CSCE 222-199 Discrete Structures for Computing – Summer 2020
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Problem Set 3

Due dates: Electronic submission of *yourLastName-yourFirstName-hw3.tex* and *yourLastName-yourFirstName-hw3.pdf* files of this homework is due on **Monday, 6/8/2020 before 11:00 p.m.** on `https://ecampus.tamu.edu`. You will see two separate links to turn in the .tex file and the .pdf file separately. Please do not archive or compress the files. **If any of the two files are missing, you will receive zero points for this homework.**

Name: [redacted] **UIN:** [redacted]

Resources. For all parts of this question, I utilized the text from Perusall and any notes I took from the videos provided during the week. I also used some youtube tutorials to help me navigate through Latex.

On my honor, as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment. Furthermore, I have disclosed all resources (people, books, web sites, etc.) that have been used to answer this homework.

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Total 100 points.

Problem 1. (15 points) Section 2.9, Exercise 2.41

Solution. Given that m and n are integers, let m not be greater than 40 and n not be greater than 60, hence $m < 40$ and $n < 60$. Any number m less than 40 and n less than 60, will have a sum thats less than 100. Hence, $m + n < 100$. Therefore, if this statement is true, then the claim that if the sum of $m+n > 100$, then $m > 40$ and $n > 60$ is also true by proof by contraposition.

Problem 2. (20 points) Section 2.9, Exercise 2.45

Solution. Begin by assuming that 42m + 70n =1000 has an integral solution. This means that $m, n \in Z$. So factoring out a 7 gives us 7(6m + 10n)= 1000.
6m + 10n =1000/7
since $m, n \in Z$ we can say that $6m, 10n \in Z$, so 6m + 10n is an integer. But since 6m + 10n =1000/7, and since 1000/7 is a rational number it contradicts the assumption of the result being an integer. Hence 42m + 70n =1000 has no integral solution.

Problem 3. (10 points) Section 3.1, Exercise 3.2

Solution. Suppose $A \subseteq B$ and $B \subseteq C$, then let $x \in A$. Since $A \subseteq B$ then $x \in B$. Then $x \in B$ and $B \subseteq C$, then $x \in C$. Then $x \in C$, thus $A \subseteq C$, by definition.

Problem 4. (10 points) Section 3.2, Exercise 3.10

Solution. We can assume that $A \subseteq B$, and by definition if $x \in A \cap B$ then it can be divided to $x \in A$ and $x \in B$, which leads to prove $A \cap B \subseteq A$. Lets consider $x \in A$, and $x \in B$ which results in $x \in A \cap B$ and ultimately proving $A \subseteq A \cap B$. This leads to the original statement A $= A \cap B$ and $x \in A$, we can assume that $x \in A \cap B$ so $x \in A$ and $x \in B$, which finally proves $A \subseteq B$.

Problem 5. (10 points) Section 3.3, Exercise 3.12 [Hint: Use the definition of set difference.]

Solution. Given $A \cap (B - C)$ and $(A \cap B) - C$, we can say that $x \in A, x \in B, x \notin C$. Assuming $x \in A \cap (B - C)$ then we can say that $x \in A$ and $x \in B - C$, furthermore leading to $x \in B$ and $x \notin C$. From this we can conclude that $x \in A \cap B) - C$, which proves that $A \cap (B - C) \subseteq (A \cap B) - C$.
To be able to further prove that the two statements are equal, if $x \in (A \cap B) - C$, then $x \notin C$ and $x \in A \cap B$, thus $x \in A$ and $x \in B$. So, $x \in A \cap (B - C)$, concluding to $(A \cap B) - C \subseteq A \cap (B - C)$.
Hence, $A \cap (B - C) = (A \cap B) - C$.

Problem 6. (15 points) Section 3.4, Exercise 3.18 [Hint: Show that the left-hand side of the equality is a subset of the right-hand side, and vice versa.]

Solution. Given $(A \cup B) \times C$, we can say $(x, y) \in (A \cup B) \times C$. This simplifies to $x \in A \cup B$ and $y \in C$. Hence $x \in A$ but $x \notin B$, and $y \in C$. Thus, $(x, y) \in A \times C$ and $(x, y) \notin B \times C$, so $(A \cup B) \times C \subseteq (A \times C) \cup (B \times C)$. However, lets take $(x, y) \in (A \times C) \cup (B \times C)$ and we can say that $(x, y) \in A \times C$ and $(x, y) \notin B \times C$. Since, $(x.y) \notin B \times C$ and $x \notin B$, but still have $y \in C$, leaves with the implication that if $x \in A \cup B$ and $y \in C$ then $(x, y) \in (A \cup B) \times C$, we can conclude that $(A \cup B) \times C$ is also a superset of $(A \times C) \cup (B \times C)$, hence $(A \cup B) \times C = (A \times C) \cup (B \times C)$.

Problem 7. (20 points) Section 3.5, Exercise 3.28

Solution. We know that in order for a function to be bijective, it must be both injective and surjective. So given that both A and B are nonempty sets. We know that if there exists an injective function f: $A \rightarrow B$, then there exists a surjective function g: $B \rightarrow A$ such that $g \circ f = \mathrm{i}_A$ proven in Proposition 3.25. And given that f: $A \rightarrow B$ is surjective, then there exists an injective function g: $B \rightarrow A$ such that $f \circ g = \mathrm{i}_B$, proven in Proposition 3.29. Utilizing these propositions, function f: $A \rightarrow B$ is proven to be both injective and surjective, and hence can be bijective given there exists g: $B \rightarrow A$ such that $g \circ f = \mathrm{i}_A$ and $f \circ g = \mathrm{i}_B$.

Checklist:

☐ Did you type in your name and UIN?
☐ Did you disclose all resources that you have used?
(This includes all people, books, websites, etc. that you have consulted)
☐ Did you sign that you followed the Aggie honor code?
☐ Did you solve all problems?
☐ Did you submit the .tex and .pdf files of your homework to the correct link on eCampus?