Vector Algebra

EE24Btech11022 - Eshan Sharma

I. MCQs with one correct answer

1) The unit vector which is orthogonal to the vector $3\hat{i}+2\hat{j}+6\hat{k}$ and is coplanar with vectors $2\hat{i}+\hat{j}+\hat{k}$ and $\hat{i}-\hat{j}+\hat{k}$ is (2004S)

   a) $\frac{2\hat{i}-6\hat{j}+\hat{k}}{\sqrt{41}}$
   b) $\frac{2\hat{i}-3\hat{j}}{\sqrt{13}}$
   c) $\frac{3\hat{i}-\hat{k}}{\sqrt{10}}$
   d) $\frac{4\hat{i}+3\hat{j}-3\hat{k}}{\sqrt{34}}$

2) A variable plane at a distance of one unit from the origin cuts the coordinate axes at $\mathbf{A}, \mathbf{B}$ and $\mathbf{C}$. If the centroid $\mathbf{D}(x, y, z)$ of triangle ABC satisfies the relation $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=k$, then the value k is (2005S)

   a) 3
   b) 1
   c) $\frac{1}{3}$
   d) 9

3) If $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are three non-zero, non-coplanar vectors and $\mathbf{b_1}=\mathbf{b}-\frac{\mathbf{b}\cdot\mathbf{a}}{|\mathbf{a}^2|}\mathbf{a}, \mathbf{b_2}=\mathbf{b}+\frac{\mathbf{b}\cdot\mathbf{a}}{|\mathbf{a}^2|}\mathbf{a}, \mathbf{c_1}=\mathbf{c}-\frac{\mathbf{c}\cdot\mathbf{a}}{|\mathbf{a}^2|}\mathbf{a}+\frac{\mathbf{b}\cdot\mathbf{c}}{|\mathbf{c}^2|}\mathbf{b_1}, \mathbf{c_2}=\mathbf{c}-\frac{\mathbf{c}\cdot\mathbf{a}}{|\mathbf{a}^2|}\mathbf{a}+\frac{\mathbf{b_1}\cdot\mathbf{c}}{|\mathbf{b_1}^2|}\mathbf{b_1}, \mathbf{c_3}=\mathbf{c}-\frac{\mathbf{c}\cdot\mathbf{a}}{|\mathbf{c}^2|}\mathbf{a}+\frac{\mathbf{b}\cdot\mathbf{c}}{|\mathbf{c}^2|}\mathbf{b_1}, \mathbf{c_4}=\mathbf{c}-\frac{\mathbf{c}\cdot\mathbf{a}}{|\mathbf{c}^2|}\mathbf{a}=\frac{\mathbf{b}\cdot\mathbf{c}}{|\mathbf{b}^2|}\mathbf{b_1}$, then the set of orthogonal vectors is (2005S)

   a) $(\mathbf{a}, \mathbf{b_1}, \mathbf{c_3})$
   b) $(\mathbf{a}, \mathbf{b_1}, \mathbf{c_2})$
   c) $(\mathbf{a}, \mathbf{b_1}, \mathbf{c_1})$
   d) $(\mathbf{a}, \mathbf{b_2}, \mathbf{c_2})$

4) A plane which is perpendicular to two planes $2x-2y+z=0$ and $x-y+2z=4$ passes through $(1,-2,1)$. The distance of the plane from the point $(1,2,2)$ is (2006 -3M,-1)

   a) 0
   b) 1
   c) $\sqrt{2}$
   d) $2\sqrt{2}$

5) Let $\mathbf{a}=\hat{i}+2\hat{j}+\hat{k}, \mathbf{b}=\hat{i}-\hat{j}+\hat{k}$ and $\mathbf{c}=\hat{i}+\hat{j}-\hat{k}$. A vector in the plane of $\mathbf{a}$ and $\mathbf{b}$ whose projection on $\mathbf{c}$ is $\frac{1}{\sqrt{3}}$, is (2006-3M,-1)

   a) $4\hat{i}-\hat{j}+4\hat{k}$
   b) $3\hat{i}+\hat{j}-3\hat{k}$
   c) $2\hat{i}+\hat{j}-2\hat{k}$
   d) $4\hat{i}+\hat{j}-4\hat{k}$

6) The number of distinct real values of λ, for which the vectors $-\lambda^2\hat{i}+\hat{j}+\hat{k}$, $\hat{i}-\lambda^2\hat{j}+\hat{k}$ and $\hat{i}+\hat{j}-\lambda^2\hat{k}$ are coplanar, is (2007 - 3marks)

   a) 1
   b) 2
   c) 3
   d) 4

7) let $\mathbf{a}, \mathbf{b}, \mathbf{c}$ be unit vectors such that $\mathbf{a}+\mathbf{b}+\mathbf{c}=\mathbf{0}$. Which of the following are correct? (2007- 3marks)

   a) $\mathbf{a}\times\mathbf{b}=\mathbf{b}\times\mathbf{c}=\mathbf{c}\times\mathbf{a}=\mathbf{0}$
   b) $\mathbf{a}\times\mathbf{b}=\mathbf{b}\times\mathbf{c}=\mathbf{c}\times\mathbf{a}\neq\mathbf{0}$
   c) $\mathbf{a}\times\mathbf{b}=\mathbf{b}\times\mathbf{c}=\mathbf{a}\times\mathbf{c}\neq\mathbf{0}$

d) $\mathbf{a}\times\mathbf{b}, \mathbf{b}\times\mathbf{c}, \mathbf{c}\times\mathbf{a}$ are mutually perpendicular

8) The edges of a parallelopiped are of unit length and are parallel to non-coplanar unit vectors $\hat{a}, \hat{b}, \hat{c}$ such that $\hat{a}\cdot\hat{b} = \hat{b}\cdot\hat{c} = \hat{c}\cdot\hat{a} = \frac{1}{2}$. Then, the volume of the parallelopiped is (2008)

a) $\frac{1}{\sqrt{2}}$
b) $\frac{1}{2\sqrt{2}}$
c) $\frac{\sqrt{3}}{2}$
d) $\frac{1}{\sqrt{3}}$

9) Let two non-collinear unit vectors $\hat{a}$ and $\hat{b}$ form an acute angle. A point **P** moves so that at any time t the position vector $\overrightarrow{OP}$ (where O is the origin) is given by $\hat{a}\cos t + \hat{b}\sin t$. When **P** is farthest from origin **O**, let **M** be the length of $\overrightarrow{OP}$ and $\hat{u}$ be the unit vector along $\overrightarrow{OP}$. Then, (2008)

a) $\hat{u} = \frac{\hat{a}+\hat{b}}{|\hat{a}+\hat{b}|}$ and $M = (1 + \hat{a}\cdot\hat{b})^{1/2}$
b) $\hat{u} = \frac{\hat{a}-\hat{b}}{|\hat{a}-\hat{b}|}$ and $M = (1 + \hat{a}\cdot\hat{b})^{1/2}$
c) $\hat{u} = \frac{\hat{a}+\hat{b}}{|\hat{a}+\hat{b}|}$ and $M = (1 + 2\hat{a}\cdot\hat{b})^{1/2}$
d) $\hat{u} = \frac{\hat{a}-\hat{b}}{|\hat{a}-\hat{b}|}$ and $M = (1 + 2\hat{a}\cdot\hat{b})^{1/2}$

10) Let $\mathbf{P}(3, 2, 6)$ be a point in space and **Q** be a point on the line
$\mathbf{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \mu(-3\hat{i} + \hat{j} + 5\hat{k}$.
Then the value of μ for which the vector $\overrightarrow{PQ}$ is parallel to the plane $x - 4y + 3z = 1$ is (2009)

a) $\frac{1}{4}$
b) $-\frac{1}{4}$
c) $\frac{1}{8}$
d) $-\frac{1}{8}$

11) If $\mathbf{a}, \mathbf{b}, \mathbf{c},$ and $\mathbf{d}$ are unit vectors such that $(\mathbf{a}\times\mathbf{b})\cdot(\mathbf{c}\times\mathbf{d}) = 1$ and $\mathbf{a}\cdot\mathbf{c} = \frac{1}{2}$, then (2009)

a) $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are non-coplanar
b) $\mathbf{b}, \mathbf{c}, \mathbf{d}$ are non-coplanar
c) $\mathbf{b}, \mathbf{d}$ are non-parallel
d) $\mathbf{a}, \mathbf{d}$ are parallel and $\mathbf{b}, \mathbf{c}$ are parallel

12) A line with positive direction cosines passes through the point $\mathbf{P}(2, -1, 2)$ and makes equal angles with the coordinate axes. The line meets the plane $2x + y + z = 9$ at point **Q**. The length of the line segment PQ equals (2009)

a) 1
b) $\sqrt{2}$
c) $\sqrt{3}$
d) 2

13) Let $\mathbf{P}, \mathbf{Q}, \mathbf{R} and \mathbf{S}$ be the points on the plane with position vectors $-2\hat{i} - \hat{j}, 4\hat{i}, 3\hat{i} + 3\hat{j} and - 3\hat{i} + 2\hat{j}$ respectively. The quadrilateral $PQRS$ must be a (2010)

a) parallelogram, which is neither a rhombus nor a rectangle
b) square
c) rectangle, but not a square
d) rhombus, but not a square

14) Equation of the plane containing the straight line $\frac{x}{2} = \frac{y}{3} = \frac{z}{4}$ and perpendicular to the plane containing the straight lines $\frac{x}{3} = \frac{y}{4} = \frac{z}{2}$ and $\frac{x}{4} = \frac{y}{2} = \frac{z}{3}$ is (2010)

a) $x + 2y - 2z = 0$
b) $3x + 2y - 2z = 0$
c) $x - 2y + z = 0$
d) $5x + 2y - 4z = 0$

15) If the distance of the point $\mathbf{P}(1, -2, 1)$ from the plane $x + 2y - 2z = \alpha$, where $\alpha > 0$, is 5, then the foot of the perpendicular from **P** to the plane is (2010)

a) $\left(\frac{8}{3}, \frac{4}{3}, \frac{-7}{3}\right)$

b) $\left(\frac{4}{3}, \frac{-4}{3}, \frac{1}{3}\right)$
c) $\left(\frac{1}{3}, \frac{2}{3}, \frac{10}{3}\right)$
d) $\left(\frac{2}{3}, \frac{-1}{3}, \frac{5}{2}\right)$