

MA-2011

EE24Btech11022 - Eshan Sharma

- 1) For the rings $L = \frac{\mathbb{R}[x]}{\langle x^2-x+1 \rangle}$; $M = \frac{\mathbb{R}[x]}{\langle x^2+x+1 \rangle}$; $N = \frac{\mathbb{R}[x]}{\langle x^2+2x+1 \rangle}$;
which of the following is **TRUE**? (ma-2011)
- L is isomorphic to M; L is not isomorphic to N; M is not isomorphic to N
 - M is isomorphic to N; M is not isomorphic to L; N is not isomorphic to L
 - L is isomorphic to M; M is isomorphic to N
 - L is not isomorphic to M; L is not isomorphic to N; M is not isomorphic to N
- 2) The time to failure(in hours) of a component is a continuous random variable **T** with the probability density function

$$f(t) = \begin{cases} \frac{1}{10}e^{-\frac{t}{10}}, & t > 0, \\ 0, & t \leq 0. \end{cases}$$

Ten of these components are installed in a system and they work independently. Then ,the probability than **NONE** of these fail before ten hours, is (ma-2011)

- e^{-10}
 - $1 - e^{-10}$
 - $10e^{-10}$
 - $1 - 10e^{-10}$
- 3) Let **X** be the normal linear space of all real sequences with finitely many non-zero terms, with supremum norm and **T** : $X \rightarrow X$ be a one to one and onto linear operator defined by

$$\mathbf{T}(x_1, x_2, x_3 \dots) = \left(x_1, \frac{x_2}{2}, \frac{x_3}{3}, \dots\right)$$

Then, which of the following is **TRUE**? (ma-2011)

- T** is bounded but \mathbf{T}^{-1} is not bounded
 - T** is not bounded but \mathbf{T}^{-1} is bounded
 - both **T** and \mathbf{T}^{-1} are bounded
 - neither **T** nor \mathbf{T}^{-1} are bounded
- 4) Let $e_i = (0, \dots, 0, 1, 0, \dots)$ (i.e., e_i is the vector with 1 at the i^{th} place and 0 elsewhere) for $i = 1, 2, \dots$. Consider the statements:
- **P**: $\{f(e_i)\}$ converges for every continuous linear functional on ℓ^2 .
 - **Q**: $\{e_i\}$ converges in ℓ^2 .
- Then, which of the following holds?
- Both **P** and **Q** are TRUE
 - P** is TRUE but **Q** is not TRUE
 - P** is not TRUE but **Q** is TRUE
 - Neither **P** nor **Q** is TRUE
- 5) For which subspace $X \subset \mathbb{R}$ with the usual topology and with $\{0, 1\} \subseteq X$, will a continuous function $f : X \rightarrow \{0, 1\}$ satisfying $f(0) = 0$ and $f(1) = 1$ exist?
- $X = [0, 1]$
 - $X = [-1, 1]$
 - $X = \mathbb{R}$
 - $[0, 1] \subset X$
- 6) Suppose X is a finite set with more than five elements. Which of the following is TRUE?
- There is a topology on X which is T_1

- b) There is a topology on X which is T_3 but not T_1
 c) There is a topology on X which is T_2 but not T_1
 d) There is no topology on X which is T_1
- 7) A massless wire is bent in the form of a parabola $z = r^2$ and a bead slides on it smoothly. The wire is rotated about the z -axis with a constant angular acceleration α . Assume that m is the mass of the bead, ω is the initial angular velocity and g is the acceleration due to gravity. Then, the Lagrangian at any time t is
- a) $\frac{m}{2} \left(\frac{dr}{dt} \right)^2 (1 + 4r^2)r^2 + (\omega + \alpha t)^2 + 2gr$
 b) $\frac{m}{2} \left(\frac{dr}{dt} \right)^2 (1 + 4r^2)r^2 + (\omega + \alpha t)^2 - 2gr$
 c) $\frac{m}{2} \left(\frac{dr}{dt} \right)^2 (1 + 4r^2)r^2 + (\omega + \alpha t)^2 - 2gr$
 d) $\frac{m}{2} \left(\frac{dr}{dt} \right)^2 (1 + 4r^2)r^2 + (\omega + \alpha t)^2 + 2gr$
- 8) On the interval $[0, 1]$, let y be a twice continuously differentiable function which is an extremal of the functional

$$J(y) = \int_0^1 \sqrt{1 + 2y'^2} \frac{y'}{x} dx$$

with $y(0) = 1$, $y(1) = 2$. Then, for some arbitrary constant c , y satisfies

- a) $y'^2(2 - cx^2) = cx^2$
 b) $y'^2(2 + cx^2) = cx^2$
 c) $y'(1 - cx^2) = cx^2$
 d) $y'(1 + cx^2) = cx^2$

Let X and Y be two continuous random variables with the joint probability density function

$$f(x, y) = \begin{cases} 2, & 0 < x + y < 1, x > 0, y > 0, \\ 0, & \text{elsewhere.} \end{cases}$$

- 9) $P(X + Y < \frac{1}{2})$ is
- a) $\frac{1}{4}$ b) $\frac{1}{2}$ c) $\frac{3}{4}$ d) 1
- 10) $E(X | Y = \frac{1}{2})$ is
- a) $\frac{1}{4}$ b) $\frac{1}{2}$ c) 1 d) 2

Let $f(z) = \frac{z}{8 - z^2}$, $z = x + iy$.

- 11) $\text{Res}_{z=2} f(z)$ is
- a) $\frac{1}{8}$ b) $\frac{1}{6}$ c) $-\frac{1}{6}$ d) $-\frac{1}{8}$
- 12) The Cauchy principal value of $\int_{-\infty}^{\infty} f(x) dx$ is
- a) $-\frac{\pi\sqrt{3}}{6}$ b) $-\frac{\pi\sqrt{3}}{8}$ c) $\frac{\pi\sqrt{3}}{6}$ d) $\pi\sqrt{3}$
- 13) Let $f_j(x) = \frac{x}{(j-1)x+j}$ and $s_n(x) = \sum_{j=1}^n f_j(x)$ for $x \in [0, 1]$. The sequence $\{s_n\}$
- a) converges uniformly on $[0, 1]$
 b) converges pointwise on $[0, 1]$ but not uniformly
 c) converges pointwise for $x = 0$ but not for $x \in (0, 1]$
 d) does not converge for $x \in [0, 1]$