CALCULUS PRACTICAL - I.

1) $$\lim_{x\to a}\left[\frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}\right]$$

$$\lim_{x\to a}\left[\frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}\times\frac{\sqrt{a+2x}+\sqrt{3x}}{\sqrt{3a+x}+2\sqrt{x}}\right]\times\frac{\sqrt{3a+x}}{\sqrt{3a+x}}$$

$$\lim_{x\to a}\left[\frac{a+2x-3x}{3a+x-4x}\right]\frac{(\sqrt{3a+x}+2\sqrt{x})}{(\sqrt{a+2x}+\sqrt{3x})}$$

$$\lim_{x\to a}\frac{(a-x)(\sqrt{3a+x}+2\sqrt{x})}{(3a-3x)(\sqrt{a+2x}+\sqrt{3x})}$$

$$\frac{1}{3}\lim_{x\to a}\frac{(a-x)(\sqrt{3a+x}+2\sqrt{x})}{(a-x)(\sqrt{a+2x}+3\sqrt{x})}$$

$$\frac{1}{3}\ \frac{\sqrt{3a+a}+2\sqrt{a}}{\sqrt{a+2a}+\sqrt{3a}}$$

$$\frac{1}{3}\times\frac{\sqrt{4a}+2\sqrt{a}}{\sqrt{3a}+\sqrt{3a}}$$

$$\frac{1}{3}\times\frac{2\sqrt{a}+2\sqrt{a}}{\sqrt{3a}+\sqrt{3a}}$$

$$=\frac{1}{3}\times\frac{\cancel{4}^{2}\sqrt{a}}{\cancel{2}\sqrt{3a}}$$

$$=\frac{2}{3\sqrt{3}}.$$

2. $$\lim_{y\to 0}\left[\frac{\sqrt{a+y}-\sqrt{a}}{y\sqrt{a+y}}.\right]$$

$$\lim_{y\to 0}\left[\frac{\sqrt{a+y}-\sqrt{a}}{y\sqrt{a+y}}\right]\times\frac{\sqrt{a+y}+\sqrt{a}}{\sqrt{a+y}+\sqrt{a}}$$

$$\lim_{y\to 0}\frac{a+y-a}{y\sqrt{a+y}\times\sqrt{a+y}+\sqrt{a}}.$$

$$\lim_{y\to 0}=\frac{\cancel{y}}{\cancel{y}\sqrt{a+y}\times\sqrt{a+y}+\sqrt{a}}.$$

$$=\frac{1}{\sqrt{a+0}\cdot\sqrt{a+0}+\sqrt{a}}$$

$$=\frac{1}{\sqrt{a}(\sqrt{a}+\sqrt{a})}$$

$$=\frac{1}{\sqrt{a}\,(2\sqrt{a})}$$

$$=\frac{1}{2a}$$

3) $$\lim_{x\to\pi/6}\frac{\cos x-\sqrt{3}\sin x}{\pi-6x}$$

By substituting $\pi-6x=h$

$x=h+$

$$\lim_{h\to 0} \frac{\cos(h+\pi/6) - \sqrt{3}\sin(h+\pi/6)}{\pi - 6(h+\pi/6)}$$

Using
$\cos(A+B) = \cos A \cos B - \sin$
$\sin(A+B) = \sin A \cos B + \cos$

$$\lim_{h\to 0} \frac{\cosh \cos\pi/6 - \sin h \cdot \sin\pi/6 - \sqrt{3}\sin h \cos\pi/6 + \cosh \cdot \sin}{\pi - 6\left(\frac{6h+\pi}{6}\right)}$$

$$\lim_{h\to 0} \cdot \frac{\cos h \cdot \frac{\sqrt{3}}{2} \cdot \sin h \cdot \frac{1}{2} - \sqrt{3}\left(\sin h \cdot \frac{\sqrt{3}}{2} + \cos h \cdot \frac{1}{2}\right)}{\cancel{\pi} - 6h + \cancel{\pi}}$$

$(\cos\pi/6 = 30^\circ = \sqrt{3}/2$
$[\sin\pi/6 = \sin 30^\circ = \frac{1}{2}$

$$\lim_{h\to 0} \frac{\cos h \cdot \frac{\sqrt{3}h}{2} - \sin\frac{h}{2} - \sin\frac{3h}{2} - \cos\frac{\sqrt{3}}{2}}{-6h}$$

$$\lim_{h\to 0} \frac{-\frac{\sin 4h}{2}}{-6h}$$

$$\lim_{h\to 0} \frac{\sin 4h}{3/2\, h}$$

$$\frac{1}{3} \lim_{h\to 0} \frac{\sin h}{h} = \frac{1}{3} \times 1 = \frac{1}{3} \;//$$

4) $$\lim_{x\to\infty} \left[\frac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}}\right]$$

By Rationalizing numerators denominator

$$\lim_{x\to\infty} \left[\frac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}}\right] \times \left[\frac{\sqrt{x^2+5} + \sqrt{x^2-3}}{\sqrt{x^2+5} + \sqrt{x^2-3}} \times \frac{\sqrt{x^2+3} + \sqrt{x^2+1}}{\sqrt{x^2+3} + \sqrt{x^2+1}}\right]$$

$$\lim_{x\to\infty} \left[\frac{x^2+5-x^2+3}{x^2+3-x^2-1} \cdot \frac{(\sqrt{x^2+3} + \sqrt{x^2+1})}{\sqrt{x^2+5} + \sqrt{x^2-3})}\right]$$

$$\lim_{x\to\infty} \frac{\overset{4}{\cancel{8}} \cdot (\sqrt{x^2+3} + \sqrt{x^2+1}}{{}_1\cancel{2}\,(\sqrt{x^2+5} + \sqrt{x^2-3})}$$

$$4 \lim_{x\to\infty} \frac{\sqrt{x^2\left(1+\frac{3}{x^2}\right)} + \sqrt{x^2\left(1+\frac{1}{x^2}\right)}}{\sqrt{x^2\left(1+\frac{5}{x^2}\right)} + \sqrt{x^2\left(1-\frac{3}{x^2}\right)}}$$

After applying limit we get,
$= 4 \;//$

5) $$f(x) = \begin{cases} \dfrac{\sin 2x}{\sqrt{1-\cos 2x}}, & \text{for } 0 < x \le \pi/2 \\ = \dfrac{\cos x}{\pi - 2x}, & \text{for } \pi/2 < x < \pi \end{cases} \text{ at } x = \pi/2$$

$$f(\pi/2) = \frac{\sin 2(\pi/2)}{\sqrt{1-\cos 2(\pi/2)}} \qquad f(\pi/2) = 0$$

Jat $x = \pi/2$ define

ii) $\lim_{x\to\pi/2^+} f(x) = \lim_{x\to\pi/2^+} \frac{\cos x}{\pi - 2x}$

By substituting method

$x - \pi/2 = h$

$x = h + \pi/2$

where $h \to 0$

$\lim_{h\to 0} \frac{\cos(h+\pi/2)}{\pi - 2(h+\pi/2)}$

$\lim_{h\to 0} \frac{\cos(h+\pi/2)}{\pi - \cancel{2}\left(\frac{2h+\pi}{\cancel{2}}\right)}$

$\lim_{h\to 0} \frac{\cos(h+\pi/2)}{-2h}$

$\lim_{h\to 0} \frac{\cos h \cdot \cos\pi/2 - \sin h \cdot \sin\pi/2}{-2h}$ [Using $\cos(A+B) = \cos A \cdot \cos B - \sin A \cdot \sin B$]

$\lim_{h\to 0} \frac{\cos h \cdot 0 - \sin h \cdot (1)}{-2h}$

$\lim_{h\to 0} \frac{-\sin h}{-2h}$

$\frac{1}{2} \lim_{h\to 0} \frac{\sin h}{h}$

$= \frac{1}{2}$

$\therefore \lim_{x\to\pi/2} f(x) = \lim_{x\to\pi/2} \frac{\sin 2x}{\sqrt{1-\cos 2x}}$

$\lim_{x\to\pi/2} - \frac{2\sin x \cdot \cos x}{\sqrt{2}\sin^2 x}$ Using $[\sin 2x = 2\sin x \cdot \cos x]$

$\lim_{x\to\pi/2} \frac{2\cancel{\sin x}\cdot\cos x}{\sqrt{2}\cancel{\sin x}}$

$\lim_{x\to\pi/2} \frac{2\cos x}{\sqrt{2}}$

$\frac{2}{\sqrt{2}} \lim_{x\to\pi/2} \cos x$

$\frac{\cancel{2}}{\cancel{\sqrt{2}}}$ $\therefore$ LHS $\neq$ RHS

$\therefore$ f is not continuous at $x = \pi/2$

ii) $f(x) = \begin{cases} \frac{x^2-9}{x-3} & 0 < x < 3 \\ = x+3 & 3 \le x \le 6 \\ = \frac{x^2-9}{x+3} & 6 \le x < 9 \end{cases}$ at $x = 3$ & $x = 6$

at $x = 3$

i) $f(3) = \frac{x^2-9}{x-3} = 0$

f at $x = 3$ define

ii) $\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} x+3$

S8

$f(3) = x+3 = 3+3 = 6$

f is define at $x = 3$

$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} (x+3) = 6$

$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} \frac{x^2-9}{x-3} = \frac{(\cancel{x-3})(x+3)}{\cancel{(x-3)}}$

$\therefore$ LHL = RHL

$\therefore$ f is continuous at $x = 3$.

for $x = 6$

$f(6) = \frac{x^2-9}{x+3} = \frac{36-9}{6+3} = \frac{27}{9} = 3$

2) $\lim_{x\to 6^+} \frac{x^2-9}{x+3}$

$\lim_{x\to 6^+} \frac{(x-3)\cancel{(x+3)}}{\cancel{(x+3)}}$

$\lim_{x\to 6^+} (x-3) = 6-3 = 3$

$\lim_{x\to 6^-} x+3 = 3+6 = 9$

$\therefore$ LHL $\neq$ RHL

function is not continuous.

6(i) $f(x) = \frac{1-\cos 4x}{x^2}$ $\left.\begin{matrix} x < 0 \\ x = 0 \end{matrix}\right\}$ at $x =$

$= K$

Solⁿ. f is continuous at $x = 0$.

$\lim_{x\to 0} f(x) = f(0)$

$\lim_{x\to 0} \frac{1-\cos 4x}{x^2} = K$

$\lim_{x\to 0} \frac{2\sin^2 2x}{x^2} = K$

$2\lim_{x\to 0} \frac{\sin^2 2x}{x^2} = K$

$2\lim_{x\to 0} \left(\frac{\sin 2x}{x}\right)^2 = K$

$2(2)^2 = K$

$\therefore K = 8$

(ii) $f(x) = (\sec^2 x)^{\cot^2 x}$ $\left.\begin{matrix} x \neq 0 \\ x = 0 \end{matrix}\right\}$ at

$= K$

Solution:- $f(x) = (\sec^2 x)^{\cot^2 x}$

Using

$\tan^2 x - \sec^2 x = 1$

$\therefore \sec^2 x = 1 + \tan^2 x$

$\cot^2 x = \frac{1}{\tan^2 x}$

$\therefore \lim_{x\to 0} (\sec^2 x)^{\cot^2 x}$

$\lim_{x\to 0} (1+\tan^2 x)^{1/\tan^2 x}$

We know that $\lim_{x\to 0} (1+px)^{\frac{1}{px}} = e$

$\therefore = e$

$\therefore K = e$

iii) $f(x) = \dfrac{\sqrt{3} - \tan x}{\pi - 3x}$ $x \neq \pi/3$ $\Big\}$ at $x = \pi/3$

$= K$ $x = \pi/3$

$x - \pi/3 = h$

$\therefore x = h + \pi/3$

where $h \to 0$

$f(\pi/3 + h) = \dfrac{\sqrt{3} - \tan(\pi/3 + h)}{\pi - 3(\pi/3 + h)}$

$\lim_{h\to 0} \dfrac{\sqrt{3} - \tan(\pi/3 + h)}{\pi - 3(\pi/3 + h)}$

Using

$\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

$\lim_{h\to 0} \dfrac{\sqrt{3} - \dfrac{\tan \pi/3 + \tan h}{1 - \tan \pi/3 \tan h}}{\pi - \pi - 3h}$

$\lim_{h\to 0} \dfrac{\dfrac{\sqrt{3}(1 - \tan \pi/3 \tan h) - (\tan \pi/3 + \tan h)}{1 - \tan \pi/3 \cdot \tan h}}{-3h}$

$\lim_{h\to 0} \dfrac{\dfrac{(\sqrt{3} - \sqrt{3} \times \sqrt{3} \cdot \tan h) - (\sqrt{3} + \tan h)}{1 - \tan \pi/3 \cdot \tan h}}{-3h}$

$\lim_{h\to 0} \dfrac{\dfrac{(\sqrt{3} - 3\tan h) - (\sqrt{3} + \tan h)}{1 - \sqrt{3} \tan h}}{-3h}$

$\lim_{h\to 0} \dfrac{\dfrac{\cancel{\sqrt{3}} - 3\tan h - \cancel{\sqrt{3}} - \tan h}{1 - \sqrt{3} \tan h}}{-3h}$

$\lim_{h\to 0} \dfrac{-4\tan h}{-3h(1 - \sqrt{3}\tan h)}$

$\lim_{h\to 0} \dfrac{4\tan h}{3h(1 - \sqrt{3}\tan h)}$

$\dfrac{4}{3} \lim_{h\to 0} \dfrac{\tan h}{h} \cdot \lim_{h\to 0} \dfrac{1}{(1 - \sqrt{3}\tan h)}$

$\dfrac{4}{3} \times \dfrac{1}{1 - \sqrt{3}(0)}$

$= \dfrac{4}{3}\left(\dfrac{1}{1}\right)$

$= \cancel{4\sqrt{3}}\ \dfrac{4}{3}$ //

Scanned with CamScanner

7.

(i) $f(x) = \left.\begin{matrix} \dfrac{1-\cos 3x}{x \tan x}, & x \neq 0 \\ = 9, & x = 0 \end{matrix}\right\}$ at $x=0$

$f(x) = \dfrac{1-\cos 3x}{x \tan x}$

$\lim_{x \to 0} \dfrac{2 \sin^2 3/2\, x}{x \tan x}$

$\lim_{x \to 0} \dfrac{2 \dfrac{\sin^2 \frac{3x}{2}}{x^2} \times x^2}{x \cdot \dfrac{\tan x}{x^2} \times x^2}$

$= 2 \lim_{x \to 0} \dfrac{\left(\frac{3}{2}\right)^2}{1} = 2 \times \dfrac{9}{4} = \dfrac{9}{2}$

$\lim_{x \to 0} f(x) = \dfrac{9}{2} \quad 9 = f(0)$

$\therefore$ f is not continuous at $x=0$

Redefine function

$f(x) = \left\{\begin{matrix} \dfrac{1-\cos 3x}{x \tan x} & x \neq 0 \\ \dfrac{9}{2} & x = 0 \end{matrix}\right\}$

Now $\lim_{x \to 0} f(x) = f(0)$

f has removable discontinuity at $x=0$

(ii) $f(x) = \left.\begin{matrix} \dfrac{(e^{3x}-1) \sin x^\circ}{x^2} & x \neq 0 \\ = \pi/6 & x = 0 \end{matrix}\right\}$ at $x=0$

$\lim_{x \to 0} \dfrac{(e^{3x}-1) \sin\left(\frac{\pi x}{180}\right)}{x^2}$

$\lim_{x \to 0} \dfrac{e^{3x}-1}{x} \lim_{x \to 0} \dfrac{\sin\left(\frac{\pi x}{180}\right)}{x}$

$\lim_{x \to 0} 3 \cdot \dfrac{e^{3x}-1}{3x} \lim_{x \to 0} \dfrac{\sin\left(\frac{\pi x}{180}\right)}{x}$

$3 \lim_{x \to 0} \dfrac{e^{3x}-1}{3x} \lim_{x \to 0} \dfrac{\sin\left(\frac{\pi x}{180}\right)}{x}$

$3 \log e \dfrac{\pi}{180_{60}} = \dfrac{\pi}{60} = f(0)$

f is continuous at $x=0$.

8) $f(x) = \dfrac{e^{x^2} - \cos x}{x^2} \quad x = 0$

is continuous at $x=0$

$\therefore$ Given f is continuous at $x=0$

$\lim_{x \to 0} f(x) = f(0)$

$$= \lim_{x\to 0} \frac{e^{x^2} - \cos x}{x^2} = f(0)$$

$$= \lim_{x\to 0} \frac{e^{x^2} - \cos x - 1 + 1}{x^2}$$

$$= \lim_{x\to 0} \frac{(e^{x^2}-1) + (1-\cos x)}{x^2}$$

$$\lim_{x\to 0} \frac{e^{x^2}-1}{x^2} + \lim_{x\to 0} \frac{1-\cos x}{x^2}$$

$$\log e + \lim_{x\to 0} \frac{2\sin^2 x/2}{x^2}$$

$$\log e + 2 \lim_{x\to 0} \left(\frac{\sin x/2}{x}\right)^2$$

Multiplying with 2 in numerator & denominator

$$= 1 + 2 \times \frac{1}{4}$$

$$= 1 + \frac{1}{2} = \frac{3}{2} = f(0)$$

9) $f(x) = \dfrac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x}$, $x \neq \pi/2$

$f(0)$ is continuous at $x = \pi/2$

$$\lim_{x\to \pi/2} \frac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x} \times \frac{\sqrt{2} + \sqrt{1+\sin x}}{\sqrt{2} + \sqrt{1+\sin x}}$$

$$= \lim_{x\to \pi/2} \frac{2 - 1 + \sin x}{\cos^2 x \,(\sqrt{2} + \sqrt{1+\sin x})}$$

$$\lim_{x\to \pi/2} \frac{1+\sin x}{1-\sin^2 x \,(\sqrt{2} + \sqrt{1+\sin x})}$$

$$= \lim_{x=\pi/2} \frac{(1+\sin x)}{(1-\sin x)(1+\sin x)(\sqrt{2} + \sqrt{1+\sin x})}$$

$$\lim_{x=\pi/2} \frac{1}{(1-\sin x)(\sqrt{2} + \sqrt{1+\sin x})}$$

$$= \frac{1}{2(\sqrt{2} + \sqrt{2})}$$

$$= \frac{1}{2(2\sqrt{2})}$$

$$= \frac{1}{4\sqrt{2}}$$

$$\therefore f(\pi/2) = \frac{1}{4\sqrt{2}} \;//$$

21/01/2020

36

PRACTICAL - 02.

Topic : Derivation.

(Q·1) show that the following function defined from $\mathbb{R}$ to $\mathbb{R}$ are differentiable

i) $\cot x$

$f(x) = \cot x$

$Df(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$

$= \lim_{x \to a} \frac{\cot x - \cot a}{x - a}$

$= \lim_{x \to a} \frac{\frac{1}{\tan x} - \frac{1}{\tan a}}{x - a}$

$\lim_{x \to a} \frac{\frac{\tan a - \tan x}{\tan x \cdot \tan a}}{x - a}$

put $x - a = h$

$x = a + h$

as $x \to a$

$Df(h) = \lim_{h \to 0} \frac{\tan a - \tan(a+h)}{(a+h-a)\tan(a+h)\tan a}$

$\lim_{h \to 0} \frac{\tan a - \tan(a+h)}{h \cdot \tan(a+h) \tan a}$

formula $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \cdot \tan B}$

$\tan A - \tan B = \tan(A-B)\} (1 + \tan A \tan B)$

$= \lim_{h \to 0} \frac{\tan((a-a-h) - (1 + \tan a \tan(a+h)))}{h \times \tan(a+h)}$

$\lim_{h \to 0} - \frac{\tan h}{h} \frac{1 + \tan a \tan(a+h)}{\tan(a+h) \tan a}$

$= -1 \times \frac{1 + \tan^2 a}{\tan^2 a}$

$= - \frac{\sec^2 a}{\tan^2 a}$

$= - \frac{1}{\cos^2 a} \times \frac{\cos^2 a}{\sin^2 a}$

$= -\operatorname{cosec}^2 a$

$\therefore$ Q $f(a) = -\cos^2 a \therefore Df(a) = -\cos^2 a$

$\therefore$ f is differentiable $\forall a \in \mathbb{R}$

ii) $\operatorname{cosec} x$

$f(x) = \operatorname{cosec} x$

$Df(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$

$\lim_{x \to a} \frac{\operatorname{cosec} x - \operatorname{cosec} a}{x - a}$

$\lim_{x \to a} \frac{\frac{1}{\sin x} - \frac{1}{\sin a}}{x - a}$

$\lim_{x \to a} \frac{\sin a - \sin x}{(x-a)\sin a \cdot \sin x}$

Put $x - a = h$

$x = a + h$

$\therefore$ as $x \to a$, $\therefore h \to 0$

$$Df(h) = \lim_{h \to 0} \frac{\sin a - \sin(a+h)}{(a+h-a) \cdot \sin a \cdot \sin(a+h)}$$

formula

$$\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$$

$$\lim_{h \to 0} \frac{2\cos\left(\frac{a+a+h}{2}\right) \cdot \sin\left(\frac{a-a-h}{2}\right)}{h \times \sin a \sin(a+h)}$$

$$\lim_{h \to 0} -\frac{\sin h/2}{h/2} \times \frac{1}{2} \frac{2\cos\left(\frac{2a+h}{2}\right)}{\sin a \cdot \sin(a+h)}$$

$$= -\frac{1}{\cancel{2}} \times \cancel{2} \frac{\cos\left(\frac{2a+0}{2}\right)}{\sin(a+0)}$$

$$= \frac{-\cos a}{\sin^2 a} = -\cot a \cdot \operatorname{cosec} a$$

ii) $\sec x$

$F(x) = \sec x$

$$Df(a) = \lim_{x \to a} \frac{F(x) - F(a)}{x - a}$$

$$\lim_{x \to a} \frac{\sec x - \sec a}{x - a}$$

$$\lim_{x \to a} \frac{\frac{1}{\cos x} - \frac{1}{\cos a}}{x - a}$$

$$= \frac{\cos a - \cos x}{(x-a)\cos x \cdot \cos a}$$

Put $x - a = h$

$\therefore x = a + h$

$\therefore$ as $x \to a$ $h \to 0$

$$Df(h) = \lim_{h \to 0} \frac{\cos a - \cos(a+h)}{h \times \cos a \cdot \cos(a+h)}$$

Formula: $-2 \cdot \sin\left(\frac{C+D}{2}\right) \cdot \sin\left(\frac{C-D}{2}\right)$

$$\lim_{h \to 0} \frac{-2\sin\left(\frac{a+a+h}{2}\right)\sin\left(\frac{a-a-h}{2}\right)}{h \cdot \cos a \cos(a+h)}$$

$$\lim_{h \to 0} \frac{-2\sin\left(\frac{2a+h}{2}\right)\sin -\frac{h}{2}}{\cos a \cos(a+h) \times -\frac{h}{2}}$$

$$= -\frac{1}{2} \times \frac{-2\sin\left(\frac{2a+0}{2}\right)}{\cos a \cos(a+0)}$$

$$= -\frac{1}{2} \times -2 \frac{\sin a}{\cos a \cdot \cos a}$$

$$= \tan a \sec a.$$

Q.27) If $f(x) = 4x+1$, $x \leq 2$
$= x^2+5$, $x > 0$ at $x=2$, then
find function is differentiable or not

Solution :

LHD :

$$Df(2^-) = \lim_{x \to 2^-} \frac{f(x)-f(2)}{x-2}$$

$$= \lim_{x \to 2^-} \frac{4x+1-(4 \times 2+1)}{x-2}$$

$$\lim_{x \to 2^-} = \frac{4x+1-9}{x-2}$$

$$= \lim_{x \to 2^-} \frac{4x-8}{x-2}$$

$$= \lim_{x \to 2^-} 4\frac{(x-2)}{(x-2)} = 4$$

$$Df(2^-) = 4$$

RHD

$$Df(2^+) = \lim_{x \to 2^+} \frac{x^2+5-9}{x-2}$$

$$\lim_{x \to 2^+} \frac{x^2-4}{x-2}$$

$$\lim_{x \to 2^+} \frac{(x-2)(x+2)}{x-2}$$

$$\lim_{x \to 2^+} x+2$$

$$2+2 = 4$$

$$Df(2^+) = 4$$

$\therefore$ RHD = LHD

$\therefore$ f is differentiable at $x = 2$

Q.3) If $f(x) = 4x+7$, $x < 3$
$= x^2+3x+1$, $x \geq 3$ at $x=3$ then
find f is differentiable or not

Solution :

RHD

$$Df(3^+) = \lim_{x \to 3^+} \frac{f(x)-f(3)}{x-3}$$

$$= \lim_{x \to 3^+} \frac{x^2+3x+1-(3^2+3(3)+1)}{x-3}$$

$$\lim_{x \to 3^+} \frac{x^2+3x+1-19}{x-3}$$

$$\lim_{x \to 3^+} \frac{x^2+3x-18}{x-3}$$

$$\lim_{x \to 3^+} \frac{x^2+6x-3x-18}{x-3}$$

$$\lim_{x \to 3^+} \frac{x(x+6)-3(x+6)}{x-3}$$

$$\lim_{x \to 3^+} \frac{(x-3)(x+6)}{x-3}$$

$$\lim_{x \to 3^+} x+6 = 3+6$$

$DF(3^+) = 4$

LHD $= DF(3^-)$

$= \lim_{x \to 3^-} \frac{f(x) - f(3)}{x-3}$

$\lim_{x \to 3^-} \frac{4x+7-19}{x-3}$

$\lim_{x \to 3^-} \frac{4x-12}{x-3}$

$\lim_{x \to 3^-} \frac{4(x-3)}{x-3}$

$= 4$

$DF(3^-) = 4$

$\therefore$ RHD $\neq$ LHD

f is not differentiable at $x = 3$

Q.4) If $f(x) = 8x-5$ $x \le 2$ at $x = 2$

$= 3x^2-4x+7$ $x>2$

then find f is differentiable or not

Soln :- $f(2) = 8(2)-5$

$= 16-5$

$= 11$

RHD

$DF(2^+) = \lim_{x \to 2^+} \frac{f(x)-f(2)}{x-2}$

$\lim_{x \to 2^+} \frac{3x^2-4x+7-11}{x-2}$

$\lim_{x \to 2^+} \frac{3x^2-4x-4}{x-2}$

$\lim_{x \to 2^+} \frac{3x^2-6x+2x-4}{x-2}$

$\lim_{x \to 2^+} \frac{3x(x-2)+2(x-2)}{x-2}$

$\lim_{x \to 2^+} \frac{(3x+2)(x-2)}{x-2}$

$\lim_{x \to 2^+} 3x+2$

$= 3(2)+2$

$= 6+2$

$= 8$

$DF(2^+) = 8$

LHD :

$DF(2^-) = \lim_{x \to 2^-} \frac{f(x)-f(2)}{x-2}$

$\lim_{x \to 2^-} \frac{8x-5-11}{x-2}$

$= \lim_{x \to 2^-} \frac{8x-16}{x-2}$

$= \lim_{x \to 2^-} \frac{8(x-2)}{x-2}$

$= 8$

$\therefore DF(2^-) = 8$

$\therefore$ LHD = RHD

f is differentiable at $x = 2$

Practical : 3

01. Topic : Application of derivatives

Q1] Find the interval in which f^n increasing or decreasing.

a) $f(x) = x^3 - 5x - 11$

Sol^n : f is increasing iff & only iff

$f'(x) > 0$

$f(x) = x^3 - 5x - 11$

$f'(x) = 3x^2 - 5$

$3x^2 - 5 > 0$

$\therefore x = \pm\sqrt{\frac{5}{3}}$

$-\infty \quad -\sqrt{\frac{5}{3}} \quad \sqrt{\frac{5}{3}} \quad \infty$

$\therefore x \in \left(-\infty, \sqrt{\frac{5}{3}}\right) \cup \left(\frac{\sqrt{5}}{3}, \infty\right)$

Now f is decreasing iff $f'(x) < 0$

$\therefore 3x^2 - 5 < 0$

$x = \pm\sqrt{\frac{5}{3}}$

$-\sqrt{\frac{5}{3}} \quad \sqrt{\frac{5}{3}} \quad \infty$

$x \in \left(-\sqrt{\frac{5}{3}}, \sqrt{\frac{5}{3}}\right)$

c) $f(x) = x^2 - 4x$

Sol^n : f is increasing iff $f'(x) > 0$

$f(x) = x - 4x$

$f'(x) = 2x - 4$

$2x - 4 > 0$

$2(x-2) > 0$

$x - 2 > 0$

$x = 2$

$-\infty \quad - \quad 2 \quad +$

$x \in (2, \infty)$

Now f is decreasing iff

$f'(x) < 0$

$2x \cdot 4 < 0$

$2(x-2) < 0$

$x - 2 < 0$

$x = 2$

$-\infty \quad 2 \quad \infty$

$x \in (-\infty, 2)$

Q) $f(x) = 2x^3 + x^2 - 20x + 4$

$\longrightarrow$ f is increasing iff

$f'(x) = 2x^3 + x^2 - 20x + 4$

$f'(x) = 6x^2 + 2x - 20$

$6x^2 + 2x - 20 > 0$

$6x^2 + 12x - 10x - 20 > 0$

$6x(x+2) - 10(x+2) > 0$

$(x+2)(6x-10) > 0$

$x = -2, \frac{5}{3}$

$-\infty \quad - \quad -2 \quad \frac{5}{3} \quad + \quad \infty$

$\therefore x \in (-\infty, -2) \cup (\frac{5}{3}, \infty)$

NOW f is decreasing iff

$f'(x) < 0$

$6x^2 + 2x - 20 < 0$

$\therefore (x+2)(6x-10) < 0$

$x = -2, 5/3$

$-\infty \quad + \quad -2 \quad - \quad \frac{5}{3} \quad + \quad \infty$

$x \in (-2, \frac{5}{3})$

d) $f(x) = x^3 - 27x + 5$

→ f is increasing iff

$f'(x) \geq 0$

$f(x) = x^3 - 27x + 5$

$f'(x) = 3x^2 - 27$

$= 3x^2 - 27 \geq 0$

$3(x^2 - 9) \geq 0$

$x^2 - 9 \geq 0$

$x = 3, -3$

$-\infty \quad -3 \quad 3 \quad + \quad \infty$

$\therefore x \in (-\infty, -3) \cup (3, \infty)$

NOW f is decreasing $f'(x) < 0$

$3x^2 - 27 < 0$

$3(x^2 - 9) < 0$

$x^2 - 9 < 0$

$x = 3, -3$

$-\infty \quad + \quad -3 \quad - \quad 3 \quad + \quad \infty$

$x \in (-3, 3)$

e) $f(x) = 69 - 24x - 9x^2 + 2x^3$

$f(x) = 69 - 24x - 9x^2 + 2x^3$

$f'(x) = -24 - 18x + 6x^2$

$-24 - 18x + 6x^2 > 0$

$6(-4 - 3x + x^2) > 0$

$x^2 - 3x - 4 = 0$

$x^2 - 4x + x - 4 > 0$

$(x-4)(x+1) > 0$

$x = 4, -1$

$-\infty \quad -1 \quad - \quad 4 \quad + \quad -\infty$

NOW f is decreasing iff

$f'(x) < 0$

$-24 - 18x + 6x^2 < 0$

$6(-4 - 3x + x^2) < 0$

$\therefore (x-4)(x+1) < 0$

$x = 4, -1$

$\infty \quad + \quad -1 \quad + \quad 4 \quad \infty$

$x \in (-1, 4)$

Q2] Find the intervals in which f^n is concave upward & concave downwords.

a] $y = 3x^2 - 2x^3$

$\rightarrow$ $y = f(x)$

$f(x) = 3x^2 - 2x^3$

$\therefore f'(x) = 6x - 6x^2$

$f''(x) = 6 - 12x$

$\therefore$ f is concave upword iff

$f''(x) > 0$

$6 - 12x > 0$

$6(1-2x) > 0$

$(1-2x) > 0$

$-12x - 12 > 0$

$-\infty \quad + \quad \frac{1}{2} \quad \infty$

f is concave downwords iff

$f''(x) < 0$

$6(1-2x) < 0$

$-12(2x-1) < 0$

$\infty \quad + \quad \frac{1}{2} \quad - \quad \infty$

$x \in (\frac{1}{2}, \infty)$

b] $y = x^4 - 6x^3 + 12x^2 + 5x + 7$

$\rightarrow$ $y = f(x)$

$f(x) = x^4 - 6x^3 + 12x^2 + 5x + 7$

$f'(x) = 4x^3 - 18x^2 + 24x + 5$

$f''(x) = 12x^2 - 36x + 24$

f is concave upwards iff

$f''(x) > 0$

$12x^2 - 36x + 24 > 0$

$12(x^2 - 3x + 2) > 0$

$x^2 - 3x + 2 > 0$

$(x-2)\cdot(x-1) > 0$

$x = 2, 1$

$-\infty \quad + \quad 1 \quad - \quad 2 \quad + \quad \infty$

f is concave downword iff

$f''(x) < 0$

$12x^2 - 36x + 24 < 0$

$12(x^3 - 3x + 2) < 0$

$x^2 - 3x + 2 < 0$

$(x-2)(x-1) < 0$

$x = 2, 1$

$-\infty \quad + \quad 1 \quad - \quad 2 \quad + \quad \infty$

$x \in (1, 2)$

c) $y = x^3 - 27x + 5$

$\rightarrow$ $y = f(x)$

$f(x) = x^3 - 27x + 5$

$f'(x) = 3x^2 - 27$

$f''(x) = 6x$

f is concave upword iff

$f''(x) > 0$

$6x > 0$

$x > 0$

$x = 0$

$-\infty \quad 0 \quad + \quad \infty$

$x \in (0, \infty)$

f is concave downword iff

$f''(x) < 0$

$6x < 0$

$x < 0$

$x < 0$

$x = 0$

$-\infty \quad - \quad 0 \quad + \quad \infty$

$\therefore x \in (-\infty, 0)$

d) $y = 69 - 24x - 9x^2 + 2x^3$

$\rightarrow$ $y = f(x)$

$f(x) = 69 - 24x - 9x^2 + 2x^3$

$f'(x) = -24 - 18x + 6x^2$

$f''(x) = -18 + 12x$

f is concave upwords iff

$f''(x) > 0$

$-18 + 12x > 0$

$6(2x - 3) > 0$

$x = 3/2$

$-\infty \quad 3/2 \quad + \quad \infty$

$x \in (+\frac{3}{2}, \infty)$

f is concave downword iff

$f''(x) < 0$

$-18 + 12x < 0$

$6(2x - 3) < 0$

$2x - 3 < 0$

$x = 3/2$

$-\infty \quad 3/2 \quad \infty$

$x \in (-\infty, \frac{3}{2})$

e] $y = 2x^3 + x^2 - 20x + 4$

$\rightarrow y = f(x)$

$f(x) = 2x^3 + x^2 + x^2 - 20x + 4$

$f'(x) = 6x^2 + 2x - 20$

$f''(x) = 12x + 2$

f is concave upwards iff

$f''(x) > 0$

$\therefore 12x + 2 > 0$

$2(6x+1) > 0$

$6x + 1 > 0$

$x = -1/6$

$-\infty \quad -\frac{1}{6} \quad \infty$

$\therefore x \in \left(\frac{-1}{6}, \infty\right)$

$f''(x) < 0$

$12x + 2 < 0$

$2(6x+1) < 0$

$x = -1/6$

$-\infty \quad -1/6 \quad \infty$

AK
05/02/2020

Practical: 4

[Q.1] Solution

$f(x) = x^2 + \frac{16}{x^2}$

$f'(x) = 2x - \frac{32}{x^3}$

For maxima and minima

$f'(x) = 0$

$2x - \frac{32}{x^3} = 0$

$2x = \frac{32}{x^3}$

$x^4 = 16$

$x = \pm 2$

$f''(x) = 2 + \frac{96}{x^4}$

$f''(2) = f''(-2) = 2 + \frac{96}{(\pm 2)^4}$

$= 2 + \frac{96}{16}$

$f(x)$ is minimum at

$x = \pm 2$

$f(2) = 8$ is the minimum value.

(ii) Sol^n

$f(x) = 3 - 5x^3 + 3x^5$

$f'(x) = 15x^4 - 15x^2$

for maxima / minima

$f'(x) = 15x^4 - 15x^2 = 0$

$x^4 - x^2 = 0$

$x^2 (x^2 - 1) = 0$

$x = 0, -1, 1$

$f''(x) = 60x^3 - 30x$

$f''(0) = 0$

$f''(-1) = -60 + 30 = -30 < 0$

$f''(1) = 60 - 30 = 30 > 0$

$f(x)$ is maxima at -1 and minimum at 1

$f(-1) = 3 + 5 - 3 = \underline{5}$

$f(1) = 3 - 5 + 3 = \underline{1}$

(iii) $f(x)\ x^3 - 3x^2 + 1$

$f'(x) = 3x^2 - 6x$

for maxima / minima

$f'(x) = 0$

$3x^2 - 6x = 0$

$x^2 - 2x = 0$

$x(x - 2x) = 0$

$x = 0, 2$

$f''(x) = 6x - 6$

$f''(0) = -6 < 0$

$f''(2) = 6 > 0$

$f(x)$ is maxima at $x = 0$ and minimum $x = 2$

$f(0) = 1$

$f(2) = -3$

(iv) $f(x) = 2x^3 - 3x^2 - 12x + 1$

$f'(x) = 6x^2 - 6x - 12$

for maxima and minima

$f'(x) = 0$

$6x^2 - 6x - 12 = 0$

$x^2 - x - 2 = 0$

$x^2 - 2x + x - 2 = 0$

$x(x - 2) + 1(x - 2) = 0$

$(x + 1)(x - 2) = 0$

$x = -1, 2$

$f''(x) = 12x - 6$

$f''(-1) = -12 - 6 = -18 < 0$

$f''(2) = 24 - 6 = 18 > 0$

$f(x)$ is maxima at $x = 1$ and minima at $x = 2$

$f(-1) = 8$

$f(2) = -19$

Q.2) $f(x) = x^3 - 3x^2 - 55x + 95$

$f'(x) = 3x^2 - 6x - 55$

$x_1 = 0$

$f(x_0) = 9.5$

$f'(x_1) = -55$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_1)}$

$= 0 - \frac{9.5}{-55}$

$x_1 = 0.1727$

$f(x_1) = 0.0828$

$f'(x_1) = -55.9467$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$= 0.1727 - \frac{0.0828}{-55.9462}$

$x_2 = 0.1712$

$f(x_2) = 0.0011$

$f'(x_2) = -55.9393$

$x_0 = x_2 - \frac{f(x_2)}{f'(x_2)}$

$= 0.1712 - \frac{0.0011}{-55.9393}$

$\therefore x_3 = 0.1712$

$\therefore$ 0.1712 is the root of the eqn.

(ii) $f(x) = x^3 - 9x - 9$

$f'(x) = 3x^2 - 4$

$f(2) = -9$

$f(3) = 6$

6 is near to 0 as the number be

$x_0 = 3$

$f(x_0) = 6$

$f'(x_0) = 23$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_1)}$

$x_1 = 2.7391$

$f(x_1) = 0.5942$

$f'(x_1) = 18.508$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$= 7.391 - 0.5942/18.508$

$x_2 = 2.707$

$f(x_2) = 0.0085$

$f'(x_2) = 17.9835$

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$

$= 2.707 - \frac{0.0085}{17.9835}$

$x_3 = 2.7065$

$f(x_3) = -0.0005$

$f'(x_3) = 17.9757$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$$

$$= 2.7063 - \frac{0.0005}{17.9757}$$

$$\boxed{x_4 = 2.7066}$$

(iii) $f(x) = x^3 - 1.8x^2 - 10x + 17 \quad [1, 2]$

$f'(x) = 3x^2 - 3.6x - 10$

$f(1) = 1 - 1.8 - 10 + 17 = 6.2$

$f(2) = -2.2$

-2.2 is lower to 0 in the number.

$x_0 = 2$

$f(x_0) = -2.2$

$f'(x_0) = -5.2$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 2 - \frac{-2.2}{-5.2}$$

$x_1 = 1.5769$

$f(x_1) = (1.5769)^3 - 1.8(1.5769)^2 - 10(1.5764) + 17$

$= 0.6762$

$f'(x_1) = -8.217$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 1.564 - \frac{0.6762}{-8.217}$$

$x_2 = 1.6592$

$f(x_2) = 0.0204$

$f'(x_2) = -7.7143$

$$\therefore x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 1.6592 - \frac{0.0204}{-7.7144}$$

$x_3 = 1.6618$

$f(x_3) = (1.6618)^3 - 1.8(1.6618)^2 - 10(1.6618) + 17$

$f(x_3) = 0$

1.6618 is the root function

Practical - 05

(i) $I = \int \frac{1}{\sqrt{x^2 - 2x - 3}}\, dx$

$= \int \frac{1}{\sqrt{x^2 + 2x + 1 - 4}}\, dx$

$= \int \frac{1}{\sqrt{(x+1)^2 - 2^2}}\, dx$

$I = \ln |x + 1 + \sqrt{x^2 + 2x - 3}| + c$

(ii) $I = \int (4e^{3x} + 1)\, dx$

$= 4\int e^{3x} dx + 1 \int dx$

$I = \frac{4e^{3x}}{3} + x + c$

(iii) $I = \int (2x^2 - 3\sin x + 5\sqrt{x})\, dx$

$= 2\int x^2 dx - 3\int \sin x\, dx + 5\int x^{1/2} dx$

$= \frac{2x^3}{3} + 3\cos x + \frac{5x^2}{3} x^{3/2} + c$

$I = \frac{2x^3}{3} + 3\cos x + \frac{10}{3} x\sqrt{x} + c$

(iv) $I = \int \frac{x^3 + 3x + 4}{\sqrt{x}}\, dx$

Put $\sqrt{x} = t$

$\frac{1}{2\sqrt{x}} = \frac{dt}{dx} \therefore \frac{dx}{\sqrt{x}} = 2dt$

$I = \int \frac{(\sqrt{x})^6 + 3(\sqrt{x})^2 + 4}{\sqrt{x}}\, dx$

$= 2\int t^6 + 3t^2 + 4\, dt$

$= 2\left(\frac{x^{7/2}}{7} + t^{3/2} + 4x^{1/2}\right) + c$

(vi) $I = \int t^7 \sin(2t^4)\, dt$

$\int t^4 \sin(2t^4) \times t^3\, dt$

Put $t^4 = x$

$4t^3 = \frac{dx}{dt}$

$t^3\, dt = \frac{1}{4}\, dx$

$I = \frac{1}{4} \int x \cdot \sin(2x)\, dx$

P.T.

$I = \frac{1}{4}\left[x\int \sin 2x\,dx - \int\left(\frac{dx}{dx}\int \sin 2x\,dx\right)dx\right]$

$= \frac{1}{4}\left[\frac{-x\cos 2x}{2} + \frac{1}{2}\int \cos 2x\,dx\right]$

$= \frac{1}{16}\sin 2x - \frac{x\cos 2x}{8} + C$

$I = \frac{1}{16}\sin 2t^4 - t^3 \frac{\cos 2t^4}{8} + C$

(vi) $I = \int \sqrt{x}\,(x^3 - 1)\,dx$

$I = \int x^3\sqrt{x}\,dx - \int \sqrt{x}\,dx$

$\int x^{7/2}\,dx - \int x^{1/2}\,dx$

$= \frac{x^{7/2}}{7/2} - \frac{x^{3/2}}{3/2} + C$

$= \frac{2}{7}x^3\sqrt{x} - \frac{2}{3}x\sqrt{x} + C.$

(vii) $I = \int \frac{1}{x^3}\sin\left(\frac{1}{x^2}\right)dx$

$1/x^2 = t$

$\frac{-2}{x^3} = \frac{dt}{dx}$

$\frac{dx}{x^3} = \frac{-1}{2}\,dt$

$I = \frac{-1}{2}\int \sin t\,dt$

$= \frac{-1}{2}\cdot -\cos t + C$

$I = \cos\frac{(1/x^2)}{2} + C$

50

(viii) $I = \int \frac{\cos x}{\sqrt[3]{\sin^2 x}}\,dx$

Put $\sin x = t$

$\cos x\,dx = dt$

$I = \int \frac{1}{t^{2/3}}\,dt$

$\int t^{-2/3}\,dt$

$= \frac{t^{-2/3+1}}{-2/3+1} + C$

$I = 3\sqrt[3]{\sin x} + C$

(ix) $I = \int e^{\cos^2 x}\sin 2x\,dx$

Put $\cos^2 x = t$

$-2\cos x \sin x = \frac{dt}{dx}$

$\therefore -\sin 2x\,dx = -dt$

$\therefore I = -\int e^t\,dt$

$= -e^t + C$

$I = -e^{\cos^2 x} + C$

Scanned with CamScanner

Q) $I = \int \left(\frac{x^2 - 2x}{x^3 - 3x^2 + 1} \right) dx$

$x^3 - 3x^2 + 1 = t$

$3x^2 - 6x = \frac{dt}{dx}$

$\cancel{3x^2 - 6x = \frac{dt}{dx}}$

$(x^2 - 2x)\, dx = \frac{dt}{3}$

$I = \frac{1}{3} \int \frac{1}{t}\, dt$

$= \frac{1}{3} \log|t| + C$

$I = \frac{1}{3} \log|x^3 - 3x^2 + 1| + C$

51

Prac - 06

3. $y = \sqrt{4 - x^2}$

$\frac{dy}{dx} = \frac{1}{2\sqrt{4 - x^2}} \times (-2x)$

$= \frac{-x}{\sqrt{4 - x^2}}$

$l = \int_{-2}^{2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

$= \int_{-2}^{2} \sqrt{1 + \frac{x^2}{4 - x^2}}\, dx$

$= \int_{-2}^{2} \sqrt{\frac{4 - x^2 + x^2}{4 - x^2}}\, dx$

$= \int_{-2}^{2} \sqrt{\frac{4}{4 - x^2}}\, dx$

$= 2 \int_{-2}^{2} \frac{1}{\sqrt{2^2 - x^2}}\, dx$

$= 2 \left[\sin^{-1}\left(\frac{x}{2}\right) \right]_{-2}^{2}$

$= 2 \left[\sin^{-1}(1) - \sin^{-1}(-1) \right]$

$$= 2\left[\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right]$$

$$l = 2\pi$$

3) $y = x^{3/2} \quad x \in [0, 4]$

$$\frac{dy}{dx} = \frac{3}{2} x^{1/2}$$

$$L = \int_0^1 \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$= \int_0^4 \sqrt{1 + \frac{9}{4} x}\, dx$$

$$= \frac{1}{2} \int_0^4 \sqrt{4 + 9x}\, dx$$

$$= \frac{1}{2} \left(\frac{(4+9x)^{3/2}}{3/2} \times \frac{1}{9} \right)_0^4$$

$$= \frac{1}{27} \left[(4+9x)^{3/2} \right]_0^4$$

$$= \frac{1}{27} \left[(4+36)^{3/2} - (4+0)^{3/2} \right]$$

$$l = \frac{1}{27} \left(40^{3/2} - 8 \right) \text{ units}$$

4. $x = 3\sin t \qquad y = 3\cos t$

$$\frac{dx}{dt} = 3\cos t \qquad \frac{dy}{dt} = -3\sin t$$

$$L = \int_0^{2\pi} \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2}\, dt$$

$$= \int_0^{2\pi} \sqrt{(3\cos t)^2 + (-3\sin t)^2}\, dt$$

$$= \int_0^{2\pi} \sqrt{9\sin^2 t + 9\cos^2 t}\, dt$$

$$= \int_0^{2\pi} 3\sqrt{1}\, dt$$

$$= 3\int_0^{2\pi} dt$$

$$= 3\left[x\right]_0^{2\pi}$$

$$= 3\left[2\pi - 0\right]$$

$$L = 6\pi \text{ units}$$

$x = \frac{1}{6} y^3 + \frac{1}{2y}$

$$\frac{dx}{dy} = \frac{y^2}{2} - \frac{1}{2y^2}$$

$$\frac{dx}{dy} = \frac{y^4 - 1}{2y^2}$$

$$L = \int_1^2 \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$$= \int_1^2 \sqrt{1 + \frac{(y^4-1)^2}{4y^4}}\, dy$$

$$= \int_1^2 \sqrt{\frac{(y^4-1) + 4 \times y^4 - 1}{4y}}\, dy \quad \left[(a-b)^2 + 4ab = (a+b)^2\right]$$

$$= \int_1^2 \sqrt{\frac{(y^4+1)^2}{(2y^2)^2}}\, dy$$

$$= \int_1^2 \frac{y^4+1}{2y^2}\, dy$$

$$= \frac{1}{2} \int_1^2 y^2\, dy + \frac{1}{2} \int_1^2 y^{-2}\, dy$$

$$\int_0^2 e^{x^2}\, dx = \frac{h}{3} \left[(y_0 + y_4) + 4(y_1 + y_3) + 2(y_2)\right]$$

$$= \frac{0.4}{3} \left[(1 + 54.5982) + 4(1.284 + 9.5137) + 2 \times 2.3 \ldots 3\right]$$

$$= \frac{0.5}{3} \left[55.5982 + 43.036 + 5.4362\right]$$

$$\int_0^2 e^{x^2}\, dx = 17.3535$$

Q) $\int_0^4 x^2\, dx$

$n = 4$

$$h = \frac{4-0}{4} = 1$$

x	0	1	2	3	4
y	0	1	4	9	16
	y_0	y_1	y_2	y_3	y_4

$$\int_0^4 x^2\, dx = \frac{1}{3} \left[(y_0 + y_4) + 4(y_1 + y_3) + 2y_2\right]$$

$$= \frac{1}{3} \left[0 + 16 + 4(1+9) + 2 \times 4\right]$$

$$= \frac{1}{3} \left[16 + 4(10) + 8\right]$$

$$= \frac{64}{3}$$

$$\int_0^4 x^2\,dx = 21.3333$$

(iv) $\int_0^{\pi/3} \sqrt{\sin x}\,dx$, $n = 6$

$$h = \frac{\pi/3 - 0}{6} = \frac{\pi}{18}$$

x	0	$\frac{\pi}{18}$	$\frac{2\pi}{18}$	$\frac{3\pi}{18}$	$\frac{4\pi}{18}$	$\frac{5\pi}{18}$	$\frac{6\pi}{18}$
y	0	0.4167	0.5848	0.7071	0.8017	0.8752	0.9306
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

$$\int \sqrt{\sin x}\,dx = \frac{h}{3}\left[y_0 + y_6 + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4)\right]$$

$$= \frac{\pi/18}{3}\left[0.4167 + 0.9306 + 4(0.4167 + 0.7071 + 0.8752) + 2(0.3848 + 0.8017)\right]$$

$$= \frac{\pi}{54}\left[1.3473 + 4(1.999) + 2(1.2271)\right]$$

$$= \frac{\pi}{54}\left(1.3473 + 7.996 + 2.773\right)$$

$$= \frac{\pi}{54} \times 12.1163$$

$$\int_0^{\pi/3} \sqrt{\sin x}\,dx = 0.7047$$

$$\frac{1}{2}\left[\frac{t^3}{3} - \frac{t^{-2}}{1}\right]_1^2$$

$$= \frac{1}{2}\left[\frac{8}{3} - \frac{1}{2} - \frac{1}{3} + 1\right]$$

$$= \frac{1}{2}\left(7/3 + 1/2\right)$$

$$= \frac{1}{2}\left[17/6\right]$$

$$L = \frac{17}{12} \text{ units}$$

[Q12]

(1) $\int_0^2 e^{x^2}\,dx$ units $n = 4$

$$h = \frac{b-a}{n} = \frac{2-0}{4} = 0.5$$

x	0	0.5	1	1.5	2
y	1	1.284	2.7183	9.5877	54.5982
	y_0	y_1	y_1	y_2	y_3

08/01/2020

Scanned with CamScanner

Practical 07

Q.1] Solve the following differential eqn :-

i) $x \frac{dy}{dx} + y = e^x$

$\frac{dy}{dx} + \frac{1}{x} y = \frac{e^x}{x}$

$P(x) = 1/x \quad Q(x) \frac{e^x}{x}$

$IF = e^{\int P(x) dx}$

$= e^{\int 1/x dx}$

$= e^{\ln x}$

$IF = x$

$y(IF) = \int Q(x)(IF) dx + C$

$yx = \int \frac{e^x}{x} \cdot x \cdot dx + c$

$= \int e^x dx +)$

$xy = e^x + c$

ii) $e^x \frac{dy}{dx} + 2e^x y = 1$

$= \frac{dy}{dx} + 2 \cdot \frac{e^x}{e^x} y = \frac{1}{e^x} \quad (\div \text{ by } ex).$

$\frac{dy}{dx} + 2y = \frac{1}{e^x}$

$\frac{dy}{dx} + 2y = e^{-x}$

$P(x) = 2 \quad Q(x) = e^{-x}$

$\int P(x) dx$

$IF = e^{\int 2 dx}$

$= e^{2x}$

$y(IF) = \int Q(x)(IF) dx + c$

$y \cdot e^{2x} \int e^{-x + 2x} dx + c$

$= \int e^x dx + c$

$y \cdot e^{2x} = e^x + c$

(iii) $x \dfrac{dy}{dx} = \dfrac{\cos x}{x} - 2y$

$x \dfrac{dy}{dx} = \dfrac{\cos x}{x} - 2y$

$\therefore \dfrac{dy}{dx} + \dfrac{2y}{x} = \dfrac{\cos x}{x^2}$

$P(x) = 2/x \quad Q(x) = \dfrac{\cos x}{x^2}$

$I.F. = e^{\int P(x)\,dx}$

$= e^{\int 2/x\,dx}$

$= e^{2\ln x}$

$= \ln x^2$

$I.F. = x^2$

$y(I.F.) = \int Q(x)(I.F.)\,dx + c$

$= \int \dfrac{\cos x}{x^2} \cdot x^2\,dx + c$

$= \int \cos x + c$

$x^2 y = \sin x + c$

(iv) $x \dfrac{dy}{dx} + 3y = \dfrac{\sin x}{x^2}$

$\dfrac{dy}{dx} + \dfrac{3y}{x} = \dfrac{\sin x}{x^3} \quad (\div x \text{ on b.s})$

$P(x) = 3/x \quad Q(x) = \sin x / x^3$

$= e^{\int P(x)\,dx}$

$= e^{\int 3/x\,dx}$

$= e^{3\ln x}$

$= e^{\ln x^3}$

$y(I.F.) = \int Q(x)(I.F.)\,dx + c$ 56

$= \int \dfrac{\sin x}{x^3}\, x^3\,dx + c$

$= \int \sin x\,dx + c$

$x^3 y = -\cos x + c$

(v) $e^{2x} \dfrac{dy}{dx} + 2e^{2x} y = 2x$

$\dfrac{dy}{dx} + 2y = \dfrac{2x}{e^{2x}}$

$P(x) = 2 \quad Q(x) = 2x/e^{2x} = 2x e^{-2x}$

$I.F. = e^{\int P(x)\,dx}$

$= e^{\int 2\,dx}$

$= e^{2x}$

$y(I.F.) = \int Q(x)(I.F.)\,dx + c$

$= \int 2x e^{-2x} e^{2x}\,dx + c$

$= \int 2x\,dx + c$

$y e^{2x} = x^2 + c$

(vi) $\sec^2 x \tan y \, dx + \sec^2 y \tan x \, dy = 0$

$\sec^2 x \cdot \tan y \, dx = -\sec^2 y \cdot \tan x \, dy$

$\frac{\sec^2 x \, dx}{\tan x} = -\frac{\sec^2 y \, dy}{\tan y}$

$\int \frac{\sec^2 x \, dx}{\tan x} = -\int \frac{\sec^2 y \, dy}{\tan y}$

$\therefore \log|\tan x| = -\log|\tan y| + c$

$\log|\tan x \cdot \tan y| = c$

$\tan x \cdot \tan y = e^c$

(vii) $\frac{dy}{dx} = \sin^2(x-y+1)$

Put $x-y+1 = v$

differentiating both sides

$x-y+1 = v$

$1 - \frac{dy}{dx} = \frac{dv}{dx}$

$1 - \frac{dv}{dx} = \frac{dy}{dx}$

$1 - \frac{dv}{dx} = \sin^2 v$

$\frac{dv}{dx} = 1 - \sin^2 v$

$\frac{dv}{dx} = \cos^2 v$

$\frac{dv}{\cos^2 v} = dx$

$\int \sec^2 v \, dv = \int dx$

$\tan v = x + c$

$\tan(x+y-1) = x + c$

(viii) $\frac{dy}{dx} = \frac{2x+3y-1}{6x+9y+6}$

Put $2x+3y = v$

$2 + \frac{3dy}{dx} = \frac{dv}{dx}$

$\frac{dy}{dx} = \frac{1}{3}\left(\frac{dv}{dx} - 2\right)$

$\frac{1}{3}\left[\frac{dv}{dx} - 2\right] = \frac{1}{3}\frac{v-1}{(v+2)}$

$\frac{dv}{dx} = \frac{v-1}{v+2} + 2$

$$\therefore \frac{dv}{dx} = \frac{v-1+2v+4}{v+2}$$

$$= \frac{3v+3}{v+2}$$

$$= 3\left(\frac{v+1}{v+2}\right)$$

$$= 3\left(\frac{v+1}{v+2}\right)$$

$$\int\left(\frac{v+2}{v+1}\right) dv = 3 / dx$$

$$= \int \frac{v+1}{v+1}\, dx + \int \frac{1}{v+1}\, dv = 3x$$

$$v + \log|v+1| = 3x + C$$

$$2x + 3y + \log|2x+3y+1| = 3x$$

$$3y = x - \log|2x+3y+1|$$

08/01/2020

PRACTICAL- 8

TOPIC : Euler's Method :

Q.1. $\frac{dy}{dx} = y + e^x - 2$ $y(0) = 2$ $h = 0.5$ find $y(2) = ??$

Sol^n :- $f(x) = y + e^x - 2$ $x_0 = 0$ $y(0) = 2$ $h = 0.5$

n	x_n	y_n	$F(x_n, y_n)$	y_{n+1}
0	0	2	1	2.5
1	0.5	2.5	2.1487	3.5743
2	1	3.5743	4.2925	5.7205
3	1.5	5.7205	8.2021	9.8215
4	2	9.8215		

$\therefore y(2) = 9.8215$

Q.2 $\frac{dy}{dx} = 1 + y^2$, $y(0) = 1$ $h = 0.2$ find $y(1) = ?$

$y_0 = 0.$ $y_0 = 0$ $h = 0.2$

n	x_n	y_n	$F(x_n, y_n)$	y_{n+1}
0	0	0	1	0.2
1	0.2	0.2	1.04	0.408
2	0.4	0.408	1.1664	0.6412
3	0.6	0.6412	1.4111	0.9234
4	0.8	0.9234	1.8526	1.2939
5	1	1.2939		

$\therefore y(1) = 1.2939$

Q.3 $\frac{dy}{dx} = \sqrt{\frac{2}{y}}$ $y(0) = 1$ $h = 0.2$ find $y(1) = ?$

$x_0 = 0$, $y(0) = 1$ $h = 0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	1	0	1
1	0.2	1	0.4472	1.0894
2	0.4	1.0894	0.6059	1.2105
3	0.6	1.2105	0.7040	1.3513
4	0.8	1.3513	0.7696	1.5051
5	1	1.5051		

$\underline{y(1) = 1.5051}$

Q.4 $\frac{dy}{dx} = 3x^2+1$ $y(1) = 2$ find $y(2)$ = $h = 0.5$, ?

$h = 0.25$

$y(0) = 2$ $x_0 = 1$ $h = 0.5$

n	x_n	y_n	$f(x_n, y_n)$
0	1	2	4
1	1.5	4	7.75
2	2	7.875	

$\therefore \underline{\underline{y(2) = 7.875}}$

Q.5 $y_0 = 2$ $x_0 = 1$ $h = 0.25$

Sol^n :-

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	3
1	1.25	3	5.6875	4.4218
2	1.5	4.4218	59.6569	19.3360
3	1.75	19.3360	1122.6426	299.9960
4	2	299.9960		

$\therefore \underline{\underline{y(2) = 299.9960}}$

Q.6 $\frac{dy}{dx} = \sqrt{xy} + 2$ $(y(1) = 1$ $h = 0.2$

$x_0 = 1$, $y_0 = 1$, $h = 0.2$

n	x_n	y_n	$f(x_n, y_n)$
0	1	1	3
1	1.2	3.6	

$\underline{\underline{y(1.2) = 3.6}}$

PRACTICAL- 9

TOPIC :- Limits & Partial order derivatives

Q.1) Evaluate the following limits

1) $$\lim_{(x,y)\to(-4,-1)} \frac{x^3-3x+y^2-1}{xy+5}$$

$$= \lim_{(x,y)\to(-4,-1)} \frac{x^3-3x+y^2-1}{xy+5}$$

Apply limit,

$$= \frac{(-4)^3-3(-4)+(-1)^2-1}{(-4)(-1)+5}$$

$$= \frac{-64+12+1-1}{4+5}$$

$$= \frac{-52}{9}$$

2) $$\lim_{(x,y)\to(2,0)} \frac{(y+1)(x^2+y^2-4x)}{x+3y}$$

$$= \lim_{(x,y)\to(2,0)} \frac{(y+1)(x^2+y^2-4x)}{x+3y}$$

Apply limit,

$$\frac{(0+1)[(2)^2+(0)^2-4(2)]}{2+3(0)}$$

$$= \frac{1(4+0-8)}{2}$$

$$= \frac{-4}{2} = -2$$

3) $$\lim_{(x,y,z)\to(1,1,1)} \frac{x^2-y^2z^2}{x^3-x^2yz}$$

$$= \lim_{(x,y,z)\to(1,1,1)} \frac{x^2-y^2z^2}{x^3-x^2yz}$$

Apply limit

$$= \frac{(1)^2-(1)^2(1)^2}{(1)^3-(1)^2(1)(1)}$$

$$= \frac{1-1}{1-1}$$

$$= 0$$

Q.2) Find f_x, f_y for each of the following

1) $f(x,y) = xy\,e^{x^2+y^2}$

$$f(x) = y\,(1\,e^{x^2+y^2}) + xy\,(e^{x^2+y^2}\cdot 2x)$$

$$= y\,e^{x^2+y^2} + 2x^2y\,e^{x^2+y^2}$$

$$= x\,(1\cdot e^{x^2+y^2}) + xy\,(e^{x^2+y^2}\cdot 2y)$$

$$= x\cdot e^{x^2+y^2} + 2xy^2\,e^{x^2+y^2}$$

$$\therefore f(x) = y\,e^{x^2+y^2} + 2x^2y\,e^{x^2+y^2}$$

$$f(y) = x\,e^{x^2+y^2} + 2xy^2\,e^{x^2+y^2}$$

ii) $f(x,y) = e^x \cos y$

$$f(x) = \cos y\; e^x$$

$$f(y) = e^x - \sin y$$

$\therefore f(y) = -\sin y \, e^x$

iii> $f(x,y) = x^3y^2 - 3x^2y + y^3 + 1$

$f_x = y^2 3x^2 - 3y2x + 0 + 0$

$= 3x^2y^2 - 6xy$

$f_y = x^3 2y - 3x^2 + 3y^2$

$= 2x^3y - 3x^2 + 3y^2$

3> Using definition, find values of f_x, f_y at $(0,0)$ for $f(x,y) = \frac{2x}{1+y^2}$

$f_x(a,b) = \lim_{h\to 0} \frac{f(a+h,b) - f(a,b)}{h}$

$\lim_{h\to 0} \frac{f(a,b+h) - f(a,b)}{h}$

given $(a,b) = (0,0)$

$f_x(0,0) = \lim_{h\to 0} \frac{f(h,0) - f(0,0)}{h}$

$= \lim_{h\to 0} \frac{2h-0}{h}$

$f(y)(0,0) = \lim_{h\to 0} \frac{f(0,h) - f(0,0)}{h}$

$= \lim_{h\to 0} \frac{0-0}{h} = 0$

$\therefore f_x = 2, \; f_y = 0$

$f_x(0,0) = \lim_{h\to 0} \frac{f(h,0) - f(0,0)}{h}$

$= \lim_{h\to 0} \frac{2h-0}{h} = 2$

$f_y(0,0) = \lim_{h\to 0} \frac{f(0,h) - f(0,0)}{h}$

$= \lim_{h\to 0} \frac{0-0}{h} = 0$

$\therefore f_x = 2 \quad f_y = 0$

Q.4> Find all second order partial derivatives of f Also verify whether $f_{xy} = f_{yx}$

i> $f(x,y) = \frac{y^2 - xy}{x^2}$

$\therefore f_{xx} = \frac{d^2f}{dx^2} \qquad f_{yy} = \frac{d^2f}{dy^2}$

Applying $\frac{u}{v}$ rule

$f_x = \frac{x^2(0-y) - (y^2 - xy)2x}{x^4}$

$= \frac{-x^2y - 2xy^2 + 2x^2y}{x^4}$

$f_x = \frac{x^2y - 2xy^2}{x^4}$

$f_{xx} = \frac{x^4(2xy - 2y^2) - (x^2y - 2xy^2)(4x^3)}{x^8}$

$$= \frac{2x^5y - 2x^4y^2 - (4x^5y - 8x^4y^2)}{x^8}$$

$$= \frac{2x^5y - 2x^4y - (4x^5y + 8x^4y^2)}{x^8}$$

$$= \frac{2x^5y - 2x^4y^2 - 4x^5y + 8x^4y^2}{x^8}$$

$$= \frac{-2x^5y + 6x^4y^2}{x^8}$$

$$f_{xx} = \frac{6y^2 - 2xy}{x^4}$$

$$f(y) = \frac{1}{x^2}(2y-x) \quad \therefore f_y = \frac{2y-x}{x^2}$$

$$\therefore f_{yy} = \frac{1}{x^2}\, 2 = \frac{2}{x^2}$$

$$f_{xy} = \frac{2y-x}{x^2}$$

$$= \frac{x^2(-1) - (2y-x)(2x)}{x^4}$$

$$= \frac{-x^2 - 4xy + 2x^2}{x^4}$$

$$= \frac{x^2 - 4xy}{x^4}$$

$$= \frac{x - 4y}{x^3}$$

$$f_{xy} = \frac{x-4y}{x^3}$$

$$\therefore f_{yx} = \frac{x^2y - 2xy}{x^4}$$

$$= \frac{x^2 - 4xy}{x^4}$$

$$= \frac{x - 4y}{x^3}$$

$$= f_{xy}$$

$$\therefore f_{yx} = f_{xy}$$

Hence verified

Q.2) $f(x,y) = x^3 + 3x^2y^2 - \log(x^2+1)$

$$f_x = \frac{d}{dx}\, x^3 + 3x^2y^2 - \log(x^2+1)$$

$$f_x = 3x^2 + 6xy^2 - \frac{2x}{x^2+1}$$

$$f_y = \frac{d}{dy}\, x^3 + 3x^2y^2 - \log(x^2+1)$$

$$= 0 + 6x^2y - 0$$

$$f_y = 6x^2y$$

$$f_{xx} = \frac{d}{dx} f_x$$

$$= \frac{d}{dx}\left(3x^2 + 6xy^2 - \frac{2x}{x^2+1}\right)$$

$$f_{xx} = 6x + 6y^2 - 4\frac{x - 2x^2 + 2}{(x^2+1)^2}$$

$f_{yy} = \frac{d}{dy} f$

$= \frac{d}{dy} 6x^2 y$

$f_{yy} = 6x^2$

$f_{xy} = \frac{dx}{dy} 6y$

$= \frac{d}{dy} \left(3x^2 E 6xy^2 = \frac{2x}{x^2+1} \right)$

$f_{xy} = 12xy$

$f_{yx} = \frac{d}{dx} f_y$

$= \frac{d}{dx} 6x^2 y$

$f_{xy} = 12xy$

$\therefore f_{xy} = f_{yx}$

$\cancel{f_{xy}} = \frac{d}{dy} f_x$

$= \frac{d}{dy} y \cdot \cos(xy) + e^x \cdot e^x$

Soln :

$f(x, y) = \sin(xy) + e^{x+y}$

$f(x, y) = \sin(xy) + e^x \cdot e^y$

$f_x = \frac{d}{dx} \left(\sin(xy) + e^x e^y \right)$

$f_x = y \cos(xy) + e^x \cdot e^y$

$f_y = \frac{d}{dy} \left(\sin(xy) + e^x e^y \right.$

$= \left(x \cos(xy) + e^x \cdot e^y \right)$

$f_{xx} = \frac{d}{dx} f_x$

$= \frac{d}{dx} y \cos(xy) + e^x e^y$

$f_{xx} = -y \sin(x, y) + e^{x+y}$

$f_{yy} = \frac{d}{dy} f_y$

$= \frac{d}{dy} \left(x \cos(xy) + e^x e^y \right)$

$\cancel{f_{yy}} = -x^2 \sin^2(xy) + e^x \cdot e^y$

E.9.

Q5

1) $f(x,y) = \sqrt{x^2+y^2}$ at $(1,1)$

2) $f(x,y) = 1 - y\sin x$ at $(\pi/2, 0)$

3) $f(x,y) = \log x + \log y$ at $(1,1)$

1) $f(x,y) = \sqrt{x^2+y^2}$ $\quad (a,b) = (1,1)$

$f(1,1) = \sqrt{1+1} = \sqrt{2}$

$f_x = \dfrac{1}{2\sqrt{x^2+y^2}} \times 2x$ $\quad f_y = \dfrac{2y}{2\sqrt{x^2+y^2}}$

$f_x = \dfrac{x}{\sqrt{x^2+y^2}}$ $\quad f_y = \dfrac{y}{\sqrt{x^2+y^2}}$

$f_x(1,1) = \dfrac{1}{\sqrt{2}}$ $\quad f_y(1,1) = \dfrac{1}{\sqrt{2}}$

$(x,y)\ f(1) + f_x(1,1)(x-1) + f_y(1,1)(y-1)$

$= \sqrt{2} + \dfrac{x-1}{\sqrt{2}} + \dfrac{y-1}{\sqrt{2}}$

$= \dfrac{\sqrt{2} + x + y - 2}{\sqrt{2}}$

$= \dfrac{2 + x - y - 2}{\sqrt{2}}$

$(x,y) = \dfrac{x+y}{\sqrt{2}}$

Q.2

$f(x,y) = 1 - x + y\sin x$ $\quad (a,b) = (\pi/2, 0)$

$f(\pi/2, 0) = 1 - \dfrac{\pi}{2} + 0 \cdot \sin \pi/2$

$f(\pi/2, 0) = \dfrac{2}{2} \cdot 2 - \dfrac{\pi}{2}$

$f_x = -1 + y\cos x$

$f_x(\pi/2, 0) = -1 + \cos \pi/2$ $\quad f_y(\pi/2,0) = \sin \pi/2$

$f_x(\pi/2, 0) = -1$ $\quad f_y = (\pi/2, 0) = 1$

$(x,y) = f(\pi/2, 0) + f_x(\pi/2, 0)(x - \pi/2) + f_y(\pi/2, 0)(y-0)$

$(x,y) = \dfrac{2-\pi}{2} + (-1)(x - \pi/2) + (y)$

$= 1 - \dfrac{\pi}{2} - x + \dfrac{\pi}{2} + y$

$L(x,y) = 1 - x + y$

Q3] $(a,b) = 1,1$

$f(x,y) = \log x + \log y$

$f(1,1) = \log 1 + \log 1$

$f(1,1) = 0$

$f_x = \frac{1}{x}$ $\qquad f_y = 1/y$

$f_x(1,1) = 1$ $\qquad f_y(1,1) = 1$

$\therefore L(x,y) = f(1,1) + f_x(1,1)(x-1) + f_y(1,1)(y-1)$

$= 0 + 1(x-1) + 1(y-1)$

$L(x,y) = x + y - 2$

AK 27/01/2020

Practical-10.

Q.1) Find the directional derivative of the following funct^n at given points and in the direction of given vector.

i) $f(x,y) = x + 2y - 3$ $\quad a = (1,-1) \quad u = 3i - j$

Here, $u = 3i - j$ is not a unit vector.

$|\bar{u}| = \sqrt{(3)^2 + (-1)^2} = \sqrt{9+1} = \sqrt{10}$

Unit vector along u is $\frac{u}{|u|} = \frac{1}{\sqrt{10}}(3,-1)$

$= \left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$f(a+hu) = f(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$f(a) = f(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$f(a) = f(1,-1) = (1) + (2)(-1) - 3 = 1 - 2 - 3$
$= -4$

$f(a+hu) = f(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$f = \left(1 + \frac{3}{\sqrt{10}}\right), \left(-1 - \frac{h}{\sqrt{10}}\right)$

$$= f\left(\left(1 + \frac{3}{\sqrt{10}}\right), \left(-1 - \frac{h}{\sqrt{10}}\right)\right)$$

$$f(a+hu) = \left(1 + \frac{3}{\sqrt{10}}\right) + 2\left(-1 - \frac{h}{\sqrt{10}}\right) - 3$$

$$= 1 + \frac{3}{\sqrt{10}} - 2 - \frac{2h}{\sqrt{10}} - 3$$

$$f(a+hu) = -4 + \frac{h}{\sqrt{10}}$$

$$D_u f(a) = \lim_{h \to 0} \frac{f(a+hu) - f(a)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{-4} + h/\sqrt{10} + \cancel{4}}{h}$$

$$D_u f(a) = \frac{1}{\sqrt{10}}$$

(ii) $f(x) = y^2 - 4x + 1$ $a = (3,4)$, $u = i + 5j$

Here $u = i + 5j$ is not a unit vector.

$$|u| = \sqrt{(1)^2 + (5)^2} = \sqrt{26}.$$

Unit vector along u is $\frac{u}{|u|} = \frac{1}{\sqrt{26}}(1, 5)$

$$= \left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$$

66

$$f(a) = f(3,4) = (4)^2 - 4(3) + 1 = 5$$

$$f(a+hu) = f(3,4) + h\left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$$

$$= f\left(3 + \frac{h}{\sqrt{26}}, 4 + \frac{5h}{\sqrt{26}}\right)$$

$$f(a+hu) = \left(4 + \frac{5h}{\sqrt{26}}\right)^2 - 4\left(3 + \frac{h}{\sqrt{26}}\right) + 1$$

$$= 16 + \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - 12 - \frac{4h}{\sqrt{26}} + 1$$

$$= \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - \frac{4h}{\sqrt{26}} + 5$$

$$= \frac{25h^2}{26} + \frac{40h - 4h}{\sqrt{26}} + 5$$

$$= \frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 5$$

$$D_u f(a) = \lim_{h \to 0} \frac{\frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + \cancel{5} - \cancel{5}}{h}$$

$$\frac{\left(\frac{25h}{26} + \frac{36}{\sqrt{26}}\right)}{\cancel{h}}$$

$D_u f(a) = \frac{25h}{26} + \frac{36}{\sqrt{26}}$

(iii) $2x+3y$, $a = (1, 2)$, $u = (3i + 4j)$

Here $u = 3i + 4j$ is not a unit vector

$|\vec{u}| = \sqrt{(3)^2 + (4)^2} = \sqrt{25} = 5$

unit vector along u is $\frac{u}{|u|} = \frac{1}{5}(3, 4)$

$= \left(\frac{3}{5}, \frac{4}{5}\right)$

$f(a) = f(1, 2) = 2(1) + 3(2) = 8$

$f(a + hu) = f(1, 2) + h\left(\frac{3}{5}, \frac{4}{5}\right)$

$= f\left(\frac{1+3h}{5}, 2 + \frac{4}{5}\right)$

$= f\left(1 + \frac{3h}{5}, 2 + \frac{4}{5}\right)$

$f(a + hu) = 2\left(1 + \frac{3h}{5}\right) + 3\left(2 + \frac{4h}{5}\right)$

$= 2 + \frac{6h}{5} + 6 + \frac{12h}{5}$

$= \frac{18h}{5} + 8$

$D_u f(a) = \lim_{h \to 0} \frac{\frac{18h}{5} + \cancel{8} - \cancel{8}}{\cancel{h}}$

$= \frac{18h}{5}$

Q.2) Find gradient vector for the following function at given point.

a) $f(x, y) = x^y + y^x$; $a = (1, 1)$

$f_x = y \cdot x^{y-1} + y^x \log y$

$f_y = x^y \log x + x y^{x-1}$

$\nabla f(x, y) = (f_x, f_y)$

$= (y x^{y-1} + y^x \log y, x^y \log x + x y^{x-1})$

$f(1, 1) = (1+0, 1+0)$

$= (1, 1)$

(iii) $f(x, y) = (\tan^{-1} x) \cdot y^2$ $a = (1, -1)$

$f(x) = \frac{1}{1+x^2} \cdot y^2$

$f_y = 2y \tan^{-1} x$

$\nabla f(x, y) = (f_x, f_y)$

$\nabla f(x,y) = (f_x, f_y)$

$= \left(\frac{y^2}{1+x^2}, 2y\tan^{-1}x\right)$

$f(1,-1) = \left(\frac{1}{2}, \tan^{-1}(1)(-2)\right)$

$= \left(\frac{1}{2}, \frac{\pi}{4}(-2)\right)$

$= \left(\frac{1}{2}, \frac{-\pi}{2}\right)$

ans) $f(x,y,z) = xyz - e^{x+y+z}$, $a = (1,-1,0)$

$f_x = yz - e^{x+y+z}$

$f_y = xz - e^{x+y+z}$

$f_z = xy - e^{x+y-z}$

$\nabla f(x,y,z) = f_x, f_y, f_z$

$= yz - e^{x+y+z}, xz - e^{x+y+z}, xy - e^{x+y+z}$

$f(1,-1,0) = (-1(0) - e^{1+(-1)+0}, (1)(0) - e^{1+(-1)+0}, \cancel{(1)(-1)}, e^{1+(-1)+0})$

$= (0 - e^0, 0 - e^0, -1 - e^0)$

$= (-1, -1, -2)$

68

1) Find the eqn of tangent & normal to each of the following curves using at given point.

$x^2\cos y + e^{xy} = 2$ at $(1,0)$

a) $f(x) = \cos y\, 2x + e^{xy}\, y$

$f(y) = x^2(-\sin y) + e^{xy}.x$

$(x_0, y_0) = (1,0) = x_0 = 1, y_0 = 0$

eqn of tangent

$f_x(x - x_0) + f_y(y - y_0) = 0$

$f_x(x_0, y_0) = \cos 0\, 2(1) + e^0 0$

$= 1(2) + 0$

$= 2$

$f_y(x_0, y_0) = (1)^2 . (-\sin 0) + e^0 (1)$

$= 0 + 1.1$

$= 1$

$2(x-1) + 1(y-0) = 0$

$2x - 2 + y = 0$

$2x + y - 2 = 0$ is the reqd eqn of tangent

8) eq^n of Normal

$= ax+by+c=0$

$= bx+ay+d=0$

$1(1)+2(y)+d=0$

$1+2y+d=0$ at $(1,0)$

$= 1+2(0)+d=0$

$d+1=0$

$\therefore d=-1$

(i) $x^2+y^2-2x+3y+2=0$ at $(2,-2)$

$f(x) = 2x+0-2+0+0$

$= 2x-2$

$f_y = 0+2y-0+3+0$

$= 2y+3$

$(x_0, y_0) = (2,-2) \therefore x_0 = 2, y_0 = -2$

$f(x)(x_0, y_0) = 2(2)-2 = 2$

$f_y(x_0, y_0) = 2(-2)+3 = -1$

eq^n of tangent

$f(x)(x-x_0)+f_y(y-y_0)=0$

$2(x-2)+(-1(y+2)) = 0$

$2(x-2)+(-1y+2))=0$

$2x-2-y-2=0$

$2x-y-4=0 \rightarrow$ it is reqd eq^n

of normal.

$= ax+by+c=0$

$= bx+ay+d=0$

$= -1(x)+2(y)+d=0$

$= -2+2(-2)+d=0$

$-2-4+d=0$

$6+d=0$

$d=6$

(14) Find the eq^n of tangent & normal line to each of the following assitant

a) $x^2 - 2yz + 3y + xz = 7$ at $(2,1,0)$

$f_x = 2x - 0+0+z$

$f_x = 2x+z$

$f_y = 0-2z+3+0$

$= 2z+3$

$f_z = 0-2y+0+x$

$= -2y+x$

$(x_0, y_0, z_0) = (2,1,0) \therefore x_0 = 2, y_0 = 1$

$z_0 = 0$

$(f_x)(x_0, y_0, z_0) = 2(2)+0 = 4$

$f_y = (x_0, y_0, z_0) = 2(0)+3 = 3$

$f_z\ (x_0, y_0, z_0) = -2(1)+2 = 0$

eq^n of tangent

$f(x)(x_0-x_0)+f_y(y_0-y_0)$

$+f_z\ (z-z_0) = 0$

69

$= 4(x-2) + 3(y-1) + 0(z-0) = 0$

$= 4x + 3y - 11 = 0 \rightarrow$ reqd eqn of tangent.

Eqn of normal at $(4, 3, -11)$

$$\frac{x-x_0}{f_x} = \frac{y-y_0}{f_y} = \frac{z-z_0}{f_z}$$

$$= \frac{x-2}{4} = \frac{y-1}{3} = \frac{z+11}{0}$$

(iv) $3xyz - x - y + z = -4$ at $(1, -1, 2)$

$3xyz - x - y + z + 4 = 0$ at $(1, -1, 2)$

$f_x = 3yz - 1 - 0 + 0 + 0 + 0$

$= 3yz - 1$

$f_y = 3xz - 0 - 1 + 0 + 0$

$= 3xz - 1$

$f_z = 3xy - 0 - 0 + 1 + 0$

$= 3xy + 1$

$(x_0, y_0, z_0) = (1, -1, 2)$, $x_0 = 1$, $y_0 = -1$

$z_0 = 2$

$f(x)$ $(x_0, y_0, z_0) = (3)(-1)(2) = -7$

$f(y)$ " $(3)(1)(2) = -5$

$f(z)$ " $3(1)(-1)+1 = -2$

70

eqn of tangent

$-7(x-1) + 5(y+1) - 2(z-2) = 0$

$= -7x + 7 + 5y + 5 - 2z + 4 = 0$

$-7x + 5y - 2z + 16 = 0 \rightarrow$ this reqd of tangent

Eqn of tangent

$-7(x-1) + 5(y+1) - 2(z-2) = 0$

$-7x + 7 + 5y + 5 - z + 4 = 0$

$-7x + 5y - 2z + 16 = 0$

eqn of tangent

eqn of normal at

$-7(x-1) + 5(y+1) - 2(z-2) = 0$

$-7x + 2 + 5y + 5 - 2z + 4 = 0$

$-7x + 5y - 2z + 16 = 0$

Eqn of normal is

$$= \frac{x-x_0}{f_x} = \frac{y-y_0}{f_y} = \frac{z-z_0}{f_z}$$

$$\frac{x-1}{-7} = \frac{y+1}{5} = \frac{z-2}{-2}$$

Q.5] Find the local maxima & minima for the following

a) $f(x,y) = 3x^2 + y^2 - 3xy + 6 - 4y$

$f_x = 6x + 0 - 3y + 6 - 0$

$= 6x - 3y + 6$

$f_y = 0 + 2y - 3x + 0 - 4$

$= 2y - 3x - 4$

$f_x = 0$

$6x - 3y + 6 = 0$

$3(2x - y + 2) = 0$

$2x - y + 2 = 0$

$2x - y = -2$ — (1)

$f_y = 0$

$2y - 3x - 4 = 0$

$2y - 3x = 4$ — (2)

~~x with (2)~~

$4x - 2y = -4$

$2y - 3x = 4$

substitute value of x in eqn

$(2)(0) - y = -2$

$f_{y=2}$

$y = 2$

Critical Points are $(0, 2)$

$r = f_{xx} = 6$

$t = f_{yy} = 2$

$s = f_{xy} = -3$

Here $r > 0$

$= rt - s^2$

$= 6(2) - (-3)^2$

$= 12 - 9$

$= 3 > 0$

f has maximum at $(0, 2)$

$3x^2 + y^2 - 3xy + 6x - 4y$ at $(0, 2)$

$3(0)^2 + (2)^2 - 3(0)(2) + 6(0) - 4(2)$

$0 + 4 - 0 + 0 - 8$

$= -4$

(ii) $f(x, y) = 2x^4 + 3x^2y - y^2$

$f_x = 8x^3 + 6xy$

$f_y = 3x^2 - 2y$

$f_x = 0$

$8x^3 + 6xy = 0$

$2x(4x^2 + y) = 0$

$4x^2 + 3y = 0$ — (1)

$f_y = 0$

$3x^2 - 2y = 0$ — (2)

× eqⁿ (1) with 3
(2) with 4

$12x^2 + 9y = 0$
$-12x^2 \mp 8y = 0$
$17y = 0$
$y = 0$

put value in (1) of y
$4x^2 + 3(0) = 0$
$4x^2 = 0$
$x = 0$

Critical point is $(0,0)$
$r = f_{xx} = 24x^2 + 6x$
$t = f_{yy} = 0 - 2 = -2$
$s = f_{xy} = 6x - 0 - 6x = 6(0) = 0$

r at $(0,0)$
$= 24(0) + 6(0) = 0$
$= 0$
$rt - s^2 = 0(-2) - (0)^2$
$= 0 = 0.0$
$r = 0$ & $rt - s^2 = 0$

Q6)
(iii) $f(x,y) = x^2 - y^2 + 2x + 8y - 70$
$f_x = 2x + 2$
$f_y = -2y + 8$
$f_x = 0$ $2x + 2 = 0$
$x = \frac{-2}{2} = -1 \therefore x = -1$

$f_y = 0$
$-2y + 8 = 0$
$y = \frac{-8}{-2} = 4$
$y = 4$

Critical point is $(-1, 4)$
$r = f_{xx} = 2$
$t = f_{yy} = -2$
$s = f_{xy} = 0$
$r > 0$
$rt - s^2 = 2(-2) - (0)^2$
$= -4 - 0$
$= -4 < 0.$

$f(x, y)$ at $(-1, 4)$

$(-1)^2 - (4)^2 + 2(-1) + 8(4) - 70$

$= 1 + 16 - 2 + 32 - 70$

$= 17 + 30 - 70$

$= 37 - 70 = \underline{-33}$

05/02/2020