20/07/2021-Shift 1

EE24BTECH11021 - Eshan Ray

1) The logical equivalence of the Boolean expression $(p \wedge \sim q) \implies (q \vee \sim p)$ is equivalent to : [Jul-2021]
   a) $q \implies p$
   b) $p \implies q$
   c) $\sim q \implies p$
   d) $p \implies \sim q$
2) Let a be a positive real number such that $\int_0^a e^{x-[x]}\, dx = 10e - 9$ where $[x]$ is the greatest integer less than or equal to x. Then a is equal to : [Jul-2021]
   a) $10 - \log_e (1 + e)$
   b) $10 + \log_e 2$
   c) $10 + \log_e 3$
   d) $10 + \log_e (1 + e)$
3) The mean of 6 numbers is 6.5 and its variance is 10.25. If 4 numbers are $2, 4, 5\, and\, 7$, then find the other two. [Jul-2021]
   a) $10, 11$
   b) $3, 18$
   c) $8, 13$
   d) $1, 20$
4) The value of the integral $\int_{-1}^{1} \log_e \left(\sqrt{1-x} + \sqrt{1+x} \right) dx$ is equal to : [Jul-2021]
   a) $\frac{1}{2} \log_e 2 + \frac{\pi}{4} - \frac{3}{2}$
   b) $2 \log_e 2 + \frac{\pi}{4} - 1$
   c) $\log_e 2 + \frac{\pi}{2} - 1$
   d) $2 \log_e 2 + \frac{\pi}{4} - \frac{1}{2}$
5) If the roots of the quadratic equation $x^2 + 3^{\frac{1}{4}} x + 3^{\frac{1}{2}} = 0$ are α and β, then the value of $\alpha^{96} \left(\alpha^{12} - 1\right) + \beta^{96} \left(\beta^{12} - 1\right)$ [Jul-2021]
   a) $50 \cdot 3^{24}$
   b) $51 \cdot 3^{24}$
   c) $52 \cdot 3^{24}$
   d) $104 \cdot 3^{24}$
6) Let $A = \begin{pmatrix} 2 & 3 \\ a & 0 \end{pmatrix}, a \in R$ can be wriiten as $P + Q$ where P is a symmetric matrix and Q is a skew symmetric matrix. If $det(Q) = 9$, then the modulus of the sum of all possible values of determinant of P is equal to : [Jul-2021]
   a) 36
   b) 24
   c) 45
   d) 18

7) If z and ω are two complex numbers such that $|z\omega| = 1$ and $\arg z - \arg \omega = \frac{3\pi}{2}$, then $\arg\left(\frac{1-2\bar{z}\omega}{1+3\bar{z}\omega}\right)$ is :
($Here\, arg\,(z)$ $denotes\, the\, principal\, argument\, of\, complex\, number\, z$) [Jul-2021]

a) $\frac{\pi}{4}$
b) $-\frac{3\pi}{4}$
c) $-\frac{\pi}{4}$
d) $\frac{3\pi}{4}$

8) In $\triangle ABC$, if $AB = 5, \angle B = \cos^{-1}\left(\frac{3}{5}\right)$ and the radius of circumcircle of triangle is 5 units, then the area ($in\, sq.\, units$) of $\triangle ABC$ is : [Jul-2021]

a) $10 + 6\sqrt{2}$
b) $8 + 2\sqrt{2}$
c) $6 + 8\sqrt{3}$
d) $4 + 2\sqrt{3}$

9) Let $[x]$ denote the greatest integer $\leq x$, where $x \in R$. If the domain of the real valued function $f(x) = \sqrt{\frac{|[x]|-2}{|[x]|-3}}$ is $(-\infty, a) \cup [b, c) \cup [4, \infty)$, $a<b<c$, then the value of $a+b+c$ is : [Jul-2021]

a) 8
b) 1
c) -2
d) -3

10) Let $y = y(x)$ be the solution of differential equation $x\tan\left(\frac{y}{x}\right) = \left(y\tan\left(\frac{y}{x}\right) - x\right)dx, -1 \leq x \leq 1, y\left(\frac{1}{2}\right) = \frac{\pi}{6}$. Then the area of the region bounded by the curves $x = 0, x = \frac{1}{\sqrt{2}}$ and $y = y(x)$ in the upper half plane is : [Jul-2021]

a) $\frac{1}{8}(\pi - 1)$
b) $\frac{1}{12}(\pi - 3)$
c) $\frac{1}{4}(\pi - 2)$
d) $\frac{1}{6}(\pi - 1)$

11) Find the coefficient of x^{256} in $(1-x)^{101}\left(x^2 + x + 1\right)^{100}$ is: [Jul-2021]

a) $\binom{100}{16}$
b) $\binom{100}{15}$
c) $-\binom{100}{16}$
d) $-\binom{100}{15}$

12) Let $A = \left[a_{ij}\right]$ be a 3×3 matrix, where $a_{ij} \begin{cases} 1 & , \text{if } i = j \\ -x & , \text{if } |i-j| = 1 \\ 2x+1 & , \text{otherwise} \end{cases}$

Let a function $f: R \to R$ be defined as $f(x) = det(A)$. Then the sum of maximum and minimum values of f on R is equal to : [Jul-2021]

a) $-\frac{20}{27}$
b) $\frac{88}{27}$
c) $\frac{20}{27}$

d) $-\frac{88}{27}$

13) Let $\overrightarrow{a} = 2\hat{i}+\hat{j}-2\hat{k}$ and $\overrightarrow{b} = \hat{i}+\hat{j}$. If $\overrightarrow{c}$ is a vector such that $\overrightarrow{a}\cdot\overrightarrow{c} = \left|\overrightarrow{c}\right|, \left|\overrightarrow{c} - \overrightarrow{a}\right| = 2\sqrt{2}$ and the angle between $\left(\overrightarrow{a} \times \overrightarrow{b}\right)$ and $\overrightarrow{c}$ is $\frac{\pi}{6}$, then the value of $\left|\left(\overrightarrow{a} \times \overrightarrow{b}\right) \times \overrightarrow{c}\right|$ is : [Jul-2021]

a) $\frac{2}{3}$
b) 4
c) 3
d) $\frac{3}{2}$

14) The number of solutions of $\tan^{-1}\sqrt{x(x+1)} + \sin^{-1}\sqrt{x^2+x+1} = \frac{\pi}{4}$ is [Jul-2021]

a) 1
b) 2
c) 4
d) 0

15) Let $y = y(x)$ be the solution of the differential equation $e^x\sqrt{1-y^2}dx + \left(\frac{y}{x}\right)dy = 0, y(1) = -1$. Then the value of $(y(3))^2$ is equal to : [Jul-2021]

a) $1 - 4e^3$
b) $1 - 4e^6$
c) $1 + 4e^3$
d) $1 + 4e^6$