Cost Minimization Using Linear Programming Model:

A Case Study of Spring Garden Tools

FORMAL REPORT

CONTENTS

EXECUTIVE SUMMARY:

The Spring Garden Tools case study highlights the manufacturing business owned by Spring family that has been supplying high quality garden tools made from steel with wooden handle to hardware stores and distributors. It specializes in 4 main products- Trowel, Hoe, Rake and Shovel. The manufacturing process has 2 stages.

- Stage 1 has 2 operations- Stamping, Drilling.
- Stage 2 has 3 operations-Assembly, Finishing and packaging.

The manufacturing company aims to know the production schedule of each tool from the perspective of minimizing cost.

Using the cost minimization model in Excel Solver along with the constraints provided, the total minimum cost was found to be $85,472.60. The critical resources or production processes were found to be- Stamping, Drilling, Packaging and the usage of steel sheet. The following table provides information on how many units of each garden tool should be produced in regular production time, overtime, and subcontracting:

Production Schedule					
	Stage1			Stage 2	
Product	Regular	Subcontracted	Overtime	Regular	Overtime
Trowel	1691.95	0.00	108.05	1800	0
Hoe	1319.54	0.00	80.46	1400	0
Rake	1600	0	0	1600	0
Shovel	933.33	866.67	0	1146.67	653.33

To minimize cost, it is recommended that Spring Company should focus on the constraint variables that have a negative shadow price as they are major players in cost reduction if they operate within their sensitivity ranges.

The following sections will delve into the solution approach to the problem of cost minimization. This includes the defining the problem, constructing the linear programming model (4-step process), assumptions, solution summary, sensitivity report analysis and recommendations.

PROBLEM DEFINITION:

There are three problems defined in this case-

- Minimization of production cost
- Deriving a production schedule of each tool
- Identifying the critical resources in the process

FORMULATION OF LINEAR PROGRAMMING MODEL:

The problem of establishing production schedule to minimize cost is a 4 - process step related to defining the following: Decision variables, objective function, model constraint, model summary.

Decision variables:

The decision variables are the number of units of each tool produced during the regular, overtime and sub contracted production in Stage 1 and Stage 2. We have a total of 20 decision variables. So, for simplicity we will define the decision variables with names:
Let p = (Product Type) so Trowel = Product 1, Hoe = Product 2, Rake = Product 3, Shovel = Product 4

STAGE - 1:
R_p = Regular production of product p in Stage 1 (p = 1, 2, 3, 4)
S_p = Subcontracted production of product p in Stage 1 (p = 1, 2, 3, 4)
X_p = Overtime production of product p in Stage 1(p = 1, 2, 3, 4)
STAGE - 2:
A_p = Regular production of product p in Stage 2 (p = 1, 2, 3, 4)
Y_p = Overtime production of product p in Stage 2(p = 1, 2, 3, 4)

This will result in 4 decision variables each for regular, subcontracted and overtime production in Stages 1 bringing it to the total of 12 decision variables during this stage of production. In Stage 2, w will get 4 decision variables for each regular and overtime production bringing the total of decision variables to 8 in this stage. Hence, for both stages of production we get 20 variables.

Objective Function:

The objective is to minimize the total cost which is the sum of regular and overtime production cost for each tool in the 2 stages of production. To this, we will also add the subcontracted production cost (in Stage 1) which is 20% of the regular production cost. Here is the objective function:

Minimize Z = $(6R_1+10R_2+8R_3+10R_4) + (7.2S_1+12S_2+9.6S_3+12S_4) + (6.2X_1+10.7X_2+8.5X_3+10.7X_4) + (3A_1+5A_2+ 4A_3+5A_4) + (3.1Y_1+5.4Y_2+ 4.3Y_3+ 5.4Y_4)$

Model Constraints:

This model has 4 set of constraints namely total production hours, availability of sheet steel, monthly contracted production volume per tool and non-negativity constraint.

Stage 1 Regular Production:
$0.04R_1 + 0.17R_2 + 0.06R_3 + 0.12R_4 \leq 500$
$0.05R_1 + 0.14R_2 + 0.14R_4 \leq 400$

Stage 1 Overtime Production:
$0.04X_1 + 0.17X_2 + 0.06X_3 + 0.12X_4 \leq 100$
$0.05X_1 + 0.14X_2 + 0.14X_4 \leq 100$

Stage 2 Regular Production:

$0.06A_1 + 0.13A_2 + 0.05A_3 + 0.10A_4 \leq 600$
$0.05A_1 + 0.21A_2 + 0.02A_3 + 0.10A_4 \leq 550$
$0.03A_1 + 0.15A_2 + 0.04A_3 + 0.15A_4 \leq 500$

Stage 2 Overtime Production:
$0.06Y_1 + 0.13Y_2 + 0.05Y_3 + 0.10Y_4 \leq 100$
$0.05Y_1 + 0.21Y_2 + 0.02Y_3 + 0.10Y_4 \leq 100$
$0.03Y_1 + 0.15Y_2 + 0.04Y_3 + 0.15Y_4 \leq 100$

Sheet Steel Availability:
$1.2R_1 + 1.6R_2 + 2.1R_3 + 2.4R_4 + 1.2X_1 + 1.6X_2 + 2.1X_3 + 2.4X_4 \leq 10{,}000$

Number of units processed in Stage 1 must be equal to number of units processed in Stage 2:
$R_1 + S_1 + X_1 = A_1 + Y_1$
$R_2 + S_2 + X_2 = A_2 + Y_2$
$R_3 + S_3 + X_3 = A_3 + Y_3$
$R_4 + S_4 + X_4 = A_4 + Y_4$

Demand to be met:
$A_1 + Y_1 = 1{,}800$
$A_2 + Y_2 = 1{,}400$
$A_3 + Y_3 = 1{,}600$
$A_4 + Y_4 = 1{,}800$

Non negativity constraints (the units produced must be positive):
$R_p, S_p, X_p, A_p, Y_p \geq 0$
There are 10 constraints under the time available category, 1 constraint in the sheet steel category, 4 constraints in the monthly contracted production and 5 non negativity constraints.

Model Summary:

The complete model for this problem is highlighted as follows:
Minimize Z = $(6R_1+10R_2+8R_3+10R_4) + (7.2S_1+12S_2+9.6S_3+12S_4) + (6.2X_1+10.7X_2+8.5X_3+10.7X_4) + (3A_1+5A_2+4A_3+5A_4) + (3.1Y_1+5.4Y_2+4.3Y_3+5.4Y_4)$

Subject to

Stage 1 Regular Production:
$0.04R_1 + 0.17R_2 + 0.06R_3 + 0.12R_4 \leq 500$
$0.05R_1 + 0.14R_2 + 0.14R_4 \leq 400$
Stage 1 Overtime Production:
$0.04X_1 + 0.17X_2 + 0.06X_3 + 0.12X_4 \leq 100$
$0.05X_1 + 0.14X_2 + 0.14X_4 \leq 100$

Stage 2 Regular Production:
$0.06A_1 + 0.13A_2 + 0.05A_3 + 0.10A_4 \leq 600$

$0.05A_1 + 0.21A_2 + 0.02A_3 + 0.10A_4 \leq 550$
$0.03A_1 + 0.15A_2 + 0.04A_3 + 0.15A_4 \leq 500$

Stage 2 Overtime Production:
$0.06Y_1 + 0.13Y_2 + 0.05Y_3 + 0.10Y_4 \leq 100$
$0.05Y_1 + 0.21Y_2 + 0.02Y_3 + 0.10Y_4 \leq 100$
$0.03Y_1 + 0.15Y_2 + 0.04Y_3 + 0.15Y_4 \leq 100$

Sheet Steel Availability:
$1.2R_1 + 1.6R_2 + 2.1R_3 + 2.4R_4 + 1.2X_1 + 1.6X_2 + 2.1X_3 + 2.4X_4 \leq 10{,}000$

Number of units processed in Stage 1 must be equal to number of units processed in Stage 2:
$R_1 + S_1 + X_1 = A_1 + Y_1$
$R_2 + S_2 + X_2 = A_2 + Y_2$
$R_3 + S_3 + X_3 = A_3 + Y_3$
$R_4 + S_4 + X_4 = A_4 + Y_4$
Demand to be met:
$A_1 + Y_1 = 1{,}800$
$A_2 + Y_2 = 1{,}400$
$A_3 + Y_3 = 1{,}600$
$A_4 + Y_4 = 1{,}800$

Non negativity constraints (the units produced must be positive):
$R_p, S_p, X_p, A_p, Y_p \geq 0$

ASSUMPTIONS:

For applying the proposed model to case of cost minimization, following assumptions were taken into consideration:

- Proportionality: The basic assumption underlying the linear programming is that any change in the constraint inequalities will have the proportional change in the objective function.
- Additive Assumption: Every function in a linear programming model (whether the objectives function or the function on the left-hand side of a functional constraint) is the sum of the individual contributions of the respective activities.
- Continuity: Another assumption of linear programming is that the decision variables are continuous. This means a combination of outputs can be used with the fractional values along with the integer values.
- Certainty: Another underlying assumption of linear programming is a certainty, i.e. the parameters of objective function coefficients and the coefficients of constraint inequalities is known with certainty.
- Finite Choices: This assumption implies that the decision maker has certain choices, and the decision variables assume non-negative values. The non-negative assumption is true in the sense, the output in the production problem can not be negative. Thus, this assumption is considered feasible.

Thus, while solving for the linear programming problem, these assumptions should be kept in mind such that the best alternative is chosen.

SOLUTION ANALYSIS:

The production schedule for each product has been explained in detail below:

In Stage 1, the plant should have regular production for all the 4 products. In addition, it should resort to overtime production of trowels & hoes in conjunction with subcontracted production of shovels to meet the demand in Stage 1.

In Stage 2, the plant should carry out regular production of all the 4 products and overtime production of shovels only to meet the demand.

The minimum cost to produce these 4 products is estimated to be $85,472.60 which is the sum product of the number of units of each tool and their respective production cost.

To determine the critical resources in the production process, it is essential to look at the sensitivity report. We can see that the shadow prices of Packaging Regular Production, Stamping Regular Production, Drilling Regular Production and available sheet steel have negative shadow prices. These resources critical in the production process but its important to keep in mind their sensitivity ranges so that the optimal mix can be maintained.

Trowel R_1:1692 Trowels must be produced in regular production in Stage 1, X_1: 108 Trowels must be produced in overtime production in Stage 1 A_1: 1800 Trowels must be produced in regular production in Stage 2 Note: No Trowels to be produced during sub contracted production in Stage 1 and overtime production in Stage 2
Hoe R_2: 1320 Hoes must be produced in regular production in Stage 1 X_2: 80 Hoes must be produced in overtime production in Stage 1 A_2 : 1400 Hoes must be produced in regular production in Stage 2 Note: No Hoes to be produced during sub contracted production in Stage 1 and overtime production in Stage 2
Rake R_3: 1600 Rakes must be produced in regular production in Stage 1 A_3: 1600 Rakes must be produced in regular production in Stage 2 Note: No Rakes to be produced subcontracted production in Stage 1and overtime production in Stages 1 and 2
Shovel R_4: 933 Shovels must be produced in regular production in Stage 1 S_4: 867 Shovels must be produced in the subcontracted production in Stage 1 A_4 : 1147 Shovels must be produced in regular production in Stage 2 Y_4: 653 Shovels must be produced in overtime production in Stage 2 Note: No Shovels must be produced in overtime production in Stage 1

REPORT ANALYSIS:

From the problem definition perspective, we will review the sensitivity report to get insights for optimization of the production process. There are 2 reports: Answer report and Sensitivity Report

- Answer Report: The report highlights the production schedule of different tools along with the total cost incurred to produce them. Like every other production, the production process is constrained by resources. As per the report, we have 19 resources which are categorized into binding and non-binding. For instance, if we change the binding constraints (packaging, stamping, drilling, steel) then the current optimal mix is no longer feasible if its outside the bracket of sensitivity ranges. For non-binding constraints (assembly, finishing, packaging and many more) changing their quantity would not change the optimal mix.

- Sensitivity Report: It consists variable and constraint report. In the variable report, it shows the variables and its coefficients which are important for analysis. The fifth column tells you the allowable increase which is the amount by which you can increase the coefficient of the objective function without causing the optimal basis to change. The allowable decrease is the amount by which you can decrease the coefficient of the objective function without causing the optimal basis to change.

  For instance, Trowel Regular shows that the coefficient of Trowel in the objective function is 6. As long, its value is within the sensitivity range 1.e. 5.95 and 6.035, then it will not impact the optimal solution.

  The constraint report is most important from the perspective of cost minimization function, as it shows how can we vary the available resources to reduce costs. From the current report, we see that resources with negative shadow price are vital as changing them will impact the cost of production. The important ones are sheet steel, stamping, drilling and production process. They are also sensitive as they have no slack or can be said to be binding. It is essential to operate or use them within the specified ranges to achieve the target. The detailed explanation has been provided as a recommendation point in the next section.

RECOMMENDATIONS:

As per the review of the constraint section of the sensitivity report, recommendations have been provided from the perspective of minimizing cost which is also the objective of the company:

- Stamping Regular Production: The additional hour of stamping time on each of the tools will decrease the total cost by $2.41 as the production facility will be able to cut down on overtime and subcontracting costs while increasing regular time costs by a smaller amount. Spring Company should focus on total hours of stamping between 497.76 hours and 504.67 hours for decreasing the labor costs.

- Drilling Regular Production: Additional hour of drilling time will decrease the total cost by $2.07 as the company can cut down on its overtime and subcontracting cost. This decrease in labor costs per hour is only relevant if the total hours of drilling are between 394.17 hours and 401.84 hours.

- Packaging Regular Production: Increasing the packaging time by an hour will result in reducing the total cost by $2.67 as it will be able to cut down on overtime & subcontracting costs. The slight increase in regular production cost will not hamper the objective. If the packaging hours are between 498 hours and 529 hours, this saving is possible.

- Sheet Steel Usage: One option that could potentially enable large savings would be to increase the amount of sheet metal that is available to the company as an additional unit of steel sheet will reduce the total cost by $0.59. This reduction in cost is possible when the total quantity of sheet steel used per month is between 9892.57 square feet and 11428.57 square feet.

- Tools to be produced: From the sensitivity report, we can ascertain the tools which will be critical to production by looking at their shadow prices. Shovel has the highest shadow price followed by Hoe, Rake and Trowel. So, adding a unit of labour for shovel production will yield additional output by 17.40 units.