18/03/2021-Shift 1

EE24BTECH11021 - Eshan Ray

1) The differential equations satisfied by the system of parabolas $y^2 = 4a(x+a)$ is :
   a) $y\left(\frac{dy}{dx}\right) + 2x\left(\frac{dy}{dx}\right) - y = 0$
   b) $y\left(\frac{dy}{dx}\right)^2 + 2x\left(\frac{dy}{dx}\right) - y = 0$
   c) $y\left(\frac{dy}{dx}\right)^2 - 2x\left(\frac{dy}{dx}\right) - y = 0$
   d) $y\left(\frac{dy}{dx}\right)^2 - 2x\left(\frac{dy}{dx}\right) + y = 0$
2) The number of integral values of m so that the abscissa of point of intersection of lines $3x + 4y = 9$ and $y = mx + 1$ is also an integer, is:
   a) 3
   b) 2
   c) 1
   d) 0
3) Let $\left(1 + x + 2x^2\right)^0 = a_0 + a_1x + a_2x^2 + \cdots + a_{40}x^{40}$. Then $a_1 + a_3 + a_5 + \cdots + a_{37}$ is equal to :
   a) $2^{20}\left(2^{20} - 21\right)$
   b) $2^{19}\left(2^{20} - 21\right)$
   c) $2^{19}\left(2^{20} + 21\right)$
   d) $2^{20}\left(2^{20} + 21\right)$
4) The solutions of the equation

$$\begin{vmatrix} 1 + \sin^2 x & \sin^2 x & \sin^2 x \\ \cos^2 x & 1 + \cos^2 x & \cos^2 x \\ 4\sin 2x & 4\sin 2x & 1 + 4\sin 2x \end{vmatrix} = 0$$

   , $(0<x<\pi)$, are:
   a) $\frac{\pi}{6}, \frac{5\pi}{6}$
   b) $\frac{7\pi}{12}, \frac{11\pi}{12}$
   c) $\frac{5\pi}{12}, \frac{7\pi}{12}$
   d) $\frac{\pi}{12}, \frac{\pi}{6}$
5) Choose the correct statement about two circles whose equations are given below:
   $x^2 + y^2 - 10x - 10y + 41 = 0$
   $x^2 + y^2 - 22x - 10y + 137 = 0$
   a) circles have no meeting point
   b) circles have two meeting points
   c) circles have only one meeting point
   d) circles have the same centre

6) Let α, β, γ be the roots of the equation, $x^3 + ax^2 + bx + c = 0, (a, b, c \in R\, and\, a, b\, and\, a, b \neq 0)$. The system of equations $(in\, u, v, w)$ given by $\alpha u + \beta v + \gamma w = 0; \beta u + \gamma v + \alpha w = 0; \gamma u + \alpha v + \beta w = 0$ has non-trivial solutions,then the value of $\frac{a^2}{b}$ is

a) 5
b) 1
c) 0
d) 3

7) The integral $\int \frac{(2x-1)\cos\sqrt{(2x-1)^2+5}}{\sqrt{4x^2-4x+6}}\, dx$ is equal to $(where\, c\, is\, a\, constant\, of\, integration)$

a) $\frac{1}{2}\sin\sqrt{(2x+1)^2+5}+c$
b) $\frac{1}{2}\sin\sqrt{(2x-1)^2+5}+c$
c) $\frac{1}{2}\cos\sqrt{(2x+1)^2+5}+c$
d) $\frac{1}{2}\cos\sqrt{(2x-1)^2+5}+c$

8) The equation of one of the straight lines which passes through the point $(1, 3)$ and makes an angles $\tan^{-1}\left(\sqrt{2}\right)$ with the straight line,$y + 1 = 3\sqrt{2}x$ is

a) $4\sqrt{2}x + 5y - \left(15 + 4\sqrt{2}\right) = 0$
b) $5\sqrt{2}x + 4y - \left(15 + 4\sqrt{2}\right) = 0$
c) $4\sqrt{2}x + 5y - 4\sqrt{2} = 0$
d) $4\sqrt{2}x - 5y - \left(5 + 4\sqrt{2}\right) = 0$

9) If $\lim_{x\to 0} \frac{\sin^{-1}x - \tan^{-1}x}{3x^3}$ is equal to L, then the value of $(6L + 1)$ is

a) $\frac{1}{6}$
b) $\frac{1}{2}$
c) 6
d) 2

10) A vector **a** has components $3p$ and 1 with respect to a rectangular Cartesian system. This system is rotated through a certain angle about the origin in the counterclockwise sense. If with respect to the new system, **a** has components $p + 1$ and $\sqrt{10}$, then a value of p is equal to:

a) 1
b) -1
c) $\frac{4}{5}$
d) $-\frac{5}{4}$

11) If the equation $a|z|^2 + \overline{\bar{\alpha}z + \alpha\bar{z}} + d = 0$ represents a circle where a, d are real constants then which of the following conditions are correct?

a) $|\alpha|^2 - ad \neq 0$
b) $|\alpha|^2 - ad > 0$ and $a \in R - \{0\}$
c) $\alpha = 0, a, d \in R^+$
d) $|\alpha|^2 - ad \geq 0$ and $a \in R$

12) For the four circles $M.N, O\, and\, P$, following four equations are given :
Circle M: $x^2 + y^2 = 1$

Circle N: $x^2 + y^2 - 2x = 0$
Circle O: $x^2 + y^2 - 2x - 2y + 1 = 0$
Circle P: $x^2 + y^2 - 2y = 0$
If the centre of circle M is joined with the centre of circle N, further the centre of circle N is joined with centre of circle O, centre of circle O is joined with centre of circle P and lastly, centre of circle P is joined with centre of circle M, then these lines form the sides of a :

a) Rhombus
b) Square
c) Rectangle
d) Parallelogram

13) If α, β are natural numbers such that $100^{\alpha} - 199\beta = (100)(100) + (99)(101) + (98)(102) + \cdots + (1)(199)$, then the slope of the line passing through (α, β) and origin is:

a) 510
b) 550
c) 540
d) 530

14) The real-valued function $f(x) = \frac{cosec^{-1}x}{\sqrt{x-[x]}}$, where $[x]$ denotes the greatest integer less than or equal to x, is defined for all x belonging to :

a) all non- integers except the interval $[-1, 1]$
b) all integers except $0, -1, 1$
c) all reals except integers
d) all reals except the interval $[-1, 1]$

15) $\frac{1}{3^2-1} + \frac{1}{5^2-1} + \frac{1}{7^2-1} + \cdots + \frac{1}{201^2-1}$ is equal to :

a) $\frac{101}{404}$
b) $\frac{101}{408}$
c) $\frac{99}{400}$
d) $\frac{25}{101}$