CSCE 222-199 Discrete Structures for Computing – Summer 2020
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Problem Set 4

Due dates: Electronic submission of *yourLastName-yourFirstName-hw4.tex* and *yourLastName-yourFirstName-hw4.pdf* files of this homework is due on **Friday, 6/12/2020 before 11:00 p.m.** on `https://ecampus.tamu.edu`. You will see two separate links to turn in the .tex file and the .pdf file separately. Please do not archive or compress the files. **If any of the two files are missing, you will receive zero points for this homework.**

Name: [redacted] **UIN:** [redacted]

Resources. For all parts of this question, I utilized the text from Perusall and any notes I took from the videos provided during the week. I also used some youtube tutorials to help me navigate through Latex.

On my honor, as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment. Furthermore, I have disclosed all resources (people, books, web sites, etc.) that have been used to prepare this homework.

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Total: 100 points.

Problem 1. (20 points) Section 7.1, Exercise 7.2.

Solution. a) $\lfloor x \rfloor$ = n if and only if $x - 1 < n \leqslant x$
$\lfloor x \rfloor$ = max $n \in Z \mid n \leqslant x$ is the largest integer less than or equal to x by the definition of floor function. So if n=x, we can assume that x is an integer, so if $n \leqslant x$. Given that x is a real number we can say that $\lfloor x \rfloor < x$ by the definition, and let $\lfloor x \rfloor$ =n and assume $x - 1 \geqslant n$. Then $n + 1 \leqslant x$, however this contradicts the original statement because n can't be the greatest integer that is less than or equal to x, and hence can conclude that $\lfloor x \rfloor$ = n if and only if $x - 1 < n \leqslant x$.

b)$\lceil x \rceil$ = n if and only if $x \leqslant n < x + 1$
$\lceil x \rceil$ = min $n \in Z \mid n \geqslant x$ is the smallest integer greater than or equal to x by the definition of ceiling function. So when x is a real number, the we can say that $\lceil x \rceil > x$. So, lets keep $\lceil x \rceil$ = n, and further assume that $n \geqslant x + 1$, and simplify to $n - 1 \geqslant x$. However, by $n - 1 \geqslant x$, we know that n can't be the least integer that's greater than or equal to x, so by contradiction we prove that $\lceil x \rceil$ = n if and only if $x \leqslant n < x + 1$
.

Problem 2. (10 points) Section 7.1, Exercise 7.4 (a) and (f).

Solution. a) In proposition 7.4 we proved that $\lfloor x + k \rfloor$ is equal to integer $\lfloor x \rfloor$ +k, because the floor function essentially just drops any numbers after the decimal. So let's consider the reverse of this statement, where x¿n. Given that x is

a real number and assuming that its greater than the integer value n then $\lfloor x \rfloor$¡ n is false.
Here's a test case, 1.5 ¿1, which is 1 and a fraction 1/2 ¿ n. Now let's convert it to a floor function where $\lfloor 1.5 \rfloor = 1$ which is not less than 1, but rather equal, hence proving our claim, $\lfloor x \rfloor < n$ if and only if $x < n$, by contradiction.
f) $n < \lfloor x \rfloor$ if and only if $n < x$. Lets say x=1.5 and n=1, then n¡x is satisfied however $\lfloor x \rfloor$ =1, which is equal to n (n= $\lfloor x \rfloor$) which is contradictory to the statement we are given, $n < \lfloor x \rfloor$.

Problem 3. (10 points) Section 7.3, Exercise 7.29.

Solution. $\lfloor \lfloor \lfloor \lfloor x/10 \rfloor /10 \rfloor /10 \rfloor /10 \rfloor$
We can see that the initial formula is x/10, stored as a floor value, and the formula is just repeated 4 times, going outward, essentially removing the last 4 digits, after the decimal. However, this can be written in a single floor function: $\lfloor x/10000 \rfloor$.

Problem 4. (20 points) Section 4.1, Exercise 4.3. Prove by induction on n.

Solution. In order to prove that the sum of the first n squares is given by

$$\sum_{k=1}^{n} k^2 = 1^2 + 2^2 + ... + n^2 = (n(n+1)(2n+1))/6$$

Base Case: Let's say n=1, so LHS=1^2=1 and the RHS=(1(1+1)(2+1))/6=1, hence the left hand side equals the right hand side, when n=1, so the base case holds.
Inductive Step: We can assume that both sides are equal to each other when n=k. Hence, $1^2 + 2^2 + ... + k^2$ =(k(k+1)(2k+1))/6. When n=k+1, then the left hand side is: $1^2 + 2^2 + ... + k^2 + (k+1)^2$=(k(k+1)(2k+1))/6+$(k+1)^2$
=((k+1)(k(2k+1)+6(k+1)))/6
=((k+1)($2k^2 + 7k + 6$))/6
The right side is equal to: ((k+1)(k+2)(2k+3))/6
=((k+1)($2k^2 + 7k + 6$))/6
Thus, ((k+1)($2k^2 + 7k + 6$))/6 =((k+1)($2k^2 + 7k + 6$))/6, so the two sides are equal when n=k+1, proving the claim.

Problem 5. (20 points) Section 4.3, Exercise 4.15. Prove by induction on n.

Solution. Proof by Induction
Base Case: n=1; f(2)=f(3)-1, so f(2)=2 and f(3)-1=2, and since both sides is 2 the base case holds.
Inductive step: We can assume that n=k to finally prove that n=k+1. So if

$$\sum (n = k) = f(2) + ... + f(2k) = f(2k+1) - 1$$

Furthermore, if

$$\sum (n = k+1) = f(2) + ... + f(2k+2) = f(2k+3) - 1$$

Pay attention to the LHS where f(2) +...+f(2k+2)
=f(2)+...+f(2k)+f(2k+2)
=f(2k+1)-1 + f(2k+2) is equal to f(2k+3)-1, which matches the LHS of the summation.

Problem 6. (20 points) Section 4.6, Exercise 4.31.

Solution. given that f(n) = $(n^3 - 3n^2 + 2n)f_{(n-3)}$
Proof by Induction
Base Case: Lets say n=4, then f(n)=(64-48+8)*(4-3)=24 and 4!=24, hence the base case holds. Inductive Step: Assuming that n=k for all values $3 \leq x \leq k$ is true then f(k)=$(k^3 - 3k^2 + 2k)f_{(k-3)}$. To prove that n=k+1, then we can assume that f(k+1)=(k+1)!. If f(k+1)=$((k+1)^3 - 3(k+1)^2 + 2(k+1))f_{(k-2)}$
= $((k^3 + 3k^2 + 3k + 1 - 3)(k^2 + 2k + 1) + 2k + 2)f_{(k-2)}$
=$(k^3 - 3k + 2k)f_{(k-2)}$
=$k(k^2 - 1)f_{(k-2)}$
=$k * (k-1) * (k+1) * (k-2)!$
= (k+1)!, which is the same as the right side of the f(k+1)=(k+1)!, which proves the claim.

Checklist:
☐ Did you type in your name and UIN?
☐ Did you disclose all resources that you have used?
(This includes all people, books, websites, etc. that you have consulted)
☐ Did you sign that you followed the Aggie honor code?
☐ Did you solve all problems?
☐ Did you submit **both** of your .tex and .pdf files of this homework to the correct link on eCampus?