1.9.30

EE24BTECH11021 - Eshan Ray

Question:
If the distances of $\mathbf{P} = (x, y)$ from $\mathbf{A} = (5, 1)$ and $\mathbf{B} = (-1, 5)$ are equal, then prove that $3x = 2y$.

Solution:

Variable	Description
$\mathbf{A}(5, 1)$	coordinates of first point
$\mathbf{B}(-1, 5)$	coordinates of second point
$\mathbf{P}(x, y)$	Equidistant point of $\mathbf{A}$ and $\mathbf{B}$

TABLE 0: Input parameters

$$\|\mathbf{B}-\mathbf{P}\|^2 = \|\mathbf{A}-\mathbf{P}\|^2 \tag{1}$$

$$\implies (\mathbf{B}-\mathbf{P})(\mathbf{B}-\mathbf{P})^\top = (\mathbf{A}-\mathbf{P})(\mathbf{A}-\mathbf{P})^\top \tag{2}$$

$$\implies \mathbf{B}^2+\mathbf{P}^2-2\mathbf{P}\mathbf{B}^\top = \mathbf{A}^2+\mathbf{P}^2-2\mathbf{P}\mathbf{A}^\top \tag{3}$$

$$\implies \mathbf{P}\left(\mathbf{A}^\top-\mathbf{B}^\top\right) = \frac{\mathbf{A}^2-\mathbf{B}^2}{2} \tag{4}$$

$$\implies \mathbf{P}\left(\begin{pmatrix}5 & 1\end{pmatrix}-\begin{pmatrix}-1 & 5\end{pmatrix}\right) = \frac{26-26}{2} \tag{5}$$

$$\implies \begin{pmatrix}x\\y\end{pmatrix}\begin{pmatrix}6 & -4\end{pmatrix} = 0 \tag{6}$$

$$\implies 6x-4y = 0 \tag{7}$$

$$\implies 3x = 2y \tag{8}$$

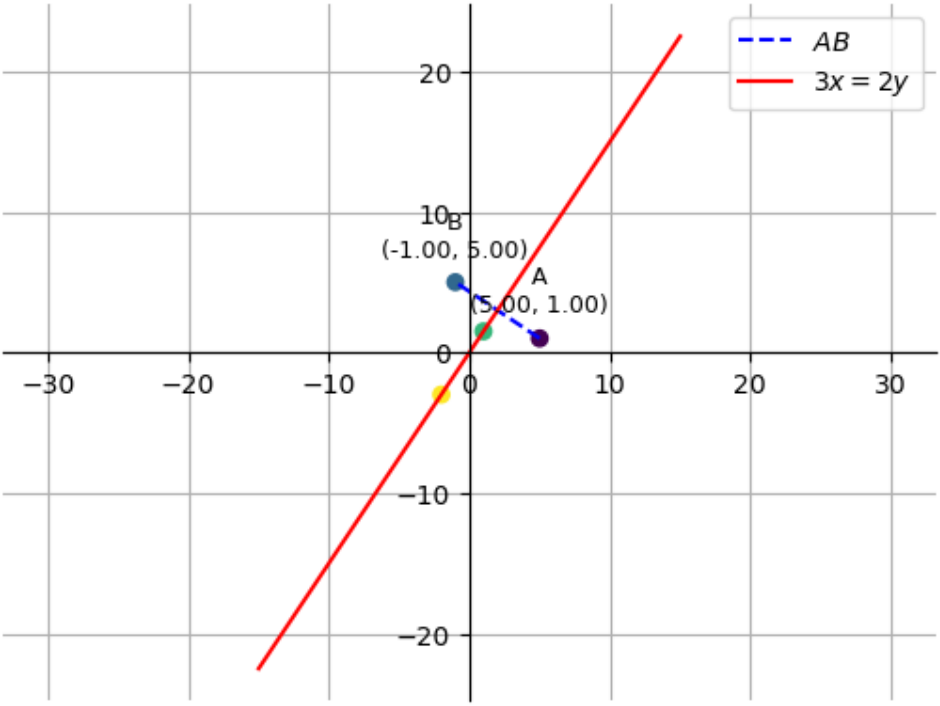

Fig. 0: Perpendicular bisector of Line AB