10.4.2.3

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Question:
Find two numbers whose sum is 27 and product is 182
Solution: Let one of the numbers be x
So, the other number is $27 - x$
Given,

$$x(27 - x) = 182 \tag{1}$$
$$27x - x^2 = 182 \tag{2}$$
$$x^2 - 27x + 182 = 0 \tag{3}$$
$$(x - 13)(x - 14) = 0 \tag{4}$$
$$\implies x = 13, 14 \tag{5}$$

So, the numbers are 13 and 14

Computational Solution:
Using Newton- Raphson Method we get,

We start by taking an initial guess and then iteratively we us the following equation to find the roots of the quadratic equation :-

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \tag{6}$$
$$f(x) = x^2 - 27x + 182 \tag{7}$$
$$f'(x) = 2x - 27 \tag{8}$$
$$x_{n+1} = x_n - \frac{x_n^2 - 27x_n + 182}{2x_n - 27} \tag{9}$$

After running the code, we obtained the following results:-

$$\text{Root-1: } 14.00000000 \tag{10}$$
$$\text{Root-2: } 13.00000000 \tag{11}$$

Alternate Method: Eigenvalues of Companion Matrix
In this method, we find the roots of any polynomial of the form $x^n + a_{n-1}x^{n-1} \ldots ax + a_0 = 0$ by finding the eigenvalues of the Companion Matrix (C) given below:-

$$C = \begin{pmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \vdots & 1 \\ -a_0 & -a_1 & -a_2 & \dots & -a_{n-1} \end{pmatrix} \tag{12}$$

For the Quadratic Equation $x^2 - 27x + 182 - 0$, we get the following companion Matrix

$$C = \begin{pmatrix} 0 & 1 \\ -182 & 27 \end{pmatrix} \tag{13}$$

Using QR Decomposition with shifts to calculate the Eigenvalues of the companion Matrix we get the following eigenvalues/roots of the equation:-

$$\text{Roots of the quadratic equation: } [14.0, 13.0] \tag{14}$$