## Functions and Recursion – 55 course points

This assignment consists of two parts. First, write a library of static methods that performs *geometric transforms* on polygons. Next, write a program that plots a *Sierpinski triangle*.

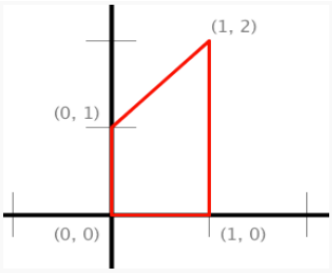
## Programming

Write 2 programs and submit on Sakai.

We provide a zip containing *PolygonTransform.java*, and *Sierpinski.java*. For each problem update and submit the corresponding file.

**DO NOT** use System.exit()
**DO NOT** add any import statements
**DO NOT** add the project or package statements
**DO NOT** change the class name
**DO NOT** change the headers of ANY of the given methods
**DO NOT** add any new class fields
**ONLY** print the result as specified by the example for each problem.
**DO NOT** print other messages, follow the examples for each problem.
**USE** StdIn, StdOut, and StdDraw libraries**.**

1. *Polygon transform (25 points)*. Write a library of static methods that performs various geometric transforms on polygons. Mathematically, a polygon is defined by its sequence of vertices (x0, y 0), (x 1, y 1), (x 2, y 2), …. In Java, we will represent a polygon by storing the x– and y-coordinates of the vertices in two parallel arrays x[] and y[].




```
// Represents the polygon with vertices (0, 0), (1, 0), (1, 2), (0, 1).
double x[] = { 0, 1, 1, 0 };
double y[] = { 0, 0, 2, 1 };
```

Three useful geometric transforms are scale, translate and rotate.

Scale the coordinates of each vertex (x i, y i) by a factor α.
x‘i = α xi
y‘i = α yi
Translate each vertex (x i, y i) by a given offset (dx, dy).
x‘i = xi + dx
y‘i = yi + dy

Rotate each vertex (x i, y i) by θ degrees counterclockwise, around the origin.

$x'_i = x_i \cos\theta - y_i \sin\theta$

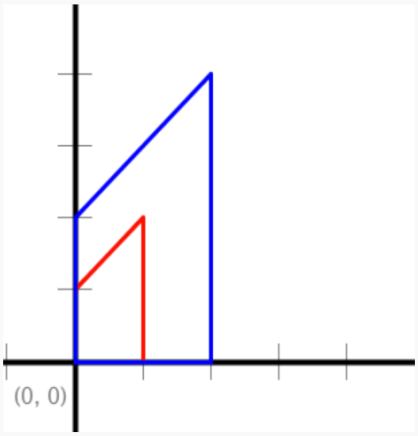
$y'_i = y_i \cos\theta + x_i \sin\theta$

Write a two-dimensional transformation library by implementing the following API:

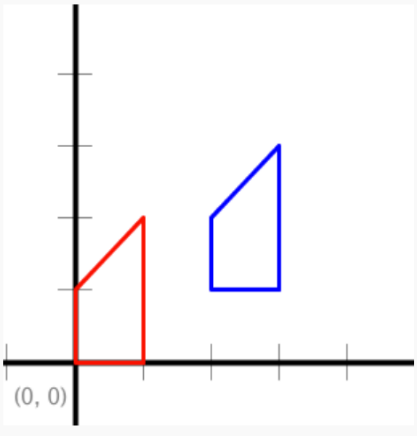
```
public class PolygonTransform {

   // Returns a new array object that is an exact copy of the given array.
   // The given array is not mutated.
   public static double[] copy(double[] array)

   // Scales the polygon by the factor alpha.
   public static void scale(double[] x, double[] y, double alpha)

   // Translates the polygon by (dx, dy).
   public static void translate(double[] x, double[] y, double dx, double dy)

   // Rotates the polygon theta degrees counterclockwise, about the origin.
   public static void rotate(double[] x, double[] y, double theta)

   // Tests each of the API methods by directly calling them.
   public static void main(String[] args)
}
```

Note that the transformation methods `scale()`, `translate()` and `rotate()` *mutate* the polygons. Here are some example test cases (tests for `copy()` are **not** shown):

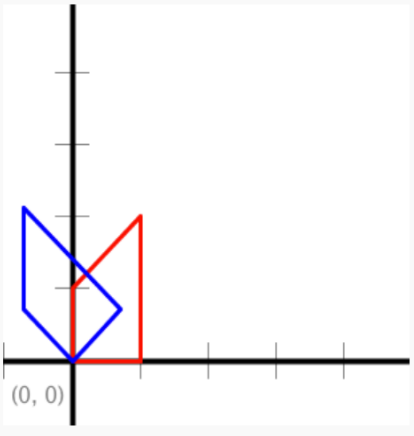
```
// Scales polygon by the factor 2.
StdDraw.setScale(-5.0, +5.0);
double[] x = { 0, 1, 1, 0 };
double[] y = { 0, 0, 2, 1 };
double alpha = 2.0;
StdDraw.setPenColor(StdDraw.RED);
StdDraw.polygon(x, y);
scale(x, y, alpha);
StdDraw.setPenColor(StdDraw.BLUE);
StdDraw.polygon(x, y);
```



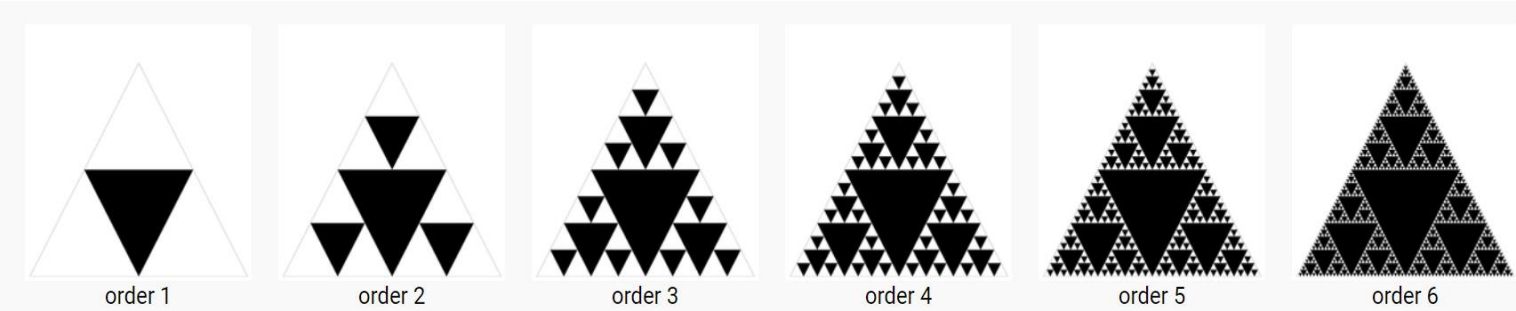

```
// Translates polygon by (2, 1).
StdDraw.setScale(-5.0, +5.0);
double[] x = { 0, 1, 1, 0 };
double[] y = { 0, 0, 2, 1 };
double dx = 2.0, dy = 1.0;
StdDraw.setPenColor(StdDraw.RED);
StdDraw.polygon(x, y);
translate(x, y, dx, dy);
StdDraw.setPenColor(StdDraw.BLUE);
StdDraw.polygon(x, y);
```




```
// Rotates polygon 45 degrees.
StdDraw.setScale(-5.0, +5.0);
double[] x = { 0, 1, 1, 0 };
double[] y = { 0, 0, 2, 1 };
double theta = 45.0;
StdDraw.setPenColor(StdDraw.RED);
StdDraw.polygon(x, y);
rotate(x, y, theta);
StdDraw.setPenColor(StdDraw.BLUE);
StdDraw.polygon(x, y);
```



3. *Sierpinski (30 points)*. The Sierpinski triangle is an example of a fractal pattern like the H-tree pattern from Section 2.3 of the textbook.




The Polish mathematician Wacław Sierpiński described the pattern in 1915, but it has appeared in Italian art since the 13th century. Though the Sierpinski triangle looks complex, it can be generated with a short recursive function. Your main task is to write a recursive function sierpinski() that plots a Sierpinski triangle of order n to standard drawing. Think recursively: sierpinski() should draw one filled equilateral triangle (pointed downwards) and then call itself recursively three times (with an appropriate stopping condition). It should draw 1 filled triangle for n = 1; 4 filled triangles for n = 2; and 13 filled triangles for n = 3; and so forth.

API specification. When writing your program, exercise modular design by organizing it into four functions, as specified in the following API:

```
public class Sierpinski {

   // Height of an equilateral triangle whose sides are of the specified length.
   public static double height(double length)

   // Draws a filled equilateral triangle whose bottom vertex is (x, y)
   // of the specified side length.
   public static void filledTriangle(double x, double y, double length)

   // Draws a Sierpinski triangle of order n, such that the largest filled
   // triangle has bottom vertex (x, y) and sides of the specified length.
   public static void sierpinski(int n, double x, double y, double length)

   // Takes an integer command-line argument n;
   // draws the outline of an equilateral triangle (pointed upwards) of length 1;
   // whose bottom-left vertex is (0, 0) and bottom-right vertex is (1, 0); and
   // draws a Sierpinski triangle of order n that fits snugly inside the outline.
   public static void main(String[] args)
}
```

*Restrictions*: You may not change either the scale or size of the drawing window.

## Before submission

1. *Collaboration policy*. Read our collaboration policy here.
2. *Update @author.* Update the @author tag of the files with your name, email and netid.
3. *Submitting the assignment*. Submit *PolygonTransform.java,* and *Sierpinski.java* separately on Sakai.