

9-9.4-3

EE24BTECH11022 - Eshan Sharma

Question:

Find the area of the circle $x^2 + y^2 = 16$ exterior to the parabola $y^2 = 6x$.

Solution:

Symbol	Value	Description
C	$x^2 + y^2 = 16$	Circle
P	$y^2 = 6x$	Parabola
I₁	$(2 \ 2\sqrt{3})$	Intersection point
I₂	$(4 \ 0)$	diametric end of the circle
A_c	16π	Area of circle
A_i	$\frac{4}{3}(4\pi + \sqrt{3})$	Area interior to the parabola
A	$\frac{4}{3}(8\pi + \sqrt{3})$	Area exterior to the parabola

TABLE 0: Variables Used

The given circle **C** can be expressed as

$$v = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, u = 0, f = -16 \quad (0.1)$$

The given parabola **P** can be expressed as

$$v = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, u = \begin{pmatrix} -3 \\ 0 \end{pmatrix}, f = 0 \quad (0.2)$$

The area exterior to the parabola, **A** is given by

$$\mathbf{A} = A_c - A_i, \text{ where} \quad (0.3)$$

$$A_c = \pi (\text{radius of C})^2 \quad (0.4)$$

$$A_i = 2 \left(\int_0^{x(I_1)} \sqrt{6x} dx + \int_{x(I_1)}^{x(I_2)} \sqrt{16 - x^2} dx \right) \quad (0.5)$$

$$\mathbf{A} = 16\pi - \frac{4}{3}(4\pi + \sqrt{3}) \text{ units} \quad (0.6)$$

$$\mathbf{A} = \frac{4}{3}(8\pi + \sqrt{3}) \text{ units} \quad (0.7)$$

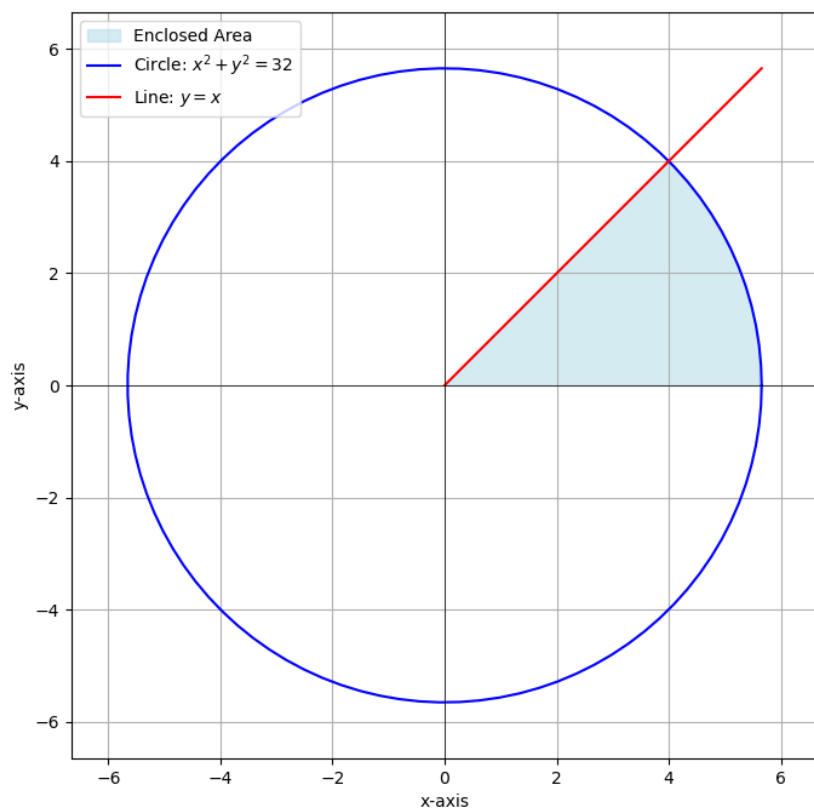


Fig. 0.1: Area enclosed in the first quadrant