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July 2, 2021

Problem 1 (CT convolution, 3 Points): Calculate and plot $y(t) = (x * h)(t)$, where $x(t) = e^{-t}u(t)$, $h(t) = u(t)$, and $u(t)$ is the CT unit step function.

$x(t) = e^{-t}u(t)$

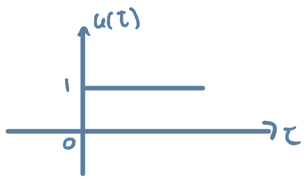
$h(t) = u(t)$

$\rightarrow$ convolution formula

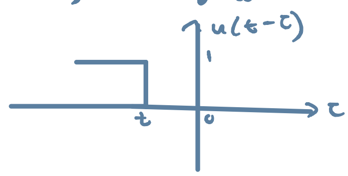
$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau$$

$$= \int_{-\infty}^{\infty} e^{-\tau} u(\tau)\,u(t-\tau)\,d\tau$$

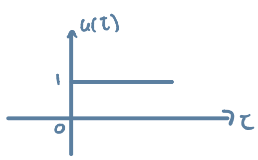
for $t<0$ $\rightarrow$ $u(\tau)\,u(t-\tau) = 0$ for every real τ



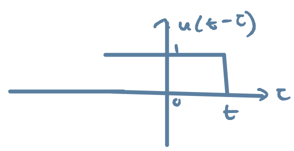
$\rightarrow$ $y(t) = \int_{-\infty}^{\infty} e^{-\tau}\cdot 0\,d\tau = \boxed{0}$



for $t>0$ $\rightarrow$ $u(\tau)\,u(t-\tau) = \begin{cases} 1, & \tau\ (0,t) \\ 0, & \text{otherwise} \end{cases}$



$\rightarrow$ $y(t) = \int_0^t e^{-\tau}\,d\tau = -e^{-\tau}\Big|_0^t = \boxed{1-e^{-t}}$



So,

$\rightarrow$ $y(t) = \begin{cases} 1-e^{-t}, & t>0 \\ 0, & t<0 \end{cases} = (1-e^{-t})(u(t))$

Problem 2 (Systems described by LCCDE, 6 Points): Consider the CT LTI system where the input and the output of the system are related by the LCCDE

$$\frac{dy(t)}{dt} + 10y(t) = x(t)$$

and under the condition of initial rest.

a) Calculate and sketch the unit impulse response $h(t)$ of the system.

b) Suppose that the input $x(t) = u(t)$, where $u(t)$ is the unit step function. Calculate and sketch the corresponding output $y(t)$.

a) given $\frac{dy(t)}{dt} + 10y(t) = x(t)$

$$\frac{dy_h(t)}{dt} + 10y(t) = 0$$

$$\frac{dy_h(t)}{dt} = -10y(t)$$

$$y_h(t) = Ae^{-10t}$$

→ since $x(t) = \delta(t) = 0$ for $t>0$ then $y_p(t) = 0$ and $y(t) = Ae^{-10t}$

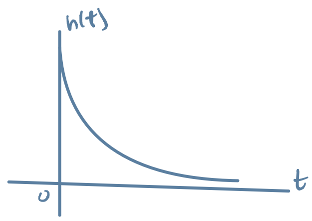
$$\int_{0^-}^{0^+} \frac{dy_h(t)}{dt} + 10\int_{0^-}^{0^+} y(t) = \int_{0^-}^{0^+} x(t) = \int_{0^-}^{0^+} \delta(t)$$

$$y(0^+) - y(0^-) + 0 = 1$$

→ since $y(0^+) = Ae^{-0}$ & $y(0^-) = 0$

$Ae^{-0} = 1$ so $A = 1$ → so $h(t) = e^{-10t}$

$h(t) = e^{-10t}u(t)$



b) $x(t) = u(t)$

$$y(t) = x(t) * h(t)$$

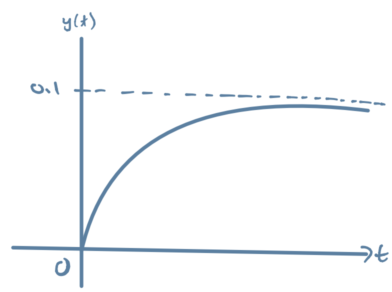
$$\hookrightarrow y(t) = u(\tau) * h(\tau)$$

$$y(t) = \int_0^t h(\tau)\, d\tau$$

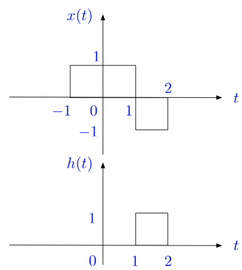
$$= \int_0^t e^{-10\tau}\, d\tau$$

$$= \left(\frac{e^{-10\tau}}{-10}\right)_0^t = -\left(\frac{e^{-10t}}{-10} - \frac{e^0}{-10}\right)$$

$$= -(0.1 - 0.1e^{-10t})$$



Problem 3 (CT convolution, 6 Points): Compute $y(t) = (x * h)(t)$ where $x(t)$ and $h(t)$ are shown as below:



Plot $y(t)$ and carefully label your plot.

$y(t) = (x * h)(t) \rightarrow y(t) = (x * \delta * h)(t)$

$y(t) = (x * u_1 * u_1 * h)(t)$

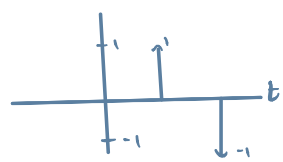
$\rightarrow$ convolution has assoc and comm. prop.

$y(t) = [(h * u_{-1}) * x] * u$

from figure
$\rightarrow h * u_{-1} = \delta_1 - \delta_2$

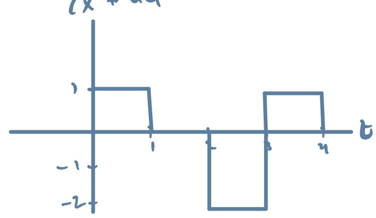
so plot for $h * u_{-1}$

$(h * u_{-1})(t)$



graph $[(h * u_{-1}) * x]$

$(x * u_{-1} * h)(t)$

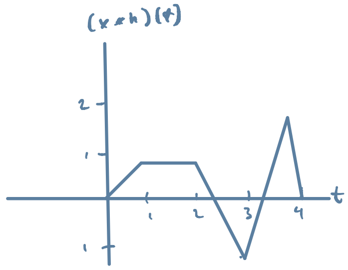


since $y(t) = [(h * u_{-1}) * x] * u$

$y(t)$ is integration of $[(h * u_{-1}) * x]$, which is also the area under the curve.

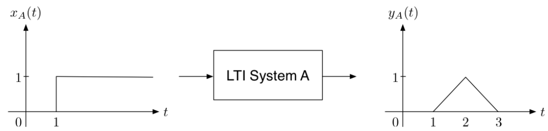
plot of $y(t)$

$(x * h)(t)$



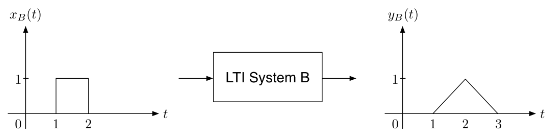
Problem 4 (CT LTI systems, 5 Points): Consider two CT LTI systems A and B.

a) For system A, the following input-output signal pair is known:



Plot the unit impulse response $h_A(t)$ of the system. Carefully label your plot.

b) For system B, the following input-output signal pair is known:



Plot the unit impulse response $h_B(t)$ of the system. Carefully label your plot.

a) $y(t) = (x_A * h_A)(t)$

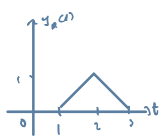
$\rightarrow y_A * \delta = (x_A * h_A) * \delta \quad \rightarrow y_A * \delta = (x_A * \delta) * h_A$

↳ $y_A * u_{-1} = (x * u_{-1}) * h_A$

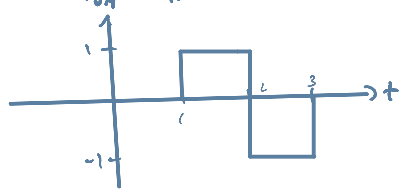
derivative of $x(t)$ $\rightarrow y_A * u_{-1} = \delta_1 * h_A$

↳ delta function means delta shift to right : $y_A * u_{-1} = h_A(t-1)$

deriv. $y_A(t)$ $\rightarrow$



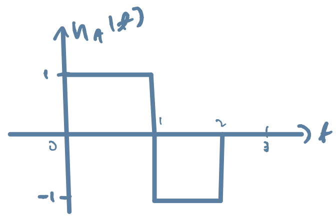
$(y_A * u_{-1})(t) = h_A(t-1)$



above func is $h_A(t-1)$

$h_A(t) = y_A * u_{-1}(t+1)$

h_A $\rightarrow$



b) $y_B(t) = (x_B * h_B)(t)$

$\rightarrow y_B * \delta = (x_B * h_B) * \delta \quad \rightarrow y_B * \delta = (x_B * \delta) * h_B$

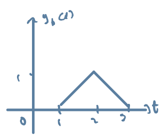
$\hookrightarrow y_B * u_{-1} = (x_B * u_{-1}) * h_B$

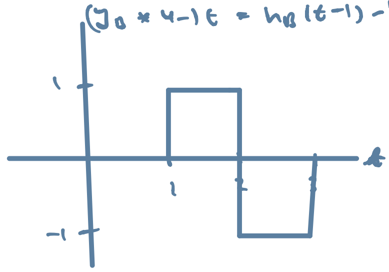
deriv of $x_B(t)$ $\rightarrow y_B * u_{-1} = (\delta_1 - \delta_2) * h_B$

$y_B * u_{-1} = h_B(t-1) - h_B(t-2)$

deriv of $y_B(t)$:

$(y_B * u_{-1})t = h_B(t-1) - h_B(t-2)$





Since h_B is combination of $h_B(t-1) - h_B(t-2)$

$h_B \rightarrow$