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EE24BTECH11021 - Eshan Ray

14) For $n \in Z$, define $c_n = \frac{1}{\sqrt{2\pi}} \int_{-\pi}^{\pi} e^{i(n-i)x}\, dx$, where $i^2 = -1$. Then $\sum_{n \in Z} |c_n|^2$ equals
    a) $\cosh(\pi)$
    b) $\sinh(\pi)$
    c) $\cosh(2\pi)$
    d) $\sinh(2\pi)$
15) If the fourth order divided difference of $f(x) = \alpha x^4 + 5x^3 + 3x + 2, \alpha \in R$, at the points $0.1, 0.2, 0.3, 0.4, 0.5$ is 5, then α equals $\dots$
16) If the quadrature rule $\int_0^2 f(x)\, dx \approx c_1 f(0) + 3f(c_2)$, where $c_1, c_2 \in R$, is exact for all polynomials of degree ≤ 1, then $c_1 + 3c_2$ equals $\dots$
17) If $u(x, y) = 1 + x + y + f(xy)$, where $f\colon R^2 \to R$ is a differential function, then u satisfies
    a) $x\frac{\partial u}{\partial x} - y\frac{\partial u}{\partial y} = x^2 - y^2$
    b) $x\frac{\partial u}{\partial x} - y\frac{\partial u}{\partial y} = 0$
    c) $x\frac{\partial u}{\partial x} - y\frac{\partial u}{\partial y} = x - y$
    d) $y\frac{\partial u}{\partial x} - x\frac{\partial u}{\partial y} = x - y$
18) The partial differential equation $x\frac{\partial^2 u}{\partial x^2} + (x - y)\frac{\partial^2 u}{\partial x \partial y} - y\frac{\partial^2 u}{\partial y^2} + \frac{1}{4}\left(\frac{\partial u}{\partial y} - \frac{\partial u}{\partial x}\right) = 0$ is
    a) hyperbolic along the line $x + y = 0$
    b) elliptic along the line $x - y = 0$
    c) elliptic along the line $x + y = 0$
    d) parabolic along the line $x + y = 0$
19) Let X and Y be topological spaces and let $f\colon X \to Y$ be a continuous surjective function. Which one of the following statements is TRUE?
    a) If X is separable, then Y is separable
    b) If X is first countable, then Y is first countable
    c) If X is Hausdorff, then Y is Hausdorff
    d) If X is regular, then Y is regular
20) Consider the topology $\mathcal{T} = \{U \subseteq Z\colon Z \backslash U\, is\, finite\, or\, 0 \neq U\}$ on Z. Then, the topological space $(Z, \mathcal{T})$ is
    a) compact but NOT connected
    b) connected but NOT compact
    c) both compact and connected
    d) neither compact nor connected
21) Let $F(x)$ be the distribution function of a random variable X. Consider the functions :
    $G_1(x) = (F(x))^3, x \in R$
    $G_2(x) = 1 - (1 - F(x))^5, x \in R$

Which of the above functions are distribution functions?

a) Neither G_1 nor G_2
b) Only G_1
c) Only G_2
d) Both G_1 and G_2

22) Let $X_1, X_2, \ldots, X_n$ $(n \geq 2)$ be independent and identically distributed random variables with finite variance σ^2 and let $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$. Then the covariance between $\bar{X}$ and $X_1 - \bar{X}$ IS

a) 0
b) $-\sigma^2$
c) $\frac{-\sigma^2}{n}$
d) $\frac{\sigma^2}{n}$

23) Let $X_1, X_2, \ldots, X_n$ $(n \geq 2)$ be a random sample from a $N\left(\mu, \sigma^2\right)$ population, where $\sigma^2 = 144$. The smallest n such that the length of the shortest 95% confidence interval for μ will not exceed 10 is …

24) Consider the linear programming problem (LPP):

Maximize $4x_1 + 6x_2$

subject to $x_1 + x_2 \leq 8,$

$2x_1 + 3x_2 \geq 18,$

$x_1 \geq 6, x_2$ *is unrestricted in sign.*

Then the LPP has

a) no optimal solution
b) only one basic feasible solution and that is optimal
c) more than one basic feasible solution and a unique optimal solution
d) infinitely many optimal solutions

25) For a linear programming problem (LPP) and its dual, which one of the following is NOT TRUE?

a) The dual of the dual is primal
b) If the primal LPP has an unbounded objective function, then the dual LPP is infeasible
c) If the primal LPP is infeasible, then the dual LPP has an unbounded objective function
d) If the primal LPP has a finite optimal solution, then the dual LPP also has a finite optimal solution

26) If U and V are the null spaces of $\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$ and $\begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & 1 & 2 & 1 \end{pmatrix}$, respectively, then the dimension of the subspace $U + V$ equals …