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EE24BTECH11021 - Eshan Ray

14) Consider a real vector space V of dimension n and a non-zero linear transformation $T\colon V \to V$. If dimension $(T(V))<n$ and $T^2 = \lambda T$, for some $\lambda \in R\setminus\{0\}$, then which of the following statements is TRUE?
   a) determinant$(T) = |\lambda|^n$
   b) There exists a non-trivial subspace V_1 of V such that $T(X) = 0$ for all $X \in V_1$
   c) T is invertible
   d) λ is the only eigenvalue of T
15) Let $S = [0,1) \cup [2,3]$ and $f\colon S \to R$ be a strictly increasing function such that $f(S)$ is connected. Which of the following statements is TRUE?
   a) f has exactly two discontinuity
   b) f has exactly two discontinuities
   c) f has infinitely many discontinuities
   d) f is continuous
16) Let $a_1 = 1$ and $a_n = a_{n-1} + 4, n \geq 2$. Then,

$$\lim_{n\to\infty}\left[\frac{1}{a_1a_2} + \frac{1}{a_2a_3} + \cdots + \frac{1}{a_{n-1}a_n}\right]$$

   is equal to ...
17) Maximum $\left\{x + y\colon\ (x,y) \in \overline{B(0,1)}\right\}$ is equal to ...
18) Let $a,b,c,d \in R$ such that $c^2 + d^2 \neq 0$. Then, the Cauchy problem

$$au_x + bu_y = e^{x+y}, \quad x, y \in R$$

$$u(x,y) = 0 \ on\ cx + dy = 0$$

   has a unique solution if
   a) $ac + bd \neq 0$
   b) $ad - bc \neq 0$
   c) $ac - bd \neq 0$
   d) $ad + bc \neq 0$
19) Let $u(x,y)$ be the d'Alembert's solution of the initial value problem for the wave equation

$$u_{tt} - c^2u_{xx} = 0$$

$$u(x,0) = f(x),\ u_t(x,0) = g(x),$$

   where c is a positive real number and f, g are smooth odd functions. Then, $u(0,1)$ is equal to ...

20) Let the probability density function of a random variable X be

$$f(x)\begin{cases} x & 0 \le x < \frac{1}{2} \\ c(2x-1)^2 & \frac{1}{2} < x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

Then, the value of c is equal to ...

21) Let V be the set of all solutions of the equation $y\prime\prime + ay\prime + by = 0$ satisfying $y(0) = y(1)$, where a, b are positive real numbers. Then, dimension(V) is equal to...

22) Let $y\prime\prime + p(x)y\prime + q(x)y = 0, x \in (-\infty, \infty)$, where $o(x)$ and $q(x)$ are continuous functions. If $y_1(x) = \sin(x) - 2\cos(x)$ and $y_2(x) = 2\sin(x) + \cos(x)$ are two linearly independent solutions of the above, equation, then $|4p(0) + 2q(1)|$ is equal to ...

23) Let $P_n(x)$ be the Legendre polynomial of degree n and $I = \int_{-1}^{1} x^k P_n(x)\, dx$, where k is a non-negative integer. Consider the following statements P and Q:
(P) : $I = 0$ if $k < n$.
(Q) : $I = 0$ if $n - k$ is an odd integer.

Which of the above statements hold TRUE?

a) both P and Q
b) only P
c) only Q
d) Neither P nor Q

24) Consider the following statements P and Q:
(P) : $x^2y\prime\prime + xy\prime + \left(x^2 - \frac{1}{4}\right)y = 0$ has two linearly independent Frobenius series solution near $x = 0$.
(Q) : $x@y\prime\prime + 3\sin(x)y\prime + y = 0$ has two linearly independent Frobenius series solution near $x = 0$.

Which of the above statements hold TRUE?

a) both P and Q
b) only P
c) only Q
d) Neither P nor Q

25) Let the polynomial x^4 be approximated by a polynomial of degree ≤ 2, which interpolates x^4 at $x = -1, 0$ and 1 .Then, the maximum absolute interpolation error over the interval $[-1, 1]$ is equal to ...

26) Let (z_n) be a sequence of distinct points in $D(0,1) = \{z \in C\colon |z| < 1\}$ with $\lim_{n\to\infty} z_n = 0$. Consider the following statements P and Q:
(P) : There exists a unique analytical function f on $D(0,1)$ such that $f(z_n) = \sin(z_n)$ for all n.
(Q) : There exists an analytical function f on $D(0,1)$ such that $f(z_n) = 0$ if n is even and $f(z_n) = 1$ if n is odd.

Which of the above statements hold TRUE?

a) both P and Q
b) only P
c) only Q
d) Neither P nor Q