11.16.4.3.3

EE24BTECH11022 - Eshan Sharma

Question: A die has two faces each with number '1', three faces each with number '2', and one face with number '3'. If the die is rolled once, determine $P(\text{not } 3)$.

Theoretical Solution: The die has 6 faces in total. The probability of rolling a number other than '3' is:

$$P(\text{not } 3) = 1 - P(3)$$

From the problem, the probability of rolling '3' is:

$$P(3) = \frac{\text{Number of faces with 3}}{\text{Total faces}} = \frac{1}{6}$$

Thus:

$$P(\text{not } 3) = 1 - \frac{1}{6} = \frac{5}{6} \approx 0.8333$$

Numerical Solution using Monte Carlo Method: The Monte Carlo method is used to estimate $P(\text{not } 3)$. This involves simulating a large number of die rolls and counting the proportion of rolls that result in a number other than '3'.

Steps of the Simulation:

1) Generate a large number N of random numbers uniformly distributed between 0 and 1.
2) Map these numbers to die outcomes based on the given probabilities:

$$\text{Face '1': Range } [0, 2/6)$$
$$\text{Face '2': Range } [2/6, 5/6)$$
$$\text{Face '3': Range } [5/6, 1)$$

3) Count the number of outcomes where the result is not '3'.
4) Estimate $P(\text{not } 3)$ as the ratio of outcomes not equal to 3 to the total number of rolls:

$$P(\text{not } 3) \approx \frac{\text{Count of outcomes not equal to 3}}{N}$$

Plots:

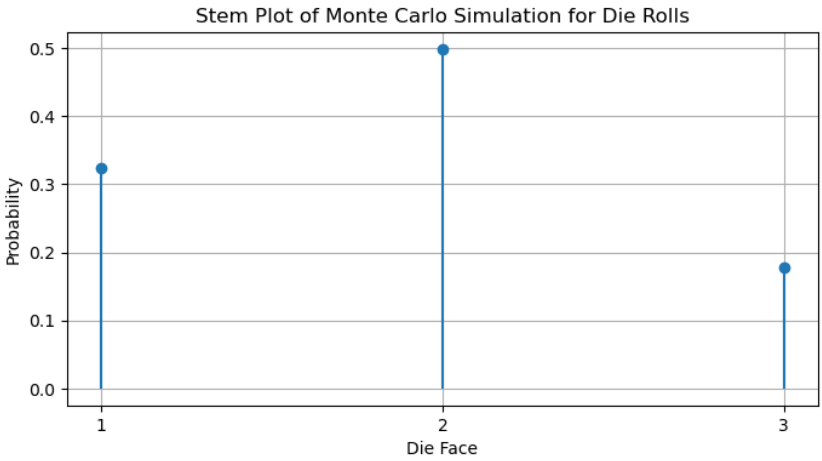

Fig. 4: Stem Plot of Monte Carlo Simulation for P(not 3).

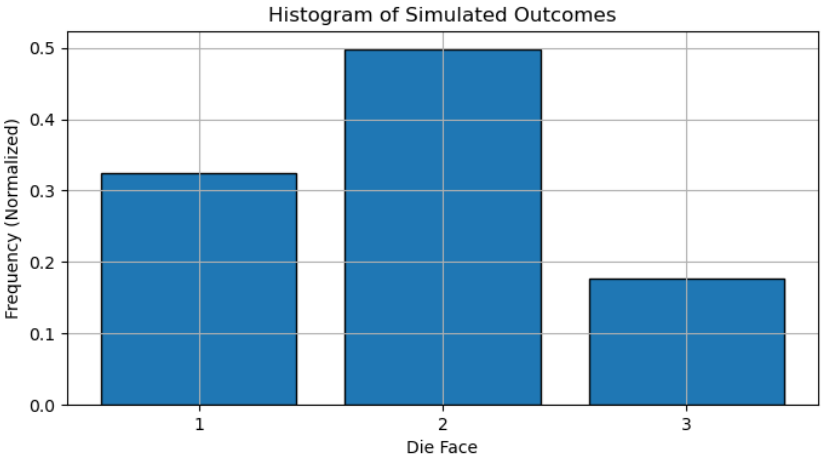

Fig. 4: Histogram of Simulated Outcomes.

Explanation of Plots:

1. Stem Plot of Monte Carlo Simulation: - **Purpose:** This plot shows the estimated probability of P(not 3) over multiple iterations of the simulation. - **X-axis:** Iteration number (or step number) in the simulation. - **Y-axis:** Cumulative estimate of P(not 3) after each iteration. - **Key Insight:** - The values fluctuate initially due to randomness. - As the number of iterations increases, the estimate converges to the theoretical value $\frac{5}{6}$. - **Usage:** This helps visualize the stability and convergence of the simulation.

2. Histogram of Simulated Outcomes: - **Purpose:** Visualizes the frequencies of

outcomes from the simulated rolls. - **X-axis:** Possible outcomes (1, 2, or 3). - **Y-axis:** Frequency of each outcome. - **Key Insight:** - The frequency of outcomes ‘1‘ and ‘2‘ combined is higher than ‘3‘, aligning with $P(\text{not } 3) = \frac{5}{6}$.
- **Usage:** Validates that the simulation adheres to the die’s probabilities.