9.2.10

EE24BTECH11021 - Eshan Ray

Question:
For the Differential Equation $x + y\frac{dy}{dx} = 0$ $(y \neq 0)$, verify that $y = \sqrt{a^2 - x^2}, x \in (-a, a)$ is a solution of the differential equation.

Solution: Solving the given D.E. , we get,

$$x + y\frac{dy}{dx} = 0 \tag{1}$$

$$\implies \frac{dy}{dx} = -\frac{x}{y} \tag{2}$$

$$\implies ydy = -xdx \tag{3}$$

Integrating both sides we get,

$$\implies \int ydy = -\int xdx \tag{4}$$

$$\implies \frac{y^2}{2} = -\frac{x^2}{2} + C \tag{5}$$

$$\implies \frac{y^2}{2} + \frac{x^2}{2} = C \tag{6}$$

$$\implies y^2 + x^2 = 2C \tag{7}$$

Substituting $2C$ with a^2 we get,

$$\implies y^2 + x^2 = a^2 \tag{8}$$

$$\implies y = \pm\sqrt{a^2 - x^2} \tag{9}$$

Thus, $y = \sqrt{a^2 - x^2}$ is a solution to the differential equation $x + y\frac{dy}{dx} = 0$.

Computational Solution:
Using classical defination of derivative we get,

$$f\prime(x) = \frac{f(x+h) - f(x)}{h} \tag{10}$$

$$\implies f(x+h) = f(x) + f\prime(x)h \tag{11}$$

For $y = f(x)$, we can get the points of the required graph by iterating the equation obtained in (11) where values of x increases in each iteration by h and obtaining the y-coordinate of it.

For,

$$x_0 = -1 \tag{12}$$
$$y_0 = 0 \tag{13}$$
$$h = 0.01 \tag{14}$$
$$\tag{15}$$

Using Euler Method, we get difference equation,

$$y_{n+1} = y_n + h\frac{dy}{dx}\bigg|_{(x_n,y_n)} \tag{16}$$
$$y_{n+1} = y_n - \left(\frac{x_n}{y_n}\right)h \tag{17}$$

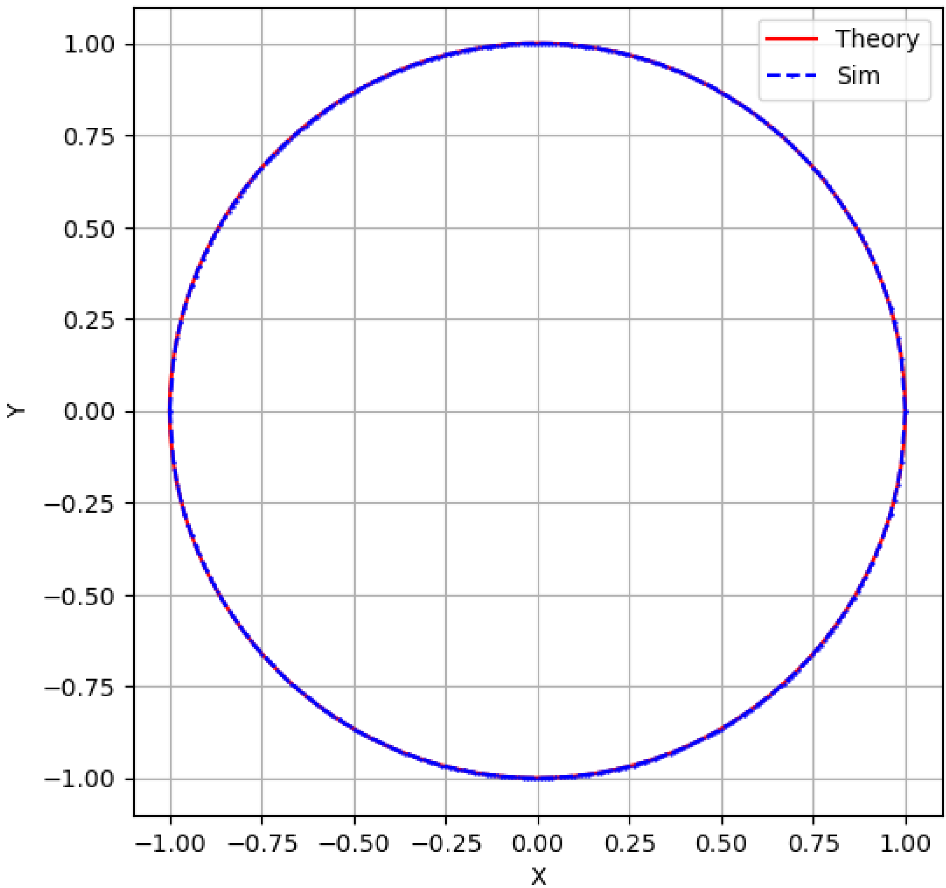

Fig. 0: Plot of the differential equation when $h = 0.01$