۱۰. مشتق گیری و انتگرال گیری عددی

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$$f'(x_0) = \lim_{\Delta x \to 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

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$$\Delta x = 1 \longrightarrow f'(x_0) = f(x_0 + 1) - f(x_0) \longrightarrow \text{diff}$$

$$f = \left[\, f_1 \underbrace{\quad , \quad}_{\Delta x} f_2 \underbrace{\quad , \quad}_{\Delta x} \dots , \; f_n \,\right]$$

$$f' = \left[\frac{f_2 - f_1}{\Delta x} , \; \frac{f_3 - f_2}{\Delta x} , \dots , \; \frac{f_n - f_{n-1}}{\Delta x} \right] = \frac{\mathrm{diff}(f)}{\Delta x}$$

$$x = [x_1, x_2, \dots, x_n]$$

$$f = [f_1, f_2, \dots, f_n]$$

$$f'_n = \frac{f_n - f_{n-1}}{x_n - x_{n-1}} = \frac{\mathrm{diff}(f)}{\mathrm{diff}(x)}$$

$$\frac{df}{dx}$$

توابع چندجمله‌ای :

polyder

$$P = 3x^5 - 2x^3 + x + 5$$

[3, 0, -2, 0, 1, 5]

↓

[15, 0, -6, 0, 1]

$$P' = 15x^4 - 6x^2 + 1$$

$F(x, y)$

$$\nabla F = \frac{\partial F}{\partial x}\,\hat{i} + \frac{\partial F}{\partial y}\,\hat{j}$$

$$f'(x_0) \simeq \frac{f(x) - f(x_0)}{x - x_0}$$

$$f(x) = f'(x_0) \cdot (x - x_0) + f(x_0)$$

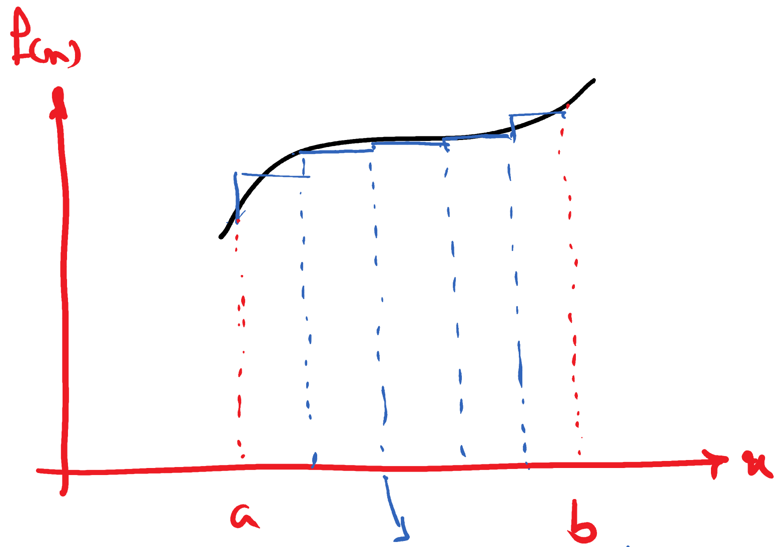
Laplacian

$u(x)$ $$L = \frac{1}{4} \frac{\partial^2 u}{\partial x^2}$$

$u(x,y)$ $$L = \frac{1}{4} \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)$$

$\longrightarrow$ <u>del2</u>

$$\int_a^b f(x)\,dx$$

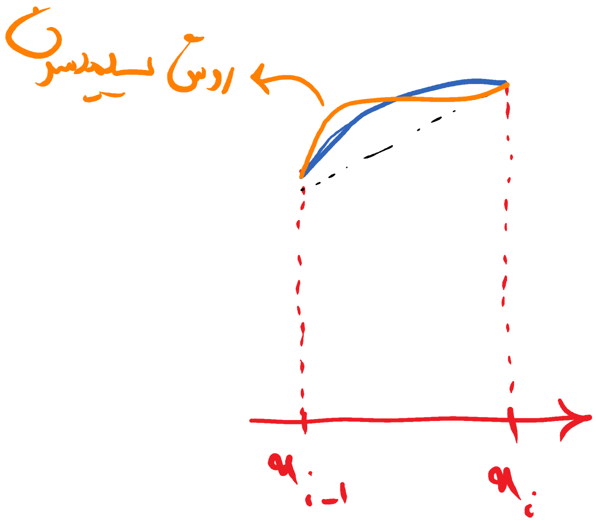


$$\int_a^b f(x)\,dx = \frac{b-a}{n}\sum_{i=0}^{n-1} f(x_i)$$

$$x_i = a + i\left(\frac{b-a}{2}\right)$$

$$\Delta x = \frac{b-a}{n}$$

روش مستطیل



روش ذوزنقه‌ای :

$$\int_a^b f(x)\,dx$$

$$= \frac{b-a}{2n} \sum_{i=0}^{n-1} \left(f(x_i) + f(x_{i+1}) \right)$$

trapz

توابع چندجمله‌ای

Polyint

$P(x) = x^2 - 1$ $\qquad [2, 0, -1]$

$\int P(x) = \frac{1}{3}x^3 - x + C$

$$I = \int_{-5}^{5} \int_{-3}^{3} (x^2 + y^2)\, dx\, dy$$

$$x(t) = \sin(2t)$$

$$y(t) = \cos(t)$$

$$z(t) = t$$

$$l = \int_a^b \sqrt{x'^2 + y'^2 + z'^2}\, dt$$

$$= \int_0^{3\pi} \sqrt{4\cos^2(2t) + \sin^2 t + 1}\, dt$$

1) $\displaystyle\int_{-5}^{0}\int_{0}^{5} (ax^2 + bx)\,dx\,dy$ $a=3$ $b=5$

2) $\displaystyle\int_{0}^{1-x}\int_{0}^{1} \frac{1}{(\sqrt{x+y})(1+x+y^2)}\,dx\,dy$

3) $$\int_{-1}^{1}\int_{-1}^{1} \frac{1}{x+y}\, dx\, dy$$

$$= \int_{-1}^{1}\int_{-1}^{-x} \frac{1}{x+y}\, dx\, dy + \int_{-1}^{1}\int_{-x}^{1} \frac{1}{x+y}\, dx\, dy$$