Vector Algebra

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I. MCQs with one correct answer

1) The unit vector which is orthogonal to the vector $3\hat{i}+2\hat{j}+6\hat{k}$ and is coplanar with vectors $2\hat{i}+\hat{j}+\hat{k}$ and $\hat{i}-\hat{j}+\hat{k}$ is (2004S)
   (a) $\frac{2\hat{i}-6\hat{j}+\hat{k}}{\sqrt{41}}$
   (b) $\frac{2\hat{i}-3\hat{j}}{\sqrt{13}}$
   (c) $\frac{3\hat{i}-\hat{k}}{\sqrt{10}}$
   (d) $\frac{4\hat{i}+3\hat{j}-3\hat{k}}{\sqrt{34}}$
2) A variable plane at a distance of the one unit from the origin cuts the coordinate axes at A,B and C. If the centroid D (x, y, z) of triangle ABC satisfies the relation $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=k$, then the value k is (2005S)
   (a) 3
   (b) 1
   (c) $\frac{1}{3}$
   (d) 9
3) If $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are three non-zero, non-coplanar vectors and $\mathbf{b_1}=\mathbf{b}-\frac{\mathbf{b}\cdot\mathbf{a}}{|\mathbf{a}^2|}\mathbf{a}, \mathbf{b_2}=\mathbf{b}+\frac{\mathbf{b}\cdot\mathbf{a}}{|\mathbf{a}^2|}\mathbf{a}, \mathbf{c_1}=\mathbf{c}-\frac{\mathbf{c}\cdot\mathbf{a}}{|\mathbf{a}^2|}\mathbf{a}+\frac{\mathbf{b}\cdot\mathbf{c}}{|\mathbf{c}^2|}\mathbf{b_1}, \mathbf{c_2}=\mathbf{c}-\frac{\mathbf{c}\cdot\mathbf{a}}{|\mathbf{a}^2|}\mathbf{a}+\frac{\mathbf{b_1}\cdot\mathbf{c}}{|\mathbf{b_1}^2|}\mathbf{b_1}, \mathbf{c_3}=\mathbf{c}-\frac{\mathbf{c}\cdot\mathbf{a}}{|\mathbf{c}^2|}\mathbf{a}+\frac{\mathbf{b}\cdot\mathbf{c}}{|\mathbf{c}^2|}\mathbf{b_1}, \mathbf{c_4}=\mathbf{c}-\frac{\mathbf{c}\cdot\mathbf{a}}{|\mathbf{c}^2|}\mathbf{a}=\frac{\mathbf{b}\cdot\mathbf{c}}{|\mathbf{b}^2|}\mathbf{b_1}$, then the set of orthogonal vectors is (2005S)
   (a) $(\mathbf{a}, \mathbf{b_1}, \mathbf{c_3})$
   (b) $(\mathbf{a}, \mathbf{b_1}, \mathbf{c_2})$
   (c) $(\mathbf{a}, \mathbf{b_1}, \mathbf{c_1})$
   (d) $(\mathbf{a}, \mathbf{b_2}, \mathbf{c_2})$
4) A plane which is perpendicular to two planes $2x-2y+z=0$ and $x-y+2z=4$ passes through $(1,-2,1)$. The distance of the plane from the point $(1,2,2)$ is (2006 -3M,-1)
   (a) 0
   (b) 1
   (c) $\sqrt{2}$
   (d) $2\sqrt{2}$
5) Let $\mathbf{a}=\hat{i}+2\hat{j}+\hat{k}, \mathbf{b}=\hat{i}-\hat{j}+\hat{k}$ and $\mathbf{c}=\hat{i}+\hat{j}-\hat{k}$. A vector in the plane of $\mathbf{a}$ and $\mathbf{b}$ whose projection on $\mathbf{c}$ is $\frac{1}{\sqrt{3}}$, is (2006-3M,-1)
   (a) $4\hat{i}-\hat{j}+4\hat{k}$
   (b) $3\hat{i}+\hat{j}-3\hat{k}$
   (c) $2\hat{i}+\hat{j}-2\hat{k}$
   (d) $4\hat{i}+\hat{j}-4\hat{k}$
6) The number of distinct real values of λ, for which the vectors $-\lambda^2\hat{i}+\hat{j}+\hat{k}$, $\hat{i}-\lambda^2\hat{j}+\hat{k}$ and $\hat{i}+\hat{j}-\lambda^2\hat{k}$ are coplanar, is (2007 - 3marks)
   (a) zero
   (b) one
   (c) two
   (d) three
7) let $\mathbf{a}, \mathbf{b}, \mathbf{c}$ be unit vectors such that $\mathbf{a}+\mathbf{b}+\mathbf{c}=\mathbf{0}$. Which of the following are correct? (2007-3marks)
   (a) $\mathbf{a}\times\mathbf{b}=\mathbf{b}\times\mathbf{c}=\mathbf{c}\times\mathbf{a}=\mathbf{0}$
   (b) $\mathbf{a}\times\mathbf{b}=\mathbf{b}\times\mathbf{c}=\mathbf{c}\times\mathbf{a}\neq\mathbf{0}$
   (c) $\mathbf{a}\times\mathbf{b}=\mathbf{b}\times\mathbf{c}=\mathbf{a}\times\mathbf{c}\neq\mathbf{0}$
   (d) $\mathbf{a}\times\mathbf{b}, \mathbf{b}\times\mathbf{c}, \mathbf{c}\times\mathbf{a}$ are mutually perpendicular
8) The edges of a parallelopiped are of unit length and are parallel to non-coplanar unit vectors $\hat{a}, \hat{b}, \hat{c}$ such that $\hat{a}\cdot\hat{b}=\hat{b}\cdot\hat{c}=\hat{c}\cdot\hat{a}=\frac{1}{2}$. Then, the volume of the parallelopiped is (2008)
   (a) $\frac{1}{\sqrt{2}}$
   (b) $\frac{1}{2\sqrt{2}}$
   (c) $\frac{\sqrt{3}}{2}$
   (d) $\frac{1}{\sqrt{3}}$
9) Let two non-collinear unit vectors $\hat{a}$ and $\hat{b}$ form an acute angle. A point P moves so that at any time t the position vector $\overrightarrow{OP}$ (where O is the origin) is given by $\hat{a}\cos t+\hat{b}\sin t$. When P is farthest from origin O, let M be the length of $\overrightarrow{OP}$ and $\hat{u}$ be the unit vector along $\overrightarrow{OP}$. Then, (2008)
   (a) $\hat{u}=\frac{\hat{a}+\hat{b}}{|\hat{a}+\hat{b}|}$ and $M=(1+\hat{a}\cdot\hat{b})^{1/2}$
   (b) $\hat{u}=\frac{\hat{a}-\hat{b}}{|\hat{a}-\hat{b}|}$ and $M=(1+\hat{a}\cdot\hat{b})^{1/2}$
   (c) $\hat{u}=\frac{\hat{a}+\hat{b}}{|\hat{a}+\hat{b}|}$ and $M=(1+2\hat{a}\cdot\hat{b})^{1/2}$
   (d) $\hat{u}=\frac{\hat{a}-\hat{b}}{|\hat{a}-\hat{b}|}$ and $M=(1+2\hat{a}\cdot\hat{b})^{1/2}$
10) Let P(3,2,6) be a point in space and Q be a point on the line
    $\mathbf{r}=(\hat{i}-\hat{j}+2\hat{k})+\mu(-3\hat{i}+\hat{j}+5\hat{k}$.
    Then the value of μ for which the vector $\overrightarrow{PQ}$

is parallel to the plane $x-4y+3z=1$ is (2009)

(a) $\frac{1}{4}$
(b) $-\frac{1}{4}$
(c) $\frac{1}{8}$
(d) $-\frac{1}{8}$

11) If $\mathbf{a}, \mathbf{b}, \mathbf{c}$, and $\mathbf{d}$ are unit vectors such that $(\mathbf{a} \times \mathbf{b}) \cdot (\mathbf{c} \times \mathbf{d}) = 1$ and $\mathbf{a} \cdot \mathbf{c} = \frac{1}{2}$, then (2009)

(a) $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are non-coplanar
(b) $\mathbf{b}, \mathbf{c}, \mathbf{d}$ are non-coplanar
(c) $\mathbf{b}, \mathbf{d}$ are non-parallel
(d) $\mathbf{a}, \mathbf{d}$ are parallel and $\mathbf{b}, \mathbf{c}$ are parallel

12) A line with positive direction cosines passes through the point P(2,-1,2) and makes equal angles with the coordinate axes. The line meets the plane $2x+y+z=9$ at point Q. The length of the line segment PQ equals (2009)

(a) 1
(b) $\sqrt{2}$
(c) $\sqrt{3}$
(d) 2

13) Let P, Q, R and S be the points on the plane with position vectors $-2\hat{i}-\hat{j}, 4\hat{i}, 3\hat{i}+3\hat{j} and -3\hat{i}+2\hat{j}$ respectively. The quadrilateral PQRS must be a (2010)

(a) parallelogram, which is neither a rhombus nor a rectangle
(b) square
(c) rectangle, but not a square
(d) rhombus, but not a square

14) Equation of the plane containing the straight line $\frac{x}{2} = \frac{y}{3} = \frac{z}{4}$ and perpendicular to the plane containing the straight lines $\frac{x}{3} = \frac{y}{4} = \frac{z}{2}$ and $\frac{x}{4} = \frac{y}{2} = \frac{z}{3}$ is (2010)

(a) $x+2y-2z=0$
(b) $3x+2y-2z=0$
(c) $x-2y+z=0$
(d) $5x+2y-4z=0$

15) If the distance of the point P(1,-2,1) from the plane $x+2y-2z=\alpha$, where $\alpha > 0$, is 5, then the foot of the perpendicular from P to the plane is (2010)

(a) $\left(\frac{8}{3}, \frac{4}{3}, \frac{-7}{3}\right)$
(b) $\left(\frac{4}{3}, \frac{-4}{3}, \frac{1}{3}\right)$
(c) $\left(\frac{1}{3}, \frac{2}{3}, \frac{10}{3}\right)$
(d) $\left(\frac{2}{3}, \frac{-1}{3}, \frac{5}{2}\right)$