Experiment 4

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1 Aim

To study and analyze the transient response of an LC circuit, determine the natural frequency (Ω_n), and calculate the damping ratio (ξ) using theoretical and experimental methods.

2 Materials and Apparatus Required

1) 100 μF Capacitor
2) Largest available inductor in the lab (denoted as L)
3) Resistor (small value for practical considerations)
4) DC Power Supply
5) Oscilloscope

3 Theory

An LC circuit consists of an inductor (L) and a capacitor (C) connected in parallel. When a charged capacitor is connected to an inductor, energy oscillates between the capacitor's electric field and the inductor's magnetic field. This oscillatory behavior is governed by the second-order differential equation:

$$L\frac{d^2q}{dt^2} + \frac{q}{C} = 0,$$

where $q(t)$ is the charge on the capacitor as a function of time.

The natural frequency of oscillation is given by:

$$\Omega_n = \frac{1}{\sqrt{LC}},$$

where: - L is the inductance in henries (H), - C is the capacitance in farads (F).

For an ideal LC circuit (no resistance), the damping ratio ($\xi = 0$) indicates purely oscillatory behavior. However, in practical circuits, resistance (R) introduces damping, and the damping ratio becomes:

$$\xi = \frac{R}{2}\sqrt{\frac{C}{L}}.$$

3.1 Theoretical Calculations for Given Values

Given:

- Capacitance, $C = 1\,\text{nF} = 1 \times 10^{-9}\,\text{F}$,
- Inductance, $L = 2.2\,\text{mH} = 2.2 \times 10^{-3}\,\text{H}$,
- Internal resistance of the inductor, $R = 25.2\,\Omega$.

3.1.1 Natural Frequency (Ω_n):

$$\Omega_n = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(2.2 \times 10^{-3})(1 \times 10^{-9})}}.$$

Simplifying:

$$\Omega_n = \frac{1}{\sqrt{2.2 \times 10^{-12}}} = \frac{1}{1.483 \times 10^{-6}} \approx 6.74 \times 10^5 \text{ rad/s}.$$

3.1.2 Damping Ratio (ξ):

$$\xi = \frac{R}{2}\sqrt{\frac{C}{L}} = \frac{25.2}{2}\sqrt{\frac{1 \times 10^{-9}}{2.2 \times 10^{-3}}}.$$

Simplifying:

$$\xi = 12.6\sqrt{\frac{10^{-9}}{2.2 \times 10^{-3}}} = 12.6\sqrt{4.545 \times 10^{-7}}.$$

Further simplification:

$$\xi = 12.6 \times 6.74 \times 10^{-4} \approx 0.0085.$$

Since $\xi \approx 0.0085$, which is much less than 1, the circuit is **underdamped**, and oscillations will decay over time.

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The transient response of the LC circuit can be classified based on the value of ξ:

1) **Underdamped** ($0 < \xi < 1$)**:** Oscillations decay over time.
2) **Critically damped** ($\xi = 1$)**:** Fastest return to equilibrium without oscillation.
3) **Overdamped** ($\xi > 1$)**:** Slow return to equilibrium without oscillation.

The voltage across the capacitor during oscillations can be expressed as:

$$V_C(t) = V_0 e^{-\xi\Omega_n t}\cos(\Omega_d t + \phi),$$

where:

- V_0 is the initial voltage,
- $\Omega_d = \Omega_n\sqrt{1 - \xi^2}$ is the damped natural frequency,
- ϕ is the phase angle.

For experimental analysis, we monitor voltage waveforms using an oscilloscope. The observed oscillation frequency can be compared with theoretical calculations to validate the model.

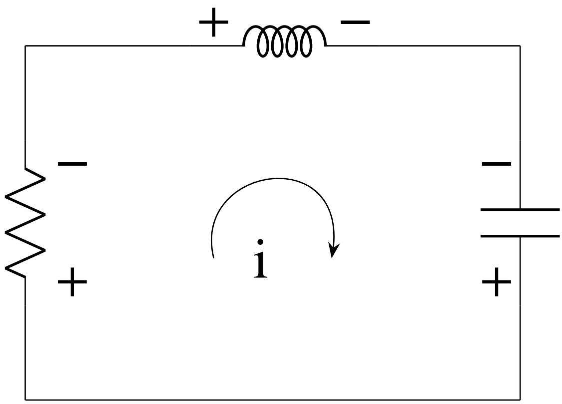
In summary:

- For a purely LC circuit: $R = 0, \xi = 0, \Omega_d = \Omega_n$.
- For an RLC circuit: $R > 0$, leading to damping and reduced oscillation frequency.

The energy exchange between components can be calculated as:

- Energy stored in capacitor: $E_C = \frac{1}{2}CV_C^2$,
- Energy stored in inductor: $E_L = \frac{1}{2}LI^2$, where $I = -C\frac{dV_C}{dt}$.

These equations form the basis for analyzing transient responses.



4 Procedure

1) Precharge the Capacitor:
   - Connect the 100 μF capacitor to a 5V DC power supply.
   - Once charged, disconnect it carefully without discharging.
2) Construct the LC Circuit:
   - Connect the charged capacitor in parallel with the largest available inductor.
   - Ensure minimal resistance in wiring.
3) Capture Transient Response:
   - Use an oscilloscope to monitor voltage across the capacitor.
   - Observe natural oscillations.
4) Calculate Theoretical Values:
   - Compute natural frequency ($\Omega_n = 1/\sqrt{LC}$).
   - Estimate damping ratio ($\xi = R/2\sqrt{\frac{C}{L}}$) if resistance is non-negligible.
5) Compare with Experimental Results:
   - Extract oscillation frequency from oscilloscope data.
   - Compare with theoretical calculations.

5 Observations

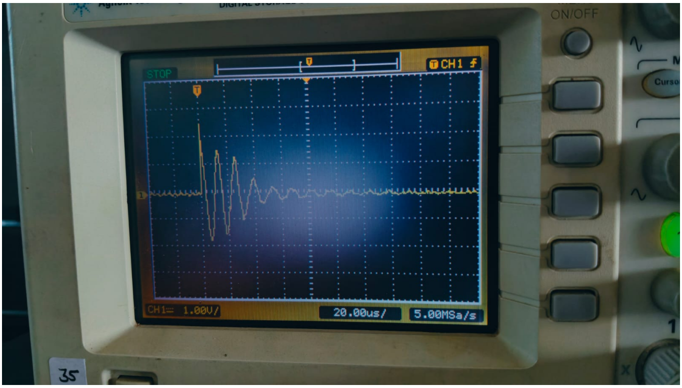


Fig. 1: Transient Response of LC