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EE24Btech11022 - Eshan Sharma

1) Let $X_1, \ldots, X_n$ be a random sample from $N(\mu, 1)$ distribution, where $\mu \in \left\{0, \frac{1}{2}\right\}$. For testing the null hypothesis $H_0 : \mu = 0$ against the alternative hypothesis $H_1 : \mu = \frac{1}{2}$, consider the critical region

$$R = \left\{(x_1, x_2, \ldots, x_n) : \frac{1}{n} \sum_{i=1}^{n} x_i > c\right\}$$

where c is some real constant. If the critical region R has size 0.025 and power 0.7054, then the value of the sample size n is equal to ________.

2) Let X and Y be independently distributed central chi-squared random variables with degrees of freedom $m(\geq 3)$ and $n(\geq 3)$, respectively. If $E\left(\frac{X}{Y}\right) = 3$ and $m + n = 14$, then $E\left(\frac{Y}{X}\right)$ is equal to

   a) $\frac{2}{7}$ b) $\frac{3}{7}$ c) $\frac{4}{7}$ d) $\frac{5}{7}$

3) Let $X_1, X_2, \ldots$ be a sequence of independent and identically distributed random variables with $P(X_1 = 1) = \frac{1}{4}$ and $P(X_1 = 2) = \frac{3}{4}$. If $X_n = \frac{1}{n} \sum_{i=1}^{n} X_i$, for $n = 1, 2, \ldots$, then $\lim_{n\to\infty} P(X_n \leq 1.8)$ is equal to ________.

4) Let $u(x, y) = 2f(y)\cos(x - 2y)$, $(x, y) \in \mathbb{R}^2$, be a solution of the initial value problem

$$\begin{aligned} 2u_{xx} + u_y &= u \\ u(x, 0) &= \cos(x). \end{aligned}$$

   Then $f(1)$ is equal to

   a) $\frac{1}{2}$ b) $\frac{e}{2}$ c) e d) $\frac{3e}{2}$

5) Let $u(x, t), x \in \mathbb{R}, t \geq 0$, be the solution of the initial value problem

$$\begin{aligned} u_{tt} &= u_{xx}, \\ u(x, 0) &= x, \\ u_t(x, 0) &= 1. \end{aligned}$$

   Then $u(2, 2)$ is equal to ______.

6) Let $W = \text{Span}\left\{\frac{1}{\sqrt{2}}(0, 0, 1, 1), \frac{1}{\sqrt{2}}(1, -1, 0, 0)\right\}$ be a subspace of the Euclidean space $\mathbb{R}^{\ntrianglerighteq}$. Then the square of the distance from the point $(1, 1, 1, 1)$ to the subspace W is equal to ______.

7) Let $T : \mathbb{R}^{\ntrianglerighteq} \to \mathbb{R}^{\ntrianglerighteq}$ be a linear map such that the null space of T is

$$\{(x, y, z, w) \in \mathbb{R}^4 : x + y + z + w = 0\}$$

   and rank of $(T - 4I_4)\, is 3$. If the minimal polynomial of $T is x\,(x - 4)^{\alpha}$, then α is equal to ______.

8) Let M be an invertible Hermitian matrix and let $x, y \in \mathbb{R}$ be such that $x^2 < 4y$. Then
   a) both $M^2 + xM + yI$ and $M^2 - xM + yI$ are singular
   b) $M^2 + xM + yI$ is singular but $M^2 - xM + yI$ is non-singular
   c) $M^2 + xM + yI$ is non-singular but $M^2 - xM + yI$ is singular

d) both $M^2 + xM + yI$ and $M^2 - xM + yI$ are non-singular

9) Let $G = \{e, x, x^2, x^3, y, xy, x^2y, x^3y\}$ with $o(x) = 4, o(y) = 2$ and $xy = yx^3$. Then the number of elements in the center of the group G is equal to

a) 1 b) 2 c) 4 d) 8

10) The number of ring homomorphisms from $\mathbb{Z}_2 \times \mathbb{Z}_2$ to $\mathbb{Z}_4$ is equal to _______.

11) Let $p(x) = 9x^5 + 10x^3 + 5x + 15$ and $q(x) = x^3 - x^2 - x - 2$ be two polynomials in $\mathbb{Q}[x]$. Then, over $\mathbb{Q}$,

a) $p(x)$ and $q(x)$ are both irreducible
b) $p(x)$ is reducible but $q(x)$ are both irreducible
c) $p(x)$ is irreducible but $q(x)$ are both reducible
d) $p(x)$ and $q(x)$ are both reducible

12) Consider the linear programming problem

$$\begin{aligned} \text{Maximise } 3x + 9y, \text{ subject to} \\ 2y - x \leq 2 \\ 3y - x \geq 0 \\ 2x + 3y \leq 10 \\ x, y \geq 0. \end{aligned}$$

Then the maximum value of objective function is equal to _______.

13) Let $S = \{\left(x, \sin\frac{1}{x}\right) : 0 < x \leq 1\}$ and $T = S \cup \{(0,0)\}$. Under the usual metric on $\mathbb{R}^2$,

a) S is closed but T is **NOT** closed.
b) T is closed but S is **NOT** closed.
c) both S and T are closed.
d) neither S nor T is closed.