

## 3.2.30

EE24BTECH11021 - Eshan Ray

### Question:

Draw a parallelogram  $ABCD$  in which  $BC = 5\text{cm}$ ,  $AB = 3\text{cm}$  and  $\angle ABC = 60^\circ$ , divide it into triangles  $ACB$  and  $ABD$  by the diagonal  $BD$ . Construct the triangle  $BD'C'$  similar to  $\triangle BDC$  with scale factor  $\frac{4}{3}$ . Draw the line segment  $D'A'$  parallel to  $DA$  where  $A'$  lies on extended side  $BA$ . Is  $A'BC'D'$  a parallelogram?

**Solution:**  $ABCD$  is a parallelogram,

$$\Rightarrow AB \parallel DC, AB = DC \quad (1)$$

$$\Rightarrow AD \parallel BC, AD = BC \quad (2)$$

$$\triangle BDC \sim \triangle BD'C' \text{ (Given)}$$

$$\text{scale factor} = \frac{4}{3}$$

$$\Rightarrow \frac{BD'}{BD} = \frac{4}{3} \quad (3)$$

$$\Rightarrow \frac{BC'}{BC} = \frac{4}{3} \quad (4)$$

$$\Rightarrow \angle BCD = \angle BC'D' \quad (5)$$

$$\Rightarrow \angle BDC = \angle BD'C' \quad (6)$$

$$\therefore CD \parallel C'D' \quad (7)$$

$$BC' \parallel BC \quad (8)$$

$$\text{Given, } A'D' \parallel AD$$

$$\text{From } \triangle BA'D' \triangle BAD, \quad (9)$$

$$\Rightarrow \angle ABD = \angle A'BD' \quad (10)$$

$$\Rightarrow \angle BDA = \angle BD'A' \quad (11)$$

$$\Rightarrow \angle BAD = \angle BA'D' \quad (12)$$

$$\therefore \triangle ABD \sim \triangle A'BD'$$

$$\Rightarrow \frac{BD'}{BD} = \frac{BA'}{BA} = \frac{A'D'}{AD} = \frac{4}{3} \quad (13)$$

In quadrilateral  $A'BC'D'$ ,

$$A'D' \parallel AD \parallel BC \quad (14)$$

$$\Rightarrow A'D' \parallel BC' \quad (15)$$

$$\Rightarrow BC' = \frac{4}{3}BC \quad (16)$$

$$\Rightarrow A'D' = \frac{4}{3}AD \quad (17)$$

$$\therefore BC' = A'D' \quad (18)$$

$$\text{Similarly, } BA' \parallel C'D', BA' = C'D' \quad (19)$$

So, quadrilateral  $A'BC'D'$  is a parallelogram.

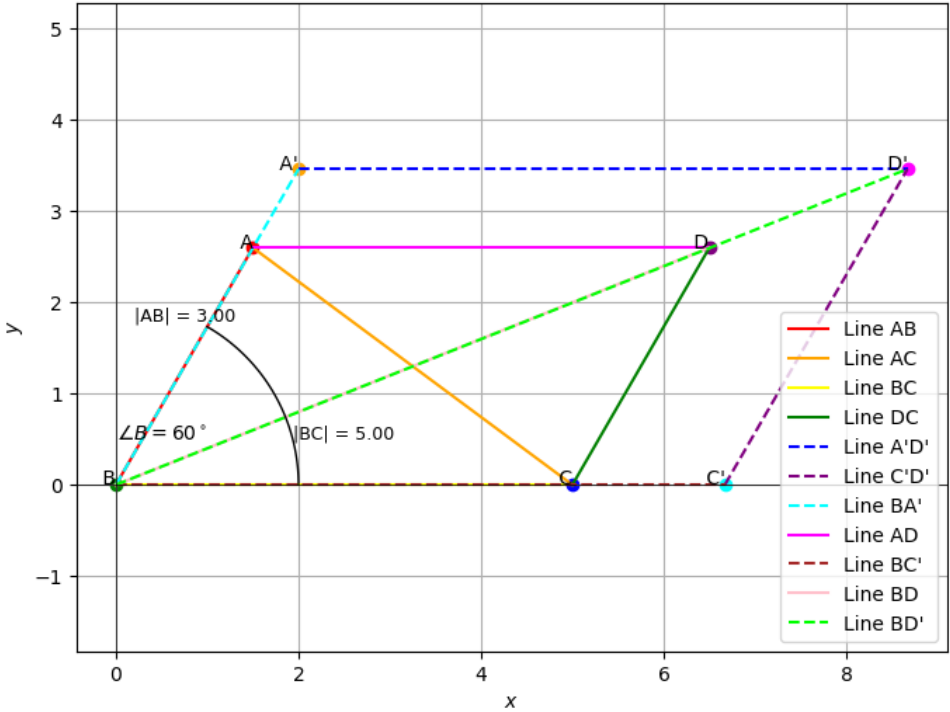


Fig. 0:  $\|^{gm}ABCD$  and  $\|^{gm}A'BC'D'$