1.9.30

EE24BTECH11021 - Eshan Ray

Question:
If the distances of $\mathbf{P} = (x, y)$ from $\mathbf{A} = (5, 1)$ and $\mathbf{B} = (-1, 5)$ are equal, then prove that $3x = 2y$.

Solution:

Variable	Description
$\mathbf{A}(5, 1)$	coordinates of first point
$\mathbf{B}(-1, 5)$	coordinates of second point
$\mathbf{P}(x, y)$	Equidistant point of $\mathbf{A}$ and $\mathbf{B}$

TABLE 0: Input parameters

$$\|\mathbf{B}-\mathbf{P}\|^2 = \|\mathbf{A}-\mathbf{P}\|^2 \tag{1}$$

$$\implies \mathbf{B}^2 + \mathbf{P}^2 - 2\mathbf{P}\mathbf{B}^\top = \mathbf{A}^2 + \mathbf{P}^2 - 2\mathbf{P}\mathbf{A}^\top \tag{2}$$

$$\implies \mathbf{P}\left(\mathbf{A}^\top - \mathbf{B}^\top\right) = \frac{\mathbf{A}^2 - \mathbf{B}^2}{2} \tag{3}$$

$$\implies \mathbf{P}\left(\begin{pmatrix}5 & 1\end{pmatrix} - \begin{pmatrix}-1 & 5\end{pmatrix}\right) = \frac{26-26}{2} \tag{4}$$

$$\implies \begin{pmatrix}x\\y\end{pmatrix}\begin{pmatrix}6 & -4\end{pmatrix} = 0 \tag{5}$$

$$\implies 6x - 4y = 0 \tag{6}$$

$$\implies 3x = 2y \tag{7}$$

$$\tag{8}$$

Hence, proved.

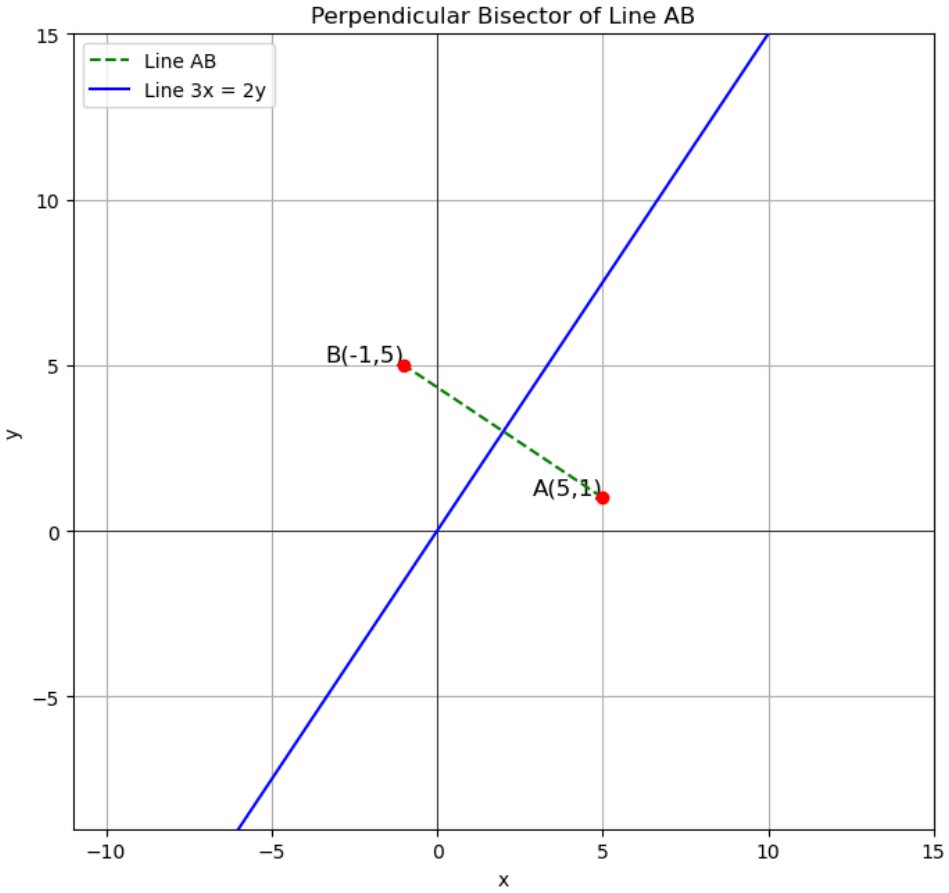
Perpendicular Bisector of Line AB
Line AB
Line 3x = 2y
B(-1,5)
A(5,1)
15
10
5
0
−5
−10
−5
0
5
10
15
x
y

Fig. 0