PH-2019

EE24Btech11022 - Eshan Sharma

1) Consider the motion of a particle along the x-axis in a potential $V(x) = F|x|$. Its ground state energy E_0 is estimated using the uncertainty principle. Then E_0 is proportional to

   a) $F^{\frac{1}{3}}$ b) $F^{\frac{1}{2}}$ c) $F^{\frac{2}{5}}$ d) $F^{\frac{2}{3}}$

2) A 3-bit analog-to-digital converter is designed to digitize analog signals ranging from 0 V to 10 V. For this converter, the binary output corresponding to an input of 6 V is

   a) 011 b) 101 c) 100 d) 010

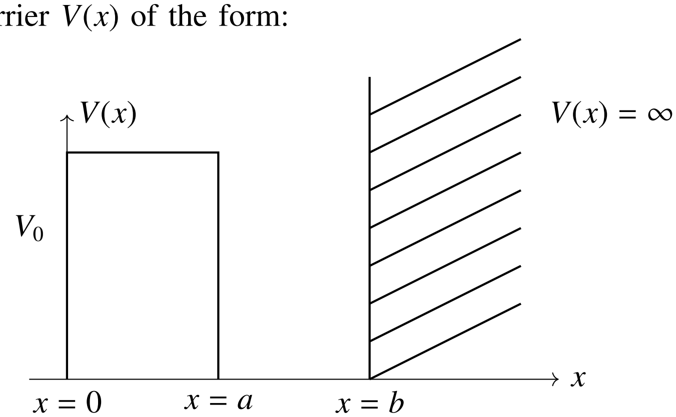
3) The Hamiltonian operator for a two-level quantum system is $H = \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix}$. If the state of the system at $t = 0$ is given by $|\psi(0)\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$, then $|\langle\psi(0)|\psi(t)\rangle|^2$ at a later time t is

   a) $\frac{1}{2}\left(1 + e^{-(E_1-E_2)t/\hbar}\right)$
   b) $\frac{1}{2}\left(1 - e^{-(E_1-E_2)t/\hbar}\right)$
   c) $\frac{1}{2}\left(1 + \cos[(E_1 - E_2)t/\hbar]\right)$
   d) $\frac{1}{2}\left(1 - \cos[(E_1 - E_2)t/\hbar]\right)$

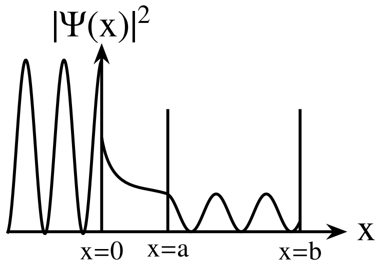
4) A particle of mass m moves in a lattice along the x-axis in a periodic potential $V(x) = V(x + d)$ with periodicity d. The corresponding Brillouin zone extends from $-k_0$ to k_0, with these two k-points being equivalent. If a weak force F in the x-direction is applied to the particle, it starts a periodic motion with time period T. Using the equation of motion $F = \frac{dp_{\text{crystal}}}{dt}$ for a particle moving in a band, where p_{crystal} is the crystal momentum of the particle, the period T is found to be (h is Planck constant)

   a) $\sqrt{\frac{2md}{F}}$ b) $2\sqrt{\frac{2md}{F}}$ c) $\frac{2h}{Fd}$ d) $\frac{h}{Fd}$

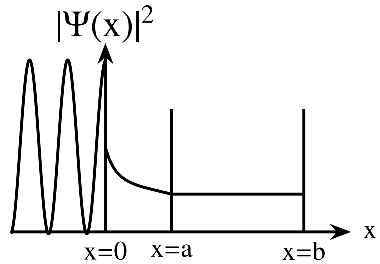
5) Consider a potential barrier $V(x)$ of the form:



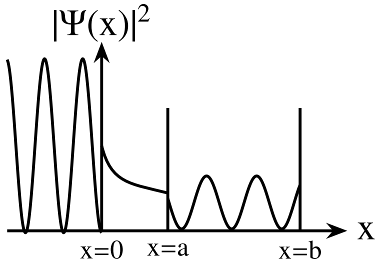
   where V_0 is a constant. For particles of energy $E < V_0$ incident on this barrier from the left, which of the following schematic diagrams best represents the probability density $|\psi(x)|^2$ as a function of x.



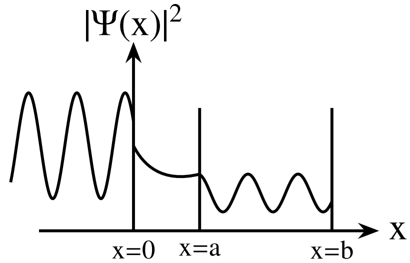
a)



c)



b)

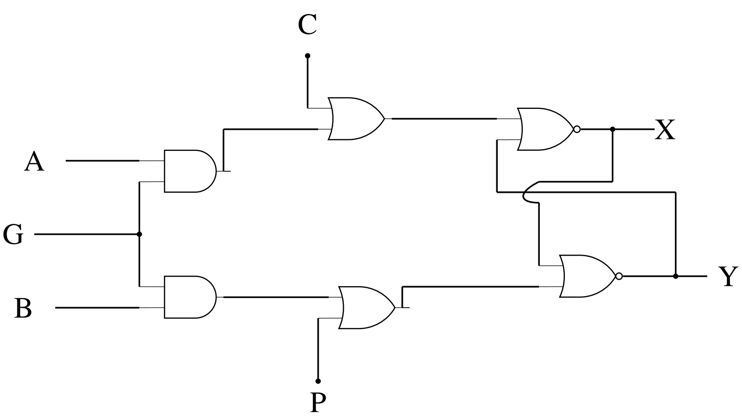


d)

6) The spin-orbit interaction term of an electron moving in a central field is written as $f(r)\mathbf{l}\cdot\mathbf{s}$, where r is the radial distance of the electron from the origin. If an electron moves inside a uniformly charged sphere, then

(A) $f(r) =$ constant
(B) $f(r) \propto r^{-1}$
(C) $f(r) \propto r^{-2}$
(D) $f(r) \propto r^{-3}$

7) For the following circuit, the correct logic values for the entries X_2 and Y_2 in the truth table are



G	A	B	P	C	X	Y
1	0	1	0	0	0	1
0	0	0	1	1	X_2	Y_2
1	0	0	0	1	0	1

a) 1 and 0
b) 0 and 0
c) 0 and 1
d) 1 and 1

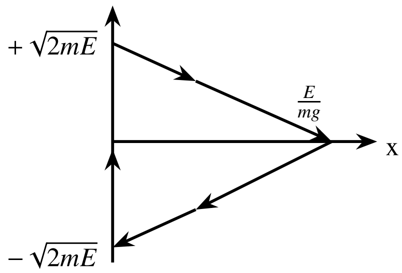
8) In a set of N successive polarizers, the m^{th} polarizer makes an angle $\left(\frac{m\pi}{2N}\right)$ with the vertical. A vertically polarized light beam of intensity I_0 is incident on two such sets with $N = N_1$ and $N = N_2$, where $N_2 > N_1$. Let the intensity of light beams coming out be $I(N_1)$ and $I(N_2)$, respectively. Which of the following statements is correct about the two outgoing beams?

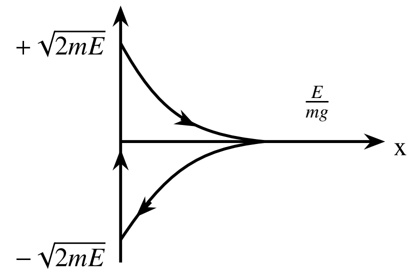
a) $I(N_2) > I(N_1)$; the polarization in each case is vertical
b) $I(N_2) < I(N_1)$; the polarization in each case is vertical
c) $I(N_2) > I(N_1)$; the polarization in each case is horizontal
d) $I(N_2) < I(N_1)$; the polarization in each case is horizontal

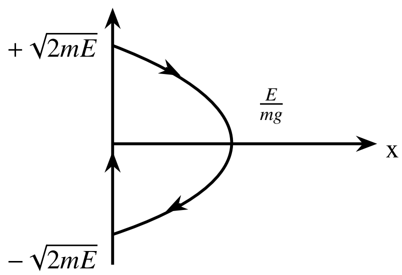
9) A ball bouncing off a rigid floor is described by the potential energy function

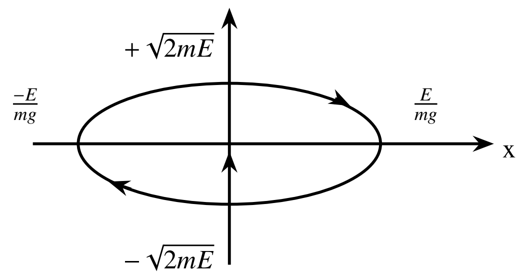
$$V(x) = mgx \text{ for } x > 0$$
$$= \infty \text{ for } x \leq 0$$

Which of the following schematic diagrams best represents the phase space plot of the ball?

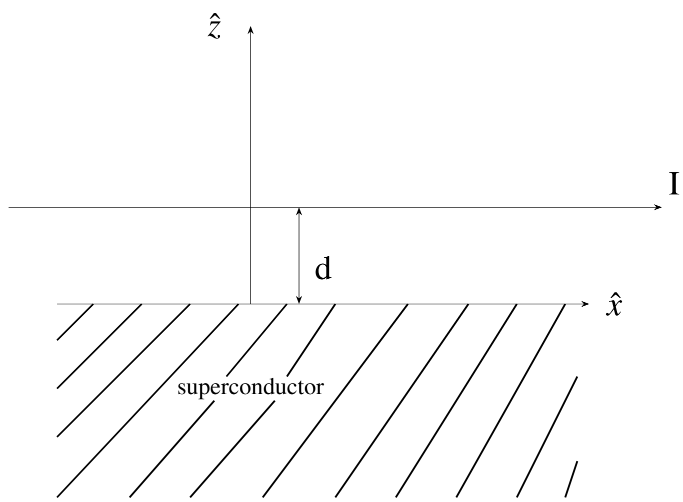
a) 

c) 

b) 

d) 

10) An infinitely long wire parallel to the $x-axis$ is kept at $z = d$ and carries a current I in the positive x direction above a superconductor filling the region $z \leq 0$ (see figure). The magnetic field $\overrightarrow{B}$ inside the superconductor is zero so that the field just outside the superconductor is parallel to its surface. The magnetic field due to this configuration at a point $(x, y, z > 0)$ is



a) $\left(\frac{\mu_0 I}{2\pi}\right) \frac{-(z-d)\hat{j}+y\hat{k}}{[y^2+(z-d)^2]}$

b) $\left(\frac{\mu_0 I}{2\pi}\right)\left(\frac{-(z-d)\hat{j}+y\hat{k}}{[y^2+(z-d)^2]} + \frac{(z+d)\hat{j}-y\hat{k}}{[y^2+(z+d)^2]}\right)$

c) $\left(\frac{\mu_0 I}{2\pi}\right)\left(\frac{-(z-d)\hat{j}+y\hat{k}}{[y^2+(z-d)^2]} - \frac{(z+d)\hat{j}-y\hat{k}}{[y^2+(z+d)^2]}\right)$

d) $\left(\frac{\mu_0 I}{2\pi}\right)\left(\frac{y\hat{j}+(z-d)\hat{k}}{[y^2+(z-d)^2]} + \frac{y\hat{j}-(z+d)\hat{k}}{[y^2+(z+d)^2]}\right)$

11) The vector potential inside a long solenoid, with n turns per unit length and carrying current I, written in cylindrical coordinates is $\overrightarrow{A}(s, \phi, z) = \frac{\mu_0 nI}{2} s\hat{\phi}$. If the term $\frac{\mu_0 nI}{2} s\left(\alpha \cos\phi\hat{\phi} + \beta \sin\phi\hat{s}\right)$, where $\alpha \neq 0$, $\beta \neq 0$, is added to $A(s, \phi, z)$, the magnetic field remains the same if

a) $\alpha = \beta$ b) $\alpha = -\beta$ c) $\alpha = 2\beta$ d) $\alpha = \frac{\beta}{2}$

$$\left(\begin{array}{l} \text{Useful formulae:} \quad \mathbf{v} = \frac{\partial t}{\partial s}\hat{s} + \frac{1}{s}\frac{\partial t}{\partial \phi}\hat{\phi} + \frac{\partial t}{\partial z}\hat{z}; \\ \nabla \times \mathbf{v} = \left(\frac{1}{s}\frac{\partial v_z}{\partial \phi} - \frac{\partial v_\phi}{\partial z}\right)\hat{s} + \left(\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s}\right)\hat{\phi} + \frac{1}{s}\left(\frac{\partial (s v_\phi)}{\partial s} - \frac{\partial v_s}{\partial \phi}\right)\hat{z} \end{array} \right)$$

12) Low energy collision (s-wave scattering) of pion (π^+) with deuteron (d) results in the production of two protons ($\pi^+ + d \to p + p$). The relative orbital angular momentum (in units of $\hbar$) of the resulting two-proton system for this reaction is

a) 0 b) 1 c) 2 d) 3

13) Consider the Hamiltonian $H(q, p) = \frac{\alpha p^2 q^4}{2} + \frac{\beta}{q^2}$, where α and β are parameters with appropriate dimensions, and q and p are the generalized coordinate and momentum, respectively. The corresponding Lagrangian $L(q, \dot{q})$ is

a) $\frac{1}{2\alpha}\frac{\dot{q}^2}{q^4} - \frac{\beta}{q^2}$ b) $\frac{2}{\alpha}\frac{\dot{q}^2}{q^4} + \frac{\beta}{q^2}$ c) $\frac{1}{\alpha}\frac{\dot{q}^2}{q^4} + \frac{\beta}{q^2}$ d) $-\frac{1}{2\alpha}\frac{\dot{q}^2}{q^4} + \frac{\beta}{q^2}$