10.3.3.1.4

EE24BTECH11021 - Eshan Ray

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Problem Statement

Solve the following system of equations:

$$\begin{aligned} 0.2x + 0.3y &= 1.3 \\ 0.4x + 0.5y &= 2.3 \end{aligned}$$

Method 1: Row Reduction

The following system of equations can be written in matrix form:-

$$Ax = b \tag{1}$$

where,

$$A = \begin{pmatrix} 0.2 & 0.3 \\ 0.4 & 0.5 \end{pmatrix} \tag{2}$$

$$b = \begin{pmatrix} 1.3 \\ 2.3 \end{pmatrix} \tag{3}$$

$$x = \begin{pmatrix} x \\ y \end{pmatrix} \tag{4}$$

$$\tag{5}$$

The augmented matrix $[A|b]$ is:

$$\begin{pmatrix} 0.2 & 0.3 & |1.3 \\ 0.4 & 0.5 & |2.3 \end{pmatrix} \tag{6}$$

Row Reduction Steps

Perform row operations:

$$R_2 \to R_2 - 2R_1$$
$$R_1 \to R_1 + 3R_2$$
$$R_1 \to \frac{R_1}{0.2}, R_2 \to \frac{R_2}{-0.1}$$

Resulting augmented matrix:

$$\begin{pmatrix} 1 & 0 & |2 \\ 0 & 1 & |3 \end{pmatrix} \tag{7}$$

Thus, the solution is $(x, y) = (2, 3)$.

Method 2: LU Decomposition

The matrix equation $A\vec{x} = \vec{b}$ can be solved using LU Decomposition:

$$A = LU \tag{8}$$

where L is a lower triangular matrix and U is an upper triangular matrix.

LU Decomposition - Step 1

Perform row reduction to decompose A:

$$\begin{pmatrix} 0.2 & 0.3 \\ 0.4 & 0.5 \end{pmatrix} \xrightarrow{R_2 = R_2 - 2R_1} \begin{pmatrix} 0.2 & 0.3 \\ 0 & -0.1 \end{pmatrix} \tag{9}$$

Let

$$L = \begin{pmatrix} 1 & 0 \\ l_{21} & 1 \end{pmatrix} \tag{10}$$

l_{21} is the multiplier used to zero a_{21}, so $l_{21} = 2$.
The lower triangular matrix L is:

$$L = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} \tag{11}$$

And the upper triangular matrix U is:

$$U = \begin{pmatrix} 0.2 & 0.3 \\ 0 & -0.1 \end{pmatrix} \tag{12}$$

Alternate LU Decomposition:DoLittle Algorithm

The LU Decomposition of matrix A can also be obtained by Doolittle's Algorithm. This gives us update equations to construct the L and U matrix.

Elements of U matrix:

For each column j,

$$U_{ij} = \begin{cases} A_{ij} & i = 0 \\ A_{ij} - \sum_{k=0}^{i-1} L_{ik} U_{kj} & i > 0 \end{cases} \tag{13}$$

Elements of L matrix:

For each row i,

$$L_{ij} = \begin{cases} \frac{A_{ij}}{U_{ij}} & j = 0 \\ \frac{A_{ij} - \sum_{k=0}^{j-1} L_{ik} U_{kj}}{U_{ij}} & j > 0 \end{cases} \tag{14}$$

LU Decomposition - Step 2

Now solve the two systems:

$$U\vec{x} = \vec{y} \tag{15}$$

$$L\vec{y} = \vec{b} \tag{16}$$

First, solve $L\vec{y} = \vec{b}$:

$$\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 1.3 \\ 2.3 \end{pmatrix} \tag{17}$$

$$\Rightarrow \vec{y} = \begin{pmatrix} 1.3 \\ -0.3 \end{pmatrix} \tag{18}$$

LU Decomposition - Step 3

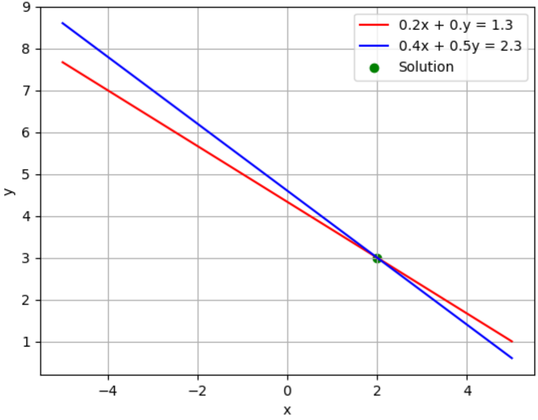
Next, solve $U\vec{x} = \vec{y}$:

$$\begin{pmatrix} 0.2 & 0.3 \\ 0 & -0.1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1.3 \\ -0.3 \end{pmatrix} \tag{19}$$

$$\Rightarrow \vec{x} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \tag{20}$$

Thus, the point of intersection is $(2, 3)$.

Plot of the Solution



Conclusion

The system of equations was solved using two methods: Row Reduction and LU Decomposition.

Both methods resulted in the solution $(2, 3)$, which is the point of intersection.

Codes

C code :
`https://github.com/eshan810/ee1003/blob/main/Assignments/6/codes/code.c`
Python code:
`https://github.com/eshan810/ee1003/blob/main/Assignments/6/codes/plot.py`