

9.3.28

EE24BTECH11021 - Eshan Ray

Question:

Find the equation of tangent to the curve $y = \sqrt{3x-2}$ which is parallel to the line $4x - 2y + 5 = 0$ and also write the equation of the normal to the curve at the contact.

Solution:

Variable	Description	value
V	Quadratic Form Matrix	$\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$
u	Linear Coefficient Vector	$\begin{pmatrix} -3 \\ 2 \\ 0 \end{pmatrix}$
f	Constant Term	2
n	normal vector of the line $4x - 2y + 5 = 0$	$\begin{pmatrix} -2 \\ 1 \end{pmatrix}$
m	slope of the line $4x - 2y + 5 = 0$ parallel to the tangent	$\begin{pmatrix} 1 \\ 2 \end{pmatrix}$
q	constant for point of intersection of the curve and the tangent	-
p_1	The eigenvector corresponding to the zero eigenvalue	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

TABLE 0: Input parameters

Since, V is not invertible, the point of contact to is given by the matrix equation:-

$$\begin{pmatrix} (u + kn)^\top \\ V \end{pmatrix} q = \begin{pmatrix} -f \\ kn - u \end{pmatrix} \quad (1)$$

$$\text{where, } k = \frac{p_1^\top u}{p_1^\top n} \quad (2)$$

$$\Rightarrow k = \frac{3}{4} \quad (3)$$

$$\Rightarrow \begin{pmatrix} \left(\begin{pmatrix} -3 \\ 2 \\ 0 \end{pmatrix} + \frac{3}{4} \begin{pmatrix} -2 \\ 1 \end{pmatrix} \right)^\top \\ \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \end{pmatrix} q = \begin{pmatrix} -2 \\ \frac{3}{4} \begin{pmatrix} -2 \\ 1 \end{pmatrix} - \begin{pmatrix} -3 \\ 2 \\ 0 \end{pmatrix} \end{pmatrix} \quad (4)$$

$$\Rightarrow \begin{pmatrix} -3 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} q = \begin{pmatrix} -\frac{41}{16} \\ 0 \\ \frac{3}{4} \end{pmatrix} \quad (5)$$

The augmented matrix can be expressed as:-

$$\left(\begin{array}{cc|c} -3 & 0 & -\frac{41}{16} \\ 0 & 0 & 0 \\ 0 & 1 & \frac{3}{4} \end{array} \right) \quad (6)$$

Performing row operations: $R_2 \leftrightarrow R_3, R_1 \rightarrow \frac{-R_1}{3}$

$$\left(\begin{array}{cc|c} -3 & 0 & -\frac{41}{16} \\ 0 & 0 & 0 \\ 0 & 1 & \frac{3}{4} \end{array} \right) \rightarrow \left(\begin{array}{cc|c} 1 & 0 & \frac{41}{48} \\ 0 & 1 & \frac{3}{4} \\ 0 & 0 & 0 \end{array} \right) \quad (7)$$

$$\Rightarrow q = \left(\begin{array}{c} \frac{41}{48} \\ \frac{3}{4} \end{array} \right) \quad (8)$$

The equation of Tangent obtained is

$$n^T (x - q) = 0 \quad (9)$$

$$\Rightarrow (-2 \quad 1)x + \frac{23}{24} = 0 \quad (10)$$

Similarly, the equation of normal from the point of contact is

$$m^T (x - q) = 0 \quad (11)$$

$$(1 \quad 2)x = \frac{113}{48} \quad (12)$$

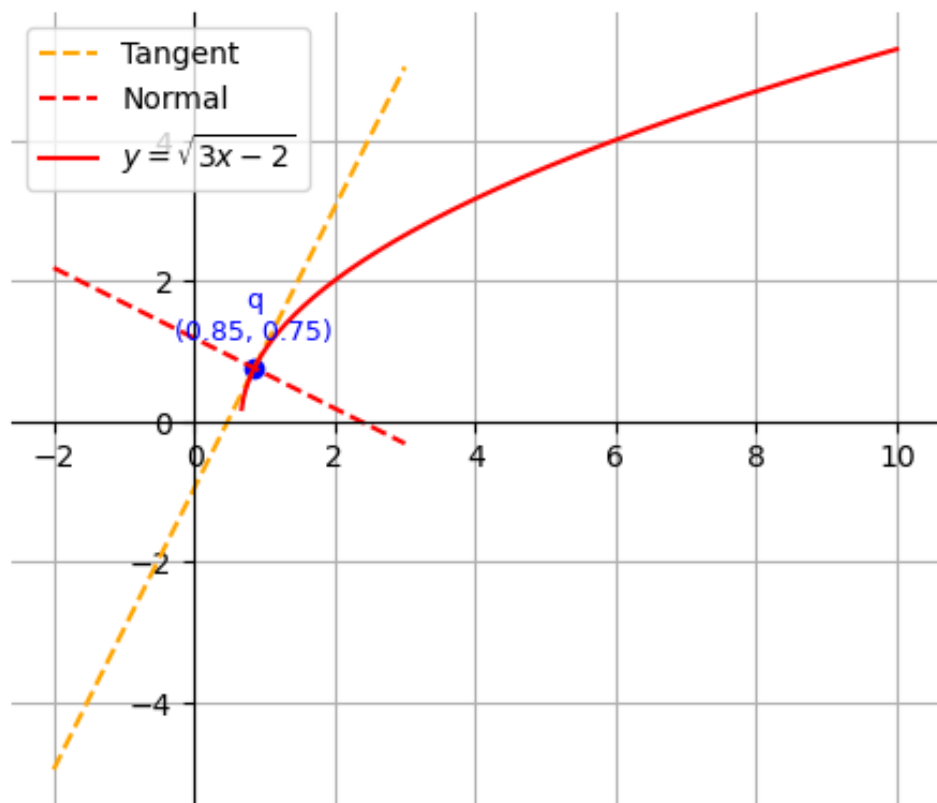


Fig. 0: Tangent and normal to parabola