CSCE 222-199 Discrete Structures for Computing – Summer 2020
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Problem Set 5

Due dates: Electronic submission of *yourLastName-yourFirstName-hw5.tex* and *yourLastName-yourFirstName-hw5.pdf* files of this homework is due on **Thursday, 6/18/2020 before 11:00 p.m.** on `https://ecampus.tamu.edu`. You will see two separate links to turn in the .tex file and the .pdf file separately. Please do not archive or compress the files. **If any of the two files are missing, you will receive zero points for this homework.**

Name: [redacted] **UIN:** [redacted]

Resources. For all parts of this question, I utilized the text from Perusall and any notes I took from the videos provided during the week. I also used some youtube tutorials to help me navigate through Latex.

On my honor, as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment. Furthermore, I have disclosed all resources (people, books, web sites, etc.) that have been used to prepare this homework.

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Total 100 points.

Problem 1. (15 points) Section 9.4, Exercise 9.23 [Requirement: Prove by induction on n.]

Solution. Base Case: When n=1 then S(n,1) or S(1,1) is equal to 1.
Inductive Step: Proposition 9.19 proves that the Stirling numbers of the second kind satisfy S(n,n)=1 for all nonnegative number, as shown in our base case S(1,1)=1. However let's consider, S(n+1,1), or n=k+1, and n is still equal to 1. So we get S(2,1) which is still equal to 1, as shown in Table 9.1 as part of the proof. Hence, S(n,1)=1, holds for n=k+1, thus proving its claim that S(n,1)=1 for all $n \geq 1$.

Problem 2. (15 points) Find lower and upper bounds to the $\sum_{k=1}^{n} k^3$ using Proposition 10.1 in Section 10.1 Monotonic Functions and integrals from calculus.

Solution. As proved in Proposition 10.1, when f is a monotonically increasing function thats defined on the interval [a-1,b+1], then

$$\int_{a-1}^{b} f(x)dx \leq \sum_{k=a}^{b} f(k) \leq \int_{a}^{b+1} f(x)dx$$

Hence, given the equation $\sum_{k=1}^{n} k^3$, with the function being k^3 gives us the inequality of:

$$\int_{0}^{n} x^3 dx \leq \sum_{k=1}^{n} k^3 \leq \int_{1}^{n+1} x^3 dx$$

If we were to simplify the inequality, or solve the integrals, we would get:

$$n^4/4 \leq \sum_{k=1}^{n} k^3 \leq (n+1)^4/4 - 1/4$$

where the lower bound is $n^4/4$(in terms of n) and upper bound is $(n+1)^4/4 - 1/4$(in terms of n).

Problem 3. (15 points) Section 11.1, Exercise 11.3

Solution. Considering proposition 11.8, then we can say that $f \sim g$ if lim($n \to \infty$) f(n)/g(n)=1. Given that f(n) $=n^2 + 2n$ and g(n)=n^2 and the lim($n \to \infty$) f(n)/g(n)= $n^2 + 2n/n^2$, we notice that n^2 is common within both functions, hence, lim($n \to \infty$) f(n)/g(n)= $n^2(1 + 2n)/n^2$. We can now cancel n^2 since its in both the numerator and denominator, which leaves us with lim($n \to \infty$) 1+2/n. solving the limit, we can conclude that lim($n \to \infty$) f(n)/g(n)=1, hence f(n) and g(n) are asymptotically equal, proving Ernie's claim and disproving Bert's.

Problem 4. (10 points $\times$ 2 = 20 points) Section 11.3, Exercise 11.14 (ii) and (iii)

Solution. ii) given that f and g are functions (defined as N) $f \asymp g$ and the sum, $m \in N$ holds a and b that are some positive real numbers, then $f(n) \leq (1/a)g(n)$ and $(1/b)g(n) \leq f(n)$ for any n greater than or equal to m. Furthermore, $(1/b)g(n) \leq f(n) \leq (1/a)g(n)$. Thus, $g \asymp f$, is proved.
iii) given that f, g and h are functions (defined as N) $f \asymp g$ and $g \asymp h$ and $m1, m2 \in N$ holds a, b, c, and d that are some positive real numbers, then $a \times c \times f(n) \leq c \times g(n) \leq h(n) \leq d \times g(n) \leq b \times d \times f(n)$ for any n greater than or equal to m and we can say that m is the max m1, m2. Hence, $a \times c \times f(n) \leq h(n) \leq b \times d \times f(n)$. Thus, $f \asymp h$ is proved when $f \asymp g$ and $g \asymp h$.

Problem 5. (20 points) Section 11.3, Exercise 11.19

Solution. Given that k is a positive integer, we want to prove that $1^k + 2^k + ... + n^k = \Theta(n^k + 1)$. Lets take that k=1 and C=1, then whenever x¿k=1, we can say that $|1^k + 2^k + ... + n^k|$=$1^k + 2^k + ... + n^k \leq n^k + n^k + ... + n^k$. We know that the equations given $1^k + 2^k + ... + n^k$ contains n amount of terms, which lets us conclude that $n^k + n^k + ... + n^k$ also contains n terms, but all terms are n^k, hence n x n^k is equal to n^{k+1} =$|n^{k+1}|$. Therefore, with k=1 and C=1, by the definition of the Big-O notation the statement $1^k + 2^k + ... + n^k = \Theta(n^k + 1)$, is proved.

Problem 6. (15 points) Section 11.6, Exercise 11.37

Solution. // search a key in an array a[1..n] of the length n
search (a,n,key)
for k in (1..n) do // execute n+1 times

if a[k]=key then // execute n times
return k // execute 1 times
end // execute 1 times
result false. // execute 1 times
The case time complexity is: T(n) = n+1+n+1+1+1, and since constant don't grow with n the worst case is: Θ(n).

Checklist:

☐ Did you type in your name and UIN?
☐ Did you disclose all resources that you have used?
(This includes all people, books, websites, etc. that you have consulted)
☐ Did you sign that you followed the Aggie honor code?
☐ Did you solve all problems?
☐ Did you submit the .tex and .pdf files of your homework to the correct link on eCampus?