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EE24BTECH11021 - Eshan Ray

1) Consider R^2 with the usual topology. Let $S = \left\{(x, y) \in R^2 : x\, is\, an\, integer\right\}$. Then S is
   a) open but NOT closed
   b) both open and closed
   c) neither open nor closed
   d) closed but not open
2) Suppose $X = \{\alpha, \beta, \gamma\}$. Let
$$\mathcal{J}_1 = \{\phi, X, \{\alpha\}, \{\alpha, \beta\}\} \; and \, \mathcal{J}_2 = \{\phi, X, \{\alpha\}, \{\beta, \gamma\}\}.$$
   Then
   a) both $\mathcal{J}_1 \cap \mathcal{J}_2$ and $\mathcal{J}_1 \cup \mathcal{J}_2$ are topologies
   b) neither $\mathcal{J}_1 \cap \mathcal{J}_2$ nor $\mathcal{J}_1 \cup \mathcal{J}_2$ is a topologies
   c) $\mathcal{J}_1 \cup \mathcal{J}_2$ is a topology but $\mathcal{J}_1 \cap \mathcal{J}_2$ is NOT a topology
   d) $\mathcal{J}_1 \cap \mathcal{J}_2$ is a topology but $\mathcal{J}_1 \cup \mathcal{J}_2$ is NOT a topology
3) For a positive integer n, let $f_n : R \to R$ be defined by
$$f_n(x) = \begin{cases} \frac{1}{4n+5}, \text{ if } 0 \le x \le n, \\ 0, \text{ otherwise} \end{cases}$$
   Then $\{f_n(x)\}$ converges to zero
   a) uniformly but NOT in L^1 norm
   b) uniformly and also in L^1 norm
   c) pointwise but NOT uniformly
   d) in L^1 norm but NOT pointwise
4) Let P_1 and P_2 be two projection operations on a vector space. Then
   a) $P_1 + P_2$ is a projection if $P_1P_2 = P_2P_1 = 0$
   b) $P_1 - P_2$ is a projection if $P_1P_2 = P_2P_1 = 0$
   c) $P_1 + P_2$ is a projection
   d) $P_1 = P_2$ is a projection
5) Consider the system of linear equations
   $x + y + z = 3$
   $x - y - z = 4$
   $x - 5y + kz = 6.$
   Then the value of k for which this system has an infinite number of solutions is
   a) $k = -5$
   b) $k = 0$
   c) $k = 1$

d) $k = 3$

6) Let

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 2 & 3 \\ x & y & z \end{pmatrix}$$

and let $V = \left\{(x, y, z) \in R^3 : det\,(A) = 0\right\}$. Then the dimension of V equals

a) infinite
b) 0
c) 1
d) 2

7) Let $S = \{0\} \cup \left\{\frac{1}{4n+7} : n = 1, 2, \ldots\right\}$. Then the number of analytic functions which vanish only on S is

a) 0
b) 1
c) 2
d) 3

8) It is given that $\sum_{n=0}^{\infty} a_n z^n$ converges at $z = 3 + i4$. Then the radius of convergence of the power series $\sum_{n=0}^{\infty} a_n z^n$ is

a) ≤ 5
b) ≥ 5
c) <5
d) >5

9) The value of α for which $G = \{\alpha, 1, 3, 9, 19, 27\}$ is a cyclic group under multiplication modulo 56 is

a) 5
b) 15
c) 25
d) 35

10) Consider Z_24 as the additive group modulo 24. Then the number of elements of order 8 in the group Z_24 is

a) 1
b) 2
c) 3
d) 4

11) Define $f : R^2 \to R$ by

$$f\,(x, y) = \begin{cases} 1, \text{ if } xy = 0, \\ 2, \text{ otherwise.} \end{cases}$$

If $S = \{(x, y) : f\, is\, a\, continuous\, at\, the\, point\ (x, y)\}$, then

a) S is open
b) S is connected
c) $S = \phi$

d) S is closed

12) Consider the linear programming problem,
$Max.z = c_1x_1 + c_2x_2, c_1, c_2>0,$
subject to
$x_1 + x_3 \leq 3$
$2x_1 + 3x_2 \leq 4$
$x_1, x_2 \geq 0$
Then,

a) the primal has an optimal solution but the dual does NOT have an optimal solution
b) both the primal and dual have optimal solutions
c) the dual has an optimal solution but the primal does NOT have an optimal solution
d) neither the primal nor the dual have optimal solutions

13) Let $f(x) = x^{10} + x - 1, x \in R$and let $x_k = k, k = 0, 1, 2, \ldots, 10$. Then the value of the divided difference $f[x_0, x_1, x_2, x_3, \ldots, x_{10}]$ is

a) -1
b) 0
c) 1
d) 10

14) Let X and Y be jointly distributed random variables having the joint probability density function

$$f(x, y) = \begin{cases} \frac{1}{\pi}, \text{ if } x^2 + y^2 \leq 1, \\ 0, \text{ otherwise}, \end{cases}$$

Then $P(Y>max(X, -X)) =$

a) $\frac{1}{2}$
b) $\frac{1}{3}$
c) $\frac{1}{4}$
d) $\frac{1}{6}$

15) Let $X_1, X_2, \ldots$ be a sequence of independent and identically distributed chi-square random variables, each having 4 degrees of freedom . Define $S_n = \sum_{i=1}^{n} X_i^2, n = 1, 2, \ldots$.
If $\frac{S_n}{n} \xrightarrow{\text{P}} \mu$, as $n \to \infty$, then $\mu =$

a) 8
b) 16
c) 24
d) 32

16) Let E_n: $n = 1, 2, \ldots$ be a decreasing sequence of Lebesgue measurable sets on R and let F be a Lebesgue measurable set on R such that $E_i \cap F = \phi$. Suppose that F has Lebesgue measure 2 and the Lebesgue measure of $E_n = \frac{2n+2}{3n+1}, n = 1, 2, \ldots$.
Then the Lebesgue measure of the set $\left(\cap_{n=1}^{\infty} E_n\right) \cup F$ equals

a) $\frac{5}{3}$
b) 2
c) $\frac{7}{3}$

d) $\frac{8}{3}$

17) The extremum for the variational problem

$$\int_0^{\frac{\pi}{8}} \left((y\prime)^2 + 2yy\prime - 16y^2\right) dx, y(0) = 0, y\left(\frac{\pi}{8}\right) = 1,$$

occurs for the curve

a) $y = \sin(4x)$
b) $y = \sqrt{2}\sin(2x)$
c) $y = 1 - \cos(4x)$
d) $y = \frac{1-\cos(8x)}{2}$