26/02/2021-Shift 2

EE24BTECH11021 - Eshan Ray

1) If vectors $\overrightarrow{a_1} = x\hat{i} - \hat{j} + \hat{k}$ and $\overrightarrow{a_2} = \hat{i} + y\hat{j} + z\hat{k}$ are collinear, then a possible unit vector parallel to the vector parallel to the vector $x\hat{i} + y\hat{j} + z\hat{k}$ is : [Feb-2021]
   a) $\frac{1}{\sqrt{2}}\left(-\hat{j} + \hat{k}\right)$
   b) $\frac{1}{\sqrt{2}}\left(\hat{i} - \hat{j}\right)$
   c) $\frac{1}{\sqrt{3}}\left(\hat{i} - \hat{j} + \hat{k}\right)$
   d) $\frac{1}{\sqrt{3}}\left(\hat{i} + \hat{j} - \hat{k}\right)$
2) Let $A = \{1, 2, 3 \ldots, 10\}$ and $f\colon A \to A$ be defined as $f(k) = \begin{cases} k+1 & \text{if k is odd} \\ k & \text{if k is even} \end{cases}$
   Then the number of possible functions $g\colon A \to A$ such that $gof = f$ is : [Feb-2021]
   a) 10^5
   b) $\binom{10}{5}$
   c) 5^5
   d) $5!$
3) Let $f\colon R \to R$ be defined as $f(x) = \begin{cases} 2\sin\left(-\frac{\pi x}{2}\right) & \text{if } x < -1 \\ \left|ax^2 + x + b\right|, & \text{if } -1 \le x \le 1 \\ \sin(\pi x) & \text{if } x > 1 \end{cases}$
   If $f(x)$ is continuous on R, then $a + b$ equals : [Feb-2021]
   a) 3
   b) -1
   c) -3
   d) 1
4) For $x>0$, if $f(x) = \int_1^x \frac{\log_e t}{1+t}\, dt$, then $f(e) + f\left(\frac{1}{e}\right)$ is equal to : [Feb-2021]
   a) $\frac{1}{2}$
   b) -1
   c) 1
   d) 0
5) A natural number has prime factorization given by $n = 2^x 3^y 5^z$, where y and z are such that $y + z = 5$ and $y^{-1} + z^{-1} = \frac{5}{6}, y>z$.Then the number of odd divisors of n, including 1, is : [Feb-2021]
   a) 11
   b) 6
   c) $6x$
   d) 12
6) Let $f(x) = \sin^{-1} x$ and $g(x) = \frac{x^2-x-2}{2x^2-x-6}$. If $g(2) = \lim_{x\to 2} g(x)$, then the domain of the function fog is : [Feb-2021]

a) $(\infty, -2] \cup \left[-\frac{3}{2}, \infty\right)$
b) $(\infty, -2] \cup [-1, \infty)$
c) $(\infty, -2] \cup \left[-\frac{4}{3}, \infty\right)$
d) $(\infty, -1] \cup [2, \infty)$

7) The triangle of maximum area which can be inscribed in a given circle of radius $'r'$ is : [Feb-2021]

a) An isosceles triangle with base equal to $2r$
b) An equilateral triangle of height $\frac{2r}{3}$
c) An equilateral triangle having each of its side of length $\sqrt{3}r$
d) A right angle triangle having two of its sides of length $2r$ and r

8) Let L be a line obtained from the intersection of two planes $x + 2y + z = 6$ and $y + 2z = 4$. If point $P(\alpha, \beta, \gamma)$ is the foot of perpendicular from $(3, 2, 1)$ on L, then the value of $21(\alpha, \beta, \gamma)$ equals : [Feb-2021]

a) 142
b) 68
c) 136
d) 102

9) Let $F_1(A, B, C) = (A \wedge \sim B) \vee [\sim C \wedge (A \vee B)] \vee \sim A$ and $F_2(A, B) = (A \vee B) \vee (B \rightarrow \sim A)$ be two logical expressions. Then: [Feb-2021]

a) F_1 is not a tautology but F_2 is a tautology
b) F_1 is a tautology but F_2 is not a tautology
c) F_1 and F_2 both are tautologies
d) Both F_1 and F_2 are not tautologies

10) Let the slope of the tangent line to a curve at any point $P(x, y)$ be given by $\frac{xy^2+y}{x}$. If the curve intersects the line $x + 2y = 4$ at $x = -2$, then the value of y, for which the point $(3, y)$ lies on the curve, is: [Feb-2021]

a) $-\frac{18}{11}$
b) $-\frac{18}{19}$
c) $-\frac{4}{3}$
d) $\frac{18}{35}$

11) If the locus of the mid-point of the line segment from the point $(3, 2)$ to a point on the circle, $x^2 + y^2 = 1$ is a circle of the radius r, then r is equal to : [Feb-2021]

a) $\frac{1}{4}$
b) $\frac{1}{2}$
c) 1
d) $\frac{1}{3}$

12) Consider the following system of equations :

$x + 2y - 3z = a$
$2x + 6y - 11z = b$
$x - 2y + 7z = c,$

Where a, b and c are real constants. Then the system of equations : [Feb-2021]

a) has a unique solution when $5a = 2b + c$

b) has infinite number of solutions when $5a = 2b + c$
c) has no solution for all a, b and c
d) has a unique solution for all a, b and c

13) If $0<a, b<1$, and $\tan^{-1} a + \tan^{-1} b = \frac{\pi}{4}$, then the value of $(a - b) - \left(\frac{a^2+b^2}{2}\right) + \left(\frac{a^3+b^3}{3}\right) - \left(\frac{a^4+b^4}{4}\right) + \dots$ is: [Feb-2021]
a) $\log_e 2$
b) $\log_e \frac{e}{2}$
c) e
d) $e^2 - 1$

14) The sum of the series $\sum_{n=1}^{\infty} \frac{n^2+6n+10}{(2n+1)!}$ is equal to : [Feb-2021]
a) $\frac{41}{8}e + \frac{19}{8}e^{-1} - 10$
b) $-\frac{41}{8}e + \frac{19}{8}e^{-1} - 10$
c) $\frac{41}{8}e - \frac{19}{8}e^{-1} - 10$
d) $\frac{41}{8}e + \frac{19}{8}e^{-1} + 10$

15) Let $f(x)$ be a differentiable function at $x = a$ with $f\prime(a) = 2$ and $f(a) = 4$. Then $\lim_{x \to a} \frac{xf(a) - af(x)}{x-a}$ equals: [Feb-2021]
a) $2a + 4$
b) $2a - 4$
c) $4 - 2a$
d) $a + 4$