

1) If vectors $\vec{a}_1 = x\hat{i} - \hat{j} + \hat{k}$ and $\vec{a}_2 = \hat{i} + y\hat{j} + z\hat{k}$ are collinear, then a possible unit vector parallel to the vector parallel to the vector $x\hat{i} + y\hat{j} + z\hat{k}$ is :

- a) $\frac{1}{\sqrt{2}}(-\hat{j} + \hat{k})$
- b) $\frac{1}{\sqrt{2}}(\hat{i} - \hat{j})$
- c) $\frac{1}{\sqrt{3}}(\hat{i} - \hat{j} + \hat{k})$
- d) $\frac{1}{\sqrt{3}}(\hat{i} + \hat{j} - \hat{k})$

2) Let $A = \{1, 2, 3, \dots, 10\}$ and $f: A \rightarrow A$ be defined as $f(k) = \begin{cases} k+1 & \text{if } k \text{ is odd} \\ k & \text{if } k \text{ is even} \end{cases}$
Then the number of possible functions $g: A \rightarrow A$ such that $g \circ f = f$ is :

- a) 10^5
- b) $\binom{10}{5}$
- c) 5^5
- d) $5!$

3) Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined as $f(x) = \begin{cases} 2 \sin\left(-\frac{\pi x}{2}\right) & \text{if } x < -1 \\ |ax^2 + x + b|, & \text{if } -1 \leq x \leq 1 \\ \sin(\pi x) & \text{if } x > 1 \end{cases}$
If $f(x)$ is continuous on \mathbb{R} , then $a + b$ equals :

- a) 3
- b) -1
- c) -3
- d) 1

4) For $x > 0$, if $f(x) = \int_1^x \frac{\log_e t}{1+t} dt$, then $f(e) + f\left(\frac{1}{e}\right)$ is equal to :

- a) $\frac{1}{2}$
- b) -1
- c) 1
- d) 0

5) A natural number has prime factorization given by $n = 2^x 3^y 5^z$, where y and z are such that $y + z = 5$ and $y^{-1} + z^{-1} = \frac{5}{6}$, $y > z$. Then the number of odd divisors of n , including 1, is :

- a) 11
- b) 6
- c) $6x$
- d) 12

6) Let $f(x) = \sin^{-1} x$ and $g(x) = \frac{x^2 - x - 2}{2x^2 - x - 6}$. If $g(2) = \lim_{x \rightarrow 2} g(x)$, then the domain of the function $f \circ g$ is :

- a) $(\infty, -2] \cup \left[-\frac{3}{2}, \infty\right)$
- b) $(\infty, -2] \cup [-1, \infty)$
- c) $(\infty, -2] \cup \left[-\frac{4}{3}, \infty\right)$
- d) $(\infty, -1] \cup [2, \infty)$

7) The triangle of maximum area which can be inscribed in a given circle of radius ' r ' is :

- a) An isosceles triangle with base equal to $2r$
- b) An equilateral triangle of height $\frac{2r}{3}$
- c) An equilateral triangle having each of its side of length $\sqrt{3}r$
- d) A right angle triangle having two of its sides of length $2r$ and r

8) Let L be a line obtained from the intersection of two planes $x + 2y + z = 6$ and $y + 2z = 4$. If point $P(\alpha, \beta, \gamma)$ is the foot of perpendicular from $(3, 2, 1)$ on L , then the value of $21(\alpha, \beta, \gamma)$ equals :

- a) 142
- b) 68
- c) 136
- d) 102

9) Let $F_1(A, B, C) = (A \wedge \sim B) \vee [\sim C \wedge (A \vee B)] \vee \sim A$ and $F_2(A, B) = (A \vee B) \vee (B \rightarrow \sim A)$ be two logical expressions. Then:

- a) F_1 is not a tautology but F_2 is a tautology
- b) F_1 is a tautology but F_2 is not a tautology
- c) F_1 and F_2 both are tautologies
- d) Both F_1 and F_2 are not tautologies

10) Let the slope of the tangent line to a curve at any point $P(x, y)$ be given by $\frac{xy^2 + y}{x}$. If the curve intersects the line $x + 2y = 4$ at $x = -2$, then the value of y , for which the point $(3, y)$ lies on the curve, is:

- a) $-\frac{18}{11}$
- b) $-\frac{18}{19}$
- c) $-\frac{4}{3}$
- d) $\frac{18}{35}$

11) If the locus of the mid-point of the line segment from the point $(3, 2)$ to a point on the circle, $x^2 + y^2 = 1$ is a circle of the radius r , then r is equal to :

- a) $\frac{1}{4}$
- b) $\frac{1}{2}$
- c) 1
- d) $\frac{1}{3}$

12) Consider the following system of equations :

$$x + 2y - 3z = a$$

$$2x + 6y - 11z = b$$

$$x - 2y + 7z = c,$$

Where a, b and c are real constants. Then the system of equations :

- a) has a unique solution when $5a = 2b + c$

- b) has infinite number of solutions when $5a = 2b + c$
 c) has no solution for all a, b and c
 d) has a unique solution for all a, b and c
- 13) If $0 < a, b < 1$, and $\tan^{-1} a + \tan^{-1} b = \frac{\pi}{4}$, then the value of $(a - b) - \left(\frac{a^2 + b^2}{2}\right) + \left(\frac{a^3 + b^3}{3}\right) - \left(\frac{a^4 + b^4}{4}\right) + \dots$ is:
 a) $\log_e 2$
 b) $\log_e \frac{e}{2}$
 c) e
 d) $e^2 - 1$
- 14) The sum of the series $\sum_{n=1}^{\infty} \frac{n^2 + 6n + 10}{(2n+1)!}$ is equal to :
 a) $\frac{41}{8}e + \frac{19}{8}e^{-1} - 10$
 b) $-\frac{41}{8}e + \frac{19}{8}e^{-1} - 10$
 c) $\frac{41}{8}e - \frac{19}{8}e^{-1} - 10$
 d) $\frac{41}{8}e + \frac{19}{8}e^{-1} + 10$
- 15) Let $f(x)$ be a differentiable function at $x = a$ with $f'(a) = 2$ and $f(a) = 4$. Then $\lim_{x \rightarrow a} \frac{xf(a) - af(x)}{x - a}$ equals:
 a) $2a + 4$
 b) $2a - 4$
 c) $4 - 2a$
 d) $a + 4$