26/02/2021-Shift 2

EE24BTECH11021 - Eshan Ray

1) Let L be a line obtained from the intersection of two planes $x+2y+z=6$ and $y+2z=4$. If point $P(\alpha,\beta,\gamma)$ is the foot of perpendicular from $(3,2,1)$ on L, then the value of $21(\alpha,\beta,\gamma)$ equals :
   a) 142
   b) 68
   c) 136
   d) 102
2) The sum of the series $\sum_{n=1}^{\infty}\frac{n^2+6n+10}{(2n+1)!}$ is equal to :
   a) $\frac{41}{8}e+\frac{19}{8}e^{-1}-10$
   b) $-\frac{41}{8}e+\frac{19}{8}e^{-1}-10$
   c) $\frac{41}{8}e-\frac{19}{8}e^{-1}-10$
   d) $\frac{41}{8}e+\frac{19}{8}e^{-1}+10$
3) Let $f(x)$ be a differentiable function at $x=a$ with $f\prime(a)=2$ and $f(a)=4$. Then $\lim_{x\to a}\frac{xf(a)-af(x)}{x-a}$ equals:
   a) $2a+4$
   b) $2a-4$
   c) $4-2a$
   d) $a+4$
4) Let $A(1,4)$ and $B(1,-5)$ be two points. Let P be a point on the circle $(x-1)^2+(y-1)^2=1$ such that $(PA)^2+(PB)^2$ have maximum value, then the points, P,A and B lie on :
   a) a parabola
   b) a straight line
   c) a hyperbola
   d) an ellipse
5) If the locus of the mid-point of the line segment from the point $(3,2)$ to a point on the circle, $x^2+y^2=1$ is a circle of the radius r, then r is equal to :
   a) $\frac{1}{4}$
   b) $\frac{1}{2}$
   c) 1
   d) $\frac{1}{3}$
6) Let the slope of the tangent line to a curve at any point $P(x,y)$ be given by $\frac{xy^2+y}{x}$. If the curve intersects the line $x+2y=4$ at $x=-2$, then the value of y, for which the point $(3,y)$ lies on the curve, is:
   a) $-\frac{18}{11}$
   b) $-\frac{18}{19}$

c) $-\frac{4}{3}$
d) $\frac{18}{35}$

7) Let A_1 be the area bounded by the curves $y = \sin x, y = \cos x$ and $y - axis$ in the first quadrant. Also, let A_2 be the area of the region bounded by the curves $y = \sin x, y = \cos x, x - axis$ and $x = \frac{\pi}{2}$ in the first quadrant.Then,

a) $A_1 = A_2$ and $A_1 + A_2 = \sqrt{2}$
b) $A_1 : A_2 = 1 : 2$ and $A_1 + A_2 = 1$
c) $2A_1 = A_2$ and $A_1 + A_2 = 1 + \sqrt{2}$
d) $A_1 : A_2 = 1 : \sqrt{2}$ and $A_1 + A_2 = 1$

8) If $0<a, b<1$, and $\tan^{-1} a + \tan^{-1} b = \frac{\pi}{4}$, then the value of $(a - b) - \left(\frac{a^2+b^2}{2}\right) + \left(\frac{a^3+b^3}{3}\right) - \left(\frac{a^4+b^4}{4}\right) + \dots$ is:

a) $\log_e 2$
b) $\log_e \frac{e}{2}$
c) e
d) $e^2 - 1$

9) Let $F_1(A, B, C) = (A \wedge \sim B) \vee [\sim C \wedge (A \vee B)] \vee \sim A$ and $F_2(A, B) = (A \vee B) \vee (B \rightarrow \sim A)$ be two logical expressions. Then:

a) F_1 is not a tautology but F_2 is a tautology
b) F_1 is a tautology but F_2 is not a tautology
c) F_1 and F_2 both are tautologies
d) Both F_1 and F_2 are not tautologies

10) Consider the following system of equations :
$x + 2y - 3z = a$
$2x + 6y - 11z = b$
$x - 2y + 7z = c,$
Where a, b and c are real constants. Then the system of equations :

a) has a unique solution when $5a = 2b + c$
b) has infinite number of solutions when $5a = 2b + c$
c) has no solution for all a, b and c
d) has a unique solution for all a, b and c

11) A seven digit number is formed using digit $3, 3, 4, 4, 4, 5, 5$. The probability, that number so formed is divisible by 2, is :

a) $\frac{6}{7}$
b) $\frac{4}{7}$
c) $\frac{3}{7}$
d) $\frac{1}{7}$

12) If vectors $\overrightarrow{a_1} = x\hat{i} - \hat{j} + \hat{k}$ and $\overrightarrow{a_2} = \hat{i} + y\hat{j} + z\hat{k}$ are collinear, then a possible unit vector parallel to the vector parallel to the vector $x\hat{i} + y\hat{j} + z\hat{k}$ is :

a) $\frac{1}{\sqrt{2}}\left(-\hat{j} + \hat{k}\right)$
b) $\frac{1}{\sqrt{2}}\left(\hat{i} - \hat{j}\right)$
c) $\frac{1}{\sqrt{3}}\left(\hat{i} - \hat{j} + \hat{k}\right)$

d) $\frac{1}{\sqrt{3}}\left(\hat{i}+\hat{j}-\hat{k}\right)$

13) For $x>0$, if $f(x)=\int_1^x \frac{\log_e t}{1+t}\,dt$, then $f(e)+f\left(\frac{1}{e}\right)$ is equal to :

a) $\frac{1}{2}$
b) -1
c) 1
d) 0

14) Let $f: R \to R$ be defined as $f(x)=\begin{cases} 2\sin\left(-\frac{\pi x}{2}\right) & \text{if } x<-1 \\ \left|ax^2+x+b\right|, & \text{if } -1 \le x \le 1 \\ \sin(\pi x) & \text{if } x>1 \end{cases}$

If $f(x)$ is continuous on R, then $a+b$ equals :

a) 3
b) -1
c) -3
d) 1

15) Let $A=\{1,2,3\ldots,10\}$ and $f: A \to A$ be defined as $f(k)=\begin{cases} k+1 & \text{if k is odd} \\ k & \text{if k is even} \end{cases}$

Then the number of possible functions $g: A \to A$ such that $gof=f$ is :

a) 10^5
b) $\binom{10}{5}$
c) 5^5
d) $5!$