31/01/2023-Shift 1

EE24BTECH11021 - Eshan Ray

1) If the maximum distance of normal to the ellipse $\frac{x^2}{4}+\frac{y^2}{b^2}=1$, $b<2$, from the origin is 1, then the eccentricity of the ellipse is :
   a) $\frac{1}{\sqrt{2}}$
   b) $\frac{\sqrt{3}}{2}$
   c) $\frac{1}{2}$
   d) $\frac{\sqrt{3}}{4}$
2) For all $z \in C$ on the curve $C_1: |z| = 4$, let the locus of the point $z+\frac{1}{z}$ be the curve C_2. Then
   a) the curve s C_1 and C_2 intersect at 4 points
   b) the curve C_1 lies inside C_2
   c) the curves C_1 and C_2 intersect at 2 points
   d) the curve C_2 lies inside C_1
3) A wire of length $20\,m$ is to be cut into two pieces. A piece of length l_1 is bent into the shape of area A_1 and the other piece if length l_2 is made into circle of area A_2. If $2A_1+3A_2$ is minimum then $(\pi l_1) : l_2$ is equal to :
   a) $6: 1$
   b) $3: 1$
   c) $1: 6$
   d) $4: 1$
4) For the system of linear equations
   $x+y+z=6$
   $\alpha x+\beta y+7z=3$
   $x+2y+3z=14,$
   which of the following is NOT true?
   a) If $\alpha=\beta=7$, then the system has no solution
   b) If $\alpha=\beta$ and $\alpha\neq 7$ then the system has a unique solution
   c) There is a unique point (α,β) on the line $x+2y+18=0$ for which the system has infinitely many solutions
   d) For every point $(\alpha,\beta)\neq(7,7)$ on the line $x-2y+7=0$, the system has infinitely many solutions
5) Let the shortest distance between the lines
   L: $\frac{x-5}{-2}=\frac{y-\lambda}{0}=\frac{z+\lambda}{1}, \lambda\geq 0$ and $L_1: x+1=y-1=4-z$ be $2\sqrt{6}$. If (α,β,γ) lies on L, then which of the following is NOT possible ?
   a) $\alpha+2\gamma=24$
   b) $2\alpha+\gamma=7$
   c) $2\alpha-\gamma=9$

d) $\alpha - 2\gamma = 19$

6) Let $y = f(x)$ represent a parabola with focus $\left(-\frac{1}{2}, 0\right)$ and directrix $y = -\frac{1}{2}$. Then $S = \left\{x \in R\colon \tan^{-1}\left(\sqrt{f(x)} + \sin^{-1}\left(\sqrt{f(x)+1}\right)\right) = \frac{\pi}{2}\right\}$:

a) contains exactly two elements
b) contains exactly one element
c) is an infinite set
d) is an empty set

7) Let $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 12 & -3 \end{pmatrix}$. Then the sum of the diagonal elements of the matrix $(A + I)^{11}$ is equal to :

a) 6144
b) 4094
c) 4097
d) 2050

8) Let R be a relation $N \times N$ defined by $(a, b)\, R\, (c, d)$ if and only if $ad\,(b - c) = bc\,(a - d)$. Then R is

a) symmetric but neither reflexive nor transitive
b) transitive but neither reflexive nor symmetric
c) reflexive and symmetric but not transitive
d) symmetric and transitive but not reflexive

9) Let
$y = f(x) = \sin^3\left(\frac{\pi}{3}\left(\cos\left(\frac{\pi}{3\sqrt{2}}\left(-4x^3 + 5x^2 + 1\right)^{\frac{3}{2}}\right)\right)\right)$
Then , at $x = 1$,

a) $2y\prime + \sqrt{3}\pi^2 y = 0$
b) $2y\prime + 3\pi^2 y = 0$
c) $\sqrt{2}y\prime - 3\pi^2 y = 0$
d) $y\prime + 3\pi^2 y = 0$

10) If sum and product of four positive consecutive terms of a $G.P.$, are 126 and 1296, respectively, then the sum of common ratios of all such GPs is :

a) 7
b) $\frac{9}{2}$
c) 3
d) 14

11) The number of real roots of the equation $\sqrt{x^2 - 4x + 3} + \sqrt{x^2 - 9} = \sqrt{4x^2 - 14x + 6}$, is :

a) 0
b) 1
c) 3
d) 2

12) Let a differentiable function f satisfy $f(x) + \int_3^x \frac{f(t)}{t}\, dt = \sqrt{x+1}, x \geq 3$. Then $12f(8)$ is equal to :

a) 34
b) 19
c) 17
d) 1

13) If the domain of the function $f(x) = \frac{[x]}{1+x^2}$, where $[x]$ is greatest integer $\leq x$, is $[2, 6)$, then its range is
a) $\left(\frac{5}{26}, \frac{2}{5}\right] - \left\{\frac{9}{29}, \frac{27}{109}, \frac{18}{89}, \frac{9}{53}\right\}$
b) $\left(\frac{5}{26}, \frac{2}{5}\right]$
c) $\left(\frac{5}{37}, \frac{2}{5}\right] - \left\{\frac{9}{29}, \frac{27}{109}, \frac{18}{89}, \frac{9}{53}\right\}$
d) $\left(\frac{5}{37}, \frac{2}{5}\right]$

14) Let $\overrightarrow{a} = 2\hat{i} + \hat{j} + \hat{k}$ and $\overrightarrow{b}$ and $\overrightarrow{c}$ be two nonzero vectors such that $\left|\overrightarrow{a} + \overrightarrow{b} + \overrightarrow{c}\right| = \left|\overrightarrow{a} + \overrightarrow{b} - \overrightarrow{c}\right|$ and $\overrightarrow{b} \cdot \overrightarrow{c} = 0$. Consider the following two statements :
(A) $\left|\overrightarrow{a} + \lambda\overrightarrow{c}\right| \geq \left|\overrightarrow{a}\right|$ for all $\lambda \in R$
(B) $\overrightarrow{a}$ and $\overrightarrow{c}$ are always parallel
a) only (B) is correct
b) neither (A) nor (B) is correct
c) only (A) is correct
d) both (A) and (B) are correct

15) Let $\alpha \in (0, 1)$ and $\beta = \log_e(1 - \alpha)$. Let $P_n(x) = x + \frac{x^2}{2} + \frac{x^3}{3} + \cdots + \frac{x^n}{n}, x \in (0, 1)$. Then, the integral $\int_0^\alpha \frac{t^{50}}{1-t} dt$ is equal to :
a) $\beta - P_{50(\alpha)}$
b) $-(\beta + P_{50}(\alpha))$
c) $P_{50}(\alpha) - \beta$
d) $\beta + P_{50}(\alpha)$