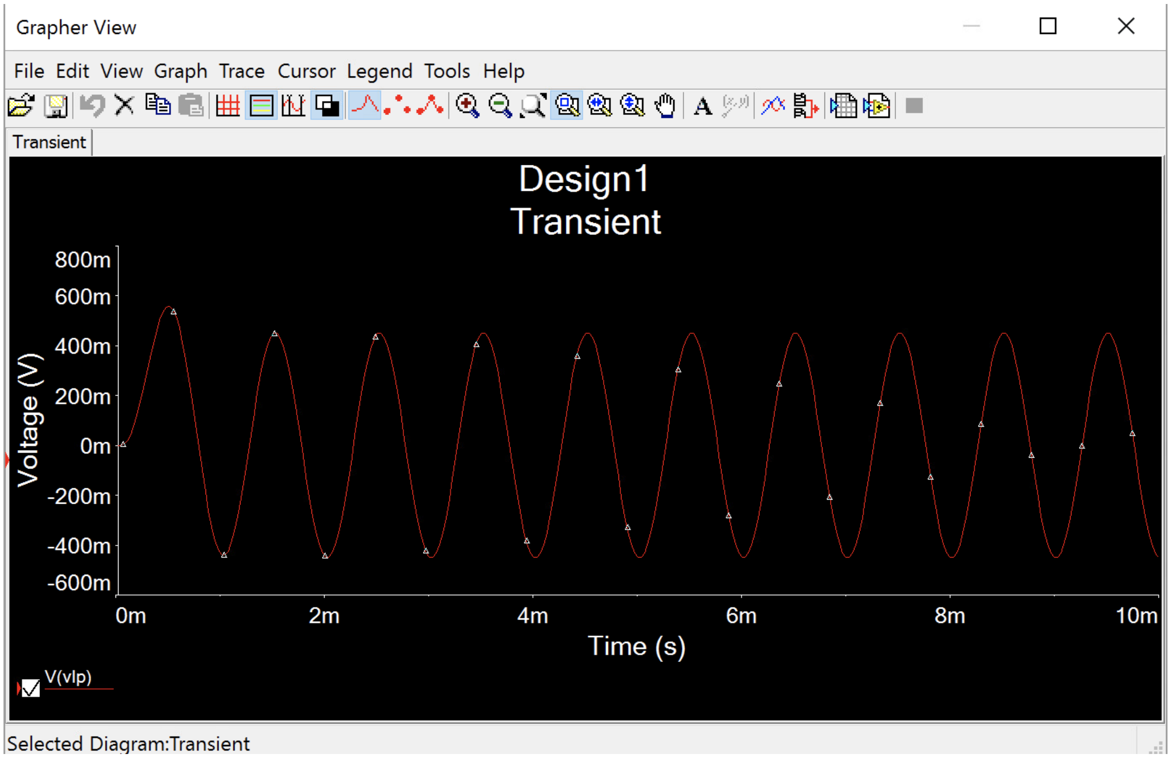
<u>Simulations (on Multisim)</u>

Lowpass Circuit

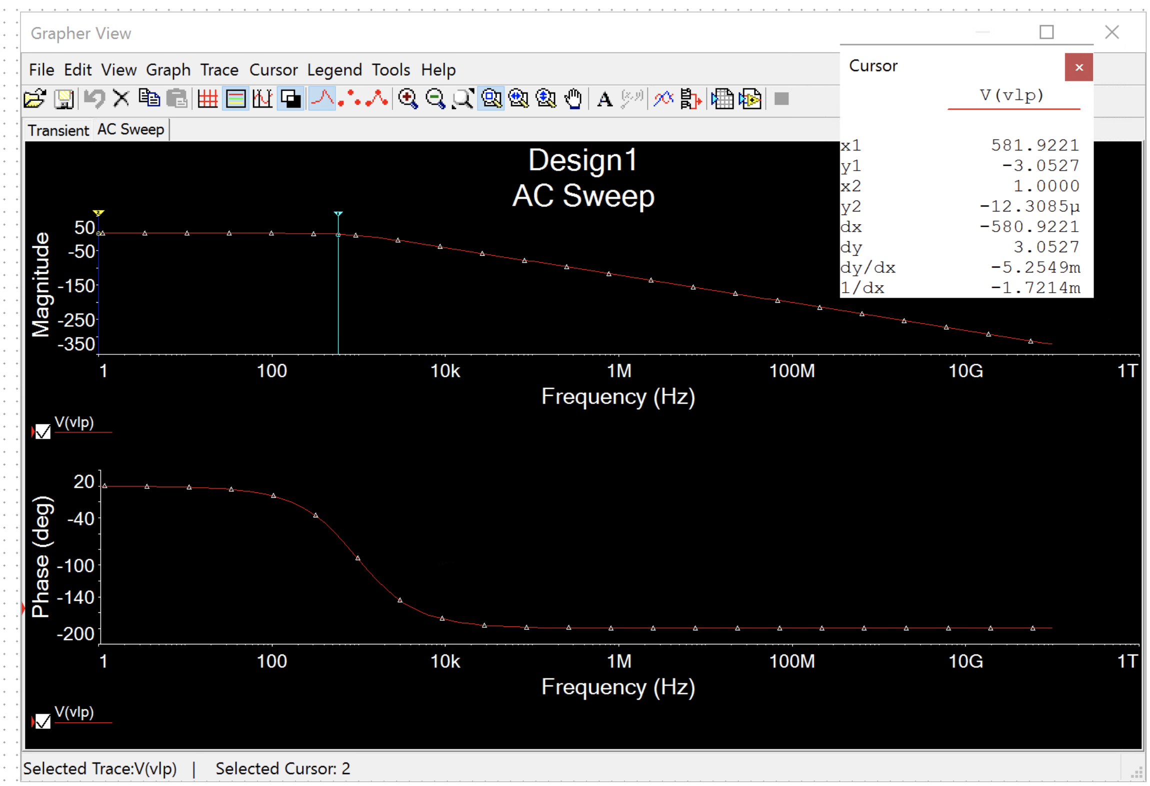
Schematic



Transient

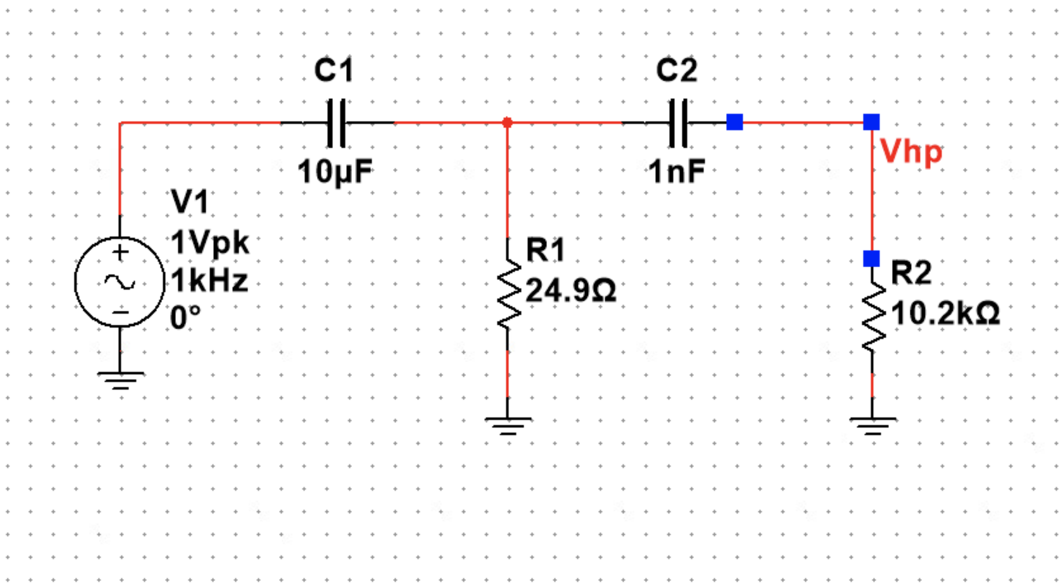


AC Sweep



Highpass Circuit

Schematics



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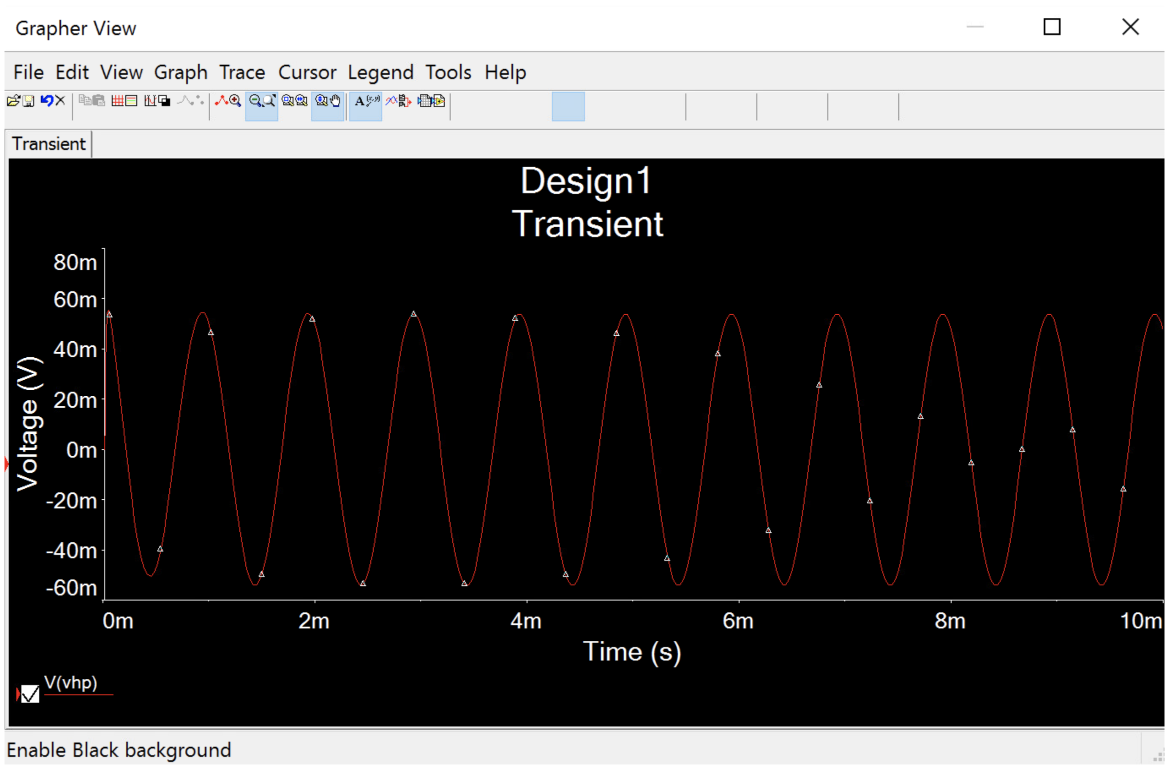

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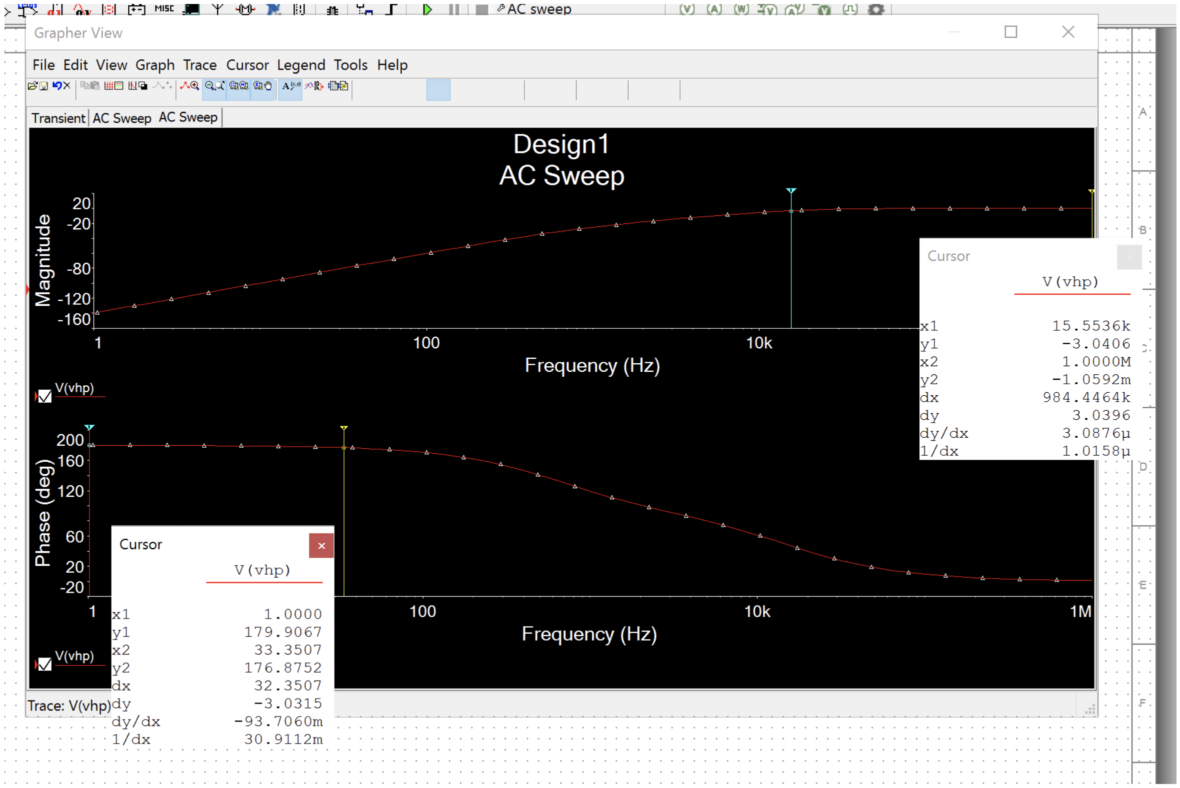
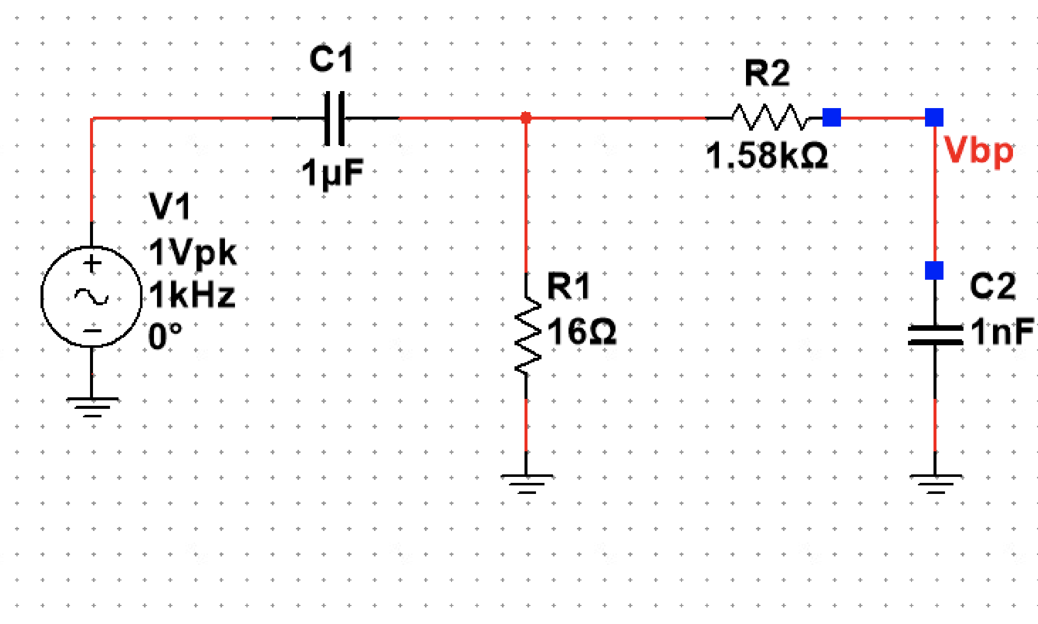
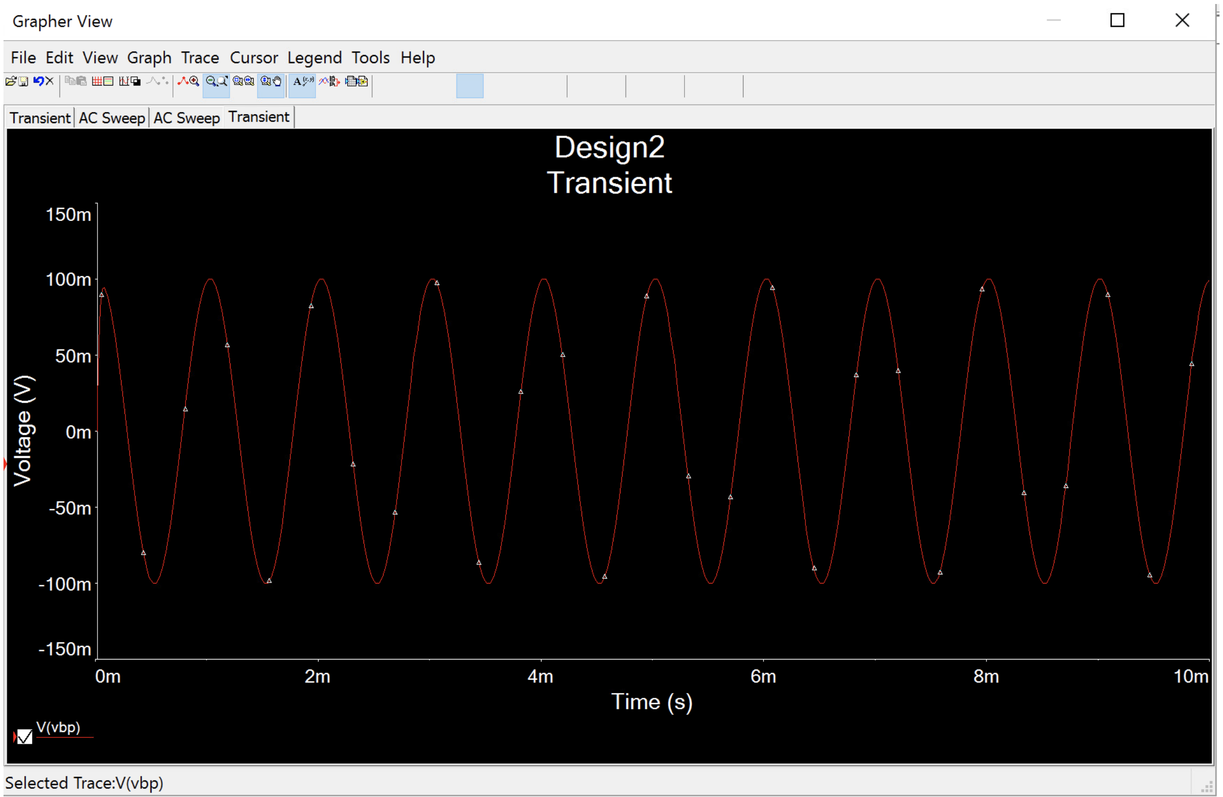


Bandpass Circuit

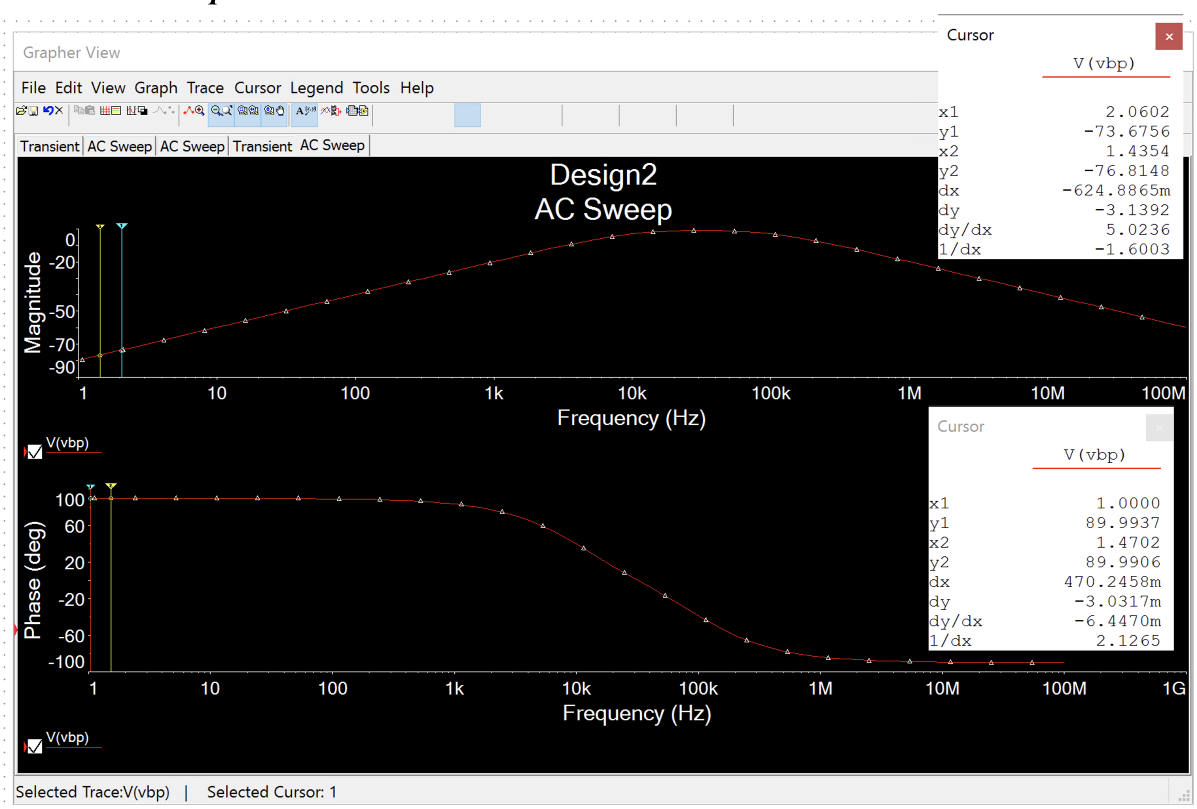
Schematic



Transient



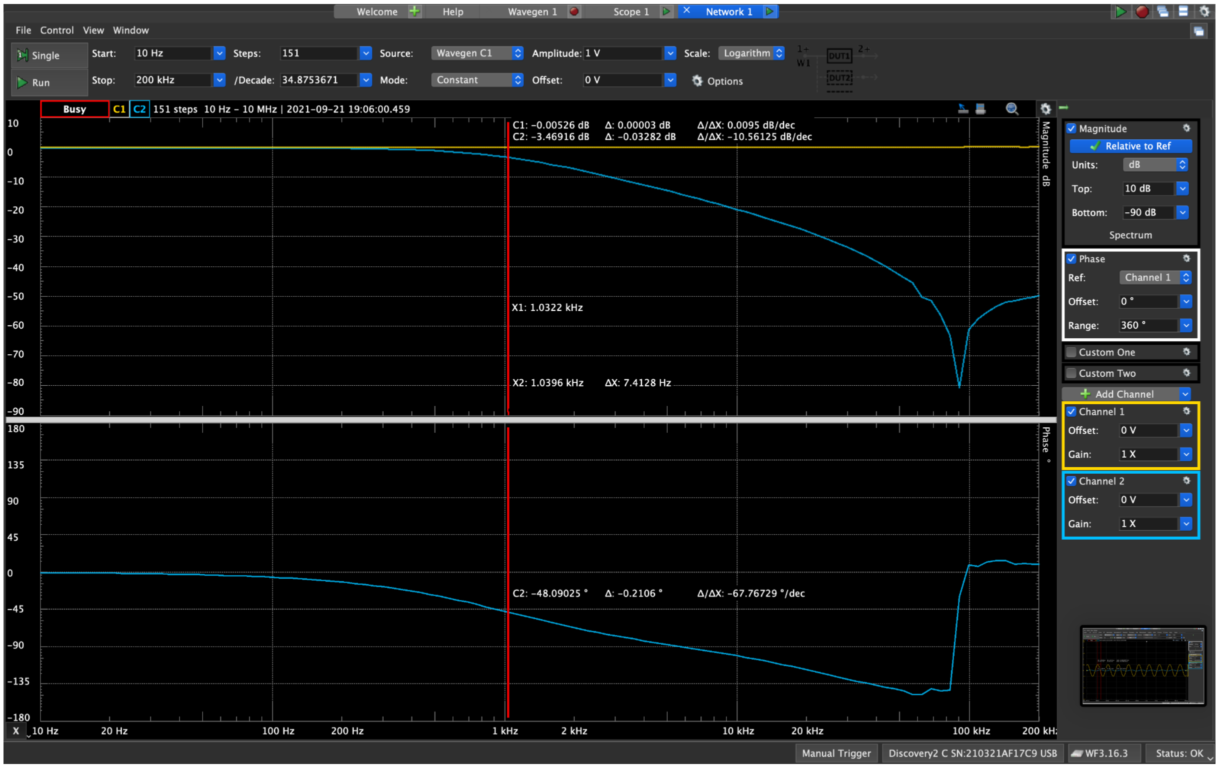
AC Sweep



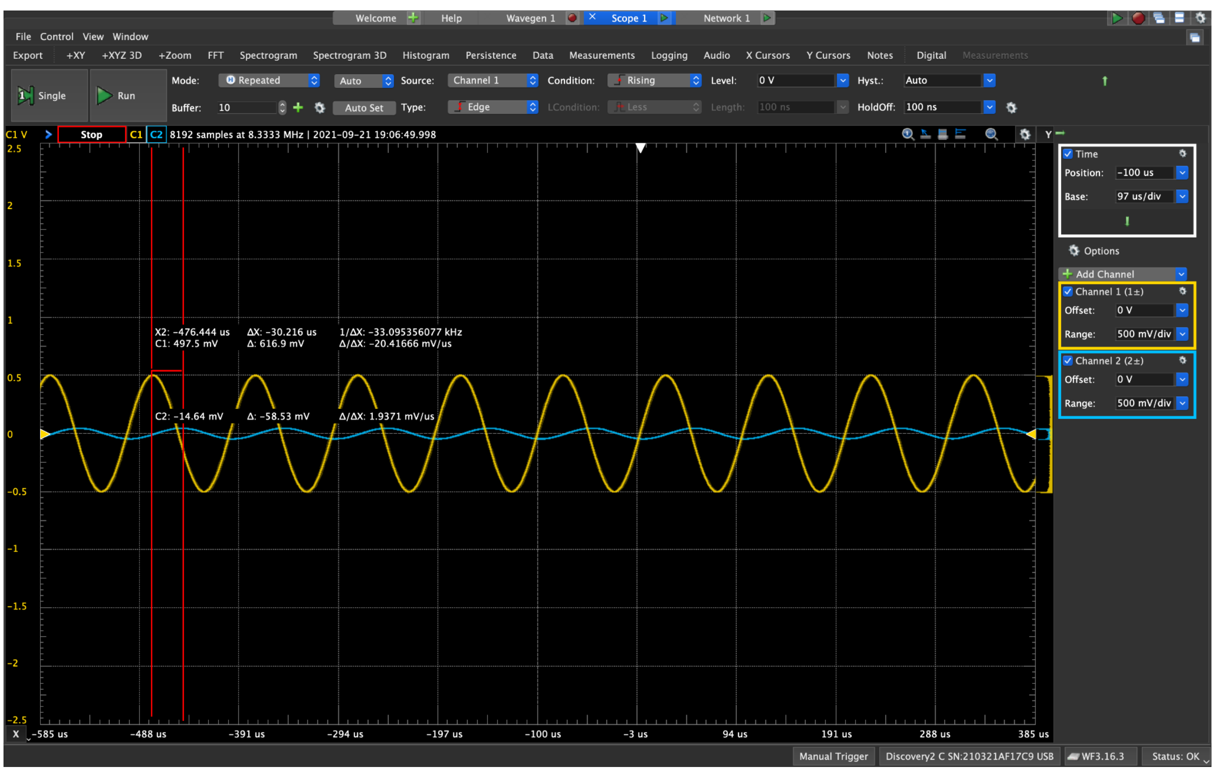
<u>Measured Waveforms</u>

Circuit A-low pass

Bode Plot

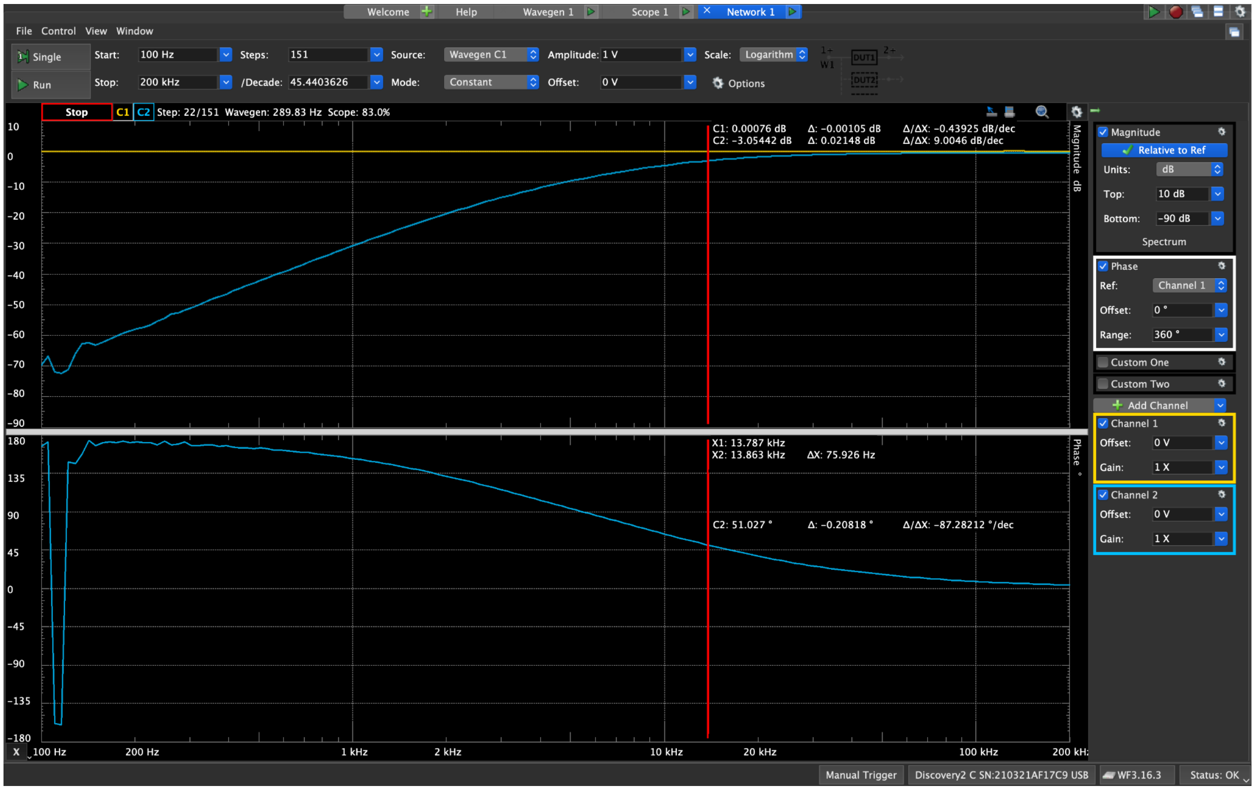


Time Domain Plot

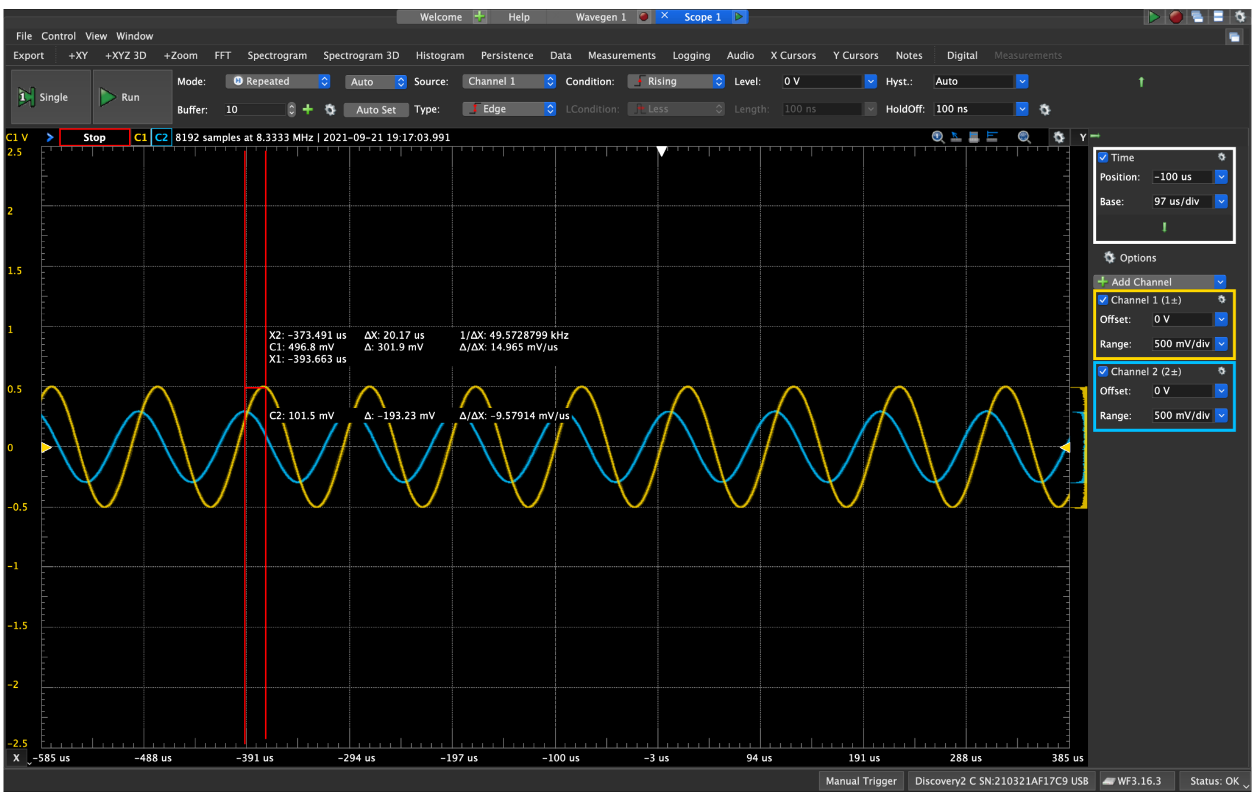


Circuit B- high pass

Bode Plot

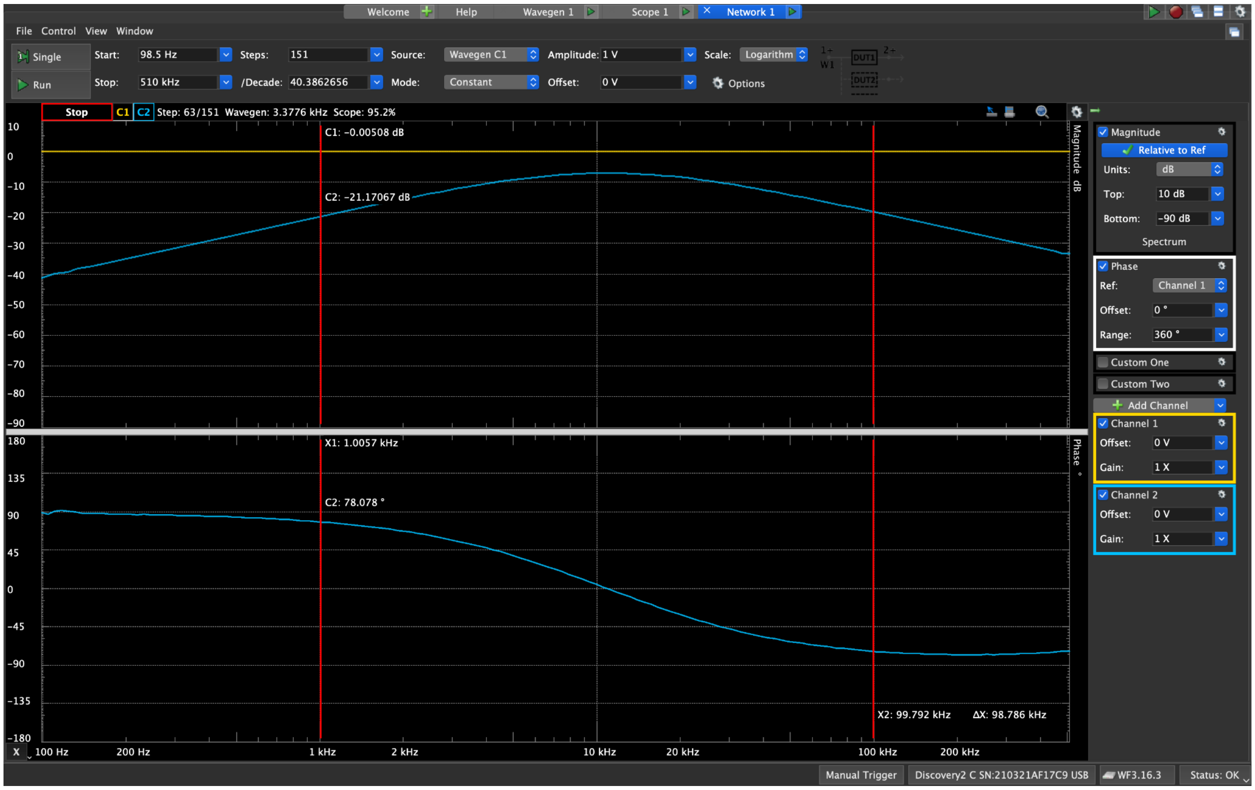


Time Domain Plot

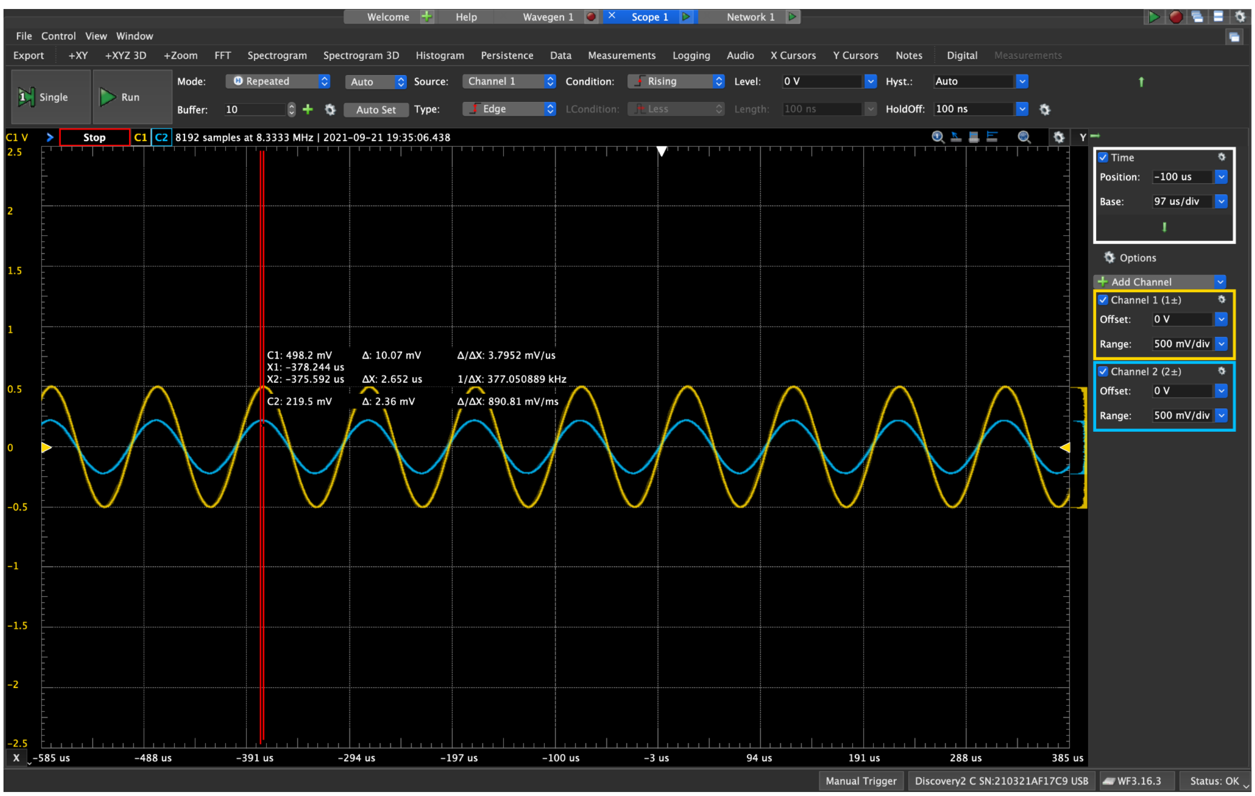


Circuit C-bandpass filter

Bode Plot



Time Domain Plot



Data Tables

	Time Difference	**∠H**	**|H|**
Calculation Circuit A	N/A	-132.37°	0.176V
Calculation Circuit B	N/A	129.28°	123mV
Calculation Circuit C	N/A	5.14°	0.497V
Simulation Circuit A	-10.08	-36.288°	394.1mV
Simulation Circuit B	15.81	56.916°	269.3mV
Simulation Circuit C	0	0°	497mV
Measurements Circuit A	-30.216	-108.778°	-14.64mV
Measurements Circuit B	20.17	72.612°	101.5mV
Measurements Circuit C	2.65	9.54°	219.5mV

Discussion

The values I got for my simulations were slightly different from my measurements, but this could be because of the breadboard, or the sensitivity of the Analog Discovery. For some, I had to make a series for the resistors or use resistors that were closest to the calculated values because of limited materials. When it came to calculating the component values, I utilized the transfer function, but the resistance values were either too small or too large to actually use a resistor of the exact resistance value, hence why the calculated component values may be different from the measurement values.