9.2.18

EE24BTECH11021 - Eshan Ray

Question:
Find the area of the region bounded by the curve $x^2 = y$ and the lines $y = x + 2$ and the $x - axis$.
Solution:

Variable	**Description**	**value**
V	Quadratic Form Matrix	$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$
u	Linear Coefficient Vector	$\begin{pmatrix} 0 \\ \frac{-1}{2} \end{pmatrix}$
f	Constant Term	0
h	Point on the line $x = 2y + 3$	$\begin{pmatrix} 0 \\ 2 \end{pmatrix}$
m	slope of the line $x = 2y + 3$	$\begin{pmatrix} 1 \\ 1 \end{pmatrix}$
k_i	constant for point of intersection of the curve and the line	$-$

TABLE 0: Input parameters

The point of intersection of the line with the parabola is $x_i = h + k_i m$,
where,k_i is calculated as follows:-

$$k_i = \frac{1}{m^\top V m}\left(-m^\top (Vh + u) \pm \sqrt{\left[m^\top (Vh + u)\right]^2 - g(h)(m^\top V m)}\right)$$

Substituting the input parameters in k_i,

$$k_i = \frac{1}{\begin{pmatrix} 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix}}\left(-\begin{pmatrix} 1 & 1 \end{pmatrix}\left(\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 2 \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{-1}{2} \end{pmatrix}\right) \pm \right.$$
$$\left.\sqrt{\left[\begin{pmatrix} 1 & 1 \end{pmatrix}\left(\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 2 \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{-1}{2} \end{pmatrix}\right)\right]^2 - g(h)\left(\begin{pmatrix} 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix}\right)}\right) \quad (1)$$

We get,

$$k_i = -1, 2$$

Substituting k_i in $x_i = h + k_i m$ we get,

$$x_1 = \begin{pmatrix}0\\2\end{pmatrix} + (-1)\begin{pmatrix}1\\1\end{pmatrix} \tag{2}$$

$$\implies x_1 = \begin{pmatrix}0\\2\end{pmatrix} + \begin{pmatrix}-1\\-1\end{pmatrix} \tag{3}$$

$$\implies x_1 = \begin{pmatrix}-1\\1\end{pmatrix} \tag{4}$$

$$x_2 = \begin{pmatrix}0\\2\end{pmatrix} + (2)\begin{pmatrix}1\\1\end{pmatrix} \tag{5}$$

$$\implies x_2 = \begin{pmatrix}0\\2\end{pmatrix} + \begin{pmatrix}2\\2\end{pmatrix} \tag{6}$$

$$\implies x_2 = \begin{pmatrix}2\\4\end{pmatrix} \tag{7}$$

The area bounded by the curve $y = x^2$ and line $y = x + 2$ is given by

$$\int_{-1}^{2} x + 2 - x^2\, dx = \left(\frac{x^2}{2} + 2x - \frac{x^3}{3}\right)_{-1}^{2} \tag{8}$$

$$= \left(\frac{4}{2} + 4 - \frac{8}{3}\right) - \left(\frac{1}{2} - 2 + \frac{1}{3}\right) \tag{9}$$

$$= \left(\frac{10}{3}\right) - \left(\frac{-7}{6}\right) \tag{10}$$

$$= \frac{9}{2} \tag{11}$$

So, the required area is 4.50 units.

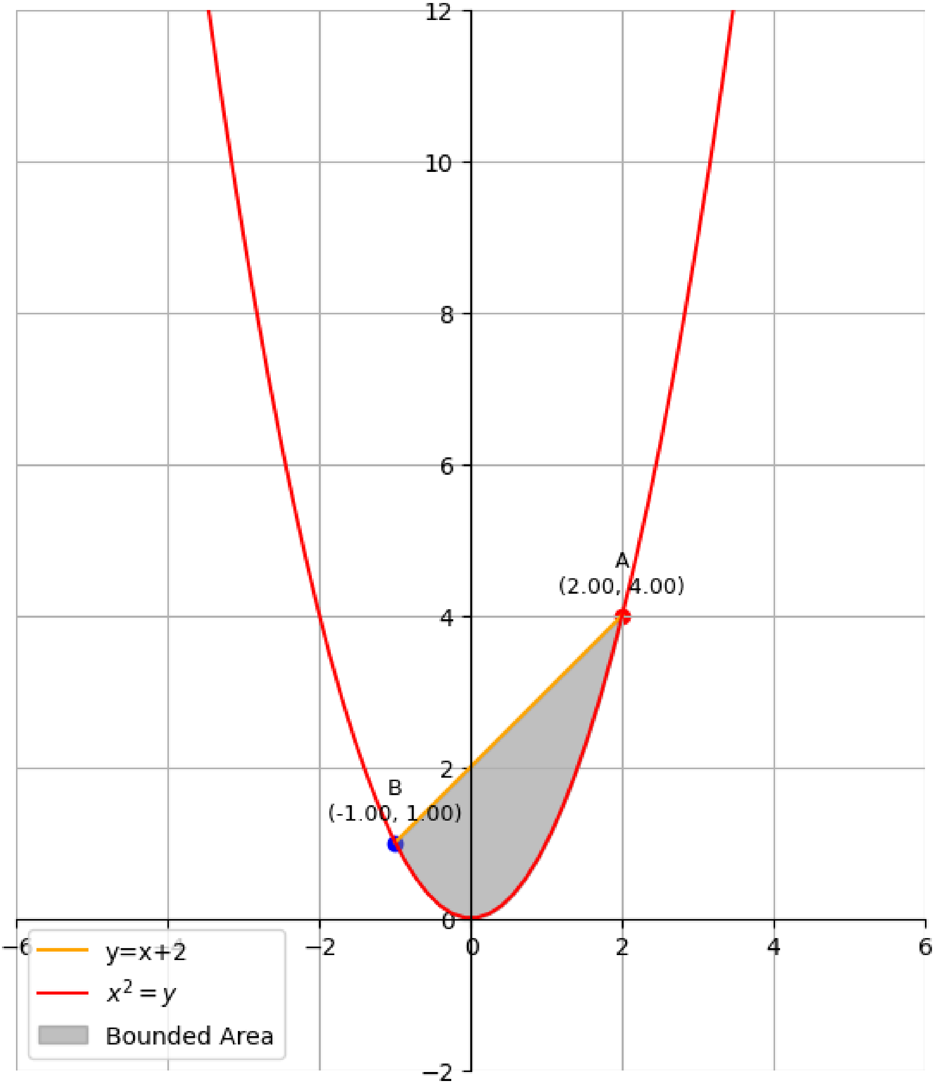

Fig. 0: Intersection of line and parabola