3.2.30

EE24BTECH11021 - Eshan Ray

Question:
Draw a parallelogram $ABCD$ in which $BC = 5cm$, $AB = 3cm$ and $\angle ABC = 60$, divide it into triangles ACB and ABD by the diagonal BD. Construct the triangle $BD'C'$ similar to $\triangle BDC$ with scale factor $\frac{4}{3}$. Draw the line segment $D'A'$ parallel to DA where A' lies on extended side BA. Is $A'BC'D'$ a parallelogram?
Solution:

Variable	**Description**	**value**
B	Vertex of parallelogram $ABCD$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$
C	Vertex of parallelogram $ABCD$	$\begin{pmatrix} 5 \\ 0 \end{pmatrix}$
BC	Side of parallelogram $ABCD$	$5cm$
A	Vertex of parallelogram $ABCD$	$\begin{pmatrix} 1.50 \\ 2.598 \end{pmatrix}$
AB	Side of parallelogram $ABCD$	$3cm$
D	Vertex of parallelogram $ABCD$	$\begin{pmatrix} 6.50 \\ 2.598 \end{pmatrix}$
$\angle ABC$	Angle subtended by sides AB and AC	60

TABLE 0: Input parameters

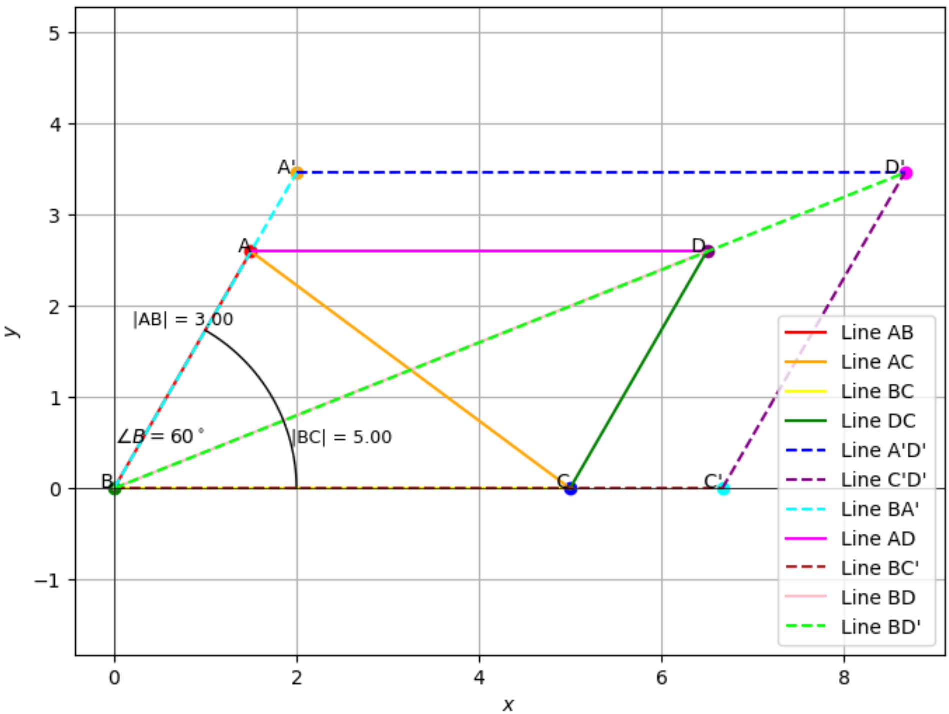

Fig. 0: $\|^{gm} ABCD$ and $\|^{gm} A'BC'D'$

$ABCD$ is a parallelogram,

$$\implies AB \parallel DC, AB = DC \tag{1}$$
$$\implies AD \parallel BC, AD = BC \tag{2}$$
$$\triangle BDC \sim \triangle BD'C' \ (Given) \tag{3}$$
$$scale factor = \frac{4}{3} \tag{4}$$
$$\implies \frac{BD'}{BD} = \frac{4}{3} \tag{5}$$
$$\implies \frac{BC'}{BC} = \frac{4}{3} \tag{6}$$
$$\implies \angle BCD = \angle BC'D' \tag{7}$$
$$\implies \angle BDC = \angle BD'C' \tag{8}$$
$$\therefore CD \parallel C'D' \tag{9}$$
$$BC' \parallel BC \tag{10}$$
$$\tag{11}$$
$$From \triangle BA'D' \triangle BAD, \tag{12}$$
$$\implies \angle ABD = \angle A'BD' \tag{13}$$
$$\implies \angle BDA = \angle BD'A' \tag{14}$$
$$\angle BAD = \angle BA'D' \tag{15}$$

$\therefore \triangle ABD \sim \triangle A'BD'$

$$\implies \frac{BD'}{BD} = \frac{BA'}{BA} = \frac{A'D'}{AD} = \frac{4}{3} \tag{16}$$

In quadrilateral $A'BC'D'$,

$$A'D' \parallel AD \parallel BC \tag{17}$$

$$\implies A'D' \parallel BC' \tag{18}$$

$$\implies BC' = \frac{4}{3}BC \tag{19}$$

$$\implies A'D' = \frac{4}{3}AD \tag{20}$$

$$\therefore BC' = A'D' \tag{21}$$

$$Similarly, BA' \parallel C'D', BA' = C'D' \tag{22}$$

So, quadrilateral $A'BC'D'$ is a parallelogram.