9.2.18

EE24BTECH11021 - Eshan Ray

Question:
Find the area of the region bounded by the curve $x^2 = y$ and the lines $y = x + 2$ and the $x - axis$.
Solution:

Variable	value
V	$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$
u	$\begin{pmatrix} 0 \\ \frac{-1}{2} \end{pmatrix}$
f	0
h	$\begin{pmatrix} 0 \\ 2 \end{pmatrix}$
m	$\begin{pmatrix} 1 \\ 1 \end{pmatrix}$
k_i	$-$

TABLE 0: Input parameters

Substituting the input parameters in:-

$$k_i = \frac{1}{m^\top Vm}\left(-m^\top (Vh + u) \pm \sqrt{\left[m^\top (Vh + u)\right]^2 - g(h)\left(m^\top Vm\right)}\right)$$

$$k_i = \frac{1}{\begin{pmatrix} 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix}}\left(-\begin{pmatrix} 1 & 1 \end{pmatrix}\left(\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 2 \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{-1}{2} \end{pmatrix}\right) \pm \right.$$
$$\left.\sqrt{\left[\begin{pmatrix} 1 & 1 \end{pmatrix}\left(\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 2 \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{-1}{2} \end{pmatrix}\right)\right]^2 - g(h)\left(\begin{pmatrix} 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix}\right)}\right) \quad (1)$$

We get,

$$k_i = -1, 2$$

Substituting k_i in $x_i = h + k_i m$ for point of intersection, we get,

$$
\begin{aligned}
x_1 &= \begin{pmatrix}0\\2\end{pmatrix} + (-1)\begin{pmatrix}1\\1\end{pmatrix} && (2)\\
\implies x_1 &= \begin{pmatrix}0\\2\end{pmatrix} + \begin{pmatrix}-1\\-1\end{pmatrix} && (3)\\
\implies x_1 &= \begin{pmatrix}-1\\1\end{pmatrix} && (4)\\
x_2 &= \begin{pmatrix}0\\2\end{pmatrix} + (2)\begin{pmatrix}1\\1\end{pmatrix} && (5)\\
\implies x_2 &= \begin{pmatrix}0\\2\end{pmatrix} + \begin{pmatrix}2\\2\end{pmatrix} && (6)\\
\implies x_2 &= \begin{pmatrix}2\\4\end{pmatrix} && (7)
\end{aligned}
$$

The area bounded by the curve $y = x^2$ and line $y = x + 2$ is given by

$$
\begin{aligned}
\int_{-1}^{2} x + 2 - x^2 \, dx &= \left(\frac{x^2}{2} + 2x - \frac{x^3}{3}\right)_{-1}^{2} && (8)\\
&= \left(\frac{4}{2} + 4 - \frac{8}{3}\right) - \left(\frac{1}{2} - 2 + \frac{1}{3}\right) && (9)\\
&= \left(\frac{10}{3}\right) - \left(\frac{-7}{6}\right) && (10)\\
&= \frac{9}{2} && (11)
\end{aligned}
$$

So, the required area is 4.50 units.

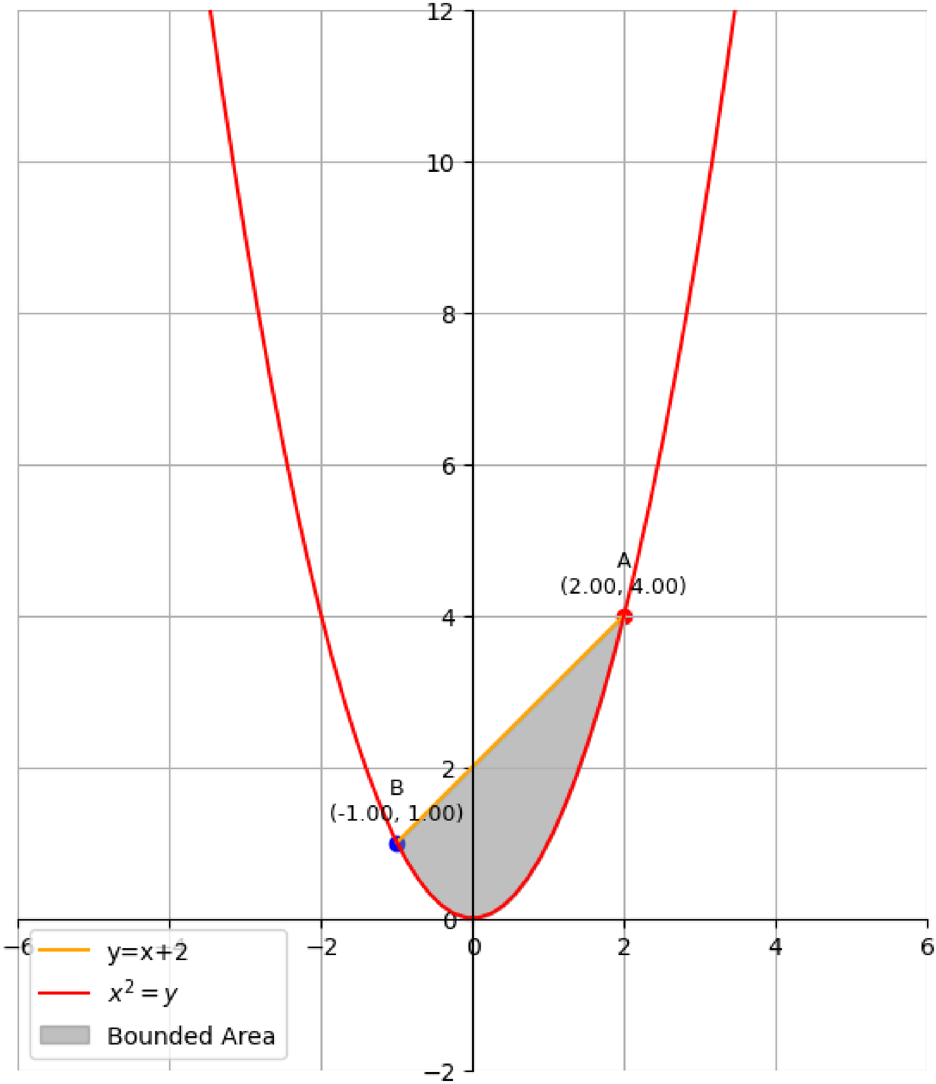

Fig. 0: Intersection of line and parabola