2015-MA- 53-65

EE24BTECH11021 - Eshan Ray

53) Let $H = \left\{(x_n) \in l_2 \colon \sum_{n=1}^{\infty} \frac{x_n}{n} = 1\right\}$. Then H
   a) is bounded
   b) is closed
   c) is a subspace
   d) has an interior point
54) Let V be a closed subspace of $L^2[0,1]$ be given by $f(x) = x$ and $g(x) = x^2$. If $V^{\perp} = Span\{f\}$ and Pg is the orthogonal projection of g on V, then $(g - Pg)(x), x \in [0,1]$, is
   a) $\frac{3}{4}x$
   b) $\frac{1}{4}x$
   c) $\frac{3}{4}x^2$
   d) $\frac{1}{4}x^2$
55) Let $p(x)$ be the polynomial of degree at most 3 that passes through the points $(-2,12), (-1,1), (0,2)$ and $(2,-8)$. Then the coefficient of x^3 in $p(x)$ is equal to ...
56) If, for some $\alpha, \beta \in R$, the integration formula

$$\int_0^2 p(x)\, dx = p(\alpha) + p(\beta)$$

   holds for all polynomials $p(x)$ of degree at most 3, then the value of $3(\alpha - \beta)^2$ is equal to ...
57) Let $y(t)$ be a continuous function on $[0, \infty)$ whose Laplace transform exists. If $y(t)$ satisfies

$$\int_0^t (1 - \cos(t - \tau))\, y(\tau)\, d\tau = t^4,$$

   then $y(1)$ is equal to ...
58) Consider the initial value problem

$$x^2 y'' - 6y = 0, \quad y(1) = \alpha, y'(1) = 6.$$

   if $y(x) \to 0$ as $x \to 0^+$, then α is equal to ...
59) Define $f_1, f_2 \colon [0,1] \to R$ by

$$f_1(x) = \sum_{n=1}^{\infty} \frac{x \sin\left(n^2 x\right)}{n^2} \; and \; f_2(x) \sum_{n=1}^{\infty} x^2 \left(1 - x^2\right)^{n-1}$$

   Then
   a) f_1 is continuous but f_2 is NOT continuous
   b) f_2 is continuous but f_1 is NOT continuous
   c) both f_1 and f_2 are continuous

d) neither f_1 nor f_2 is continuous

60) Consider the unit sphere $S = \left\{(x,y,z) \in R^3 : x^2 + y^2 + z^2 = 1\right\}$ and the unit norm vector $\hat{n} = (x,y,z)$ at each point (x,y,z) on S.The value of the surface integral

$$\int\int_S \left\{\left(\frac{2x}{\pi} + \sin\left(y^2\right)\right)x + \left(e^z - \frac{y}{\pi}\right)y + \left(\frac{2z}{\pi} + \sin^2 y\right)z\right\} d\sigma$$

is equal to

61) Let $D = \left\{(x,y) \in R^2 : 1 \le x \le 1000, 1 \le y \le 1000\right\}$. Define

$$f(x,y) = \frac{xy}{2} + \frac{500}{x} + \frac{500}{y}$$

Then the minimum value of f on D is equal to ...

62) Let $D = \{z \in C : |z| < 1\}$. Then there exists a non-constant analytic function f on D such that for all $n = 2,3,4,\ldots$

a) $f\left(\frac{\sqrt{-1}}{n}\right) = 0$

b) $f\left(\frac{1}{n}\right) = 0$

c) $f\left(1 - \frac{1}{n}\right) = 0$

d) $f\left(\frac{1}{2} - \frac{1}{n}\right) = 0$

63) Let $\sum_{n=-\infty}^{\infty} a_n z^n$ be the Laurent series expansion of $f(z) = \frac{1}{2z^2-13z+15}$ in the annulus $\frac{3}{2} < |z| < 5$. Then $\frac{a_1}{a_2}$ is equal to ...

64) The value of $\frac{i}{4-\pi}\int_{|z|=4} \frac{dz}{z\cos(x)}$ is equal to ...

65) Suppose that among all continuously differentiable functions $y(x), x \in R$, with $y(0) = 0$ and $y(1) = \frac{1}{2}$, the function $y_0(x)$ minimizes the functional

$$\int_0^1 \left(e^{-(y\prime - x)} + (1+y)\,y\prime\right) dx.$$

Then $y_0\left(\frac{1}{2}\right)$ is equal to

a) 0

b) $\frac{1}{8}$

c) $\frac{1}{4}$

d) $\frac{1}{2}$