[redacted] [redacted]

Jun 23, 2021

Problem 1 (Basic system properties, 15 Points): Determine whether the following systems are i) causal, ii) TI, iii) linear. Carefully justify your answers.

a) $y[n] = 2x[n]$;

b) $y(t) = x(0.5t)$;

c) $y[n] = (-1)^n x[n]$;

d) $y[n] = \begin{cases} (-1)^n x[n], & \text{if } x[n] \geq 0 \\ 2x[n], & \text{if } x[n] < 0 \end{cases}$;

e) $y[n] = \sum_{k=n}^{\infty} x[k]$.

a) $y[n] = 2x[n]$

i) for causal: the system's output depends upon the current value of the input thus the system is <u>causal</u>

ii) for TI:

$y[n-n_0] = f(x[n-n_0])$

$f(x[n-n_0]) = 2x[n-n_0]$

$y[n-n_0] = 2x[n-n_0]$

→ its satisfied, the system is <u>Time Invarient</u>

iii) for linear:

$y_1[n] = 2x_1[n]$

$y_2[n] = 2x_2[n]$

$y_3[n] = 2x_3[n]$

→ Three system

↳ $y_1[n] + y_2[n] + y[n] = 2x_1[n] + 2x_2[n] + 2x_3[n]$

$y_1[n] + y_2[n] + y[n] = 2(x_1[n] + x_2[n] + x_3[n])$

→ system holds linearity property, the system is <u>Linear</u>

b) $y(t) = x(0.5t)$

i) for causal: The output of the system depends upon the past value, the system is <u>non-causal</u>

ii) for TI:

$f(x[t-t_0]) = x[0.5t - t_0]$

$y[t-t_0] = x[0.5(t-t_0)]$

so, $y[t-t_0] \neq f(x[0.5(t-t_0)])$ → so system is <u>Time Variant</u>

iii) for Linear:

$y_1[t] = x_1[0.5t]$

$y_2[t] = x_2[0.5t]$

$y_3[t] = x_3[0.5t]$

→ $y_1[n] + y_2[n] + y[n] = f(x_1[0.5t] + x_2[0.5t] + x_3[0.5t])$

↳ Thus, it can be said that the given system is <u>linear</u>

c) $y[n] = (-1)^n x[n]$

i) for causal: the system's output depends upon the current value of the input thus the system is <u>causal</u>

ii) for TI:

$y[n-n_0] = (-1)^{n-n_0} x[n-n_0]$

$y[n-n_0] = \frac{(-1)^n}{(-1)^{n_0}} x[n-n_0]$

$y[t-t_0] \neq f(x[n-n_0])$

→ The system is <u>Time Varient</u>

iii) for linear: suppose, $f(ax[n]) = a(-1)^n x[n] = ay[n]$

↳ The system is <u>linear</u>

d) $y[n] = \begin{cases} (-1)^n x[n] & x[n] \geq 0 \\ 2x[n] & x[n] < 0 \end{cases}$

i) for causal: In both case, if $x[n] \geq 0$ and $x[n] < 0$, the output depends upon the present input value, thus the system is causal.

ii) for TI: $x[n] \geq 0$, $y[t - t_0] \neq f(x[n-n_0])$ for $x[n] \geq 0$ → system is same as problem 1c, thus the function is non-time variant, although the $x[n] < 0$ is time variant

iii) for Linear: for $x[n] \geq 0$

$f(ax[n]) = a(-1)^n x[n] = ay[n]$

$x[n] < 0$

$f(ax[n]) = a2x[n] = ay[n]$

since all region $f(ax[n] = ay[n])$ is satisfied thus system is linear.

e) $y[n] = \sum_{k=n}^{\infty} x[k]$

i) for causal: $y[n]$ depends upon the future value of n, so the system is non-causal

ii) for TI: $f(x[n-n_0]) = \sum_{k=n}^{\infty} x[k-n_0]$

suppose, $k - n_0 = a$, then $k = a + n_0$

so, $f([n-n_0]) = \sum_{k=a+n_0}^{\infty} x[a]$

$f([n-n_0]) = \sum_{k=n+n_0}^{\infty} x[n]$

$f([n-n_0]) \neq y[n-n_0]$ → system is Time Variant

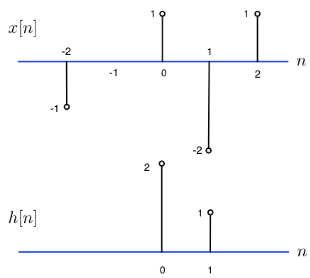
iii) for linear: let a be a constant

$f(ax[n]) = \sum_{k=n}^{\infty} ax[k]$

$f(ax[n]) = a \sum_{k=n}^{\infty} ax[k]$ → system is linear

$f(ax[n]) = ay[n]$

Problem 2 (DT convolution, 5 Points): Two DT signals $x[n]$ and $h[n]$ are given below:



a) Express $x[n]$ and $h[n]$ as linear combinations of the DT unit impulse function $\delta[n]$ and its time shifts.

b) Calculate $y[n] = x[n] * h[n]$ numerically.

a) $x[n]$ as linear combination of function $\delta[n]$ and time shift : $x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k]$

$x[n] = x[-2]\delta[n+2] + x[-1]\delta[n+1] + x[0]\delta[n] + x[1]\delta[n-1] + x[2]\delta[n-2]$

$$\boxed{x[n] = -1\delta[n+2] + \delta[n] - 2\delta[n-1] + 1\delta[n-2]}$$

$h[n]$ can be : $h[n] = \sum_{k=-\infty}^{\infty} h[k]\delta[n-k]$

$h[n] = h[1]\delta[n-1] + h[2]\delta[n-2]$

$$\boxed{h[n] = 2\delta[n-1] + \delta[n-2]}$$

b) For $(x * h)[n]$ we have

$$(x * h)[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

$$y[-2] = \sum_{k=-\infty}^{\infty} x[k]h[-2-k]$$

$y[-2] = -1 \times 2 = -2$

$y[-1] = \sum_{k=-\infty}^{\infty} x[k]h[-1-k]$

$y[-1] = 1 \times -1 = -1$

$y[0] = \sum_{k=-\infty}^{\infty} x[k]h[-k]$

$y[0] = 2 \times 1 = 2$

$y[1] = \sum_{k=-\infty}^{\infty} x[k]h[1-k]$

$y[1] = 2 \times -2 + 1 \times 1 = -3$

$y[2] = \sum_{k=-\infty}^{\infty} x[k]h[2-k]$

$y[2] = -2 \times 1 + 1 \times 2 = 0$

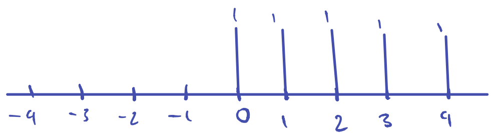
$y[3] = \sum_{k=-\infty}^{\infty} x[k]h[3-k]$

$y[3] = 1 \times 1 = 1$

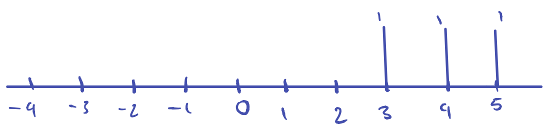
Problem 3 (DT convolution, 5 Points): Let $x[n] = u[n] - u[n-3]$ and $h[n] = u[n]$, where $u[n]$ is the DT unit step function. Plot $x[n]$ and $h[n]$ and carefully label your plots. Calculate $y[n] = x[n] * h[n]$ using the simplest method that you can think of. Plot $y[n]$ and carefully label your plot.

$x[n] = u[n] - u[n-3]$

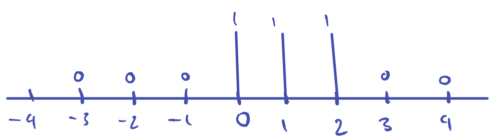
$u[n] =$



$u[n-3] =$



$x[n] =$



$h[n] = u[n]$

so

$y[0] = 1 \times 1 = 1$

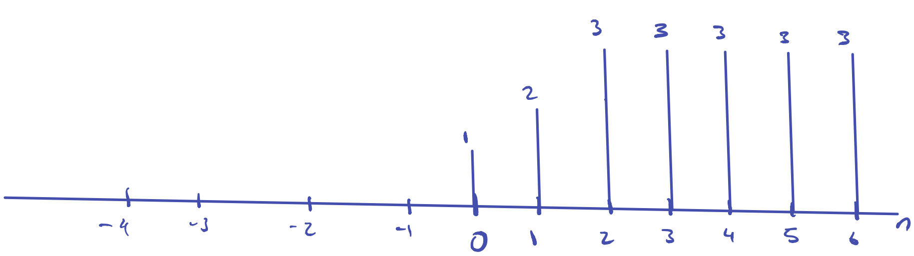
$y[1] = 1 \times 1 + 1 \times 1 = 2$

$y[2] = 1 \times 1 + 1 \times 1 + 1 \times 1 = 3$

$y[3] = 1 \times 1 + 1 \times 1 + 1 \times 1 = 3$

$y[4] = 1 \times 1 + 1 \times 1 + 1 \times 1 = 3$

$y[n] \rightarrow$ plot



Problem 4 (DT convolution, 5 Points): A DT signal $f[n]$ is said to be *supported* on a subset of integers $\mathcal{S}$ if $f[n] = 0$ for any $n \notin \mathcal{S}$. Let $y[n] = (x * h)[n]$ where $x[n]$ is supported on $\{a_1, a_1 + 1, \ldots, b_1\}$ and $h[n]$ is supported on $\{a_2, a_2 + 1, \ldots, b_2\}$. For what values of n the signal $y[n]$ can *possibly* take *nonzero* values?

$$y[n] = (x * h)(n) = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

$$= \sum_{k=-\infty}^{\infty} h[k]\, x[n-k]$$

given $x[n]$ supported by $\{a_1, a_1 + 1, \ldots b_1\}$

and $h[n]$ supported by $\{a_2, a_2 + 1, \ldots b_2\}$

then $y[n] = (x * h)(n)$ supported by

$\{a_1 + a_2, a_1 + a_2 + 1, \ldots b_1 + b_2\}$

so $y[n]$ sequence can be non zero

from $\boxed{a_1 + a_2 \text{ to } b_1 + b_2}$