

3.2.30

EE24BTECH11021 - Eshan Ray

Question:

Draw a parallelogram $ABCD$ in which $BC = 5\text{cm}$, $AB = 3\text{cm}$ and $\angle ABC = 60^\circ$, divide it into triangles ACB and ABD by the diagonal BD . Construct the triangle $BD'C'$ similar to $\triangle BDC$ with scale factor $\frac{4}{3}$. Draw the line segment $D'A'$ parallel to DA where A' lies on extended side BA . Is $A'BC'D'$ a parallelogram?

Solution:

Variable	value
B	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$
C	$\begin{pmatrix} 5 \\ 0 \end{pmatrix}$
BC	5cm
A	$\begin{pmatrix} 1.50 \\ 2.598 \end{pmatrix}$
AB	3cm
D	$\begin{pmatrix} 6.50 \\ 2.598 \end{pmatrix}$
$\angle ABC$	60

TABLE 0: Input parameters

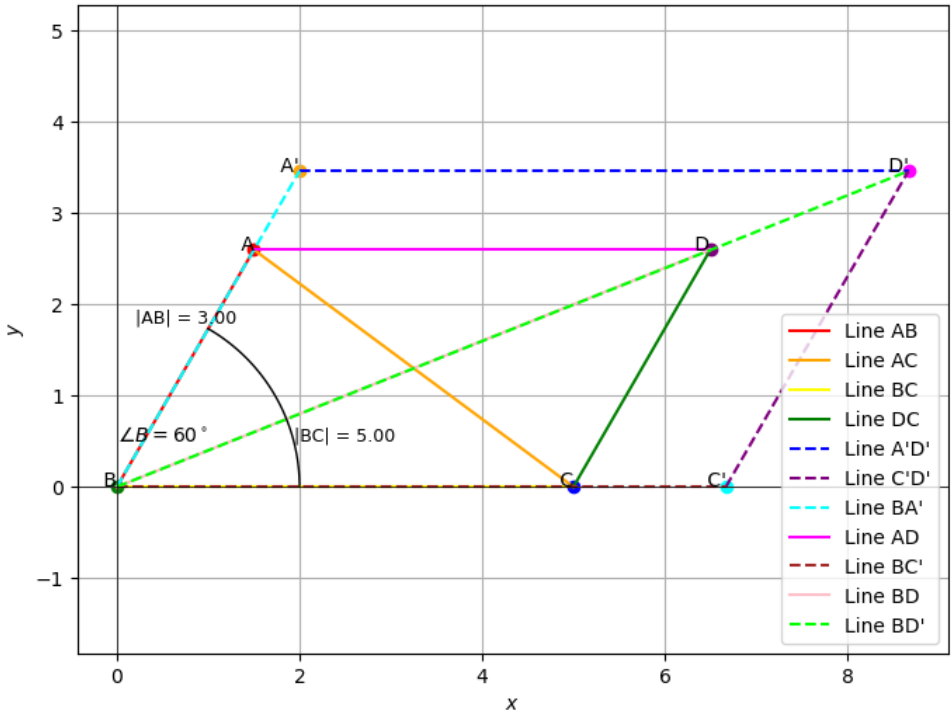


Fig. 0: $\|^{gm}ABCD$ and $\|^{gm}A'BC'D'$

$ABCD$ is a parallelogram,

$$\Rightarrow AB \parallel DC, AB = DC \quad (1)$$

$$\Rightarrow AD \parallel BC, AD = BC \quad (2)$$

$$\triangle BDC \sim \triangle BD'C' \text{ (Given)} \quad (3)$$

$$\text{scale factor} = \frac{4}{3} \quad (4)$$

$$\Rightarrow \frac{BD'}{BD} = \frac{4}{3} \quad (5)$$

$$\Rightarrow \frac{BC'}{BC} = \frac{4}{3} \quad (6)$$

$$\Rightarrow \angle BCD = \angle BC'D' \quad (7)$$

$$\Rightarrow \angle BDC = \angle BD'C' \quad (8)$$

$$\therefore CD \parallel C'D' \quad (9)$$

$$BC' \parallel BC \quad (10)$$

$$\text{From } \triangle BA'D' \triangle BAD, \quad (11)$$

$$\Rightarrow \angle ABD = \angle A'BD' \quad (12)$$

$$\Rightarrow \angle BDA = \angle BD'A' \quad (13)$$

$$\angle BAD = \angle BA'D' \quad (14)$$

$$\therefore \triangle ABD \sim \triangle A'BD'$$

$$\Rightarrow \frac{BD'}{BD} = \frac{BA'}{BA} = \frac{A'D'}{AD} = \frac{4}{3} \quad (16)$$

In quadrilateral $A'BC'D'$,

$$A'D' \parallel AD \parallel BC \quad (17)$$

$$\Rightarrow A'D' \parallel BC' \quad (18)$$

$$\Rightarrow BC' = \frac{4}{3}BC \quad (19)$$

$$\Rightarrow A'D' = \frac{4}{3}AD \quad (20)$$

$$\therefore BC' = A'D' \quad (21)$$

$$\text{Similarly, } BA' \parallel C'D', BA' = C'D' \quad (22)$$

So, quadrilateral $A'BC'D'$ is a parallelogram.