CSCE 222-199 Discrete Structures for Computing – Summer 2020
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**Problem Set 7**

**Due dates:** Electronic submission of *yourLastName-yourFirstName-hw7.tex* and *yourLastName-yourFirstName-hw7.pdf* files of this homework is due on **Friday, 6/26/2020 before 11:00 p.m.** on `https://ecampus.tamu.edu`. You will see two separate links to turn in the .tex file and the .pdf file separately. Please do not archive or compress the files. **If any of the two files are missing, you will receive zero points for this homework.**

**Name:** [redacted] **UIN:** [redacted]

**Resources.** For all parts of this question, I utilized the text from Perusall and any notes I took from the videos provided during the week. I also used some youtube tutorials to help me navigate through Latex, and a calculator.

On my honor, as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment. Furthermore, I have disclosed all resources (people, books, web sites, etc.) that have been used to prepare this homework.

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Total: 100 points.

**Problem 1.** (15 points) Section 13.2, Exercise 13.8

**Solution.** We already know that 4 $\frac{1}{1-x}$ is the generating function for the sequence (1,1,1,1,1......), So given then (1,-1,1,-1,1,-1....) then g(x) = $1x^0 - 1x^1 + 1x^2 - 1x^3 + ..... + (-1)^{n+1}x^n$, so g(x) = $(1+x)^{-1}$, hence the generating function is $\frac{1}{1+x}$.

**Problem 2.** (15 points) Section 14.1, Exercise 14.3

[Hint: Write down the first ten terms of the sequence such as $(m_0, m_1, \ldots, m_9, \ldots) =$ . Then, think of how you can define $m_k$ in terms of some previous term(s). Also, what are the initial conditions?]

**Solution.** Given that $m_k = \lfloor k/3 \rfloor$ then $m_0 = 0$, $m_1 = 0$, $m_2 = 0$, $m_3 = 1$, $m_4 = 1$, $m_5 = 1$, $m_6 = 2$, $m_7 = 2$, $m_8 = 2$,$m_9 = 3$, $m_{10} = 3$...
so for $m_k$ the recurrence relation should be $m_{k-1} + 1$ if k=3q for some $q \in \mathbb{Z}$ and $m_{k-1}$ if $k \neq 3q$

**Problem 3.** (15 points) Section 14.7, Exercise 14.30

**Solution.** The given equation is $g_n - 7g_{n-1} + 12g_{n-2}$ and we know that $g_1 = 1$ and $g_0 = 2$. We can factor out $g_{n-2}$, which gives us $g_{n-2}(n^2 - 7n + 12) = 0$. Factoring $(n^2 - 7n + 12)$ = (n-3)(n-4)=0, so the roots are 3 and 4. So to solve the recurrence relation we get $g_n = C_1 3^n + C_2 4^n$, where $C_1$ and $C_2$ are constants. So g(0)= $C_1$ +$C_2$ = 2 and g(1)= $3C_1$ +$4C_2$=1, hence solving the system if equations $C_1$ =7 and $C_2$ = -5, hence $g(n) = 7(3^n) - 5(4^n)$.

**Problem 4.** (15 points) Section 15.1, Exercise 15.5

[Hint: Use the Handshaking Lemma.]

**Solution.** We can start by partitioning the vertices in to even and odd degrees by: $\sum_{v \in V} = \sum_{d(v) is even} d(v) + \sum_{d(v) is odd} d(v)$. Using the handshaking lemma, lets start with the lefthand side which equals twice the number of edges, which is always even. On the right hand side the first sum of even number of edges, which would be the sum of even values and the second sum of odd number of edges must also be of even values, however since its entirely a sum of odd values it must have a even number of values. Hence, there cannot be a odd number of vertices that have an odd degree.

**Problem 5.** (10 points) Section 17.1, Exercise 17.1

**Solution.** i) an empty string is considered an in {0,1}* is an episilon.
ii) A string consisting of only 0s and 1s is in {0,1}*
iii) This string includes a 2 in it which is not part of {0,1}*
iv) A string consisting of only ones is part of {0,1}*

**Problem 6.** (15 points) Section 17.2, Exercise 17.6

**Solution.** The rule $S \to 0S0$ gives us $0^k S 0^k$, where $k \geq 1$. So then we can say that $0^k S 0^k \to 0^m A 0^m$ using $S \to 0A0$, where $m \geq 2$. Then $0^m A 0^m \to 0^m 1^n 0^m$, with $A \to 1A$, $A \to 1$ and $m \geq 2$, $n \geq 1$. Hence we can say L(G) is: L(G) = { $0^m 1^n 0^m | m \geq 2, n \geq 1$ }.

**Problem 7.** (15 points) Section 17.2, Exercise 17.8

**Solution.** Given L(G) = { $0^n 1^n | n \geq 1$ }
G= (N,T,P,S) T= {0,1}, N={S} and P = $\{S \to 0S1, S \to x | x = \emptyset\}$

**Checklist:**
☐ Did you type in your name and UIN?
☐ Did you disclose all resources that you have used?
(This includes all people, books, websites, etc. that you have consulted)
☐ Did you sign that you followed the Aggie honor code?
☐ Did you solve all problems?
☐ Did you submit the .tex and .pdf files of your homework to the correct link on eCampus?