PreLab 9: Power Transfer in AC Circuits

ECEN 214 - 517

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A. For the circuit shown in Figure 9.5, find the expression for the load resistance that will maximize the power delivered to the load resistor. Note, your result should depend on the

frequency of the input voltage source. If the AC input is a 10kHz sine wave with a peak-to-peak value of 8Volts, find the maximum power delivered to the load, and the component value of the resistor that produces maximum power dissipation. Show your derivations.

A) derivations

$$P = \left(\frac{V_{in}^2}{(Z_s + R_L)} \right)^2 RL$$

$$\frac{dP}{dL} = 0$$

$$\frac{d}{dR_L} \left(\frac{V_{in}^2}{(Z_s + R_L)^2} \right) = 0$$

$$\frac{d}{dR_L} \left(\left(\frac{R_1}{(Z_s + R_L)^2} \right) \right) = 0$$

$$\frac{(Z_s + R_L)^2 \frac{dR_L}{dR_L} - R_L - \frac{d}{dR_L}(Z_s + R_L)^2 = 0}{(Z_s + R_L)^2}$$

$$(Z_s + R_L)^2 \cdot 1 \cdot - R_L - 2(Z_s + R_L) = 0$$

$$(Z_s + R_L)(Z_s + R_L - 2R_L) = 0$$

$$Z_s + R_L - 2R_L = 0$$

$$R_L = Z_s$$

$$V_{in}(t) = \frac{4}{\sqrt{2}} \sin(\omega t)$$

$$\omega = 2\pi f = 2\pi \times 10 \times 10^3$$
$$= 62{,}831.85 \text{ rad/sec}$$

$$X_L = 2\pi f L = 2\pi \cdot 10 \cdot 10^3 \cdot 0.12$$
$$= 7.539 \text{ k}\Omega$$

$$\longrightarrow Z_s = 2.2 + j\,7.5398 \text{ k}\Omega$$

$$|\bar{Z}_s| = \sqrt{(2200)^2 + (7539.8)^2} = 7854.2\,\Omega$$

$$R_L = Z_s = 7854.2$$

$$P_{max} = \frac{V_{in}^2}{4 \cdot R_L} = \frac{(4/\sqrt{2})^2}{4 \cdot 7854.2}$$

$$P_{max} = 254.64 \ \mu \text{ watts}$$

B. Now suppose we add a shunt capacitor to the circuit as shown in Figure 9.6. Assuming that we use the load resistance that you calculated in Part A, find the value of the shunt capacitance that will maximize the power delivered to the load resistor. As before, assume Figure 9.5 – An AC circuit with a variable load resistance *RL* 2.2*k*Ω 0.12H *Vin*(*t*) + _ *C C* Figure 9.6 – An AC circuit with a variable shunt capacitance. 2.2*k*Ω 0.12H *Vin*(*t*) + _ *RL* the input is a 10kHz sine wave with a peak-to-peak value of 8Volts. What is the power delivered to the load and how does it compare with your results in Part A. Show your derivations.

B) $R_{Th} = (RL)/x_L$ $\qquad X_L = jwL$

$X_L = \frac{1}{jwC}$

$w = 2\pi f \qquad f = 10 \times 10^3 \qquad w = (2 \times 10^4)\pi$

$X_L = j(2 \times 10^4 \times \pi) \times 0.12 = j(7540)\Omega$

$X_L = \frac{1}{jwC} = \frac{1}{j(2\times10^4)\pi C}$

$$R_{Th} = \frac{(2200 + j7540)\left(\frac{1}{j(2\pi\times10^4)C}\right)}{(2200 + j7540) + \frac{1}{j(2\pi\times10^4)C}}$$

$R_{Th} = R_L$

$$= \frac{(2200) + j(7540)}{(2200) + j(7540)\, j(2\pi\times10^4)C + 1} = 7854$$

$2200 + j(7540) - 7854 = C(-3.72\times10^{12} + 1.08\times10^{12})$

$\underline{C = 1.52\,nF}$ (real) $\qquad$ & $\underline{C = 6.9\,nF}$ (imaginary)

theres $\underline{2 \text{ values}}$

C. Finally, consider the case where both the load resistance and the shunt capacitance are adjustable as shown in Figure 9.7. Assuming the input is a 10kHz sine wave with a peak-to-peak value of 8Volts, find the values of the load resistance and shunt capacitance that maximize the power delivered to the load resistor. Compute the power dissipated in this case and compare it with the results from Part A and B. Show your derivations. Hint: You may find it algebraically easier to solve 1 *ZL* = 1 *Zs* ∗ then to solve *ZL* = *Zs**

c) $= 2.2k + j\omega 0.12$

$\omega L = 2\pi f = 2\pi \times 10 \times 10^3 = 6283.185$

$Z_S = 2200 + j\ 7539.82\ \Omega$

$V_{0c} = 4V$

$Z_L = Z_S^* = 2200 - j\ 7539.82$

$\frac{V_{oc}^2}{4R_L} = \frac{4^2}{4 \times 2200} = 1.818 \times 10^{-3}\ W$

$R_L = 2200\ \Omega$

$X_L = 7539.82\ \Omega = \frac{1}{2\pi f C} = C = \frac{1}{2\pi \times 10 \times 10^3 \times 7539.82} = 2.11 \times 10^{-9} F$

$P_{max} = 1.818 \times 10^{-3}\ W$

$R_L = 2200\ \Omega\ ,\ C = 2.11 \times 10^{-9}\ F$

$Z_s = R_s + j\omega L = 2.2k + j\omega 0.12 = 2200 + j\ 7539.82$

$Z_L = R_L \frac{1}{j\omega C} = \frac{R_L}{j\omega R_L C + 1}$

$Z_S^* = Z_L = 2200 - j\ 7539.82$

$Re[Z_L] = Re[Z_S]\ ,\ Im[Z_L] = -Im[Z_S]$

$R_L = 2200\ \Omega \qquad C = 2.11 \times 10^{-9}$