PreLab 8: Sinusoidal Steady State Response of a 2nd Order Circuit

ECEN 214 - 517
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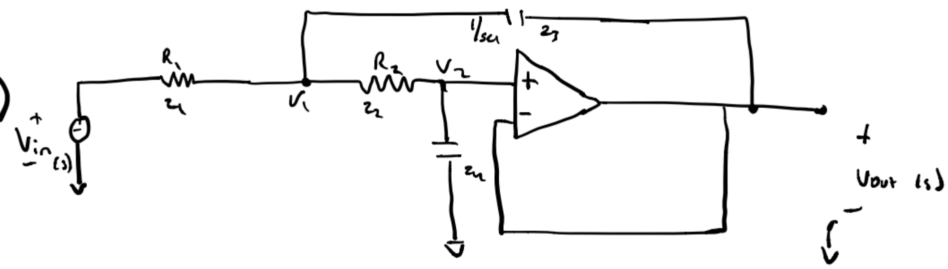
A. Design a Sallen-Key circuit as shown in Figure 8.1. Choose component values so that the circuit produces an underdamped response with a Q-factor of Q = 1.6 and a resonant radian frequency of ωo = 500π rad/sec (fo = 250Hz). Refer to the previous lab to refresh your memory on these quantities if needed. Be sure to choose component values that are available to you in your lab kit. Also, try to avoid using very small resistors (less than a few hundred Ohms) as that may cause bad behavior from your op-amp. Provide a plot of the magnitude and phase response of your circuit similar to the ones provided in Figure 8.3. In your plot, the frequency should be on a logarithmic scale and should range from 10Hz to 10kHz.

A)



$$\frac{V_1 - V_{in}}{Z_1} + \frac{V_1 - V_2}{Z_2} + \frac{V_1 - V_{out}(s)}{Z_3} = 0$$

$V^+ = V^-$

$V_2 = V_{out}$

$$\frac{V_1 - V_{in}(s)}{Z_1} + \frac{V_1 - V_{out}(s)}{Z_2} + \frac{V_1 - V_{out}(s)}{Z_3} = 0 \qquad -(1)$$

$$\frac{V_{in}(s) - V_1}{Z_1} = \frac{V_1 - V_{out}(s)}{Z_2} + \frac{V_1 - V_{out}(s)}{Z_3}$$

$$V_1 = V_{out}\left[1 + \frac{Z_2}{Z_4}\right] \rightarrow \frac{V_{in}(s) - V_{out}\left(1+\frac{Z_2}{Z_4}\right)}{Z_1}$$

$$= \frac{V_{out}\left[1 + \frac{Z_2}{Z_4}\right] - V_{out}(s)}{Z_2} + \frac{V_{out}\left(1 + \frac{Z_2}{Z_4}\right) - V_{out}(s)}{Z_3}$$

$$\frac{V_{out}}{V_{in}} = \frac{Z_3 Z_4}{Z_1 Z_2 + Z_3(Z_1+Z_2) + Z_3 Z_4} \quad — \quad ①$$

$$Z_1 = R_1 \ , \ Z_2 = R_2 \ , \ Z_3 = \frac{1}{sC_1} \ , \ Z_4 = \frac{1}{sC_2}$$

$$\frac{V_{out}}{V_{in}} = \frac{\frac{1}{sC_1} \cdot \frac{1}{sC_2}}{R_1 R_2 + \frac{1}{sC_1}(R_1+R_2) + \frac{1}{sC_1} \cdot \frac{1}{sC_2}}$$

$$H(s) = \frac{1}{1 + s(R_1+R_2)C_2 + s^2 R_1 R_2 C_1 C_2}$$

$$\hookrightarrow \frac{1}{(1+s)\left(\frac{1}{\omega_0 Q}\right) + s^2 \omega_0^{\,2}}$$

$$\omega_0^{\,2} = \frac{1}{R_1 R_2 C_1 C_2} \qquad \frac{1}{\omega_0 Q} = (R_1+R_2)C_2$$

$$\omega_0^{\,2} = \frac{1}{R^2 C_1 C_2} \qquad C_1 = 4C_2 Q^2$$

$R^2 4C_2$

$$C_1 = \frac{2Q}{R\omega_0} \ , \qquad C_2 = \frac{1}{2RQ\omega_0}$$

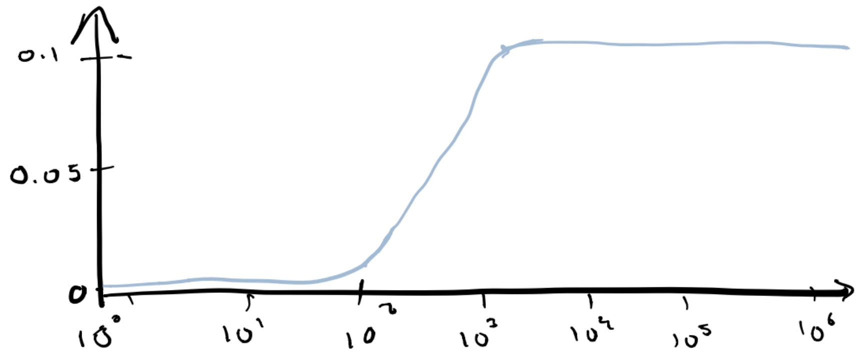
$$R_1 = R_2 = R = 2.2\,k\Omega \ , \ Q = 1.6, \ f_0 = 250\ Hz$$

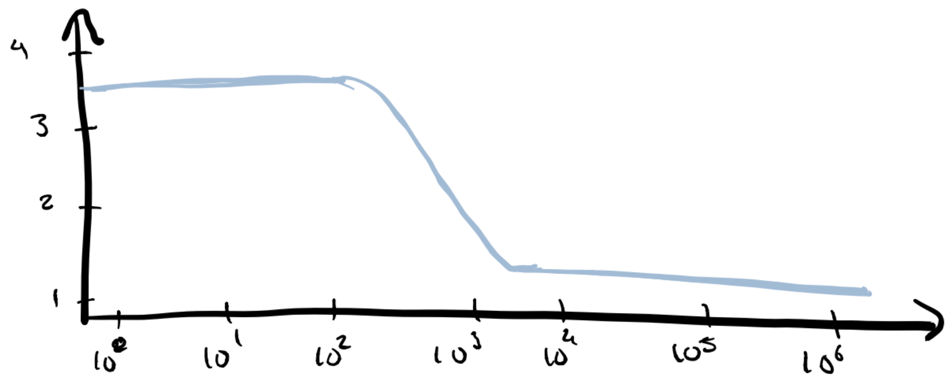
$$C_1 = \frac{2(1.6)}{(2.2\times10^3)(2\pi)(250)} = \underline{0.925\,\mu F}$$

$$C_2 = \frac{1}{(2)(2.2\times10^3)(1.6)(2\pi)(250)} = \underline{90.42\,nF}$$

$$\underline{C_1 = 0.925\,\mu F \ , \ C_2 = 90.42\,nF}$$

magnitude and phase plots





B. For the component values that you chose for your circuit, find the cutoff frequency. That is the frequency where $|H(\omega)| = 1/\sqrt{2}$. Express your cutoff frequency in Hz.

B)

$$f_c = \frac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}}$$

$$= \frac{1}{2\pi\sqrt{R^2 C_1 C_2}}$$

$$= \frac{1}{(2\pi)(2.2\times10^3)\left(\sqrt{0.925\times10^{-6}\cdot 90.42\times10^{-9}}\right)}$$

$$= \boxed{250.146\ \text{Hz}}$$