29/01/2024-Shift 1

EE24BTECH11021 - Eshan Ray

16) Let PQR be a triangle with $R(-1,4,2)$. Suppose $M(2,1,2)$ is the mid-point of PQ. The distance of the centroid of $\triangle PQR$ from the point of intersection of the lines $\frac{x-2}{0} = \frac{y}{2} = \frac{z+3}{-1}$ and $\frac{x-1}{1} = \frac{y+3}{-3} = \frac{z+1}{1}$ is
   a) $\sqrt{99}$
   b) 9
   c) $\sqrt{69}$
   d) 69
17) Let $\overrightarrow{a}, \overrightarrow{b}$ and $\overrightarrow{c}$ be three non-zero vectors such that $\overrightarrow{b}$ and $\overrightarrow{c}$ are non-collinear. If $\overrightarrow{a} + 5\overrightarrow{b}$ is collinear with $\overrightarrow{c}$, $\overrightarrow{b} + 6\overrightarrow{c}$ is collinear with $\overrightarrow{a}$ and $\overrightarrow{a} + \alpha\overrightarrow{b} + \beta\overrightarrow{c} = \overrightarrow{0}$, then $\alpha + \beta$ is equal to
   a) -25
   b) 35
   c) -30
   d) 30
18) If $z = \frac{1}{2} - 2i$ is such that $|z+1| = \alpha z + \beta(1+i)$, $i = \sqrt{-1}$ and $\alpha, \beta \in R$, then $\alpha + \beta$ is equal to
   a) -1
   b) -4
   c) 2
   d) 3
19) Let O be the origin and the position vectors of A and B be $2\hat{i} + 2\hat{j} + \hat{k}$ and $2\hat{i} + 4\hat{j} + 4\hat{k}$ respectively. If the internal bisector of $\angle AOB$ meets the line AB at C, then the length of OC is
   a) $\frac{3}{2}\sqrt{34}$
   b) $\frac{3}{2}\sqrt{31}$
   c) $\frac{2}{3}\sqrt{34}$
   d) $\frac{2}{3}\sqrt{31}$
20) If the value of the integral $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\frac{x^2 \cos x}{1+\pi^x} + \frac{1+\sin^2 x}{1+e^{\sin x^{2023}}}\right) dx = \frac{\pi}{4}(\pi + a) - 2$, then the value of a is
   a) 2
   b) $-\frac{3}{2}$
   c) $\frac{3}{2}$
   d) 3
21) A line with direction ratio $2, 1, 2$ meets the lines $x = y + 2 = z$ and $x + 2 = 2y = 2z$ respectively at points P and Q. If the length of the perpendicular from the point $(1, 2, 12)$ to the line PQ is l, then l^2 is ...

22) The area ($in\, sq.\, units$) of the part of the circle $x^2+y^2=169$ which is below the line $5x-y=13$ is $\frac{\pi\alpha}{2\beta}-\frac{65}{2}+\frac{\alpha}{\beta}\sin^{-1}\left(\frac{12}{13}\right)$, where α,β are coprime numbers. Then $\alpha+\beta$ is equal to ...
23) If the solution curve $y=y(x)$ to the differential equation $\left(1+y^2\right)(+\log_e x)\,dx+xdy=0, x>0$ passes through the point $(1,1)$ and $y(e)=\frac{\alpha-\tan\left(\frac{3}{2}\right)}{\beta+\tan\left(\frac{3}{2}\right)}$, then $\alpha+2\beta$ is ...
24) If the mean and variance of the data $65,68,58,44,48,45,60,\alpha,\beta,60 where \alpha>\beta$ are 56 and 66.2 respectively, then $\alpha^2+\beta^2$ is equal to...
25) If $\frac{\binom{11}{1}}{2}+\frac{\binom{11}{2}}{3}+\cdots+\frac{\binom{11}{9}}{10}=\frac{n}{m}$with $gcd\,(m,n)=1$, then $m+n$ is equal to ...
26) If the points of intersection of two conics $x^2+y^2=4b$ and $\frac{x^2}{16}+\frac{y^2}{b^2}=1$ lie on the curve $y^2=3x^2$, then $3\sqrt{3}$ times the area of the rectangle formed by the intersection points is ...
27) Let α,β be the roots of the equation $x^2-x+2=0$ with $Im\,(\alpha)>Im\,(\beta)$. Then $\alpha^6+\alpha^4+\beta^4-5\alpha^2$ is equal to ...
28) Equations of two diameters of a circle are $2x-3y=5$ and $3x-4y=7$. The line joining the points $\left(-\frac{22}{7},-4\right)$ and $\left(-\frac{1}{7},3\right)$ intersects the circle at only one point $P\,(\alpha,\beta)$. Then, $17\beta-\alpha$ is equal to ...
29) All the letters of the word "$GTWENTY$" are written in all possible ways with or without meaning and these words are written as in a dictionary. The serial number of the word "$GTWENTY$" is ...
30) Let $f\,(x)=2^x-x^2, x\in R$. If m and n are respectively the number of points t which the curves $y=f\,(x)$ and $y=f\prime\,(x)$ intersect the $x-axis$ then the value of $m+n$ is ...