9.2.18

EE24BTECH11021 - Eshan Ray

Question:
Find the area of the region bounded by the curve $x^2 = y$ and the lines $y = x + 2$ and the $x - axis$.
Solution:

Variable	**value**
V	$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$
u	$\begin{pmatrix} 0 \\ \frac{-1}{2} \end{pmatrix}$
f	0
h	$\begin{pmatrix} 0 \\ 2 \end{pmatrix}$
m	$\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

TABLE 0: Input parameters

Substituting the input parameters in:-

$$k_i = \frac{1}{m^\top Vm} \left(-m^\top (Vh + u) \pm \sqrt{[m^\top (Vh + u)]^2 - g(h)(m^\top Vm)} \right)$$

$$k_i = \frac{1}{\begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix}} \left(- \begin{pmatrix} 1 & 1 \end{pmatrix} \left(\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 2 \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{-1}{2} \end{pmatrix} \right) \pm \right.$$
$$\left. \sqrt{\left[\begin{pmatrix} 1 & 1 \end{pmatrix} \left(\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 2 \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{-1}{2} \end{pmatrix} \right) \right]^2 - g(h) \left(\begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right)} \right) \tag{1}$$

We get,

$$k_i = -1, 2$$

Substituting k_i in $x_i = h + k_i m$ for point of intersection, we get,

$$\implies x_1 = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \tag{2}$$

$$\implies x_2 = \begin{pmatrix} 2 \\ 4 \end{pmatrix} \tag{3}$$

The area bounded by the curve $y = x^2$ and line $y = x + 2$ is given by

$$\int_{-1}^{2} x + 2 - x^2 \, dx = \left(\frac{x^2}{2} + 2x - \frac{x^3}{3} \right)_{-1}^{2} \tag{4}$$

$$= \left(\frac{4}{2} + 4 - \frac{8}{3} \right) - \left(\frac{1}{2} - 2 + \frac{1}{3} \right) \tag{5}$$

$$= \left(\frac{10}{3} \right) - \left(\frac{-7}{6} \right) \tag{6}$$

$$= \frac{9}{2} \tag{7}$$

So, the required area is 4.50 units.

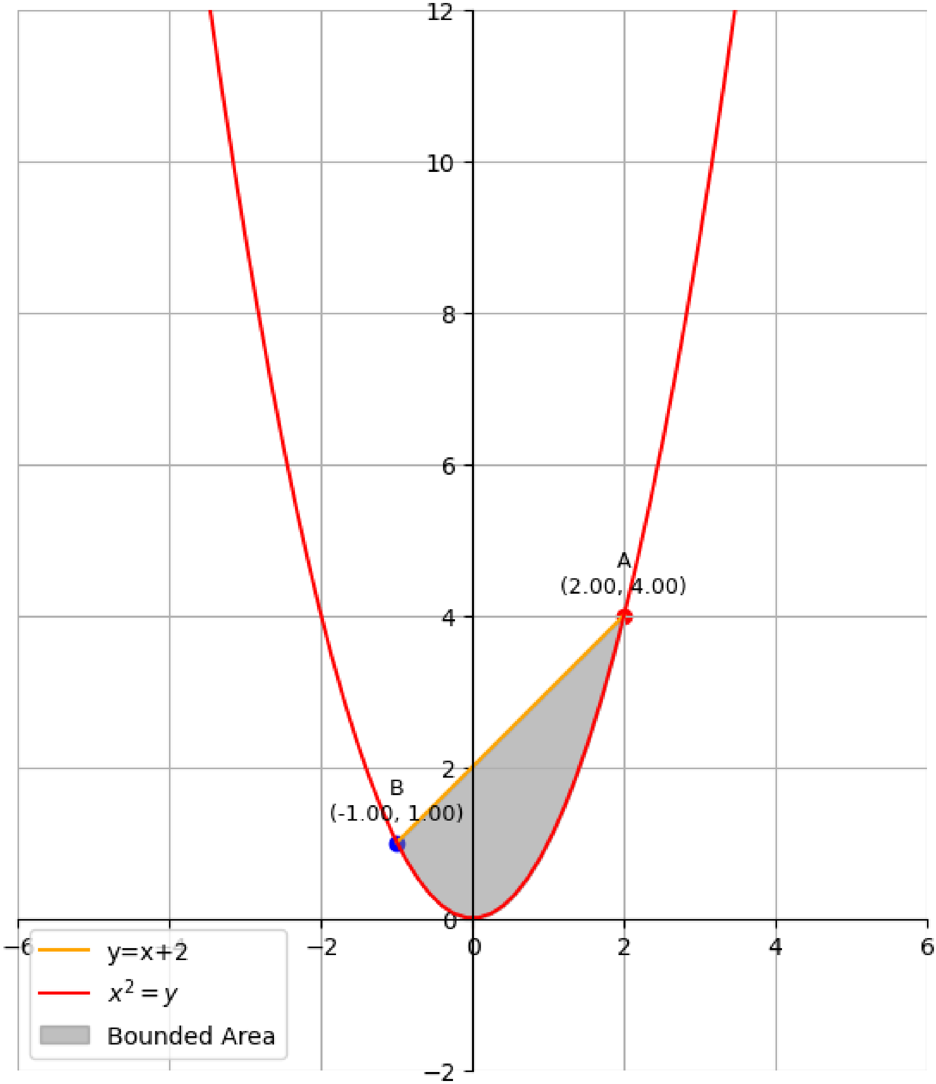

Fig. 0: Intersection of line and parabola