29/01/2024-Shift 1

EE24BTECH11021 - Eshan Ray

16) Let PQR be a triangle with $R(-1,4,2)$. Suppose $M(2,1,2)$ is the mid-point of PQ. The distance of the centroid of $\triangle PQR$ from the point of intersection of the lines $\frac{x-2}{0} = \frac{y}{2} = \frac{z+3}{-1}$ and $\frac{x-1}{1} = \frac{y+3}{-3} = \frac{z+1}{1}$ is [Jan-2024]
   a) $\sqrt{99}$
   b) 9
   c) $\sqrt{69}$
   d) 69
17) Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three non-zero vectors such that $\vec{b}$ and $\vec{c}$ are non-collinear. If $\vec{a} + 5\vec{b}$ is collinear with $\vec{c}$, $\vec{b} + 6\vec{c}$ is collinear with $\vec{a}$ and $\vec{a} + \alpha\vec{b} + \beta\vec{c} = \vec{0}$, then $\alpha + \beta$ is equal to [Jan-2024]
   a) -25
   b) 35
   c) -30
   d) 30
18) If $z = \frac{1}{2} - 2i$ is such that $|z+1| = \alpha z + \beta(1+i)$, $i = \sqrt{-1}$ and $\alpha, \beta \in R$, then $\alpha + \beta$ is equal to [Jan-2024]
   a) -1
   b) -4
   c) 2
   d) 3
19) Let O be the origin and the position vectors of A and B be $2\hat{i} + 2\hat{j} + \hat{k}$ and $2\hat{i} + 4\hat{j} + 4\hat{k}$ respectively. If the internal bisector of $\angle AOB$ meets the line AB at C, then the length of OC is [Jan-2024]
   a) $\frac{3}{2}\sqrt{34}$
   b) $\frac{3}{2}\sqrt{31}$
   c) $\frac{2}{3}\sqrt{34}$
   d) $\frac{2}{3}\sqrt{31}$
20) If the value of the integral $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\frac{x^2 \cos x}{1+\pi^x} + \frac{1+\sin^2 x}{1+e^{\sin x^{2023}}} \right) dx = \frac{\pi}{4}(\pi + a) - 2$, then the value of a is [Jan-2024]
   a) 2
   b) $-\frac{3}{2}$
   c) $\frac{3}{2}$
   d) 3
21) A line with direction ratio $2,1,2$ meets the lines $x = y + 2 = z$ and $x + 2 = 2y = 2z$ respectively at points P and Q. If the length of the perpendicular from the point $(1,2,12)$ to the line PQ is l, then l^2 is ... [Jan-2024]

22) The area ($in\, sq.\, units$) of the part of the circle $x^2 + y^2 = 169$ which is below the line $5x - y = 13$ is $\frac{\pi\alpha}{2\beta} - \frac{65}{2} + \frac{\alpha}{\beta}\sin^{-1}\left(\frac{12}{13}\right)$, where α, β are coprime numbers. Then $\alpha + \beta$ is equal to ... [Jan-2024]

23) If the solution curve $y = y(x)$ to the differential equation $\left(1 + y^2\right)(+\log_e x)\, dx + x dy = 0, x > 0$ passes through the point $(1, 1)$ and $y(e) = \frac{\alpha - \tan\left(\frac{3}{2}\right)}{\beta + \tan\left(\frac{3}{2}\right)}$, then $\alpha + 2\beta$ is ... [Jan-2024]

24) If the mean and variance of the data $65, 68, 58, 44, 48, 45, 60, \alpha, \beta, 60 where \alpha > \beta$ are 56 and 66.2 respectively, then $\alpha^2 + \beta^2$ is equal to... [Jan-2024]

25) If $\frac{\binom{11}{1}}{2} + \frac{\binom{11}{2}}{3} + \cdots + \frac{\binom{11}{9}}{10} = \frac{n}{m}$ with $gcd(m, n) = 1$, then $m + n$ is equal to ... [Jan-2024]

26) If the points of intersection of two conics $x^2 + y^2 = 4b$ and $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$ lie on the curve $y^2 = 3x^2$, then $3\sqrt{3}$ times the area of the rectangle formed by the intersection points is ... [Jan-2024]

27) Let α, β be the roots of the equation $x^2 - x + 2 = 0$ with $Im(\alpha) > Im(\beta)$. Then $\alpha^6 + \alpha^4 + \beta^4 - 5\alpha^2$ is equal to ... [Jan-2024]

28) Equations of two diameters of a circle are $2x - 3y = 5$ and $3x - 4y = 7$. The line joining the points $\left(-\frac{22}{7}, -4\right)$ and $\left(-\frac{1}{7}, 3\right)$ intersects the circle at only one point $P(\alpha, \beta)$. Then, $17\beta - \alpha$ is equal to ... [Jan-2024]

29) All the letters of the word "$GTWENTY$" are written in all possible ways with or without meaning and these words are written as in a dictionary. The serial number of the word "$GTWENTY$" is ... [Jan-2024]

30) Let $f(x) = 2^x - x^2, x \in R$. If m and n are respectively the number of points t which the curves $y = f(x)$ and $y = f\prime(x)$ intersect the $x - axis$ then the value of $m + n$ is ... [Jan-2024]