Pre-Lab 2: Second-Order Circuits

ECEN 325 - 511

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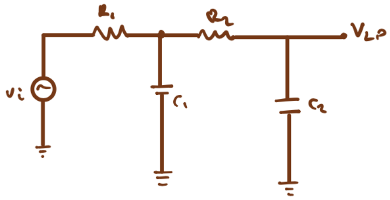
Due Date: September 14, 2021

Calculations

part 1 → find transfer functions

circuit A:



$$\frac{V - V_i}{R_1} + \frac{V}{Z_{C_1}} + \frac{V - V_{LP}}{R_2} = 0$$

$$V\left(\frac{1}{R_1} + C_1 S_1 + \frac{1}{R_2}\right) = \frac{V_i}{R_1} + \frac{V_{LP}}{R_2}$$

$$= V(R_1 + R_2 + C_1 S_1 R_1 R_2) = R_2 V_i + V_{LP} R$$

$$= \frac{R_2 V_i + V_{LP} R_1}{R_1 + R_2 + C_1 S_1 R_1 R_2}$$

and , $\frac{V_{LP}}{Z_{C_2}} + \frac{V_{LP} - V}{R_2} = 0$

$$S C_2 V_{LP} + \frac{V_{LP}}{R_2} = V \Rightarrow (S C_2 R_2 + 1) V_{LP} = V$$

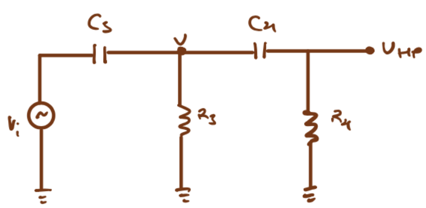
$$= V_{LP}(S C_2 R_2 + 1) = \frac{R_2 V_i + V_{LP} R_i}{R_1 + R_2 + C_1 S_1 R_1 R_2}$$

$$= V_{LP}(R_1 + R_2 + C_1 S_1 R_1 R_2 + R_1 R_2 C_2 S + S R_2^2 C_2 + R_1 R_2^2 C_1 C_2 S^2)$$
$$= R_2 V_i + V_{LP} R_i$$

$$= V_{LP}(R_2 + C_1 S_1 R_1 R_2 + R_1 R_2 C_2 S + S R_2^2 C_2 + R_1 R_2^2 C_1 C_2 S^2) = R_2 V_i$$

$$\boxed{\frac{V_{LP}}{V_i} = \frac{1}{R_1 + C_1 S_1 R_1 + C_2 R_1 S + S R_2 C_2 + R_1 R_2 C_1 C_2 S^2}}$$

circuit B:



$$\frac{V_0 - V_i}{Z_{C_3}} + \frac{V}{R_3} + \frac{V - V_{HP}}{Z_{C_4}} = 0$$

$$V\left(\frac{1}{1/SC_3} + 1/R_3 + 1/(1/SC_4)\right) = SC_3 V_i + SC_4 V_{HP}$$

$$V\left(SC_3 + \frac{1}{R_3} + SC_4\right) = SC_3 V_i + SC_4 V_{HP}$$

$$V(SC_3 R_3 + 1 + R_3 SC_4) = R_3 SC_3 V_i + R_3 SC_4 V_{HP}$$

$$V = \frac{R_3 S C_3 V_i + R_3 S C_4 V_{HP}}{(SC_3 R_3 + 1 + R_3 SC_4)}$$

and , $\frac{V_{HP}}{R_4} + \frac{V - V_0}{1/SC_4} = 0$

$$\frac{V_{HP}}{R_4} \quad V_{HP}\left(\frac{1}{R_4} + SC_4\right) = SC_4 V$$

$$V_{HP}(1 + R_4 S C_4) = S C_4 R_4 V$$

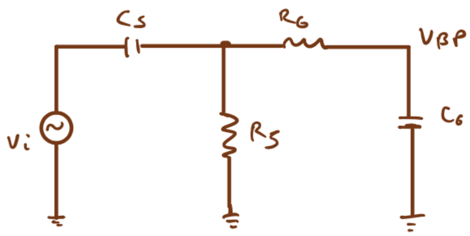
$$V_{HP}(1 + R_4 S C_4) = S R_4 C_4 \left(\frac{R_3 S C_3 V_i + S R_3 C_4 V_{HP}}{S C_3 R_3 + 1 + R_3 S C_4}\right)$$

$$V_{HP}(1 + R_4 s C_4)(s C_3 R_3 + 1 + R_3 s C_4) = s^2 (R_3 R_4 C_3 C_4 V_i + \cancel{R_3 R_4 C_4^2} V_{HP})$$

$$\hookrightarrow \; s C_3 R_3 + 1 + R_3 s C_4 + s^2 R_3 C_3 R_4 C_4 + C_4 R_4 + \cancel{s^2 R_3 R_4 C_4^2})$$

$$\boxed{\frac{V_{HP}}{V_i} = \frac{s^2 R_3 R_4 C_3 C_4}{1 + s(C_3 R_4 + C_3 R_3 + R_4 C_4) + s^2 R_3 R_4 C_3 C_4}}$$

Circuit C:



$$\frac{V - V_i}{ZC_5} + \frac{V}{R_5} + \frac{V - V_{BP}}{R_6} = 0$$

$$V\left(sC_5 + \frac{1}{R_5} + 1/R_6\right) = V_i s C_5 + \frac{V_{BP}}{R_6}$$

$$V\left(\frac{sC_5 R_5 R_6 + R_6 + R_5}{R_5 R_6}\right) = \frac{R_6 V_i s C_5 + V_{BP}}{R_6}$$

$$V(sC_5 R_5 R_6 + R_6 + R_5) = R_5 R_6 s C_5 V_i + V_{BP} R_5$$

$$V = \frac{s R_5 R_6 V_i + V_{BP} R_5}{sC_5 R_5 R_6 + R_6 + R_5}$$

and, $\dfrac{V_{BP}}{ZC_6} + \dfrac{V_{BP} - V}{R_6} = 0$

$$= V_{BP}(sC_6 + 1/R_6) = V/R_6$$

$$= V_{BP}(sC_6 R_6 + 1) = V$$

$$= V_{BP}(sC_6 R_6 + 1) = \frac{s R_5 R_6 C_5 V_i + V_{BP} R_5}{sC_5 R_5 R_6 + R_6 + R_5}$$

$$V_{BP}(1 + sC_6 R_6)(sC_5 R_5 R_6 + R_6 + R_5) = s R_5 R_6 C_5 V_i + V_{BP} R_5$$

$$V_{BP}(1 + sC_5 R_5 + s^2 C_6 C_5 R_5 R_6 + sC_6 R_6 + sC_6 R_5) = s R_5 C_5 V_i$$

$$\boxed{\frac{V_{BP}}{V_i} = \frac{s R_5 C_5}{1 + s(C_5 R_5 + C_6 R_6 + C_6 R_5) + s^2 C_6 C_5 R_5 R_6}}$$

part 2 → find values for resistors and capacitors

circuit A: $H_{LP} = \frac{1}{1+\frac{s}{2\pi f_1}} \cdot \frac{1}{1+\frac{s}{2\pi f_2}} = \frac{1}{1 + s\left(\frac{1}{2\pi f_1} + \frac{1}{2\pi f_2}\right) + s^2\left(\frac{1}{4\pi^2 f_1 f_2}\right)}$

lets say $C_1 = 47nF$ and $C_2 = 1nF$

then $\frac{1}{2\pi f_1} + \frac{1}{2\pi f_2} = C_1R_1 + C_2R_2 + R_1C_2$

$\frac{f_1 + f_2}{2\pi f_1 f_2} = R_1(C_1 + C_2) + C_2R_2$

$= \frac{11000}{2\pi 10000000} = R_1[47\times10^{-9}] + 1\times10^{-9} R_2$

$R_2 = \frac{1.75\times10^{-4}}{1\times10^{-9}} - \frac{(47\times10^{-9})R_1}{1\times10^{-9}}$ $R_2 = 1.75\times10^{-5} - 47R_1$

$\frac{1}{4\pi^2 f_1 f_2} = R_1R_2C_1C_2 \rightarrow 2.533\times10^{-9} = (1.75\times10^{5} - 47R_1)47\times10^{-9}R_1$

$5.39\times10^{-2} = 1.75\times10^{5}R_1 - 47R_1^2$

$47R_1^2 - 1.75\times10^{5}R_1 + 5.39\times10^{-2} = 0$

$$\boxed{R_1 = 3723\Omega, \quad R_2 = 171277\Omega, \quad C_1 = 47nF, \quad C_2 = 1nF}$$

Circuit B: $H_{HP}(s) = \frac{s}{s+2\pi f_3} \cdot \frac{s}{s+2\pi f_4} = \frac{s^2/4\pi^2 f_3 f_4}{1 + \frac{2\pi(f_3+f_4)}{4\pi^2 f_3 f_4} + \frac{s^2}{4\pi^2 f_3 f_4}}$

$C_4R_4 + C_4R_3 + R_3C_3 = \frac{f_3+f_4}{2\pi(f_3 f_4)}$ let $C_3 = 10\mu F$ and $C_4 = 1nF$

then, $1\times10^{-9}R_4 + 1\times10^{-9}R_3 + 10\times10^{-6}R_3 = 3.5\times10^{-4}$

$= 1\times10^{-9}R_4 + 10\times10^{-6}R_3 = 3.5\times10^{-4}$

$= 1\times10^{-3}R_4 + R_3 = 35$

so, $\frac{1}{4\pi^2 f_3 f_4} = R_3R_4C_3C_4$

$= \frac{1}{4\pi^2\cdot10^{7}} = R_4(35 - 1\times10^{-3}R_4)1\times10^{-14}$

$$\boxed{R_3 = 24.77\Omega \quad R_4 = 10223.3\Omega \quad C_3 = 10\mu F \quad C_4 = 1nF}$$

circuit C: $\frac{s}{s+2\pi f_5} \cdot \frac{1}{1+\frac{s}{2\pi f_6}} = \frac{s/2\pi f_5}{\frac{s}{2\pi f_5}+1} \cdot \frac{1}{1+s/2\pi f_6} = \frac{s/2\pi f_5}{1+\left(\frac{1}{2\pi f_5}+\frac{1}{2\pi f_6}\right)s + s^2\left(\frac{1}{4\pi^2 f_5 f_6}\right)}$

$C_5 R_5 + C_6 R_6 + C_6 R_5 = \frac{1}{2\pi}\left(\frac{1}{f_5}+\frac{1}{f_6}\right)$

$R_5(C_6 + C_5) + C_6 R_6 = \frac{1}{2\pi}\left(\frac{1}{1k}+\frac{1}{100k}\right)$

let $C_5 = 1\mu F$ and $C_6 = 1nF$

$C_5 R_5 + C_6 R_6 = \frac{1}{2\pi}\left(\frac{101000}{100000000}\right)$

$1\times10^{-6} R_5 + 1\times10^{-9} R_6 = 1.607\times10^{-5}$

$R_5 = 1607 + 1\times10^{-9} R_6$

so, $\frac{1}{4\pi^2 f_5 f_6} = C_6 C_5 R_5 R_6$

$= \frac{2.575\times10^{-11}}{1\times10^{-15}} = (16.07 + 1\times10^{-9} R_6)$

$25330 = (16.07 + 1\times10^{-9} R_6) R_6$

$\boxed{R_5 = 16.07\Omega \quad R_6 = 1576\Omega \quad C_5 = 1\mu F \quad C_6 = 1nF}$

part 3 → plots

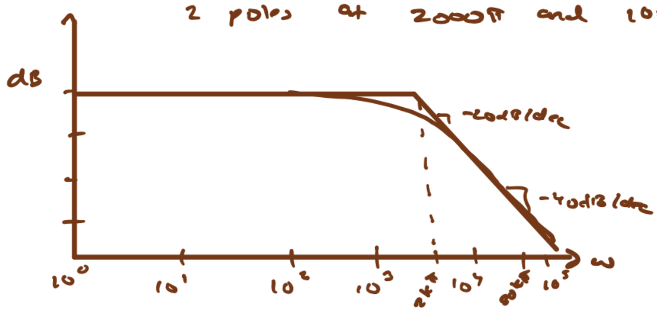
circuit A: $f_1 = 1kHz$ $f_2 = 10kHz$

$H_{LP} = \frac{1}{1+\frac{s}{2000\pi}} \cdot \frac{1}{1+\frac{s}{20000\pi}}$

$\lim_{s\to0} (H_{LP}) = 1$

$\lim_{s\to\infty} = 0$

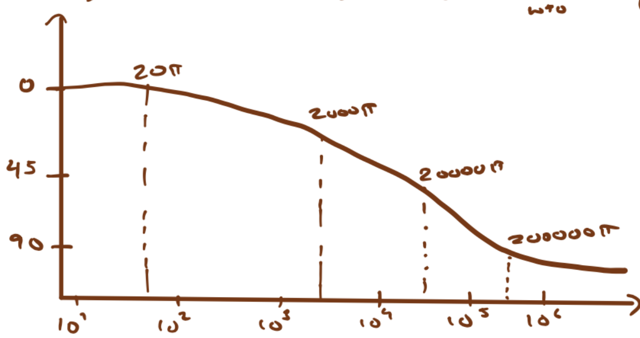
2 poles at 2000π and 10000π



magnitude response

$[20\pi, 20000\pi]$ & $[2000\pi, 200000\pi]$

$$-\lim_{\omega\to 0} \tan^{-1}\left(\frac{\omega}{2000\pi}\right) + \lim_{\omega\to 0} \tan^{-1}\left(\frac{\omega}{\text{[illegible]}}\right)$$



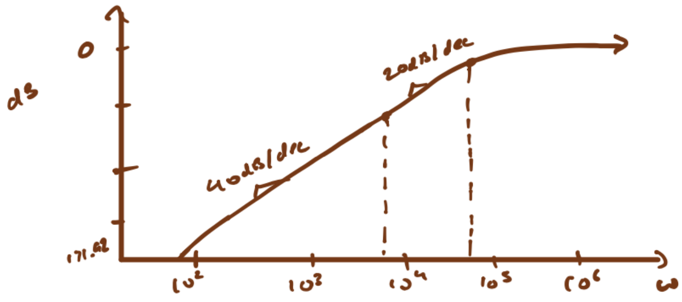
phase response

Circuit B: $H_{HP} = \frac{s}{s+2\pi f_3} \cdot \frac{s}{s+2\pi f_4}$ since $f_3 = 1kHz$, $f_4 = 10kHz$ $= \frac{s}{s+2000\pi} \cdot \frac{s}{s+20000\pi}$

there's two poles at 2000π, 20000π and 2 zeros at 0

$$\lim_{s\to 0} |H_{HP}| = 10\log(s)^2 + 10\log_{10}(s)^2 - 10\log_{10}(2000\pi)^2 - 10\log_{10}(20000\pi)^2$$

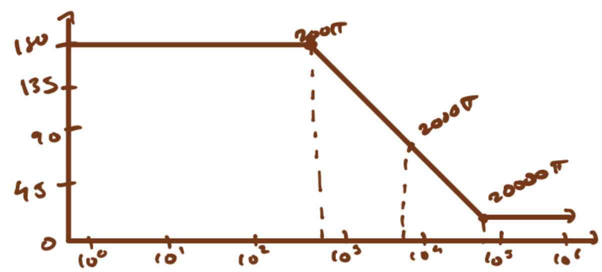
$$= -75.96 - 95.96 = -171.92$$



magnitude Response

$$\lim_{\omega\to 0} \text{phase } |H_{HP}| = \tan^{-1}\left(\frac{s}{0}\right) + \tan^{-1}\left(\frac{s}{0}\right) - \tan^{-1}\left(\frac{s}{2000\pi}\right) - \tan^{-1}\left(\frac{s}{20000\pi}\right)$$

$$= 90 + 90 = 180°$$

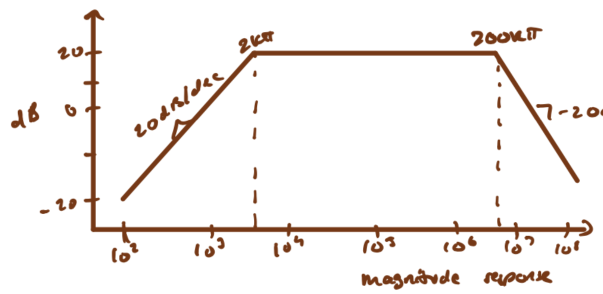


phase response

circuit 1: $\frac{s}{s+2\pi f_s} \cdot \frac{1}{1+s/2\pi f_b}$ $\quad f_s = 1\text{kHz}$, $f_b = 100\text{kHz}$ $= \frac{s}{s+2000\pi} + \frac{1}{1+\frac{s}{200k\pi}}$

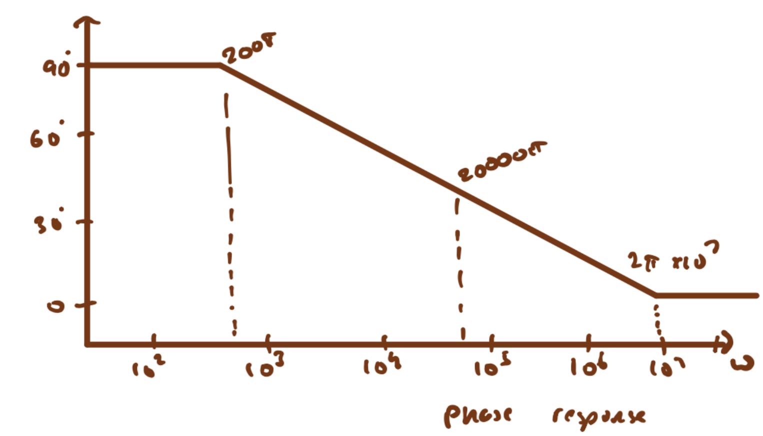
there's 0's on 0 and poles on 2000π and 200kπ

$$\lim_{\omega\to 0} |H_{BP}| = 10\log_{10}(s^2) - 10\log(2000\pi)^2 - 20\log_{10}(200k\pi)^2$$
$$= -24.42 \text{ dB}$$



$$\lim_{\omega\to 0} \text{phase}|H_{BP}| = \tan^{-1}\left(\frac{s}{0}\right) - \tan^{-1}\left(\frac{s}{2000\pi}\right) - \tan^{-1}\left(\frac{s}{200k\pi}\right) = 90°$$

[200π, 20000π] [20000π, 20000000π]



part 4 → find V_o

circuit A: Here $V_i = 0.5\sin(2\pi\, 10000t)$
$V_i = 0.5\angle 0^\circ$

Now $H_{LP} = \dfrac{1}{1+\frac{j\omega}{2000\pi}} \cdot \dfrac{1}{1+\frac{j\omega}{20000\pi}}$

$$H_{LP}(2\pi\,10000) = \frac{1}{1+j\frac{2\cancel{0000\pi}}{2\cancel{000\pi}}} \cdot \frac{1}{1+j\frac{\cancel{20000\pi}}{\cancel{20000\pi}}}$$

$= \dfrac{1}{1+10j} \cdot \dfrac{1}{1+j} = -0.024 - 0.026i$

$= 0.035\angle -132.376$

so,

$V_{LP} = H_{LP} \cdot 0.5\angle 0$

$= 0.0176\angle -132.376 =$ $\boxed{V_{LP} = .0176\sin(2\pi\,10000 - 132.376)}$

Circuit B: $H_{HP} = \dfrac{s}{s+2\pi f_3} \cdot \dfrac{s}{s+2\pi f_4}$ $\quad f_3 = 1\,kHz$, $f_4 = 10\,kHz$

$= \dfrac{s}{s+2\pi\,1000} \cdot \dfrac{s}{s+2\pi\,10000} = \dfrac{j\omega}{j\omega + 2000\pi} \cdot \dfrac{j\omega}{j\omega + 20000\pi}$

$H_{HP}(10000) = \dfrac{2\pi\,10000\pi j}{20000\pi + 2000\pi} + \dfrac{20000\pi j}{20000\pi + 20000\pi}$

$= \dfrac{10j}{10+j} \cdot \dfrac{j}{1+j} = 0.70\angle 129.28$

$V_{HP} = H_{HP} \cdot 0.5\angle 0 = 0.70\angle 129.28 \cdot 0.5\angle 0 = .35\angle 129.28$

$\boxed{V_{HP} = 0.35\sin(2\pi\,10000t + 129.28)}$

circuit c: $V_{BP} = \frac{s}{s+2\pi f_S} \cdot \frac{1}{1+\frac{s}{2\pi f_6}}$

$f_S = 1\text{kHz}$

$f_6 = 100\text{kHz}$

$= \frac{j\omega}{j\omega + 2000\pi} \cdot \frac{1}{1+\frac{j\omega}{200k\pi}}$

where $\omega = 2\pi \cdot 10000$

$$H_{BP}(10000) = \frac{j20000\pi}{j20000\pi + 2000\pi} \cdot \frac{1}{1+\frac{2\cancel{0000\pi} j}{200\cancel{0000\pi}}} = \frac{10j}{10j+1} \cdot \frac{1}{1+j/100}$$

$$= \frac{10j}{10j+1} \cdot \frac{100}{100+j} = 0.995\angle 5.138$$

$$v_{BP} = H_{BP} v_i$$

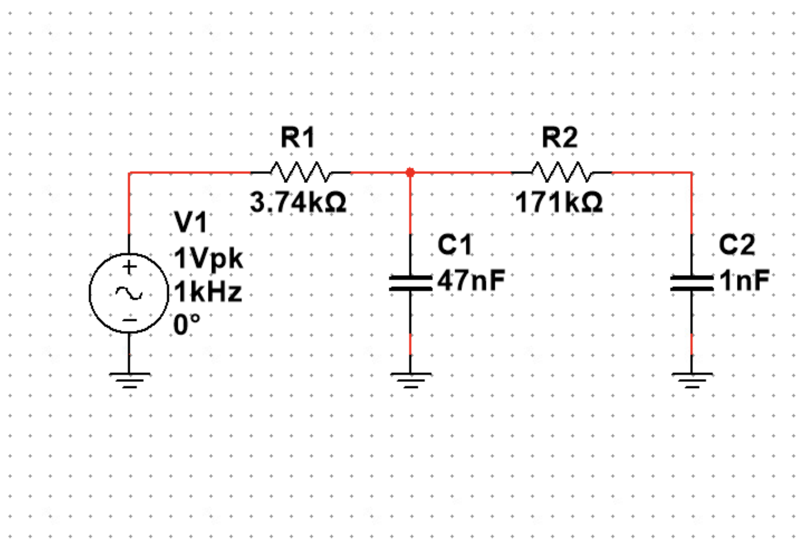
$$= 0.995\angle 5.138 \cdot 0.5\angle 0 = 0.497\angle 5.138$$

$$\boxed{V_{BP} = 0.497\sin(2\pi\, 10000t + 5.138)}$$

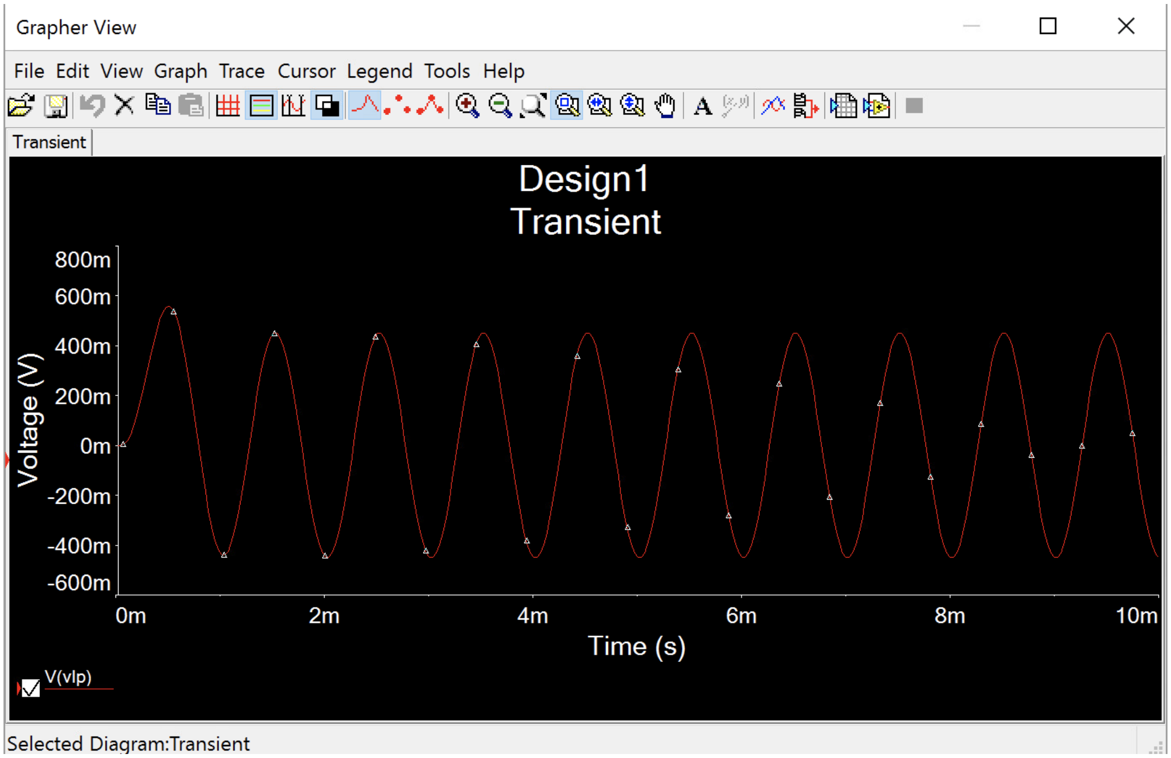
<u>Simulations (on Multisim)</u>

Lowpass Circuit

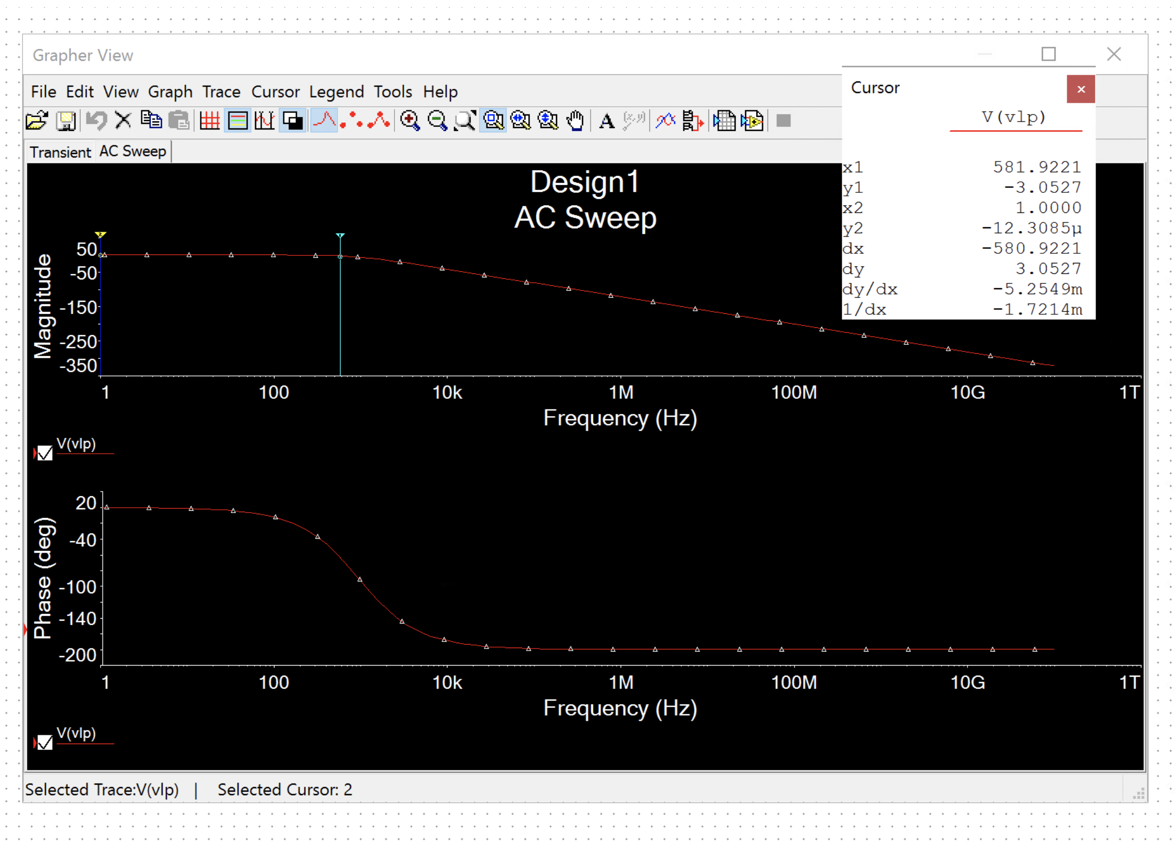
Schematic



Transient

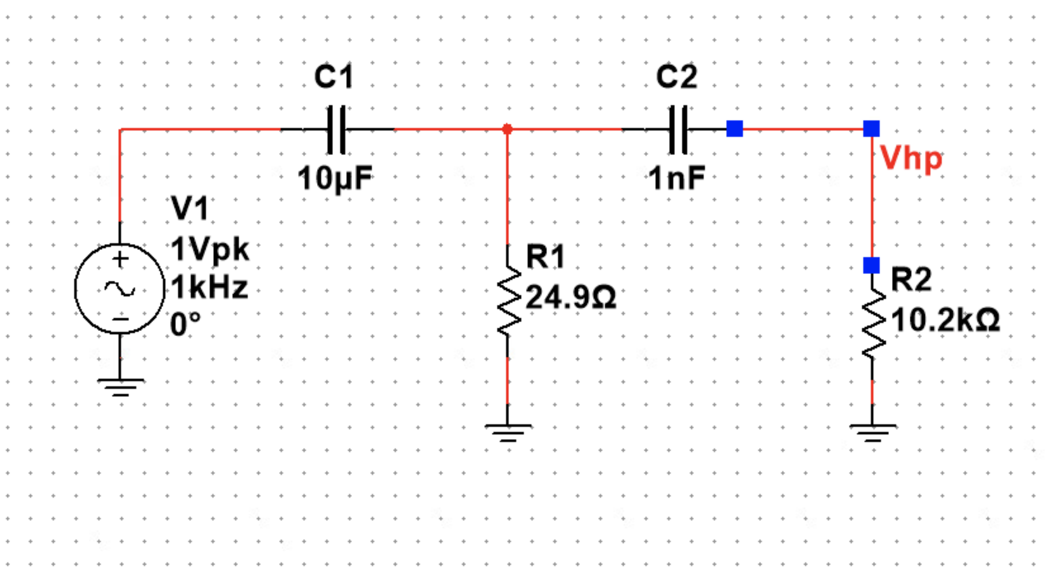


AC Sweep

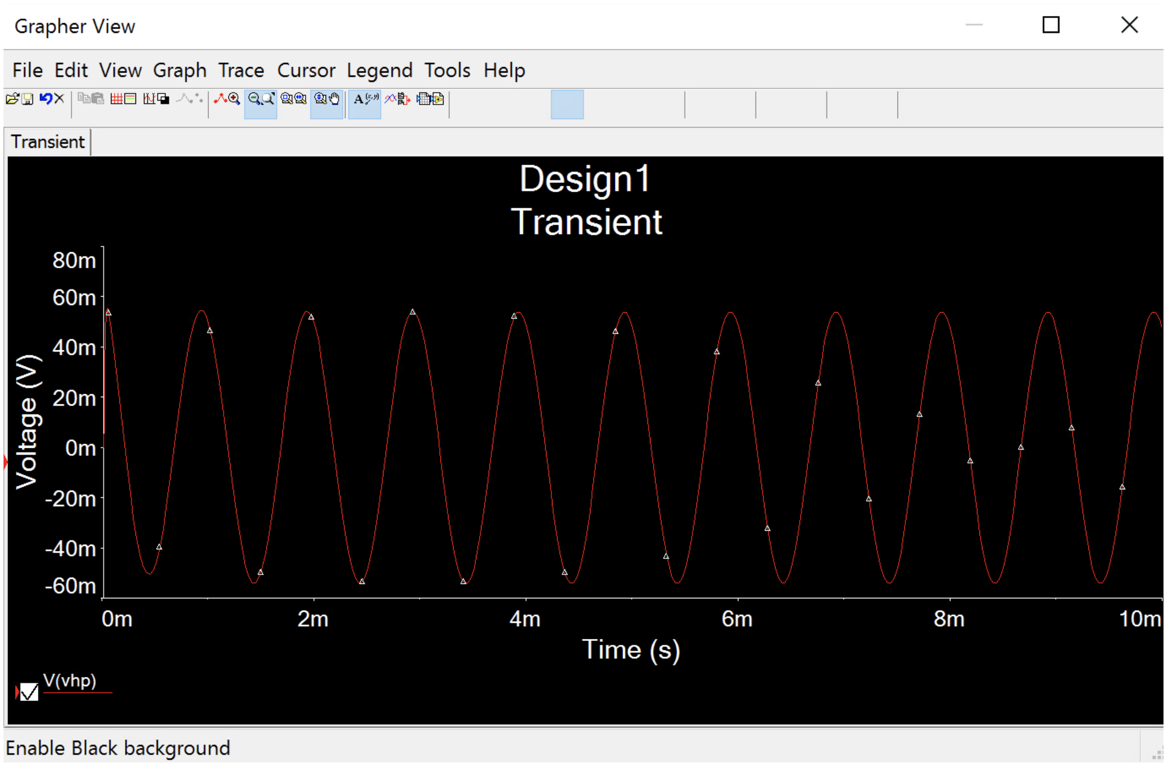


Highpass Circuit

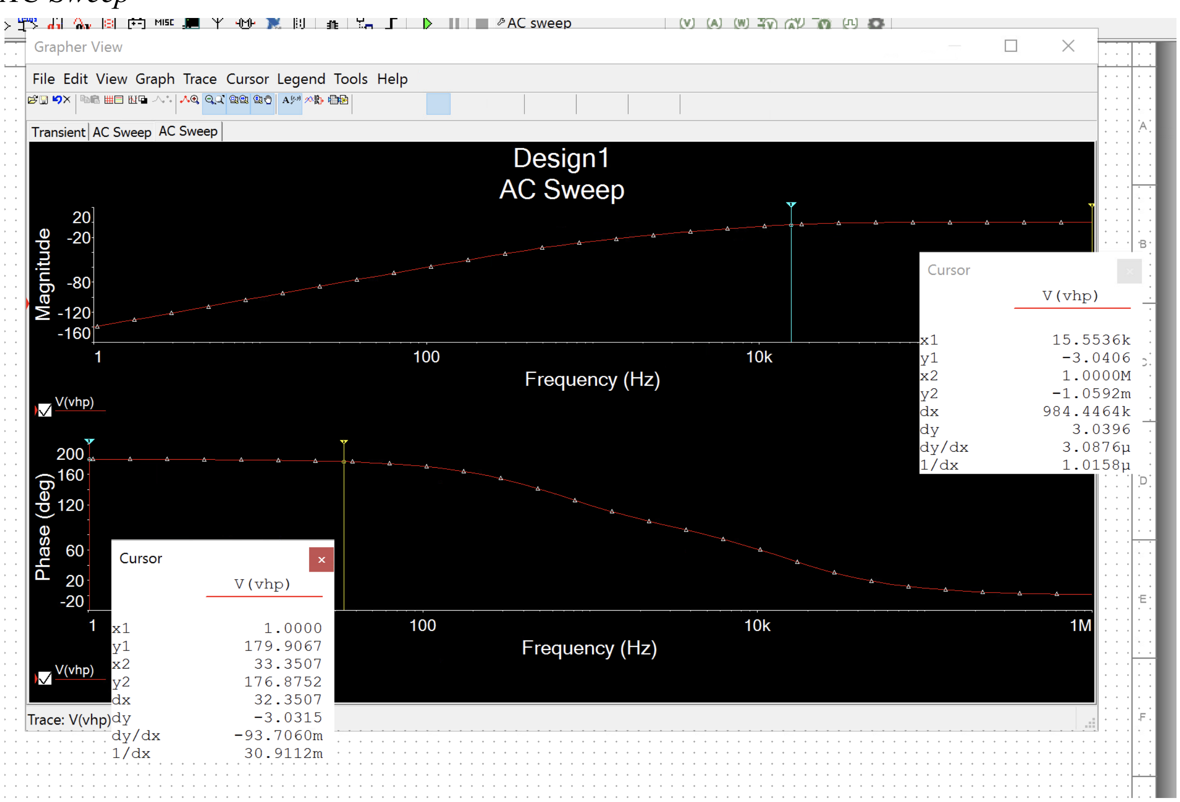
Schematics



Transient

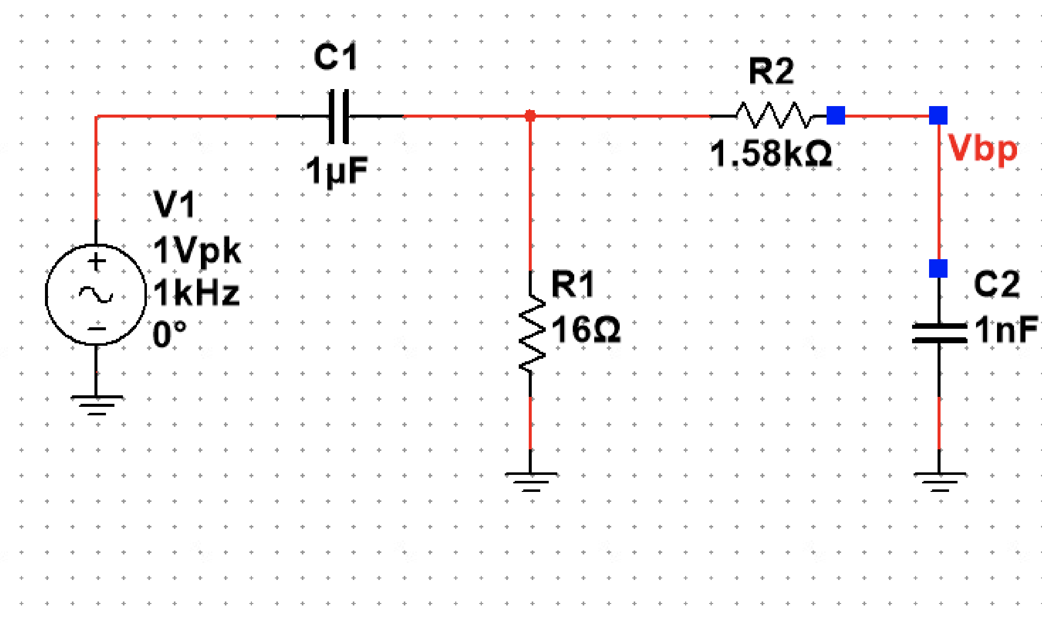


AC Sweep

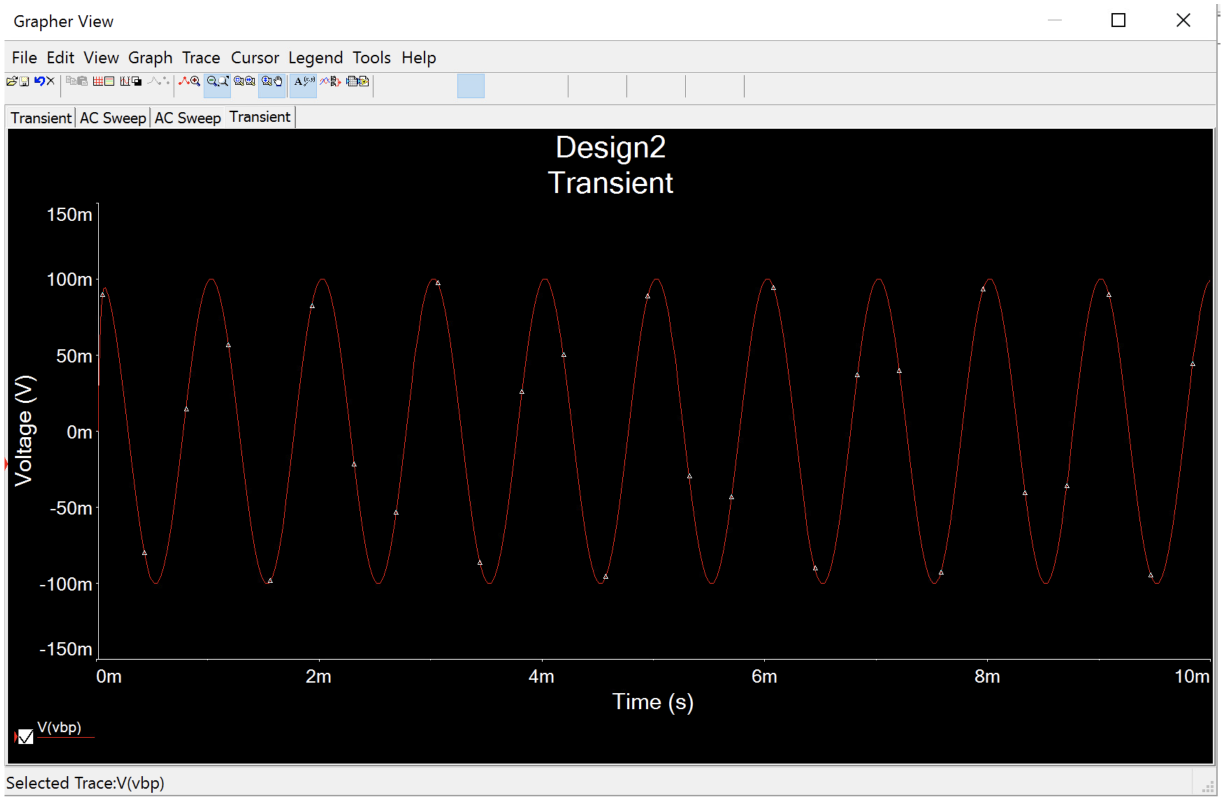


Bandpass Circuit

Schematic



Transient



AC Sweep