MA-2011

EE24Btech11022 - Eshan Sharma

1) For the rings L $= \frac{\mathbb{R}[x]}{<x^2-x+1>}$; M $= \frac{\mathbb{R}[x]}{<x^2+x+1>}$; N $= \frac{\mathbb{R}[x]}{<x^2+2x+1>}$;
which of the following is **TRUE**? (ma-2011)
   a) L is isomorphic to M; L is not isomorphic to N; M is not isomorphic to N
   b) M is isomorphic to N; M is not isomorphic to L; N is not isomorphic to L
   c) L is isomorphic to M; M is isomorphic to N
   d) L is not isomorphic to M; L is not isomorphic to N; M is not isomorphic to N
2) The time to failure(in hours) of a component is a continuous random variable **T** with the probability density function

$$f(t) = \begin{cases} \frac{1}{10}e^{-\frac{t}{10}}, & t > 0, \\ 0, & t \leq 0. \end{cases}$$

Ten of these components are installed in a system and they work independently. Then ,the probability than **NONE** of these fail before ten hours, is (ma-2011)

   a) e^{-10}
   b) $1 - e^{-10}$
   c) $10e^{-10}$
   d) $1 - 10e^{-10}$

3) Let **X** be the normal linear space of all real sequences with finitely many non-zero terms, with supremum norm and $\mathbf{T} : X \to X$ be a one to one and onto linear operator defined by

$$\mathbf{T}(x_1, x_2, x_3 \ldots) = \left(x_1, \frac{x_2}{2}, \frac{x_3}{3}, \ldots\right)$$

Then, which of the following is **TRUE**? (ma-2011)

   a) **T** is bounded but $\mathbf{T}^{-1}$ is not bounded
   b) **T** is not bounded but $\mathbf{T}^{-1}$ is bounded
   c) both **T** and $\mathbf{T}^{-1}$ are bounded
   d) neither **T** nor $\mathbf{T}^{-1}$ are bounded

4) Let $e_i = (0, \ldots, 0, 1, 0, \ldots)$ (i.e., e_i is the vector with 1 at the i^{th} place and 0 elsewhere) for $i = 1, 2, \ldots$.
   Consider the statements:
   - **P**: $\{f(e_j)\}$ converges for every continuous linear functional on l^2.
   - **Q**: $\{e_j\}$ converges in l^2.

   Then, which of the following holds?
   a) Both **P** and **Q** are TRUE
   b) **P** is TRUE but **Q** is not TRUE
   c) **P** is not TRUE but **Q** is TRUE
   d) Neither **P** nor **Q** is TRUE
5) For which subspace $X \subset \mathbb{R}$ with the usual topology and with $\{0, 1\} \subseteq X$, will a continuous function $f : X \to \{0, 1\}$ satisfying $f(0) = 0$ and $f(1) = 1$ exist?
   a) $X = [0, 1]$
   b) $X = [-1, 1]$
   c) $X = \mathbb{R}$
   d) $[0, 1] \subset X$
6) Suppose X is a finite set with more than five elements. Which of the following is TRUE?
   a) There is a topology on X which is T_1

b) There is a topology on X which is T_3 but not T_1
c) There is a topology on X which is T_2 but not T_1
d) There is no topology on X which is T_1

7) A massless wire is bent in the form of a parabola $z = r^2$ and a bead slides on it smoothly. The wire is rotated about the z-axis with a constant angular acceleration α. Assume that m is the mass of the bead, ω is the initial angular velocity and g is the acceleration due to gravity. Then, the Lagrangian at any time t is
a) $\frac{m}{2}\left(\frac{dr}{dt}\right)^2(1+4r^2)r^2+(\omega+\alpha t)^2+2gr$
b) $\frac{m}{2}\left(\frac{dr}{dt}\right)^2(1+4r^2)r^2+(\omega+\alpha t)^2-2gr$
c) $\frac{m}{2}\left(\frac{dr}{dt}\right)^2(1+4r^2)r^2+(\omega+\alpha t)^2-2gr$
d) $\frac{m}{2}\left(\frac{dr}{dt}\right)^2(1+4r^2)r^2+(\omega+\alpha t)^2+2gr$

8) On the interval $[0,1]$, let y be a twice continuously differentiable function which is an extremal of the functional

$$J(y)=\int_0^1 \sqrt{1+2y'^2}\frac{y'}{x}\,dx$$

with $y(0)=1$, $y(1)=2$. Then, for some arbitrary constant c, y satisfies
a) $y'^2(2-cx^2)=cx^2$
b) $y'^2(2+cx^2)=cx^2$
c) $y'(1-cx^2)=cx^2$
d) $y'(1+cx^2)=cx^2$

Let X and Y be two continuous random variables with the joint probability density function

$$f(x,y)=\begin{cases}2, & 0<x+y<1,\ x>0,\ y>0,\\ 0, & \text{elsewhere.}\end{cases}$$

9) $P\left(X+Y<\frac{1}{2}\right)$ is

a) $\frac{1}{4}$ b) $\frac{1}{2}$ c) $\frac{3}{4}$ d) 1

10) $E\left(X \mid Y=\frac{1}{2}\right)$ is

a) $\frac{1}{4}$ b) $\frac{1}{2}$ c) 1 d) 2

Let $f(z)=\frac{z}{8-z^2}$, $z=x+iy$.

11) $\mathrm{Res}_{z=2}\, f(z)$ is

a) $\frac{1}{8}$ b) $\frac{1}{6}$ c) $-\frac{1}{6}$ d) $-\frac{1}{8}$

12) The Cauchy principal value of $\int_{-\infty}^{\infty} f(x)\,dx$ is

a) $-\frac{\pi\sqrt{3}}{6}$ b) $-\frac{\pi\sqrt{3}}{8}$ c) $\frac{\pi\sqrt{3}}{6}$ d) $\pi\sqrt{3}$

13) Let $f_j(x)=\frac{x}{(j-1)x+j}$ and $s_n(x)=\sum_{j=1}^{n} f_j(x)$ for $x\in[0,1]$. The sequence $\{s_n\}$
a) converges uniformly on $[0,1]$
b) converges pointwise on $[0,1]$ but not uniformly
c) converges pointwise for $x=0$ but not for $x\in(0,1]$
d) does not converge for $x\in[0,1]$