7.2.29

EE24BTECH11021 - Eshan Ray

Question:
A circle drawn with origin as the centre passes through $\left(\frac{13}{2}, 0\right)$. The point which does not lie in the interior of the circle is

1) $\left(\frac{-3}{4}, 1\right)$
2) $\left(2, \frac{7}{3}\right)$
3) $\left(5, \frac{-1}{2}\right)$
4) $\left(-6, \frac{-5}{2}\right)$

Solution:

Variable	**value**
A	$\begin{pmatrix} \frac{13}{2} \\ 0 \end{pmatrix}$
O	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$
B	$\begin{pmatrix} \frac{-3}{4} \\ 1 \end{pmatrix}$
C	$\begin{pmatrix} 2 \\ \frac{7}{3} \end{pmatrix}$
D	$\begin{pmatrix} 5 \\ \frac{13}{2} \end{pmatrix}$
E	$\begin{pmatrix} -6 \\ \frac{-5}{2} \end{pmatrix}$

TABLE 4: Input parameters

As A lies on circle,

$$\|A\|^2 + 2u^\top A + f = 0 \tag{1}$$

$$\implies f = -\left(\left(\frac{13}{2}\right)^2\right) - 2\begin{pmatrix} 0 & 0 \end{pmatrix}\begin{pmatrix} \frac{13}{2} \\ 0 \end{pmatrix} \tag{2}$$

$$\implies f = -\frac{169}{4} \tag{3}$$

$\therefore$ The equation of the circle is $\|x\|^2 = \frac{169}{4}$.

A point is inside the circle if $\|P - O\|^2 < r^2$
$r^2 = \frac{169}{4} = 42.25$

For $B\left(\frac{-3}{4}, 1\right)$

$$
\begin{aligned}
||B-O||^2 &= \left\|\begin{pmatrix}\frac{-3}{4}\\ 1\end{pmatrix} - \begin{pmatrix}0\\0\end{pmatrix}\right\|^2 && (4)\\
\implies ||B-O||^2 &= \left\|\begin{pmatrix}\frac{-3}{4}\\ 1\end{pmatrix}\right\|^2 && (5)\\
\implies ||B-O||^2 &= \left(\frac{-3}{4}\right)^2 + 1^2 && (6)\\
\implies ||B-O||^2 &= \frac{25}{16} < \frac{169}{4} && (7)\\
\implies ||B-O||^2 &< r^2 && (8)
\end{aligned}
$$

So, $B\left(\frac{-3}{4}, 1\right)$ lies inside the circle.

For $C\left(2, \frac{7}{3}\right)$

$$
\begin{aligned}
||C-O||^2 &= \left\|\begin{pmatrix}2\\ \frac{7}{3}\end{pmatrix} - \begin{pmatrix}0\\0\end{pmatrix}\right\|^2 && (9)\\
\implies ||C-O||^2 &= \left\|\begin{pmatrix}2\\ \frac{7}{3}\end{pmatrix}\right\|^2 && (10)\\
\implies ||C-O||^2 &= 2^2 + \left(\frac{7}{3}\right)^2 && (11)\\
\implies ||C-O||^2 &= \frac{85}{9} < \frac{169}{4} && (12)\\
\implies ||C-O||^2 &< r^2 && (13)
\end{aligned}
$$

So, $C\left(2, \frac{7}{3}\right)$ lies inside the circle.

For $D\left(5, \frac{-1}{2}\right)$

$$
\begin{aligned}
||D-O||^2 &= \left\|\begin{pmatrix}5\\ \frac{-1}{2}\end{pmatrix} - \begin{pmatrix}0\\0\end{pmatrix}\right\|^2 && (14)\\
\implies ||D-O||^2 &= \left\|\begin{pmatrix}5\\ \frac{-1}{2}\end{pmatrix}\right\|^2 && (15)\\
\implies ||D-O||^2 &= 5^2 + \left(\frac{-1}{2}\right)^2 && (16)\\
\implies ||D-O||^2 &= \frac{101}{4} < \frac{169}{4} && (17)\\
\implies ||D-O||^2 &< r^2 && (18)
\end{aligned}
$$

So, $D\left(5, \frac{-1}{2}\right)$ lies inside the circle.

For $E\left(-6, \frac{-5}{2}\right)$

$$\|E - O\|^2 = \left\| \begin{pmatrix} -6 \\ \frac{-5}{2} \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right\|^2 \tag{19}$$

$$\implies \|E - O\|^2 = \left\| \begin{pmatrix} -6 \\ \frac{-5}{2} \end{pmatrix} \right\|^2 \tag{20}$$

$$\implies \|E - O\|^2 = (-6)^2 + \left(\frac{-5}{2} \right)^2 \tag{21}$$

$$\implies \|E - O\|^2 = \frac{169}{4} = \frac{169}{4} \tag{22}$$

$$\implies \|E - O\|^2 = r^2 \tag{23}$$

So, only point $E\left(-6, \frac{-5}{2}\right)$ lies on the circle.

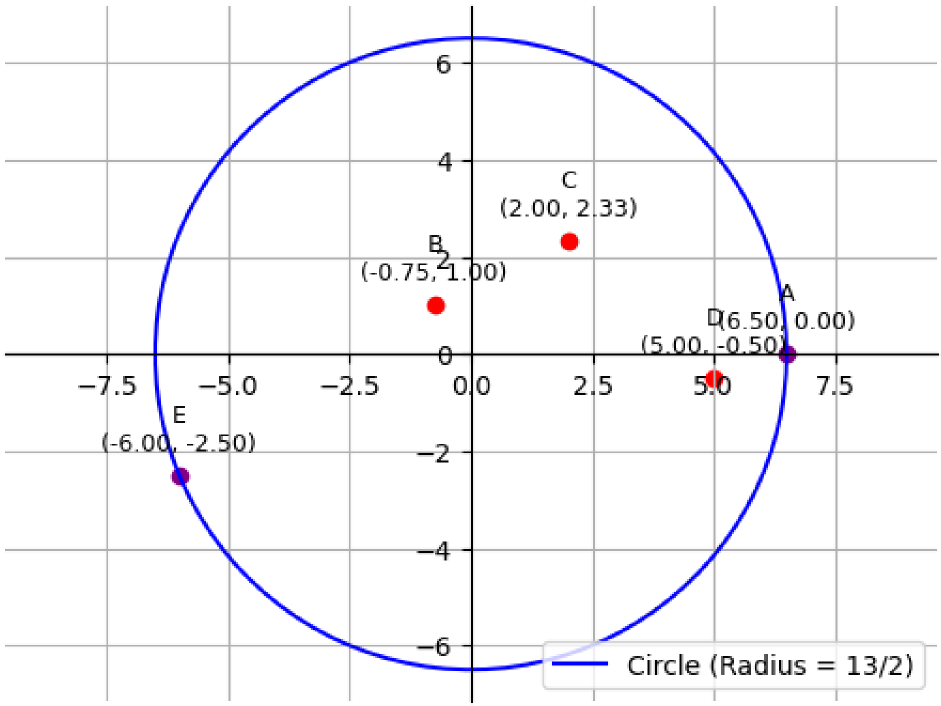

Fig. 4: Point E lies on the circle