# 9.3.28

EE24BTECH11021 - Eshan Ray

**Question:**
Find the equation of tangent to the curve $y = \sqrt{3x-2}$ which is parallel to the line $4x - 2y + 5 = 0$ and also write the equation of the normal to the curve at the contact.
**Solution:**

| Variable | Description | value |
|---|---|---|
| $V$ | Quadratic Form Matrix | $\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$ |
| $u$ | Linear Coefficient Vector | $\begin{pmatrix} \frac{-3}{2} \\ 0 \end{pmatrix}$ |
| $f$ | Constant Term | 2 |
| $n$ | normal vector of the line $4x - 2y + 5 = 0$ | $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$ |
| $m$ | slope of the line $4x - 2y + 5 = 0$ parallel to the tangent | $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ |
| $q$ | constant for point of intersection of the curve and the tangent | $-$ |
| $p_1$ | The eigenvector corresponding to the zero eigenvalue | $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ |

TABLE 0: Input parameters

Since, $V$ is not invertible, the point of contact to is given by the matrix equation:-

$$\begin{pmatrix} (u + kn)^\top \\ V \end{pmatrix} q = \begin{pmatrix} -f \\ kn - u \end{pmatrix} \tag{1}$$

$$where, k = \frac{p_1^\top u}{p_1^\top n} \tag{2}$$

$$\implies k = \frac{3}{4} \tag{3}$$

$$\implies \begin{pmatrix} \left(\begin{pmatrix} \frac{-3}{2} \\ 0 \end{pmatrix} + \frac{3}{4}\begin{pmatrix} -2 \\ 1 \end{pmatrix}\right)^\top \\ \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \end{pmatrix} q = \begin{pmatrix} -2 \\ \frac{3}{4}\begin{pmatrix} -2 \\ 1 \end{pmatrix} - \begin{pmatrix} \frac{-3}{2} \\ 0 \end{pmatrix} \end{pmatrix} \tag{4}$$

$$\implies \begin{pmatrix} -3 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} q = \begin{pmatrix} -\frac{41}{16} \\ 0 \\ \frac{3}{4} \end{pmatrix} \tag{5}$$

The augmented matrix can be expressed as:-

$$\begin{pmatrix} -3 & 0 \mid & -\frac{41}{16} \\ 0 & 0 \mid & 0 \\ 0 & 1 \mid & \frac{3}{4} \end{pmatrix} \tag{6}$$

Performing row operations: $R_2 \leftrightarrow R_3, R_1 \rightarrow \frac{-R_!}{3}$

$$\begin{pmatrix} -3 & 0 \mid & -\frac{41}{16} \\ 0 & 0 \mid & 0 \\ 0 & 1 \mid & \frac{3}{4} \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 \mid & \frac{41}{48} \\ 0 & 1 \mid & \frac{3}{4} \\ 0 & 0 \mid & 0 \end{pmatrix} \tag{7}$$

$$\implies q = \begin{pmatrix} \frac{41}{48} \\ \frac{3}{4} \end{pmatrix} \tag{8}$$

The equation of Tangent obtained is

$$n^\top (x - q) = 0 \tag{9}$$

$$\implies \begin{pmatrix} -2 & 1 \end{pmatrix} x + \frac{23}{24} = 0 \tag{10}$$

Similarly, the equation of normal from the point of contact is

$$m^\top (x - q) = 0 \tag{11}$$

$$\begin{pmatrix} 1 & 2 \end{pmatrix} x = \frac{113}{48} \tag{12}$$

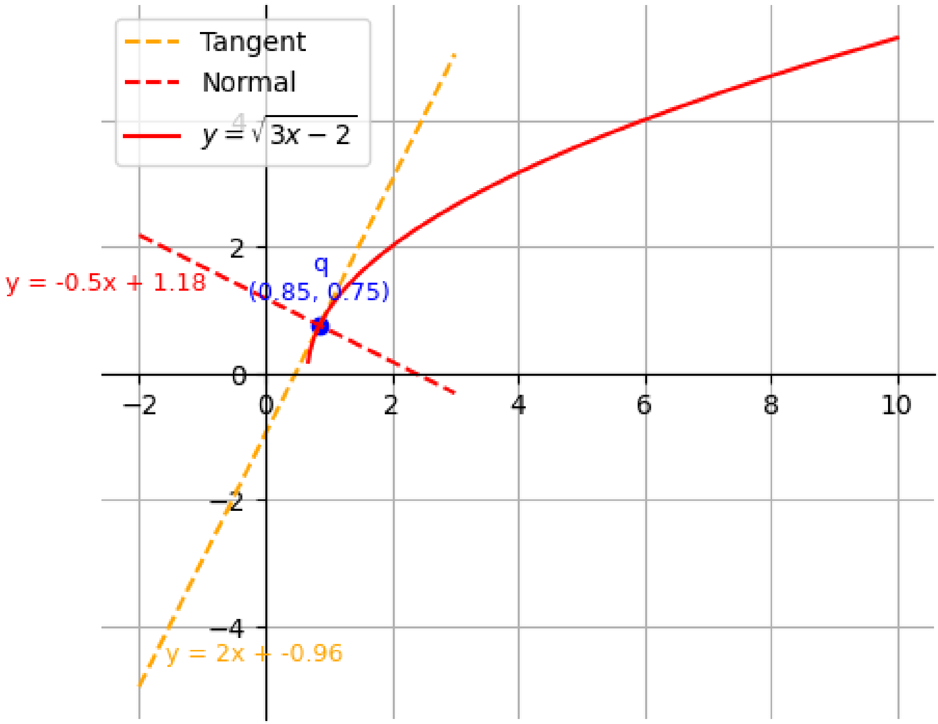


Fig. 0: Tangent and normal to parabola