9.2.18

EE24BTECH11021 - Eshan Ray

Question:
Find the area of the region bounded by the curve $y = \sqrt{x}$ and the lines $x = 2y + 3$ and the $x - axis$.
Solution:

Variable	value
V	$\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$
u	$\begin{pmatrix} \frac{-1}{2} \\ 0 \end{pmatrix}$
f	0
h	$\begin{pmatrix} 3 \\ 0 \end{pmatrix}$
m	$\begin{pmatrix} 2 \\ 1 \end{pmatrix}$
k_i	$-$

TABLE 0: Input parameters

The point of intersection of the line with the parabola is $x_i = h + k_i m$,
where,k_i is a constant and is calculated as follows:-

$$k_i = \frac{1}{m^\top V m} \left(-m^\top (Vh + u) \pm \sqrt{[m^\top (Vh + u)]^2 - g(h)(m^\top V m)} \right)$$

$$k_i = \frac{1}{\begin{pmatrix} 2 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \end{pmatrix}} \left(-\begin{pmatrix} 2 & 1 \end{pmatrix} \left(\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 3 \\ 0 \end{pmatrix} + \begin{pmatrix} \frac{-1}{2} \\ 0 \end{pmatrix} \right) \pm \right.$$
$$\left. \sqrt{\left[\begin{pmatrix} 2 & 1 \end{pmatrix} \left(\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 3 \\ 0 \end{pmatrix} + \begin{pmatrix} \frac{-1}{2} \\ 0 \end{pmatrix} \right) \right]^2 - g(h) \left(\begin{pmatrix} 2 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \end{pmatrix} \right)} \right) \quad (1)$$

We get,

$$k_i = -1, 3$$

$$x_1 = \begin{pmatrix}3\\0\end{pmatrix} + (-1)\begin{pmatrix}2\\1\end{pmatrix} \tag{2}$$

$$\implies x_1 = \begin{pmatrix}3\\0\end{pmatrix} + \begin{pmatrix}-2\\-1\end{pmatrix} \tag{3}$$

$$\implies x_1 = \begin{pmatrix}1\\-1\end{pmatrix} \tag{4}$$

$$x_2 = \begin{pmatrix}3\\0\end{pmatrix} + (3)\begin{pmatrix}2\\1\end{pmatrix} \tag{5}$$

$$\implies x_2 = \begin{pmatrix}3\\0\end{pmatrix} + \begin{pmatrix}6\\3\end{pmatrix} \tag{6}$$

$$\implies x_2 = \begin{pmatrix}9\\3\end{pmatrix} \tag{7}$$

As, y cannot be negative, so the point of intersection of the line and the parabola is $(9, 3)$.
$\therefore$ The area bounded by the curve $y = \sqrt{x}$ and line $x = 2y + 3$ is given by

$$Area = \int_0^3 \sqrt{x}\,dx + \int_3^9 \sqrt{x} - \frac{x-3}{2}\,dx \tag{8}$$

$$Area = \frac{2}{3}\left[x^{\frac{3}{2}}\right]_0^9 - \frac{1}{2}\left(\left(\frac{9^2}{2} - 27\right) - \left(\frac{3^2}{2} - 9\right)\right) \tag{9}$$

$$Area = \frac{2}{3}\left((27) - (0)\right) - \frac{1}{2}\left(\frac{27}{2} - \frac{-9}{2}\right) \tag{10}$$

$$Area = 18 - 9 \tag{11}$$

$$Area = 9 \tag{12}$$

So, the required area is 9 units.

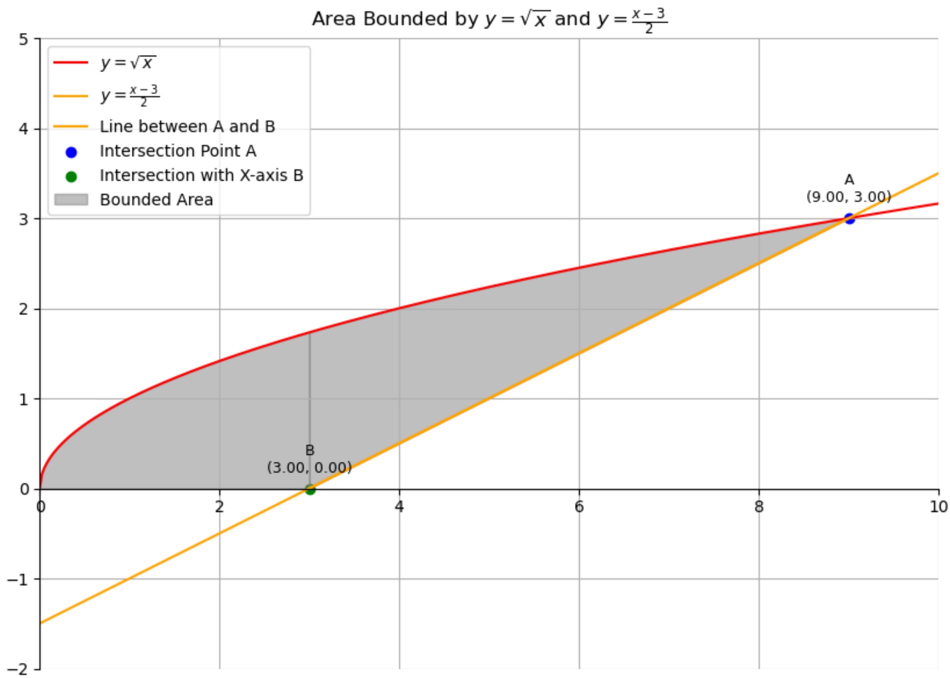

Fig. 0: Intersection of line and parabola