

# 9.2.36

EE24BTECH11021 - Eshan Ray

## Question:

Find the area of the region bounded by the curve  $y = \sqrt{x}$  and the lines  $x = 2y + 3$  and the  $x$ -axis.

## Solution:

Variable	value
$V$	$\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$
$u$	$\begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix}$
$f$	0
$h$	$\begin{pmatrix} 3 \\ 0 \end{pmatrix}$
$m$	$\begin{pmatrix} 2 \\ 1 \end{pmatrix}$
$k_i$	—

TABLE 0: Input parameters

The point of intersection of the line with the parabola is  $x_i = h + k_i m$ , where,  $k_i$  is a constant and is calculated as follows:-

$$k_i = \frac{1}{m^\top V m} \left( -m^\top (V h + u) \pm \sqrt{[m^\top (V h + u)]^2 - g(h) (m^\top V m)} \right)$$

$$k_i = \frac{1}{\begin{pmatrix} 2 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \end{pmatrix}} \left( -\begin{pmatrix} 2 & 1 \end{pmatrix} \left( \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 3 \\ 0 \end{pmatrix} + \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix} \right) \pm \sqrt{\left[ \begin{pmatrix} 2 & 1 \end{pmatrix} \left( \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 3 \\ 0 \end{pmatrix} + \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix} \right) \right]^2 - g(h) \left( \begin{pmatrix} 2 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \end{pmatrix} \right)} \right) \quad (1)$$

We get,

$$k_i = -1, 3$$

$$x_1 = \begin{pmatrix} 3 \\ 0 \end{pmatrix} + (-1) \begin{pmatrix} 2 \\ 1 \end{pmatrix} \quad (2)$$

$$\Rightarrow x_1 = \begin{pmatrix} 3 \\ 0 \end{pmatrix} + \begin{pmatrix} -2 \\ -1 \end{pmatrix} \quad (3)$$

$$\Rightarrow x_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad (4)$$

$$x_2 = \begin{pmatrix} 3 \\ 0 \end{pmatrix} + (3) \begin{pmatrix} 2 \\ 1 \end{pmatrix} \quad (5)$$

$$\Rightarrow x_2 = \begin{pmatrix} 3 \\ 0 \end{pmatrix} + \begin{pmatrix} 6 \\ 3 \end{pmatrix} \quad (6)$$

$$\Rightarrow x_2 = \begin{pmatrix} 9 \\ 3 \end{pmatrix} \quad (7)$$

As, y cannot be negative, so the point of intersection of the line and the parabola is (9, 3).  
 $\therefore$  The area bounded by the curve  $y = \sqrt{x}$  and line  $x = 2y + 3$  in the 1<sup>st</sup> - *quadrant* is given by:-

$$Area = \int_0^3 \sqrt{x} dx + \int_3^9 \sqrt{x} - \frac{x-3}{2} dx \quad (8)$$

$$Area = \frac{2}{3} \left[ x^{\frac{3}{2}} \right]_0^9 - \frac{1}{2} \left( \left( \frac{9^2}{2} - 27 \right) - \left( \frac{3^2}{2} - 9 \right) \right) \quad (9)$$

$$Area = \frac{2}{3} ((27) - (0)) - \frac{1}{2} \left( \frac{27}{2} - \frac{-9}{2} \right) \quad (10)$$

$$Area = 18 - 9 \quad (11)$$

$$Area = 9 \quad (12)$$

So, the required area is 9 units.

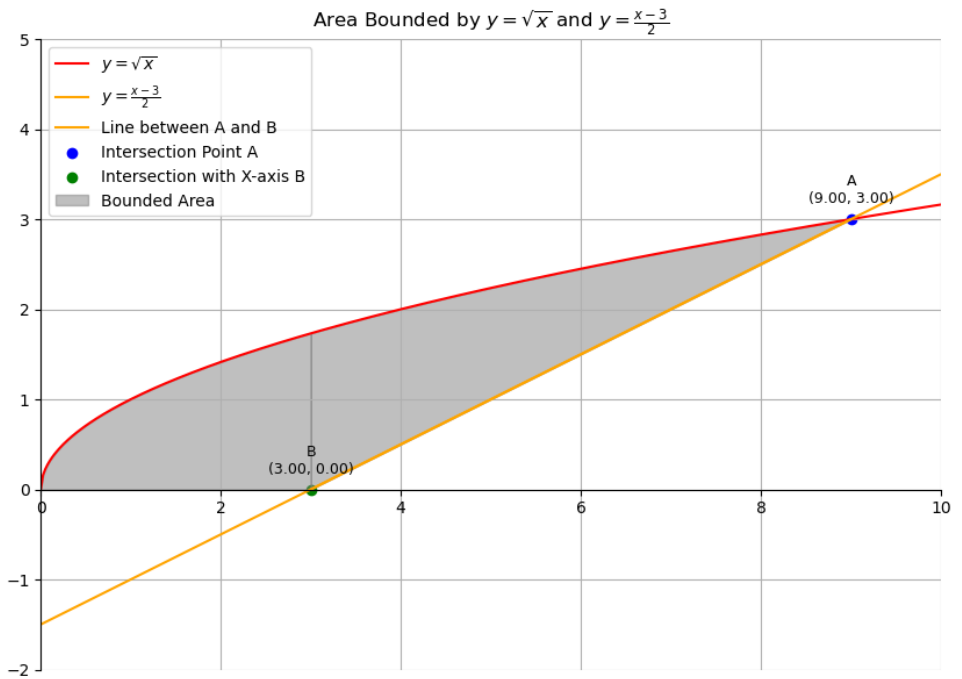


Fig. 0: Intersection of line and parabola