Lab 1: First Order Circuits

ECEN 325 - 511

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Due Date: September 16, 2021

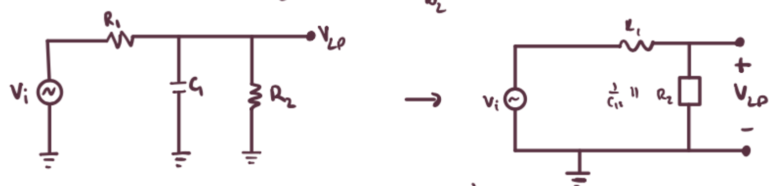
Purpose

The objective of this lab was to find the frequency response of first-order circuits and go over the basics of circuit analysis. In order to do this, students analyzed the first order low and high pass filter, consisting of some resistors and capacitors. We derive the transfer function for high and low pass filters and calculate the values for the resistors and capacitors. We were also supposed to calculate the output voltages. Finally, we run the simulations, and collect the measurement, by building the circuit on both Multisim and on a breadboard.

Calculations

part 1 → derive transfer functions & express ω_L, ω_H, k_L, k_H in terms of resistors and capacitors:

Figure 3a: $H_{LP}(s) = \frac{V_{LP}}{V_i}(s) = k_L \frac{1}{1+\frac{s}{\omega_L}}$



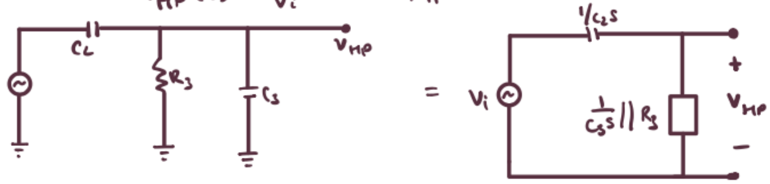
$$C_1 = \frac{1}{C_1 s} \rightarrow \frac{1}{C_1 s} \parallel R_2 = \frac{\frac{1}{C_1 s}\cdot R_2}{\frac{1}{C_1 s}+R_2} = \frac{\frac{R_2}{C_1 s}}{\frac{R_2 C_1 s+1}{C_1 s}} = \frac{R_2}{R_2 C_1 s+1}$$

$$V_{LP} = \frac{\frac{R_2}{R_2 C_1 s+1}}{R_1+\frac{R_2}{R_2 C_1 s+1}} V_i \Rightarrow \frac{V_{LP}}{V_i} = \frac{\frac{R_2}{R_2 C_1 s+1}}{R_1+\frac{R_2}{R_2 C_1 s+1}} = \frac{R_2}{R_1+R_2+R_1 R_2 C_1 s}$$

$$H_{LP}(s) = k_L \frac{1}{1+\frac{s}{\omega_L}} = \frac{R_2}{R_1+R_2+R_1 R_2 C_1 s} = \frac{R_2}{R_1+R_2}\left(\frac{1}{1+\frac{R_1 R_2 C_1 s}{R_1+R_2}}\right)$$

$$\boxed{k_L = \frac{R_2}{R_1+R_2} \qquad \omega_L = \frac{R_1+R_2}{R_1 R_2 C_1}}$$

Figure 3b: $H_{HP}(s) = \frac{V_{HP}}{V_i}(s) = k_H \frac{s}{s+\omega_H}$



$$\frac{1}{C_3 s} \,||\, R_3 = \frac{R_3}{1+R_3C_3 s}$$

$$V_{HP} = V_i \frac{\frac{R_3}{1+R_3C_3 s}}{\frac{1}{C_2 s} + \frac{R_3}{1+R_3C_3 s}} \Rightarrow \frac{V_{HP}}{V_i} = \frac{R_3 C_2 s}{1 + R_3C_3 s + R_3 C_2 s}$$

$$H_{HP}(s) = k_H \frac{s}{s+\omega_H} = \frac{R_3C_2 s}{1+R_3C_3 s+R_3C_2 s} = \frac{C_2}{C_2+C_3} \; \frac{s}{s+\frac{1}{R_3(C_2+C_3)}}$$

$$\boxed{k_H = \frac{C_2}{C_2+C_3} \qquad \omega_H = \frac{1}{R_3(C_2+C_3)}}$$

part 2 → Calculating values for all resistors & capacitors

$$k_L = 0.5 = \frac{R_2}{R_1+R_2} \Rightarrow R_1 = R_2$$

$$\omega_L = 2\pi f_L = 2\pi(5000) = \frac{R_1+R_2}{R_1R_2C_1}$$

$$= \frac{2R_1}{R_1^2 C_1}$$

$$= {}^2/_{R_1C_1} = 2\pi(5000)$$

$$= R_1C_1 = {}^1/_{5000\pi} \approx 63.66 \times 10^{-6}$$

$$= \boxed{R_1 = R_2 = 1\,\Omega \;;\; C_1 = 63.66\,\mu F}$$

$$k_H = 0.5 = \frac{C_2}{C_2+C_3} \Rightarrow C_2 = C_3$$

$$\omega_H = 2\pi f_H = 2\pi(5000) = \frac{1}{R_3(C_2+C_3)}$$

$$10000\pi = {}^1/_{2R_3C_2}$$

$$= R_3C_2 = {}^1/_{20000\pi} \approx 15.9 \times 10^{-6}$$

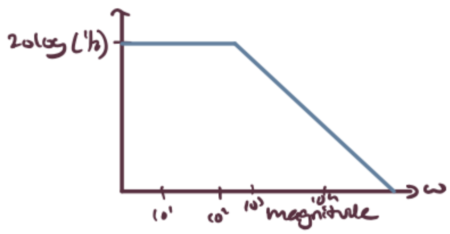
$$\boxed{R_3 = 1\,\Omega \;;\; C_2 = C_3 = 15.9\,\mu F}$$

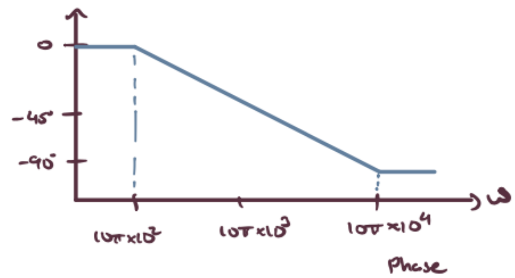
part 3 → magnitude / phase Bode plots

f. 3a) $H_{LP}(s) = K_L \dfrac{1}{1+\frac{s}{\omega_L}} = \dfrac{0.5}{1+\frac{0.5}{2\pi \cdot 5000}} = \dfrac{5000\pi}{10000\pi + s}$

$P_1 = -10000\pi$

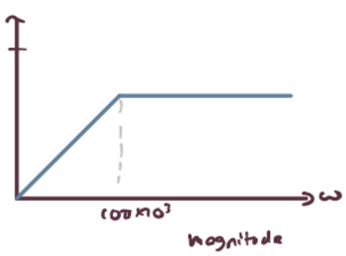
$s=0, \; H_{LP}(0) = 1/2$

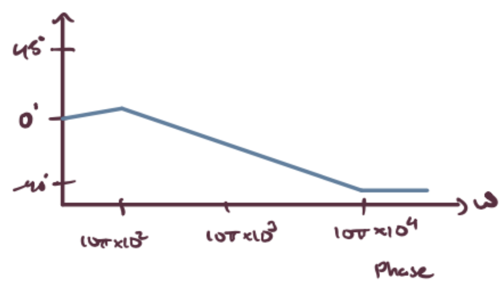




f. 3b) $H_{HP}(s) = K_H \dfrac{s}{s+\omega_H} = \dfrac{0.5s}{s+10000\pi}$

$z_1 = 0, \quad P_1 = -10000\pi \qquad s=0 \Rightarrow 0$





part 4 → $V_{LP}(t)$? $V_{HP}(t)$? for $0.4\sin(2\pi\,4000t)$

f. 3a) $V_i = 0.4\sin(2\pi\,4000t)$

$= 0.4\angle 0°$

$\omega = 8000\pi$

$$H_{LP}(s) = \frac{5000\pi}{10000\pi + s}$$

$$H_{LP}(\omega) = \frac{5000\pi}{10000\pi + j\omega} = \frac{5000\pi}{10000\pi + j(8000\pi)}$$

$$|H_{LP}| = \frac{5000\pi}{\sqrt{(10\pi\times10^3)^2 + (8\pi\times10^3)^2}} = \frac{5}{\sqrt{164}} \approx 0.468$$

$$\angle H_{LP} = -\tan^{-1}\left(\frac{8\pi\times10^3}{10\pi\times10^3}\right) \approx -38.66°$$

$$V_{LP}(t) = 0.4 \cdot 0.468 \cdot \sin(8000\pi t - 38.66°)$$

$$\boxed{V_{LP}(t) = 0.194\sin(8000\pi t - 38.66°)}$$

f. 3b) $H_{HP}(s) = \dfrac{0.5s}{s + 10000\pi} \Rightarrow \dfrac{j0.5\omega}{j\omega + 10000\pi}$

$$H_{HP}(j\omega) = \left(\frac{j \cdot 0.5 \cdot 8000\pi}{j(8000\pi) + 10000\pi}\right) \cdot 0.4\angle 0°$$

$= (0.195 + 0.244j)(0.4\angle 0°)$

$= (0.312\angle 51.3°)(0.4\angle 0°)$

$= (0.125\angle 51.34°)$

$$\boxed{V_{HP}(t) = 0.125\sin(8000\pi t + 51.34°)}$$

part 5 → $V_i(t) = 0.3\sin(12000\pi t) = 0.3\angle 0^\circ$

f. 3a) $V_{LP}(j\omega) = \dfrac{5000\pi}{10000\pi + 12000\pi j} \cdot 0.3\angle 0^\circ$

$= (0.246 + 0.205j)(0.3\angle 0^\circ)$

$= 0.096\angle 39.8^\circ$

$$\boxed{V_{LP}(t) = 0.096\sin(12000\pi + 39.8^\circ)}$$

f. 3b) $V_{HP}(j\omega) = \dfrac{(0.5)(12000\pi)j}{(12000\pi)j + 10000\pi} (0.3\angle 0^\circ)$

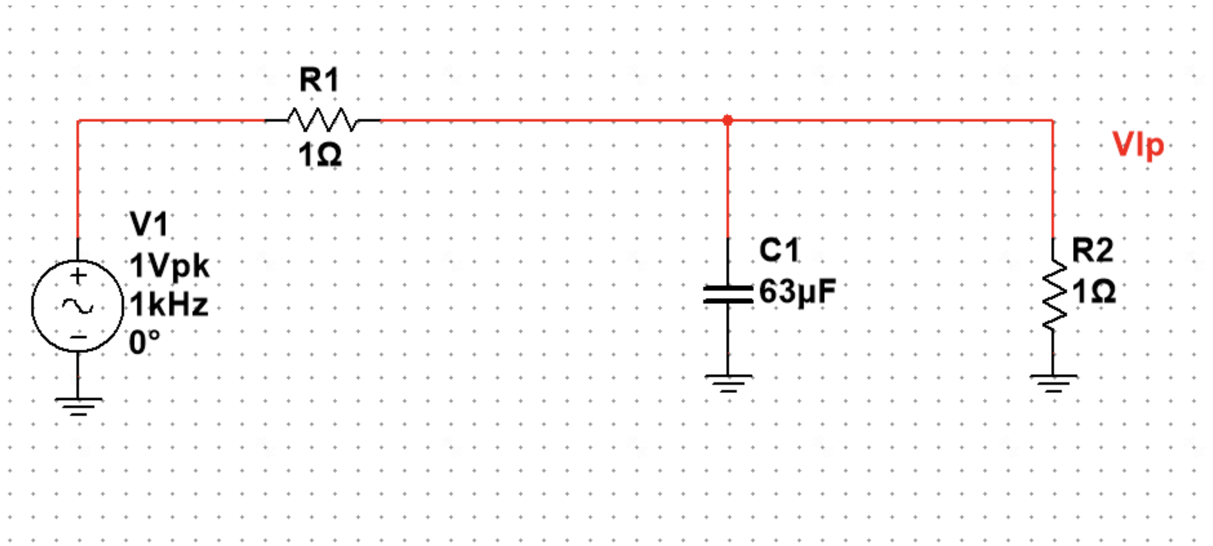
$= (0.295 + 0.246j)(0.3\angle 0^\circ)$

$= 0.115\angle 39.8^\circ$

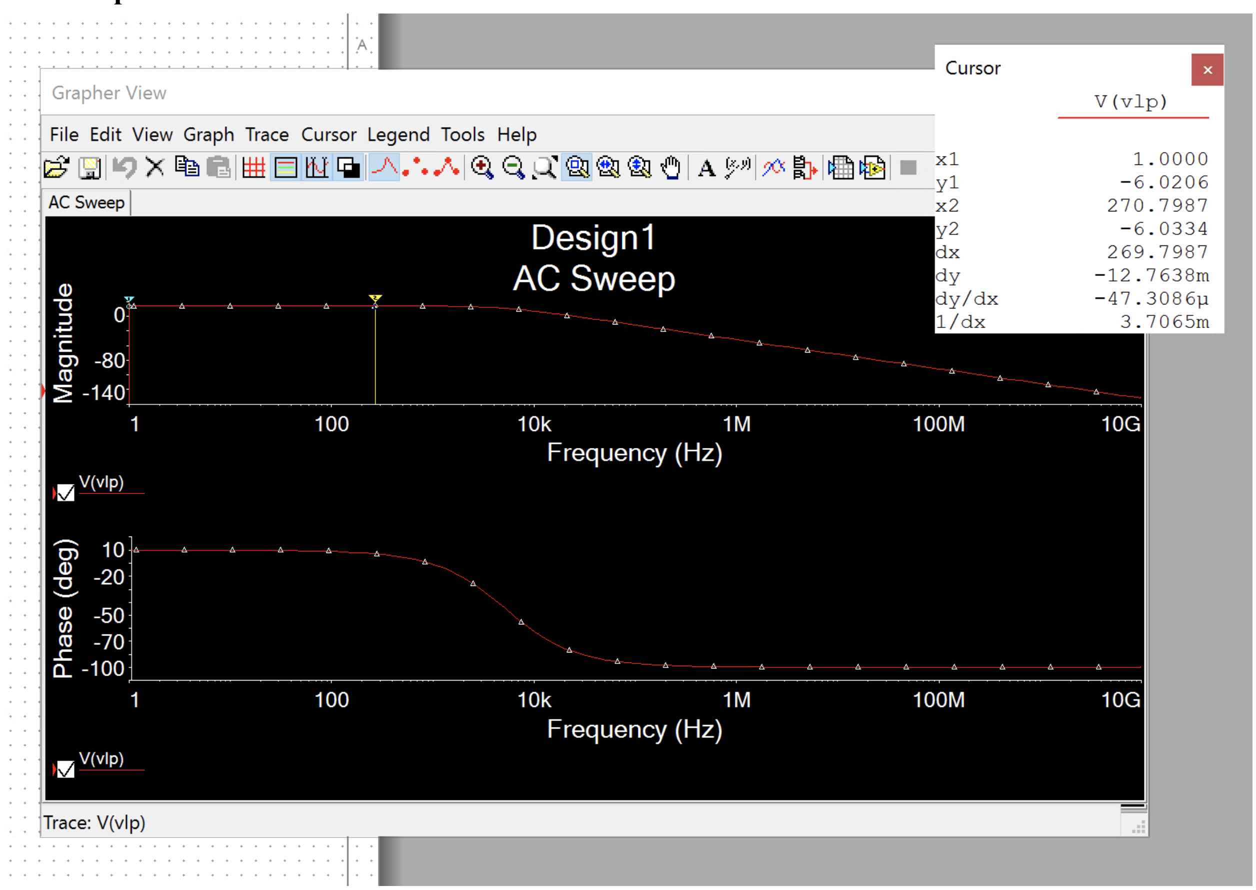
$$\boxed{V_{HP}(t) = 0.115\sin(12000\pi + 39.8^\circ)}$$

Schematics / Simulations (on Multisim)

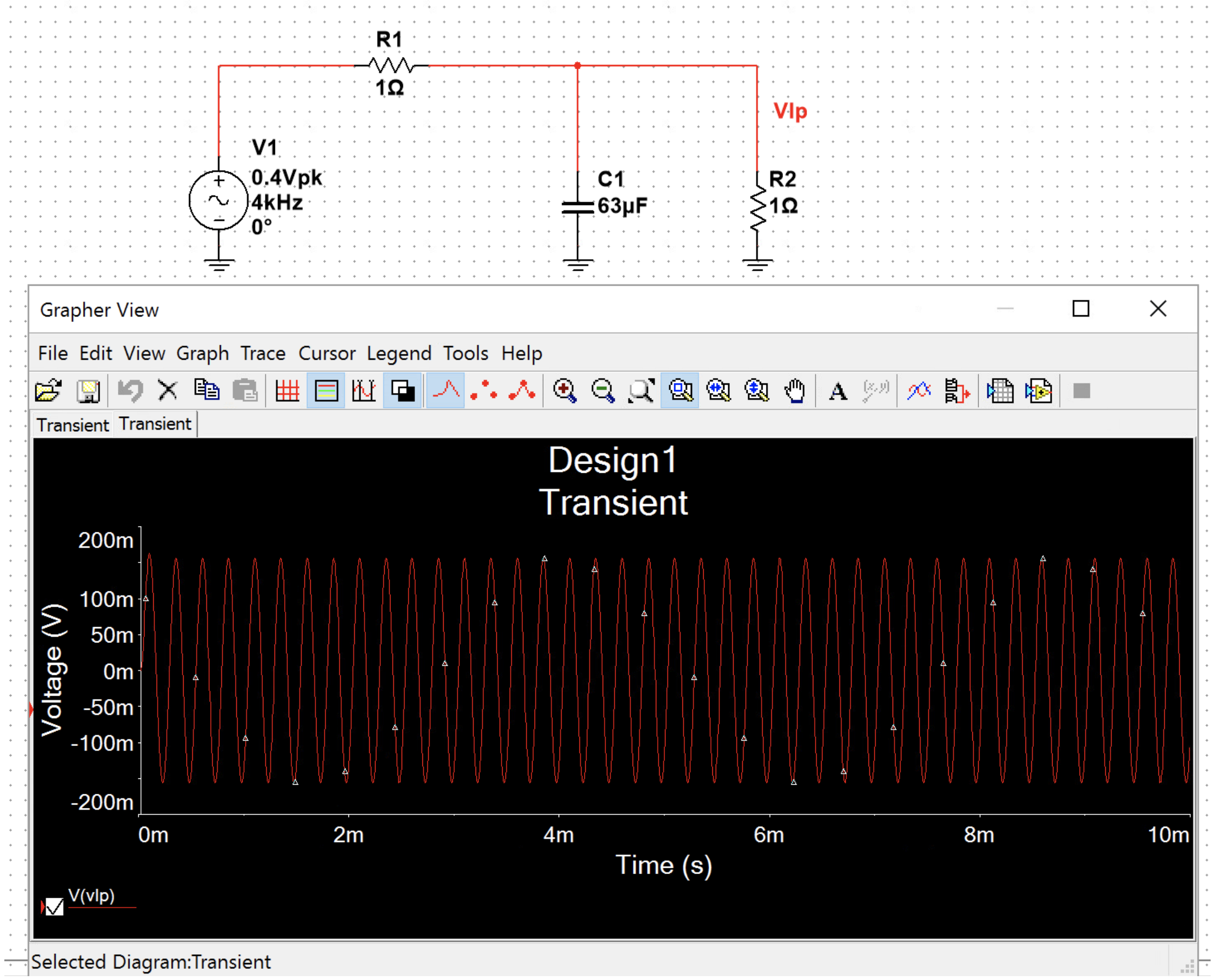
Circuit A Schematic



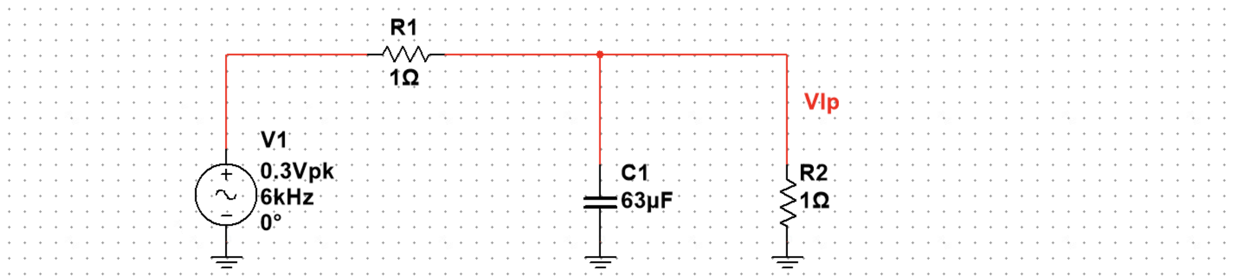
AC Sweep Circuit A

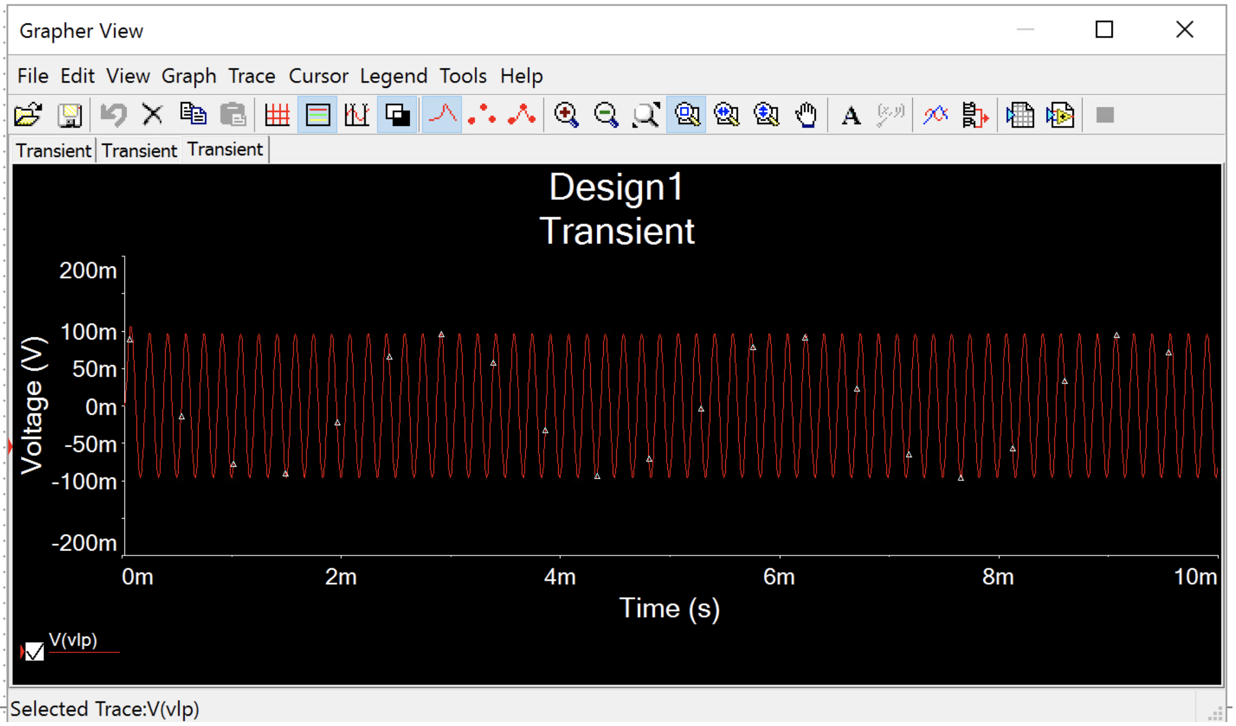


Transient (4K) Circuit A

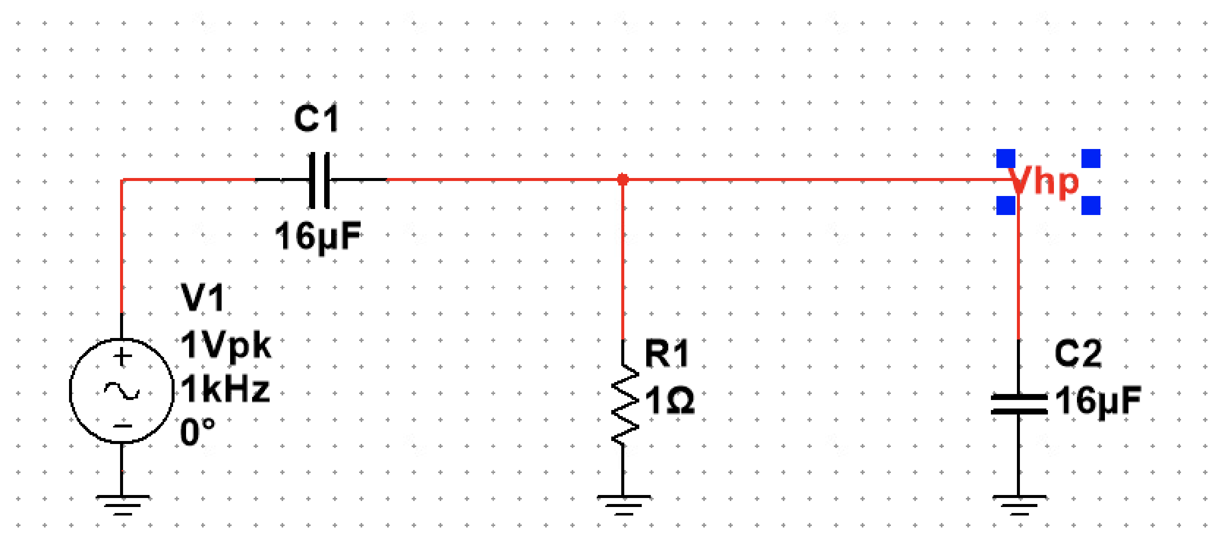


Transient(6K) Circuit A

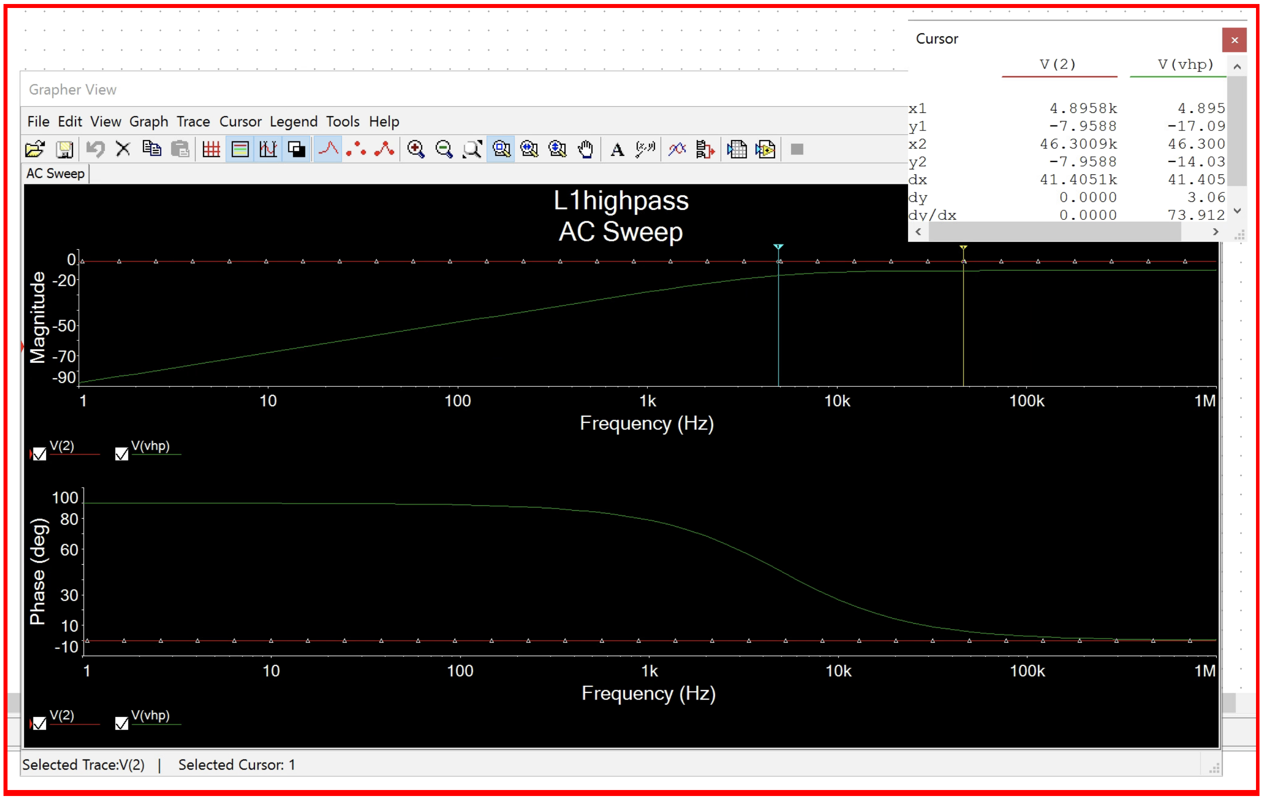




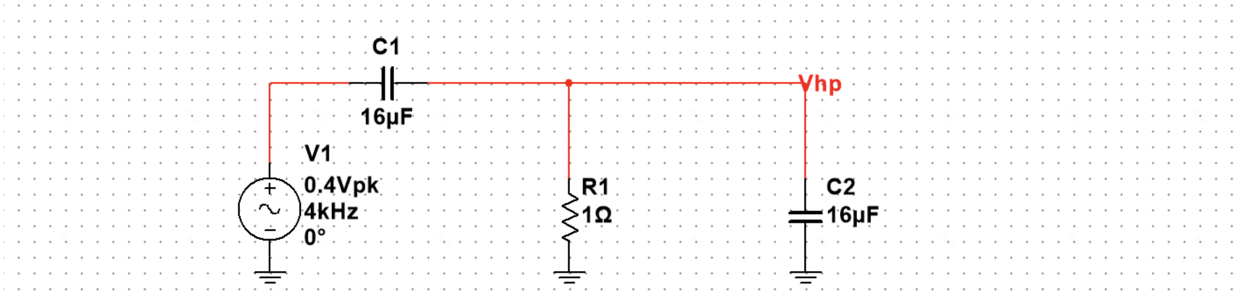
Circuit B Schematic

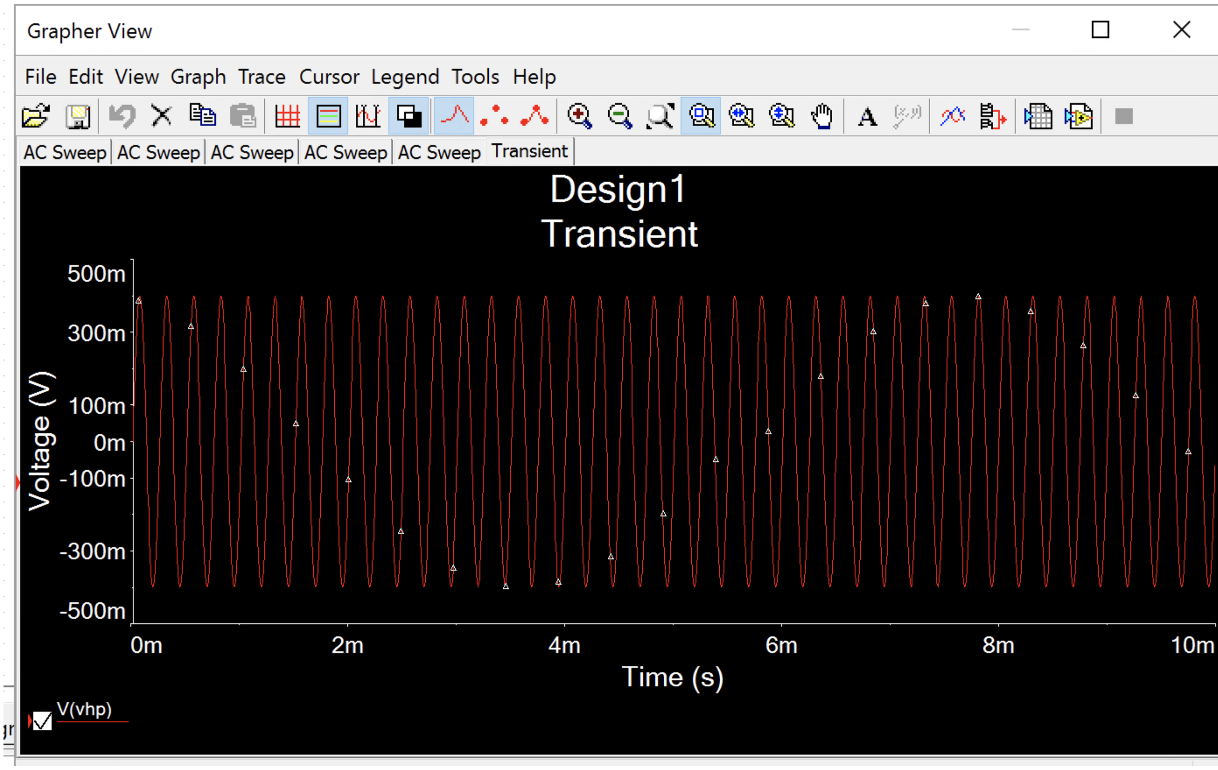


AC Sweep Circuit B

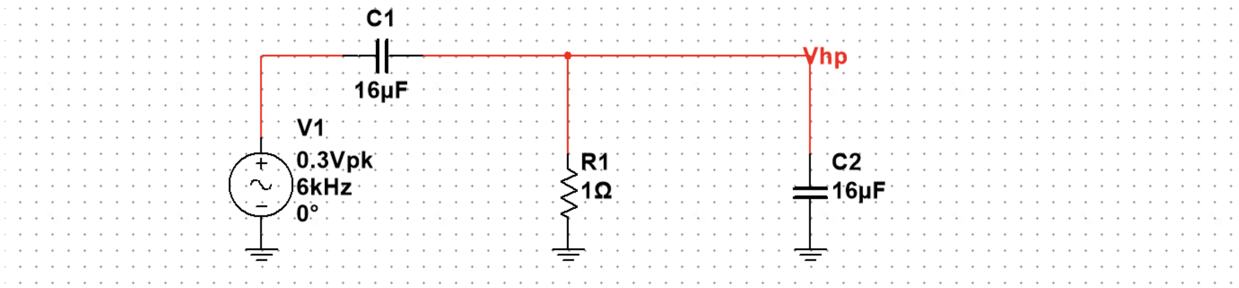


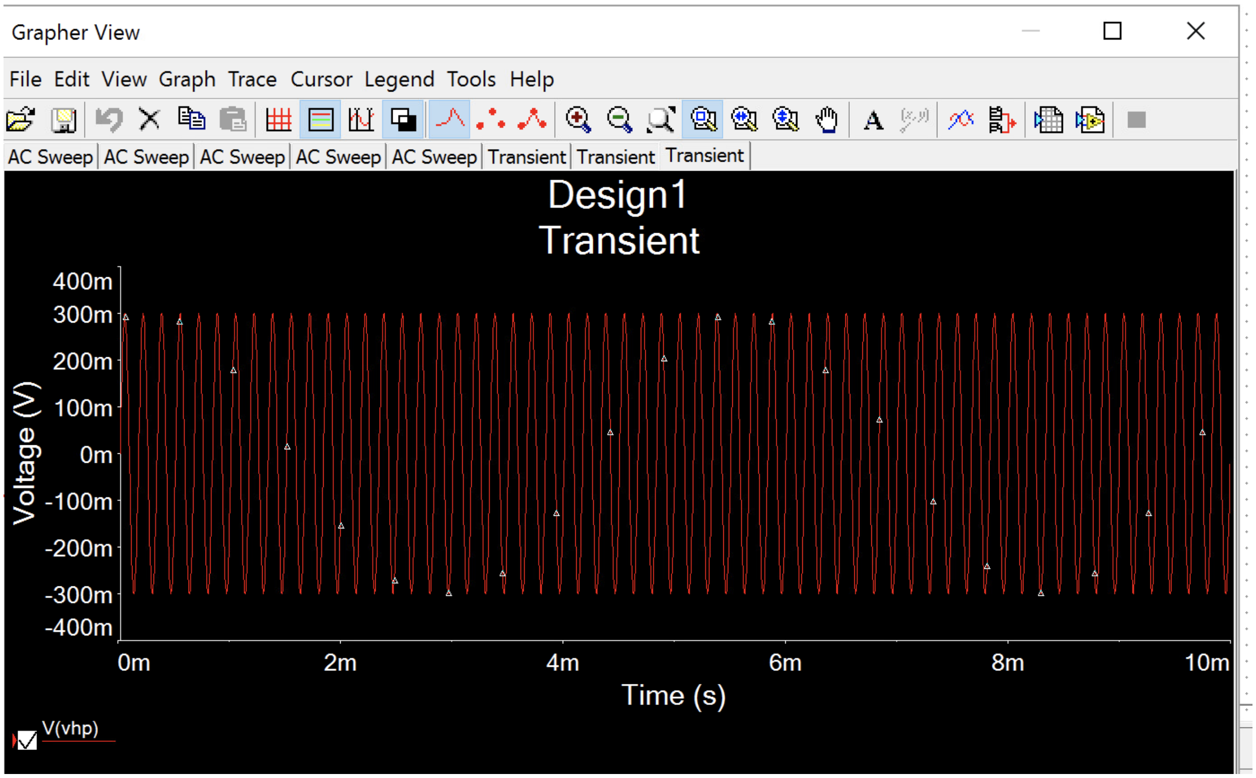
Transient (4K) Circuit B





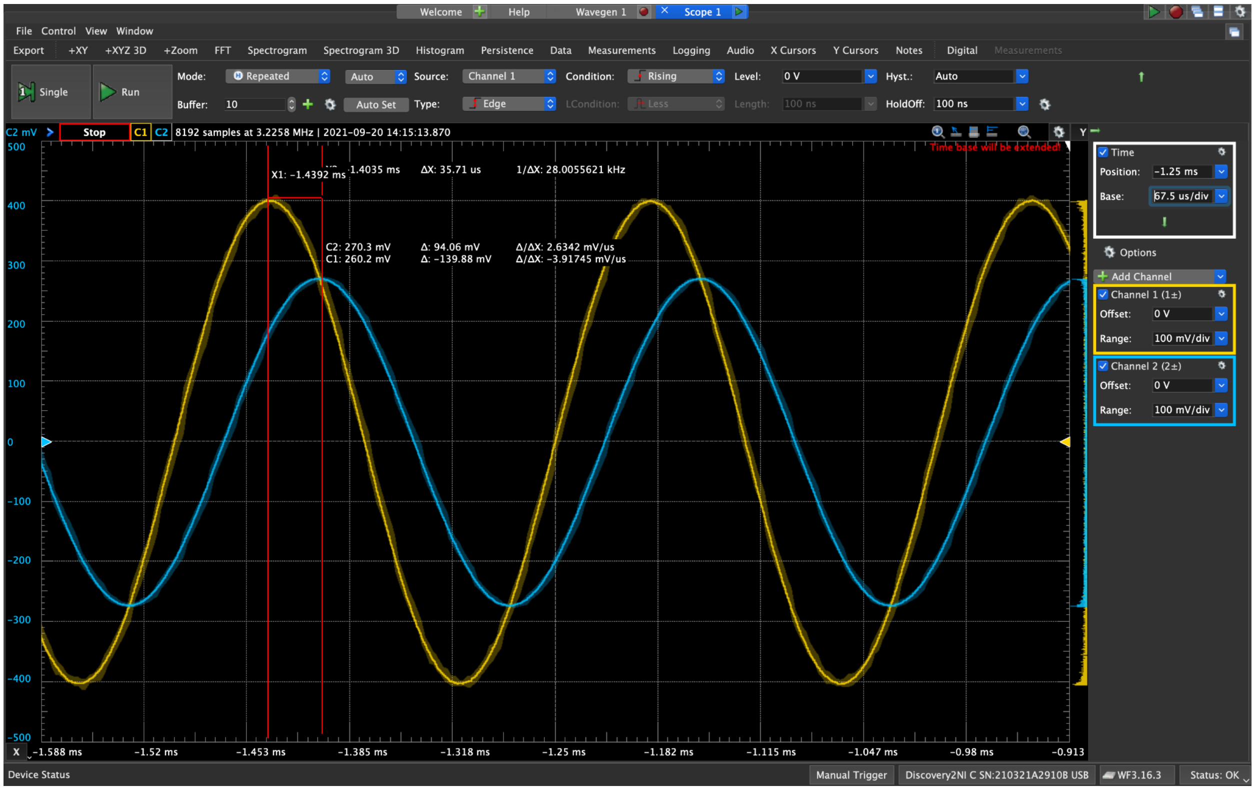
Transient(6K) Circuit B



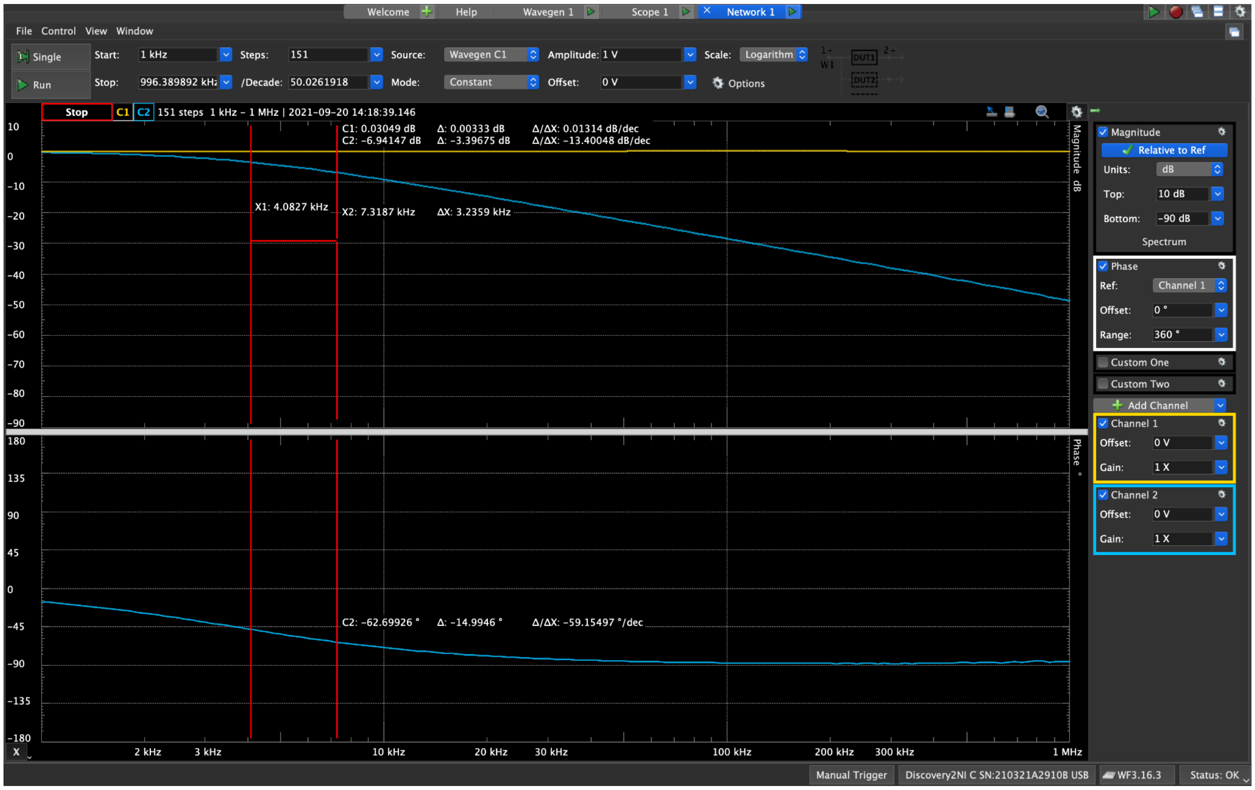


Measured Waveforms

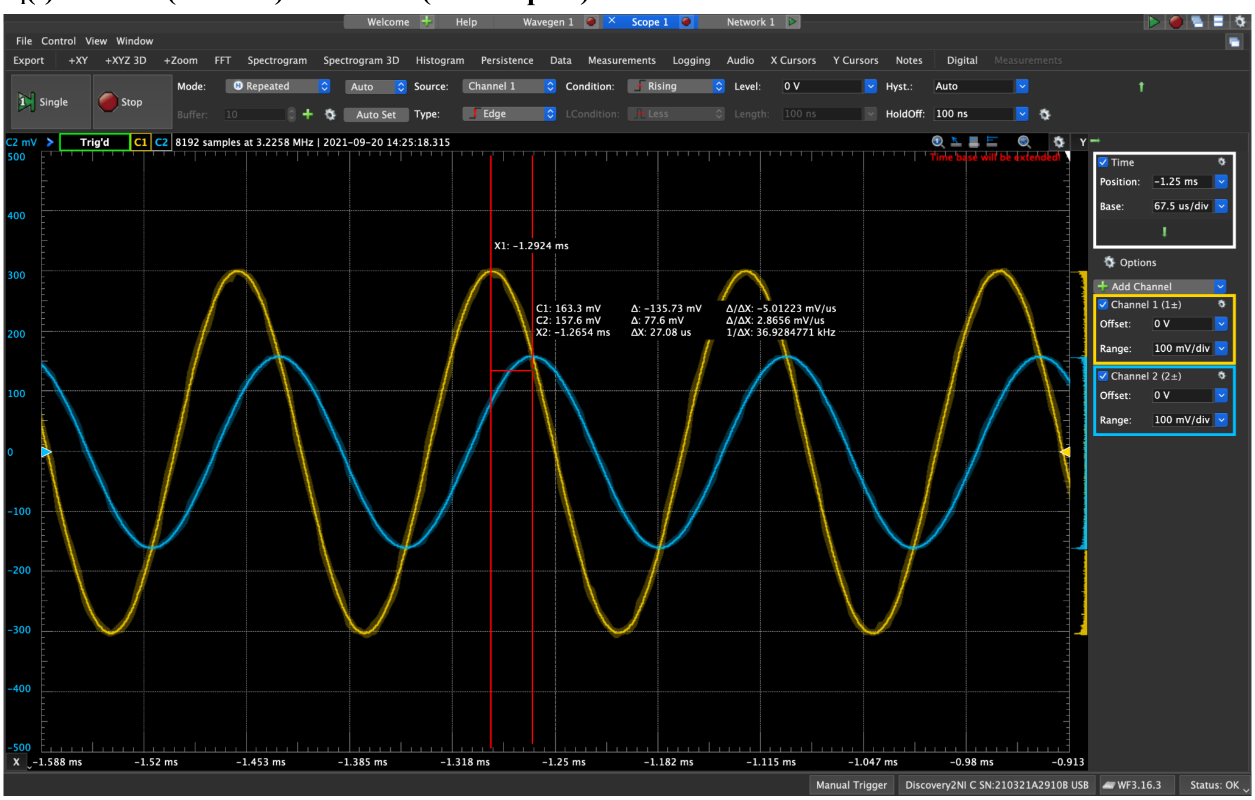
$V_i(t) = 0.4 \sin(2\pi 4000t)$ Circuit A (4K low pass)



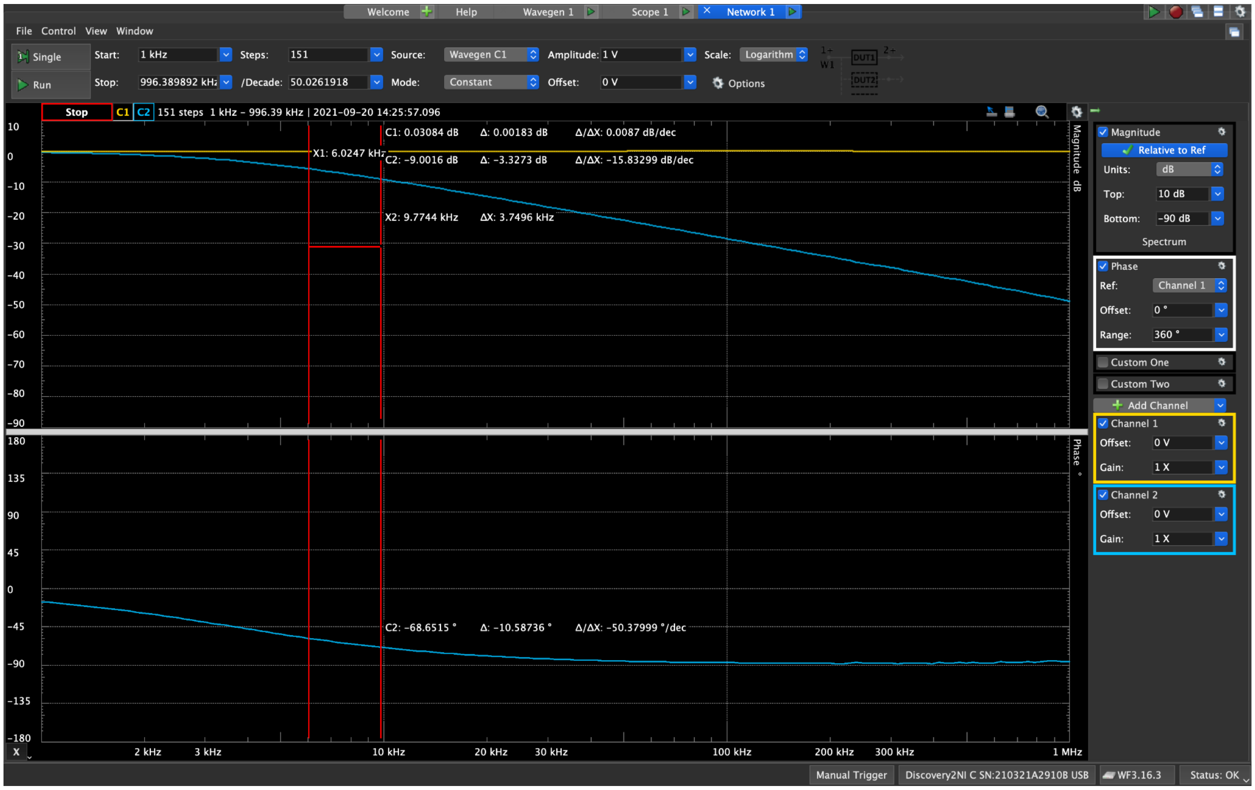
$V_i(t) = 0.4 \sin(2\pi 4000t)$ Bode Plot Circuit A (4K low pass bode)



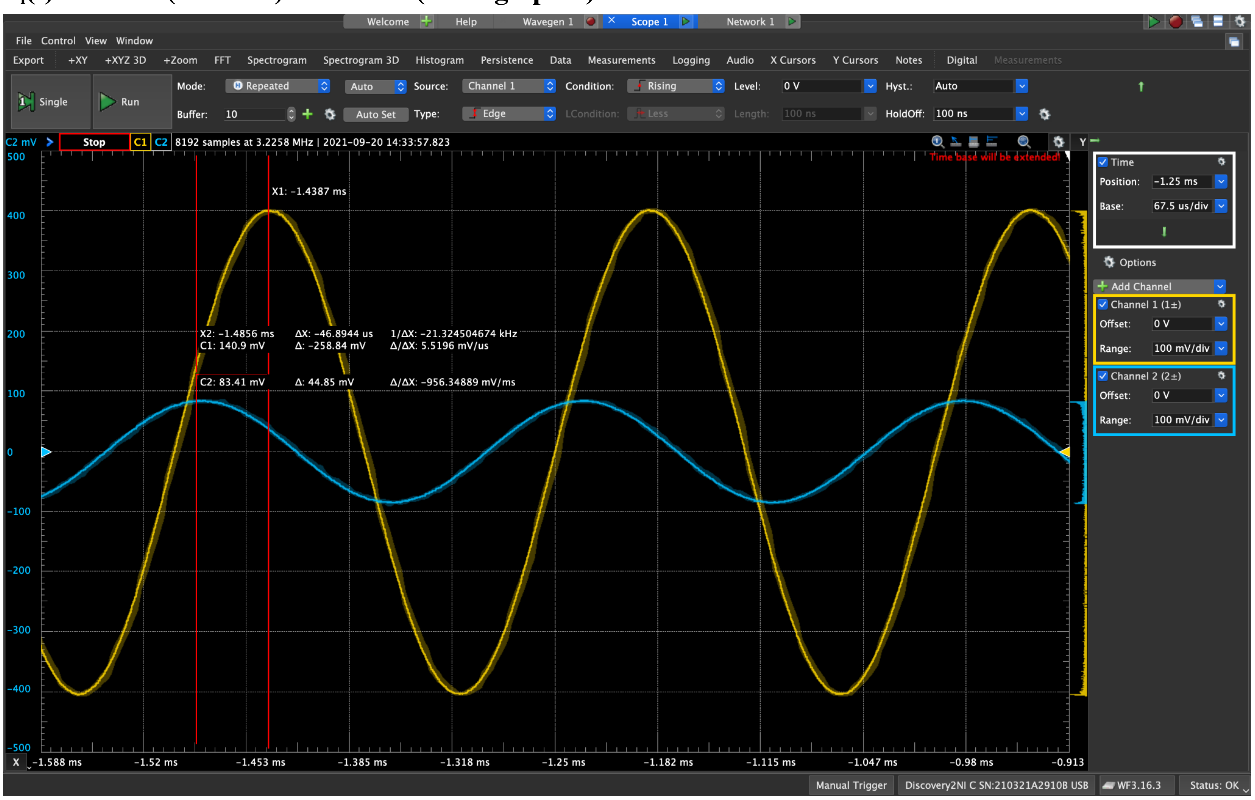
$V_i(t) = 0.3 \sin(2\pi 6000t)$ Circuit A (6K low pass)



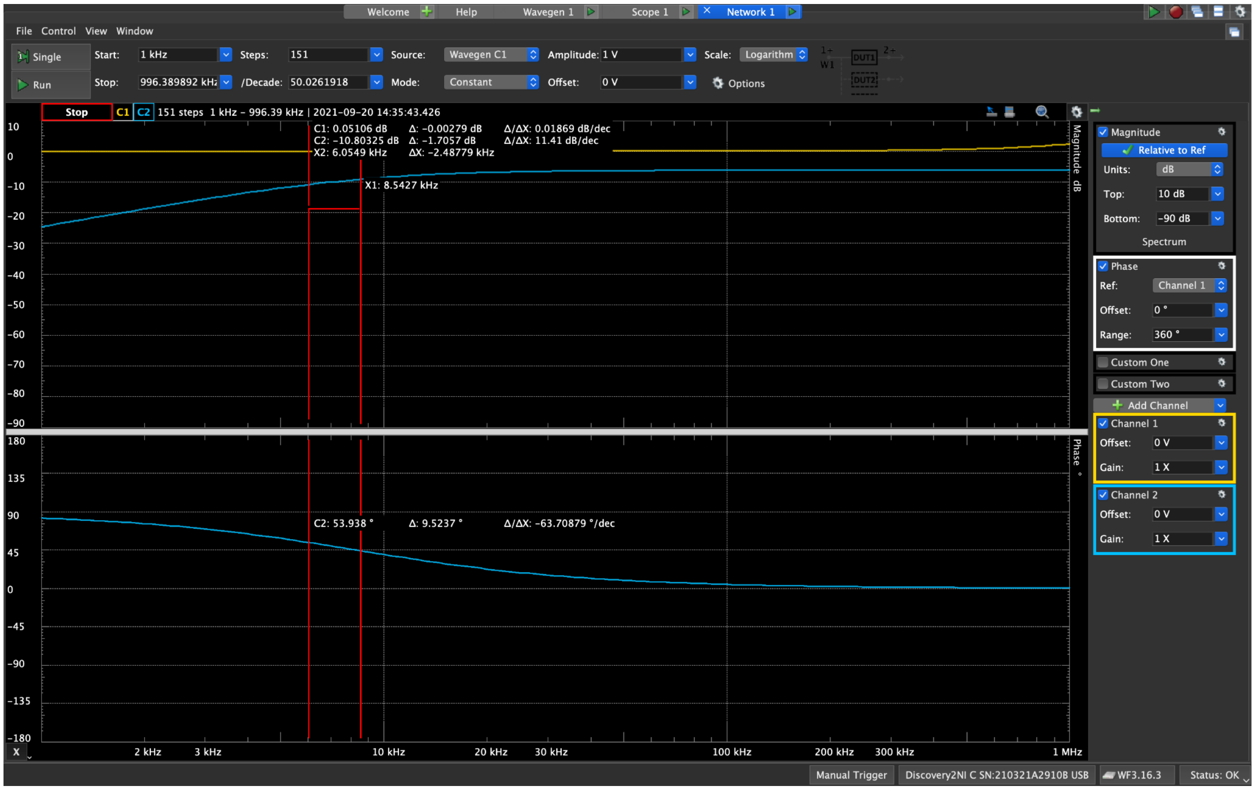
$V_i(t) = 0.3 \sin(2\pi 6000t)$ Bode Plot Circuit A (6K low pass bode)



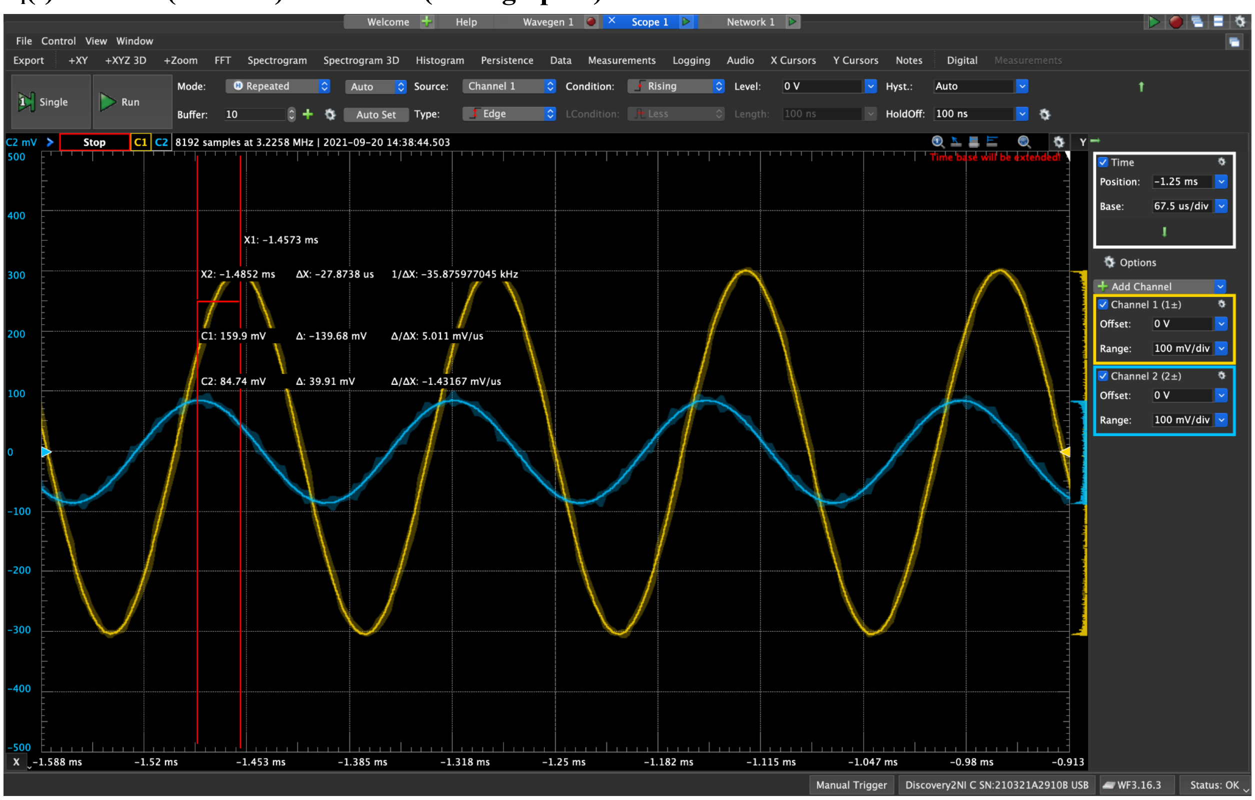
$V_i(t) = 0.4 \sin(2\pi 4000t)$ Circuit B (4K high pass)



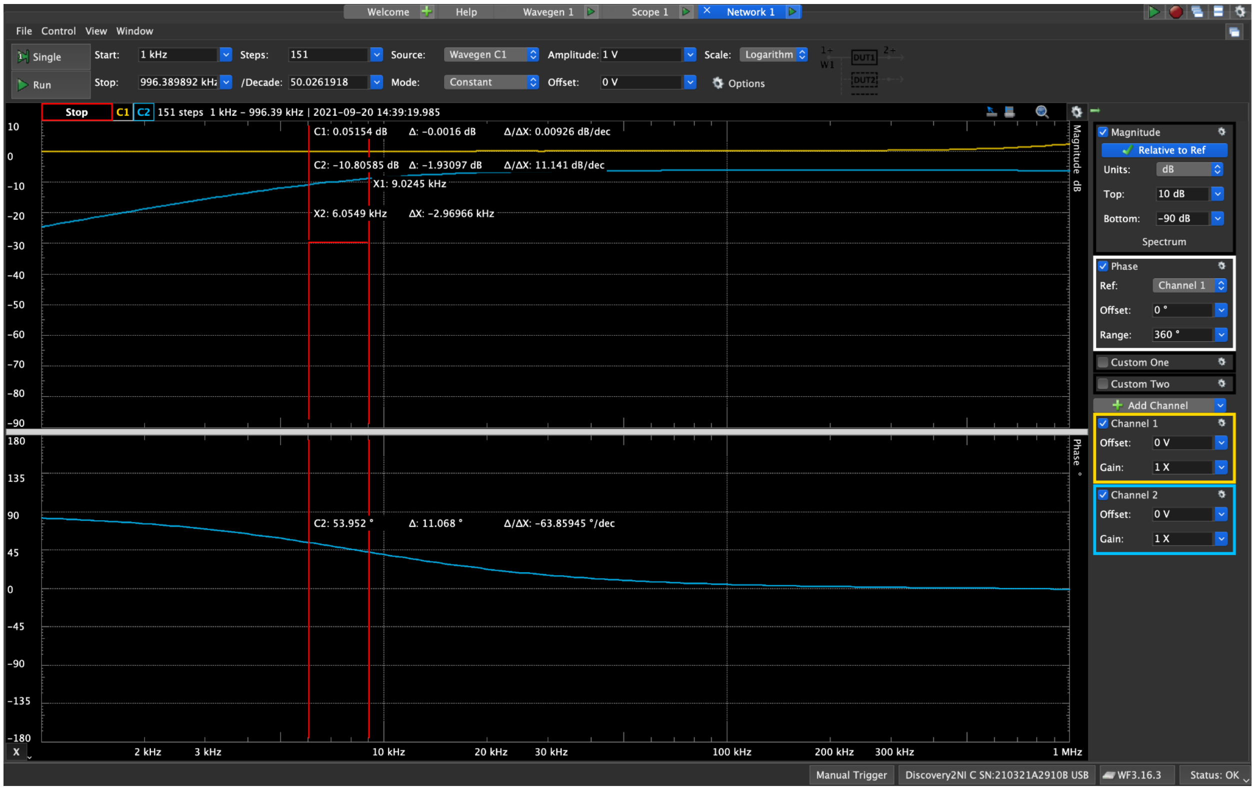
$V_i(t) = 0.4 \sin(2\pi 4000t)$ Bode Plot Circuit B (4K high pass bode)



$V_i(t) = 0.3 \sin(2\pi 6000t)$ Circuit B (6K high pass)



$V_i(t) = 0.3 \sin(2\pi 6000t)$ Bode Plot Circuit B (6K high pass bode)



<u>Data Tables</u>

$V_i(t) = 0.4 \sin(2\pi 4000t)$

	Frequency	Time Difference	∠H	|H|
Calculations Circuit A	4k	none	-38.66°	0.194
Calculations Circuit B	4k	none	51.34°	0.125
Simulations Circuit A	4k	-29.4	-42.36°	0.121
Simulations Circuit B	4k	34.2	121.3°	0.132
Measurements Circuit A	4k	-35.71	-51.45°	0.271
Measurements Circuit B	4k	46.89	67.52°	0.141

$V_i(t) = 0.3 \sin(2\pi 6000t)$

	Frequency	Time Difference	∠H	|H|
Calculations Circuit A	6k	none	39.8°	0.096
Calculations Circuit B	6k	none	39.8°	0.115
Simulations Circuit A	6k	-23.6	-50.97°	0.112
Simulations Circuit B	6k	19.2	41.472°	0.109
Measurements Circuit A	6k	-27.08	-58.49°	0.158
Measurements Circuit B	6k	27.87	-60.19°	0.159

Discussion

In the calculation, and simulations part of the lab, I used 1Ω resistors for R1, R2, and R3. For the capacitors, I used 63.66µF for C1 and 15.9µF for C2 and C3.

For the measurements part, I used 1kΩ resistors for R1, R2, and R3, and 4.7nF capacitors for C1 and 10nF capacitors for C2, and C3, since those were the closest value capacitors provided in the lab kit.