18/03/2021-Shift 1

EE24BTECH11021 - Eshan Ray

1) If the functions are defined as $f(x) = \sqrt{x}$ and $g(x) = \sqrt{1-x}$, then what is the common domain of the following functions: $f+g, f-g, \frac{f}{g}, \frac{g}{f}, g-f$ where $(f \pm g)(x) = f(x) = f(x) \pm g(x), \left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$
   a) $0<x \leq 1$
   b) $0 \leq x<1$
   c) $0 \leq x \leq 1$
   d) $0<x<1$
2) Let α, β, γ be the roots of the equation, $x^3 + ax^2 + bx + c = 0, (a, b, c \in R \, and \, a, b \, and \, a, b \neq 0)$. The system of equations $(in \, u, v, w)$ given by $\alpha u + \beta v + \gamma w = 0; \beta u + \gamma v + \alpha w = 0; \gamma u + \alpha v + \beta w = 0$ has non-trivial solutions,then the value of $\frac{a^2}{b}$ is
   a) 5
   b) 1
   c) 0
   d) 3
3) If the equation $a|z|^2 + \overline{\bar{\alpha}z + \alpha\bar{z}} + d = 0$ represents a circle where a, d are real constants then which of the following conditions are correct?
   a) $|\alpha|^2 - ad \neq 0$
   b) $|\alpha|^2 - ad>0$ and $a \in R - \{0\}$
   c) $\alpha = 0, a, d \in R^+$
   d) $|\alpha|^2 - ad \geq 0$ and $a \in R$
4) $\frac{1}{3^2-1} + \frac{1}{5^2-1} + \frac{1}{7^2-1} + \cdots + \frac{1}{201^2-1}$ is equal to :
   a) $\frac{101}{404}$
   b) $\frac{101}{408}$
   c) $\frac{99}{400}$
   d) $\frac{25}{101}$
5) The number of integral values of m so that the abscissa of point of intersection of lines $3x + 4y = 9$ and $y = mx + 1$ is also an integer, is:
   a) 3
   b) 2
   c) 1
   d) 0
6) The solutions of the equation

$$\begin{vmatrix} 1+\sin^2 x & \sin^2 x & \sin^2 x \\ \cos^2 x & 1+\cos^2 x & \cos^2 x \\ 4\sin 2x & 4\sin 2x & 1+4\sin 2x \end{vmatrix} = 0$$

, $(0<x<\pi)$, are:

a) $\frac{\pi}{6}, \frac{5\pi}{6}$
b) $\frac{7\pi}{12}, \frac{11\pi}{12}$
c) $\frac{5\pi}{12}, \frac{7\pi}{12}$
d) $\frac{\pi}{12}, \frac{\pi}{6}$

7) If $f(x) = \begin{cases} \frac{1}{|x|} & ; |x| \geq 1 \\ ax^2 + b & ; |x| < 1 \end{cases}$ is differentiable at every point of the domain,then the values of a and b are respectively :

a) $\frac{5}{2}, -\frac{3}{2}$
b) $-\frac{1}{2}, \frac{3}{2}$
c) $\frac{1}{2}, \frac{1}{2}$
d) $\frac{1}{2}, -\frac{3}{2}$

8) A vector **a** has components $3p$ and 1 with respect to a rectangular Cartesian system. This system is rotated through a certain angle about the origin in the counterclockwise sense. If with respect to the new system, **a** has components $p+1$ and $\sqrt{10}$, then a value of p is equal to:

a) 1
b) -1
c) $\frac{4}{5}$
d) $-\frac{5}{4}$

9) The sum of all the $4-digit$ distinct numbers that can be formed with the digits $1, 2, 2$ and 3 is:

a) 26664
b) 122664
c) 122234
d) 22264

10) Choose the correct statement about two circles whose equations are given below:
$x^2 + y^2 - 10x - 10y + 41 = 0$
$x^2 + y^2 - 22x - 10y + 137 = 0$

a) circles have no meeting point
b) circles have two meeting points
c) circles have only one meeting point
d) circles have the same centre

11) If α, β are natural numbers such that $100^\alpha - 199\beta = (100)(100) + (99)(101) + (98)(102) + \cdots + (1)(199)$, then the slope of the line passing through (α, β) and origin is:

a) 510
b) 550
c) 540
d) 530

12) The value of $3 + \cfrac{1}{4 + \cfrac{1}{3 + \cfrac{1}{4 + \cfrac{1}{3 + \ldots\infty}}}}$ is equal to :

a) $3 + 2\sqrt{3}$
b) $4 + \sqrt{3}$
c) $2 + \sqrt{3}$
d) $1.5 + \sqrt{3}$

13) The integral $\int \frac{(2x-1)\cos\sqrt{(2x-1)^2+5}}{\sqrt{4x^2-4x+6}}\,dx$ is equal to ($where\, c\, is\, a\, constant\, of\, integration$)
a) $\frac{1}{2}\sin\sqrt{(2x+1)^2+5}+c$
b) $\frac{1}{2}\sin\sqrt{(2x-1)^2+5}+c$
c) $\frac{1}{2}\cos\sqrt{(2x+1)^2+5}+c$
d) $\frac{1}{2}\cos\sqrt{(2x-1)^2+5}+c$

14) The differential equations satisfied by the system of parabolas $y^2 = 4a(x+a)$ is :
a) $y\left(\frac{dy}{dx}\right)+2x\left(\frac{dy}{dx}\right)-y=0$
b) $y\left(\frac{dy}{dx}\right)^2+2x\left(\frac{dy}{dx}\right)-y=0$
c) $y\left(\frac{dy}{dx}\right)^2-2x\left(\frac{dy}{dx}\right)-y=0$
d) $y\left(\frac{dy}{dx}\right)^2-2x\left(\frac{dy}{dx}\right)+y=0$

15) The real-valued function $f(x) = \frac{cosec^{-1}x}{\sqrt{x-[x]}}$, where $[x]$ denotes the greatest integer less than or equal to x, is defined for all x belonging to :
a) all non- integers except the interval $[-1, 1]$
b) all integers except $0, -1, 1$
c) all reals except integers
d) all reals except the interval $[-1, 1]$