25/06/2022-Shift 1

EE24BTECH11021 - Eshan Ray

1) Let a circle C touch the lines $L_1: 4x-3y+K_1 = 0$ and $L_2: 4x-3y+K_2 = 0, K_1, K_2 \in R$. If a line passing through the centre of the circle C intersects L_1 at $(-1, 2)$ and L_2 at $(3, -6)$, then the equation of circle C is [Jun-2022]
   a) $(x-1)^2 + (y-2)^2 = 4$
   b) $(x+1)^2 + (y-2)^2 = 4$
   c) $(x-1)^2 + (y+2)^2 = 16$
   d) $(x-1)^2 + (y-2)^2 = 16$
2) The value of $\int_0^{\pi} \frac{e^{\cos x} \sin x}{(1+\cos^2 x)(e^{\cos x}+e^{-\cos x})}\, dx$ is equal to [Jun-2022]
   a) $\frac{\pi^2}{4}$
   b) $\frac{\pi^2}{2}$
   c) $\frac{\pi}{4}$
   d) $\frac{\pi}{2}$
3) Let $a, b\, and\, c$ be the length of sides of triangle ABC such that $\frac{a+b}{7} = \frac{b+c}{8} = \frac{c+a}{9}$. If r and R are the radius of incircle and radius of circumcircle of the triangle ABC, respectively then the value of $\frac{R}{r}$ is equal to [Jun-2022]
   a) $\frac{5}{2}$
   b) 2
   c) $\frac{3}{2}$
   d) 1
4) Let $f: N \to R$ be a function such that $f(x+y) = 2f(x)f(y)$ for natural numbers x and y. If $f(1) = 2$, then the value of α for which

$$\sum_{k=1}^{10} f(\alpha + k) = \frac{512}{3}\left(2^{20} - 1\right)$$

   holds, is [Jun-2022]
   a) 2
   b) 3
   c) 4
   d) 6
5) Let A be a 3×3 real matrix such that $A\begin{pmatrix}1\\1\\0\end{pmatrix} = \begin{pmatrix}1\\1\\0\end{pmatrix}; A\begin{pmatrix}1\\0\\1\end{pmatrix} = \begin{pmatrix}-1\\0\\1\end{pmatrix}$ and $A\begin{pmatrix}0\\0\\1\end{pmatrix} = \begin{pmatrix}1\\1\\2\end{pmatrix}$. If $X = (x_1, x_2, x_3)^\top$ and I is an identity matrix of order , then the system $(A-2I)X = \begin{pmatrix}4\\1\\1\end{pmatrix}$ has [Jun-2022]

a) no solution
b) infinitely many solutions
c) unique solution
d) exactly two solutions

6) Let $f: R \rightarrow R$ be defined as $f(x) = x^3 + x - 5$. If $g(x)$ is a function such that $f(g(x)) = x, \forall x \in R$, then $g\prime(63)$ is equal to [Jun-2022]
a) $\frac{1}{49}$
b) $\frac{3}{49}$
c) $\frac{43}{49}$
d) $\frac{91}{49}$

7) Consider the following two propositions :
$P1: \sim (p \rightarrow \sim q)$
$P2: (p \wedge \sim q) \wedge ((\sim p) \vee q)$
If the proposition $p \rightarrow ((\sim p) \vee q)$ is evaluated as FALSE, then : [Jun-2022]
a) $P1$ is TRUE and $P2$ is FALSE
b) $P1$ is FALSE and $P2$ is TRUE
c) Both $P1$ and $P2$ are FALSE
d) Both $P1$ and $P2$ are TRUE

8) If $\frac{1}{2\cdot 3^{10}} + \frac{1}{2^2\cdot 3^9} + \cdots + \frac{1}{2^{10}\cdot 3} = \frac{K}{2^{10}\cdot 3^{10}}$, then the remainder when K is divided by 6 is [Jun-2022]
a) 1
b) 2
c) 3
d) 5

9) Let $f(x)$ be a polynomial function such that $f(x) + f\prime(x) + f\prime\prime(x) = x^5 + 64$. Then, the value of $\lim_{x\to 1} \frac{f(x)}{x-1}$ [Jun-2022]
a) -15
b) -60
c) 60
d) 15

10) Let E_1 and E_2 be two events such that the conditional probabilities
$P(E_1 \mid E_2) = \frac{1}{2}, P(E_2 \mid E_1) = \frac{3}{4}$ and $P(E_1 \cap E_2) = \frac{1}{8}$ Then : [Jun-2022]
a) $P(E_1 \cap E_2) = P(E_1) \cdot P(E_2)$
b) $P(E\prime_1 \cap E\prime_2) = P(E\prime_1) \cdot P(E\prime_2)$
c) $P(E_1 \cap E\prime_2) = P(E_1) \cdot P(E_2)$
d) $P(E\prime_1 \cap E_2) = P(E_1) \cdot P(E_2)$

11) Let $A = \begin{pmatrix} 0 & -2 \\ 2 & 0 \end{pmatrix}$.If M and N are two matrices given by $M = \sum_{k=1}^{10} A^{2k}$ and $N = \sum_{k=1}^{10} A^{2k-1}$ then MN^2 is [Jun-2022]
a) a non-identity symmetric matrix
b) a skew-symmetric matrix
c) neither symmetric and skew-symmetric matrix
d) an identity matrix

12) Let $g: (0,\infty) \to R$ be a differentiable function such that

$$\int \left(\frac{x(\cos x - \sin x)}{e^x + 1} + \frac{g(x)(e^x + 1 - xe^x)}{(e^x + 1)^2} \right) dx = \frac{xg(x)}{e^x + 1} + c,$$

for all $x>0$, where c is an arbitrary constant. Then, [Jun-2022]

a) g is decreasing in $\left(0, \frac{\pi}{4}\right)$
b) $g\prime$ is increasing in $\left(0, \frac{\pi}{4}\right)$
c) $g + g\prime$ is increasing in $\left(o, \frac{\pi}{2}\right)$
d) $g - g\prime$ is increasing in $\left(0, \frac{\pi}{2}\right)$

13) Let $f: R \to R$ and $g: R \to R$ be two functions defined by $f(x) = \log_e\left(x^2 + 1\right) - e^{-x} + 1$ and $g(x) = \frac{1-2e^{2x}}{e^x}$.Then, for which of the following range of α, the inequality

$$f\left(g\left(\frac{(\alpha - 1)^2}{3}\right)\right) > f\left(g\left(\alpha - \frac{5}{3}\right)\right) \; holds?$$

[Jun-2022]

a) $(2, 3)$
b) $(-2, -1)$
c) $(1, 2)$
d) $(-1, 1)$

14) Let $\overrightarrow{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}\, a_i>0, i = 1, 2, 3$ be a vector which makes equal angles with the coordinate axes OX, OY and OZ. Also, let the projection of $\overrightarrow{a}$ on the vector $3\hat{i} + 4\hat{j}$ be 7. Let $\overrightarrow{b}$ be a vector obtained by rotating $\overrightarrow{a}$ with 90. If $\overrightarrow{a}, \overrightarrow{b}$ and $x - axis$ are coplanar, then the projection of a vector $\overrightarrow{b}$ on $3\hat{i} + 4\hat{j}$ is equal to [Jun-2022]

a) $\sqrt{7}$
b) $\sqrt{2}$
c) 2
d) 7

15) Let $y = y(x)$ be the solution of differential equation $(x + 1)y\prime - y = e^{3x}(x + 1)^2$, with $y(0) = \frac{1}{3}$. Then, the point $x = -\frac{4}{3}$ for the curve $y = y(x)$ is : [Jun-2022]

a) not a critical point
b) a point of local maxima
c) a point of local minima
d) a point of inflection