

1/02/2024-Shift 1

EE24BTECH11021 - Eshan Ray

- 16) Let $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a > b$ be an ellipse, whose eccentricity is $\frac{1}{\sqrt{2}}$ and the length of latus rectum is $\sqrt{14}$. Then the square of the eccentricity of $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is :
- 3
 - $\frac{7}{3}$
 - $\frac{3}{5}$
 - $\frac{5}{2}$
- 17) Let $3, a, b, c$ be in $A.P.$ and $3, a-1, b+1, c+9$ be in $G.P.$ Then, the arithmetic mean of a, b and c is :
- 4
 - 1
 - 13
 - 11
- 18) Let $C: x^2 + y^2 = 4$ and $C': x^2 + y^2 - 4\lambda x + 9 = 0$ be two circles. If the set of all values of λ so that the circles C and C' intersect at two distinct points, is $R - [a, b]$, then the point $(8a + 12, 16b - 20)$ lies on the curve :
- $x^2 + 2y^2 - 5x + 6y = 3$
 - $5x^2 - y = -11$
 - $x^2 - 4y^2 = 7$
 - $6x^2 + y^2 = 42$
- 19) Let $5f(x) + 4f\left(\frac{1}{x}\right) = x^2 - 2, \forall x \neq 0$ and $y = 9x^2 f(x)$, then y is strictly increasing in :
- $\left(0, \frac{1}{\sqrt{5}}\right) \cup \left(\frac{1}{\sqrt{5}}, \infty\right)$
 - $\left(-\frac{1}{\sqrt{5}}, 0\right) \cup \left(\frac{1}{\sqrt{5}}, \infty\right)$
 - $\left(-\frac{1}{\sqrt{5}}, 0\right) \cup \left(0, \frac{1}{\sqrt{5}}\right)$
 - $\left(-\infty, -\frac{1}{\sqrt{5}}\right) \cup \left(0, \frac{1}{\sqrt{5}}\right)$
- 20) If the shortest distance between the lines $\frac{x-\lambda}{-2} = \frac{y-2}{1} = \frac{z-1}{1}$ and $\frac{x-\sqrt{3}}{1} = \frac{y-1}{-2} = \frac{z-2}{1}$ is 1, then the sum of all possible values of λ is :
- 0
 - $2\sqrt{3}$
 - $3\sqrt{3}$
 - $-2\sqrt{3}$
- 21) If $x = x(t)$ is the solution of the differential equation $(t+1)dx = (2x + (t+1)^4)dt, x(0) = 2$, then, $x(1)$ equals ...

- 22) The number of elements in the set $S = \{(x, y, z) : x, y, z \in \mathbb{Z}, x + 2y + 3z = 42, x, y, z \geq 0\}$ equals ...
- 23) If the coefficient of x^{30} in the expansion of $\left(1 + \frac{1}{x}\right)^6 (1 + x^2)^7 (1 - x^3)^8$; $x \neq 0$ is α then $|\alpha|$ equals ...
- 24) Let 3, 11, 15, ..., 403 and 2, 5, 8, 11, ..., 404 be two arithmetic progressions. Then the sum, of the common terms in them, is equal to ...
- 25) Let $\{x\}$ denote the fractional part of x and $f(x) = \frac{\cos^{-1}(1 - \{x\}^2) \sin^{-1}(1 - \{x\})}{\{x\} - \{x\}^3}$, $x \neq 0$. If L and R respectively denote the left and right hand limit of $f(x)$ at $x = 0$, then $\frac{32}{\pi^2} (L^2 + R^2)$ is equal to ...
- 26) Let the line $L: \sqrt{2}x + y = \alpha$ passes through the point of the intersection P (in the first quadrant) of the circle $x^2 + y^2 = 3$ and the parabola $x^2 = 2y$. Let the line L touch two circles C_1 and C_2 of equal radius $2\sqrt{3}$. If the centres Q_1 and Q_2 of the circles C_1 and C_2 lie on the y -axis, then the square of the area of triangle PQ_1Q_2 is equal to ...
- 27) Let $P = \{z \in \mathbb{C} : |z + 2 - 3i| \leq 1\}$ and $Q = \{z \in \mathbb{C} : |z(1 + i) + \bar{z}(1 - i)| \leq -8\}$. Let in $P \cap Q$, $|z - 3 + 2i|$ be maximum and minimum at z_1 and z_2 respectively. If $|z_1|^2 + 2|z_2|^2 = \alpha + \beta\sqrt{2}$, where α, β are integers, then $\alpha + \beta$ equals ...
- 28) If $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{8\sqrt{2}\cos x}{(1 + e^{\sin x})(1 + \sin^4 x)} dx = \alpha\pi + \beta \log_e(3 + 2\sqrt{2})$, where α, β are integers, then $\alpha^2 + \beta^2$ equals ...
- 29) Let the line of shortest distance between the lines $L_1: \vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$ and $L_2: \vec{r} = (4\hat{i} + 5\hat{j} + 6\hat{k}) + \mu(\hat{i} + \hat{j} - \hat{k})$ intersect L_1 and L_2 at P and Q respectively. If (α, β, γ) is the midpoint of the line segment PQ , then $2(\alpha + \beta + \gamma)$ is equal to ...
- 30) Let $A = \{1, 2, 3, \dots, 20\}$. Let R_1 and R_2 be two relation on A such that $R_1 = \{(a, b) : b \text{ is divisible by } a\}$ $R_2 = \{(a, b) : a \text{ is an integral multiple of } b\}$. Then, the number of elements in $R_1 - R_2$ is equal to ...