EE24BTECH11021 - Eshan Ray

Question:

If the distances of $\mathbf{P} = (x, y)$ from $\mathbf{A} = (5, 1)$ and $\mathbf{B} = (-1, 5)$ are equal, then prove that 3x = 2y.

Solution:

Variable	Description
A (5, 1)	coordinates of first point
B(-1,5)	coordinates of second point
$\mathbf{P}(x,y)$	Equidistant point of A and B

TABLE 0: Input parameters

$$\|\mathbf{B} - \mathbf{P}\|^2 = \|\mathbf{A} - \mathbf{P}\|^2 \tag{1}$$

$$\implies \|\mathbf{B} - \mathbf{P}\|^2 = (\mathbf{B} - \mathbf{P})(\mathbf{B} - \mathbf{P})^{\mathsf{T}} \tag{2}$$

$$\implies \mathbf{B}^2 + \mathbf{P}^2 - 2\mathbf{P}\mathbf{B}^\top = \mathbf{A}^2 + \mathbf{P}^2 - 2\mathbf{P}\mathbf{A}^\top$$
 (3)

$$\implies \mathbf{P}(\mathbf{A}^{\top} - \mathbf{B}^{\top}) = \frac{\mathbf{A}^2 - \mathbf{B}^2}{2} \tag{4}$$

$$\implies \mathbf{P}(\begin{pmatrix} 5 & 1 \end{pmatrix} - \begin{pmatrix} -1 & 5 \end{pmatrix}) = \frac{26 - 26}{2} \tag{5}$$

$$\implies \binom{x}{y} (6 - 4) = 0 \tag{6}$$

$$\implies 6x - 4y = 0 \tag{7}$$

$$\implies 3x = 2y \tag{8}$$

Hence, proved.

1

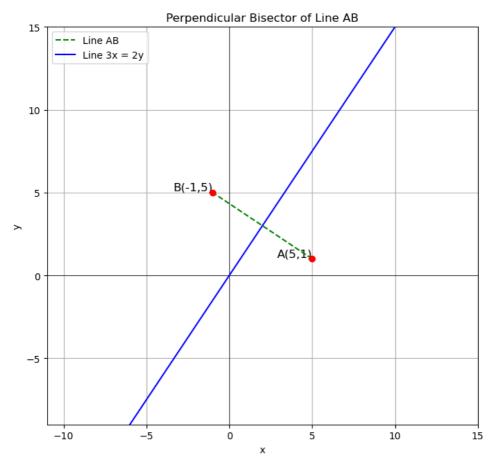


Fig. 0