

9.2.18

EE24BTECH11021 - Eshan Ray

Question:

Find the area of the region bounded by the curve $x^2 = y$ and the lines $y = x + 2$ and the $x - axis$.

Solution:

Variable	value
V	$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$
u	$\begin{pmatrix} 0 \\ -\frac{1}{2} \end{pmatrix}$
f	0
h	$\begin{pmatrix} 0 \\ 2 \end{pmatrix}$
m	$\begin{pmatrix} 1 \\ 1 \end{pmatrix}$
k_i	$-$

TABLE 0: Input parameters

The point of intersection of the line with the parabola is $x_i = h + k_i m$, where, k_i is a constant and is calculated as follows:-

$$k_i = \frac{1}{m^T V m} \left(-m^T (V h + u) \pm \sqrt{[m^T (V h + u)]^2 - g(h) (m^T V m)} \right)$$

Substituting the input parameters in k_i ,

$$k_i = \frac{1}{\begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix}} \left(-\begin{pmatrix} 1 & 1 \end{pmatrix} \left(\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 2 \end{pmatrix} + \begin{pmatrix} 0 \\ -\frac{1}{2} \end{pmatrix} \right) \pm \sqrt{\left[\begin{pmatrix} 1 & 1 \end{pmatrix} \left(\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 2 \end{pmatrix} + \begin{pmatrix} 0 \\ -\frac{1}{2} \end{pmatrix} \right) \right]^2 - g(h) \left(\begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right)} \right) \quad (1)$$

We get,

$$k_i = -1, 2$$

Substituting k_i in $x_i = h + k_i m$ we get,

$$x_1 = \begin{pmatrix} 0 \\ 2 \end{pmatrix} + (-1) \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad (2)$$

$$\Rightarrow x_1 = \begin{pmatrix} 0 \\ 2 \end{pmatrix} + \begin{pmatrix} -1 \\ -1 \end{pmatrix} \quad (3)$$

$$\Rightarrow x_1 = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \quad (4)$$

$$x_2 = \begin{pmatrix} 0 \\ 2 \end{pmatrix} + (2) \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad (5)$$

$$\Rightarrow x_2 = \begin{pmatrix} 0 \\ 2 \end{pmatrix} + \begin{pmatrix} 2 \\ 2 \end{pmatrix} \quad (6)$$

$$\Rightarrow x_2 = \begin{pmatrix} 2 \\ 4 \end{pmatrix} \quad (7)$$

The area bounded by the curve $y = x^2$ and line $y = x + 2$ is given by

$$\int_{-1}^2 x + 2 - x^2 dx = \left(\frac{x^2}{2} + 2x - \frac{x^3}{3} \right)_{-1}^2 \quad (8)$$

$$= \left(\frac{4}{2} + 4 - \frac{8}{3} \right) - \left(\frac{1}{2} - 2 + \frac{1}{3} \right) \quad (9)$$

$$= \left(\frac{10}{3} \right) - \left(\frac{-7}{6} \right) \quad (10)$$

$$= \frac{9}{2} \quad (11)$$

So, the required area is 4.50 units.

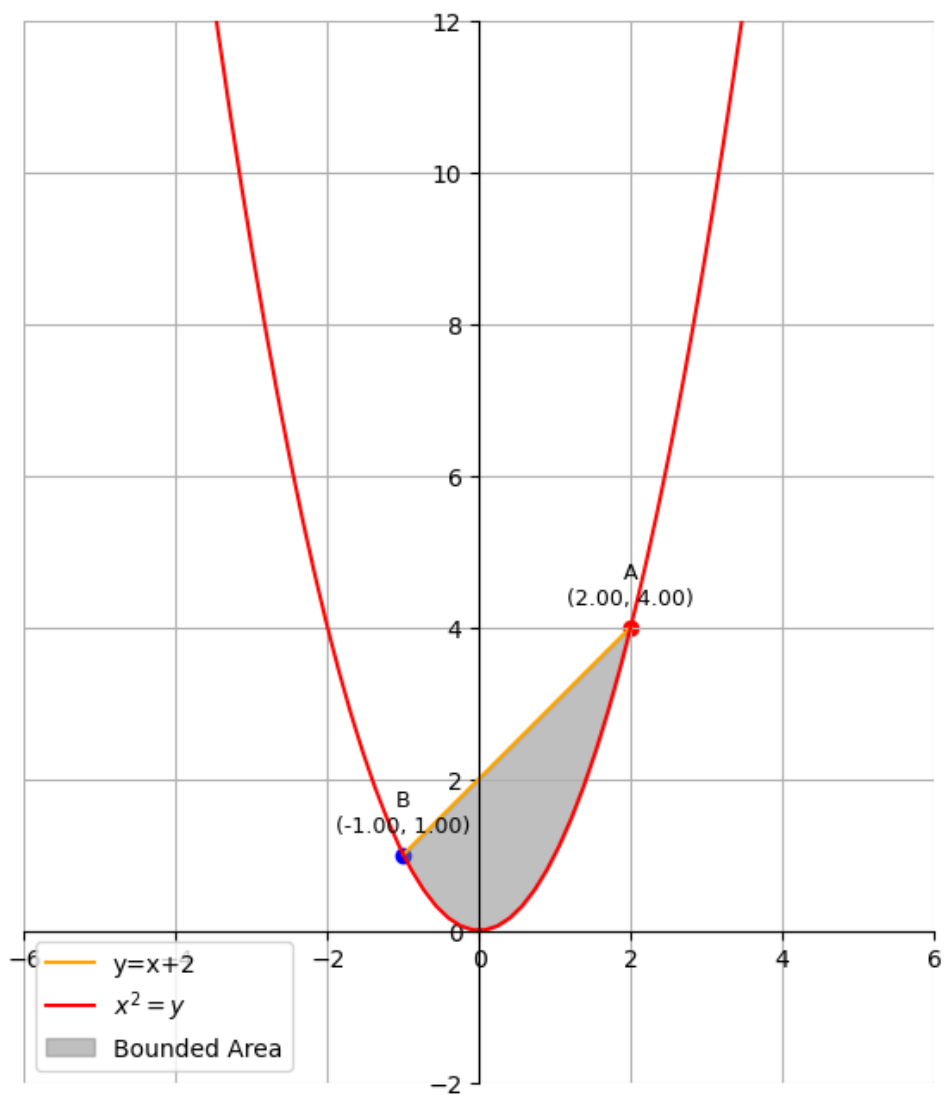


Fig. 0: Intersection of line and parabola