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EE24BTECH11021 - Eshan Ray

27) Let $f, g\colon\ [0,1] \to R$ be defined by

$$f(x) = \begin{cases} x \text{ if} x = \frac{1}{n}\, for\, n \in N \\ 0 \text{ otherwise} \end{cases}$$

and

$$g(x) \begin{cases} 1 \text{ if } x \in Q \cap\ [0,1] \\ 0 \text{ otherwise} \end{cases}$$

Then

a) Both f and g are Riemann integral
b) f is a Riemann integrable and g is Lebesgue integrable
c) g is Riemann integrable and f is Lebesgue integrable
d) Neither f nor g is Riemann integrable

28) Consider the following linear programming problem:

$$\begin{array}{lr} \text{Maximize} & x + 3y + 6z - w \\ \text{subject to} & 5x + y + 6z + 7w \le 20, \\ & 6x + 2y + 2z + 9w \le 40, \\ & x \ge 0,\ y \ge 0,\ z \ge 0,\ w \ge 0. \end{array}$$

Then the optimal value is ...

29) Suppose X is real-valued real random variable. Which of the following values *CANNOT* be attained by $E[X]$ and $E\left[X^2\right]$, respectively?

a) 0 and 1
b) 2 and 3
c) $\frac{1}{2}$ and $\frac{1}{3}$
d) 2 and 5

30) Which of the following subsets of R^2 is NOT compact?

a) $\left\{(x,y) \in R^2\colon\ -1 \le x \le 1,\ y = \sin x\right\}$
b) $\left\{(x,y) \in R^2\colon\ -1 \le y \le 1,\ y = x^8 - x^3 - 1\right\}$
c) $\left\{(x,y) \in R^2\colon\ y = 0,\ \sin(e)^{-x} = 0\right\}$
d) $\left\{(x,y) \in R^2\colon\ x{>}0, y = \sin\left(\frac{1}{x}\right)\right\} \cap \left\{(x,y) \in R^2\colon\ x{>}0, y = \frac{1}{x}\right\}$

31) Let M be the real space vector of 2×3 matrices with real entries. Let $t\colon M \to M$ be defined by

$$T\left(\begin{pmatrix} x_1 & x_2 & x_3 \\ x_4 & x_5 & x_6 \end{pmatrix}\right) = \begin{pmatrix} -x_6 & x_4 & x_1 \\ x_3 & x_5 & x_2 \end{pmatrix}$$

The determinant of T is ...

32) Let $\mathcal{H}$ be a Hilbert space and let $\{e_n : n \geq 1\}$ be an orthonormal basis of $\mathcal{H}$. Suppose $T : \mathcal{H} \to \mathcal{H}$ is a bounded linear operator. Which of the following *CANNOT* be true?

a) $T(e_n) = e_1$ for all $n \geq 1$
b) $T(e_n) = e_{n+1}$ for all $n \geq 1$
c) $T(e_n) = \frac{n+1}{n} e_n$ for all $n \geq 1$
d) $T(e_n) = e_{n-1}$ for all $n \geq 2$ and $T(e_1) = 0$

33) The value of the limit

$$\lim_{n\to\infty} \frac{2^{-n^2}}{\sum_{k=n+1}^{\infty} 2^{-k^2}}$$

is

a) 0
b) some $c \in (0,1)$
c) 1
d) ∞

34) Let $f : C \setminus \{3i\} \to C$ be defined by $f(z) = \frac{z-i}{iz+3}$. Which of the following statements about f is FALSE ?

a) f is conformal on $C \setminus \{3i\}$
b) f maps circles in $C \setminus \{3i\}$ onto circles in C
c) All the fixed points of f are in the region $\{z \in C : Im(z) > 0\}$
d) There is no straight line in $C \setminus \{3i\}$ which is mapped onto a straight line in C by f

35) The matrix $A = \begin{pmatrix} 1 & 2 & 0 \\ 1 & 3 & 1 \\ 0 & 1 & 3 \end{pmatrix}$ can be decomposed uniquely into the product $A = LU$, where $L = \begin{pmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{pmatrix}$ and $U = \begin{pmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{pmatrix}$. The solution of the system $LX = \begin{pmatrix} 1 & 2 & 2 \end{pmatrix}^t$ is

a) $\begin{pmatrix} 1 & 1 & 1 \end{pmatrix}^t$
b) $\begin{pmatrix} 1 & 1 & 0 \end{pmatrix}^t$
c) $\begin{pmatrix} 0 & 1 & 1 \end{pmatrix}^t$
d) $\begin{pmatrix} 1 & 0 & 1 \end{pmatrix}^t$

36) Let $S = \left\{ x \in R : x \geq 0, \sum_{n=1}^{\infty} x^{\sqrt{n}} < \infty \right\}$. Then the supremum of S is

a) 1
b) $\frac{1}{e}$
c) 0
d) ∞

37) The image of the region $\{z \in C : Re(z) > Im(z) > 0\}$ under the mapping $z \mapsto e^{z^2}$ is

a) $\{w \in C : Re(w) > 0, Im(w) > 0\}$
b) $\{w \in C : Re(w) > 0, Im(w) > 0, |w| > 1\}$
c) $\{w \in C : |w| > 1\}$
d) $\{w \in C : Im(w) > 0, |w| > 1\}$

38) Which of the following groups contain a unique normal subgroup of order four?

a) $Z_2 \oplus Z_4$
b) The dihedral group, D_4, of order eight
c) The quaternion group, Q_8
d) $Z_2 \oplus Z_2 \oplus Z_2$

39) Let B be a real symmetric positive-definite $n \times n$ matrix. Consider the inner product on R^n defined by $\langle X, Y \rangle = Y^t BX$. Let A be an $n \times n$ real matrix and let $T : R^n \to R^n$ be the linear operator defined by $T(X) = AX$ for all $X \in R^n$. If S is the adjoint of T, then $S(X) = CX$ for all $X \in R^n$, where C is the matrix

a) $B^{-1}A^tB$
b) BA^tB^{-1}
c) $B^{-1}AB$
d) A^t