

9.7.2.4

EE24BTECH11021 - Eshan Ray

Question:

For the Differential Equation $(x^2 + y^2) \frac{dy}{dx} - xy = 0$, verify that $x^2 = 2y^2 \log y$ is a solution of the differential equation.

Solution: Solving the given D.E. , we get,

$$(x^2 + y^2) \frac{dy}{dx} - xy = 0 \quad (1)$$

$$\Rightarrow \frac{dy}{dx} = \frac{xy}{x^2 + y^2} \quad (2)$$

$$(3)$$

Substituting, $y = vx$, we get,

$$\Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx} \quad (4)$$

$$\Rightarrow v + x \frac{dv}{dx} = \frac{x(vx)}{x^2 + (vx)^2} \quad (5)$$

$$\Rightarrow v + x \frac{dv}{dx} = \frac{vx^2}{x^2(1 + v^2)} \quad (6)$$

$$\Rightarrow x \frac{dv}{dx} = \frac{v}{1 + v^2} - v \quad (7)$$

$$\Rightarrow x \frac{dv}{dx} = \frac{v - v - v^3}{1 + v^2} \quad (8)$$

$$\Rightarrow x \frac{dv}{dx} = \frac{-v^3}{1 + v^2} \quad (9)$$

$$\Rightarrow -\frac{(1 + v^2) dv}{v^3} = \frac{dx}{x} \quad (10)$$

$$\Rightarrow -\frac{dv}{v^3} - \frac{v^2 dv}{v^3} = \frac{dx}{x} \quad (11)$$

$$(12)$$

Integrating both sides, we get,

$$\Rightarrow -\int \frac{dv}{v^3} - \int \frac{dv}{v} = \int \frac{dx}{x} \quad (13)$$

$$\Rightarrow \frac{1}{2v^2} - \log v = \log x + C \quad (14)$$

$$(15)$$

putting $v = \frac{y}{x}$ we get,

$$\Rightarrow \frac{x^2}{2y^2} - \log\left(\frac{y}{x}\right) = \log x + C \quad (16)$$

$$\Rightarrow \frac{x^2}{2y^2} = \log y - \log x + \log x + C \quad (17)$$

$$\Rightarrow x^2 = 2y^2 \log y + 2Cy^2 \quad (18)$$

Putting $x = 0, y = 1$ we get,

$$\Rightarrow 0 = 2(1)^2(0) + 2C(1)^2 \quad (19)$$

$$\Rightarrow C = 0 \quad (20)$$

$$\Rightarrow x^2 = 2y^2 \log y \quad (21)$$

Computational Solution:

Using classical definition of derivative we get,

$$f'(x) = \frac{f(x+h) - f(x)}{h} \quad (22)$$

$$\Rightarrow f(x+h) = f(x) + f'(x)h \quad (23)$$

For $y = f(x)$, we can get the points of the required graph by iterating the equation obtained in (23) where values of x increases in each iteration by h and obtaining the y -coordinate of it.

For,

$$x_0 = -3 \quad (24)$$

$$y_0 = 3.04878 \quad (25)$$

$$h = 0.01 \quad (26)$$

$$(27)$$

Using Euler Method, we get difference equation,

$$y_{n+1} = y_n + h \frac{dy}{dx} \Big|_{(x_n, y_n)} \quad (28)$$

$$y_{n+1} = y_n - \left(\frac{x_n y_n}{x_n^2 + y_n^2} \right) h \quad (29)$$

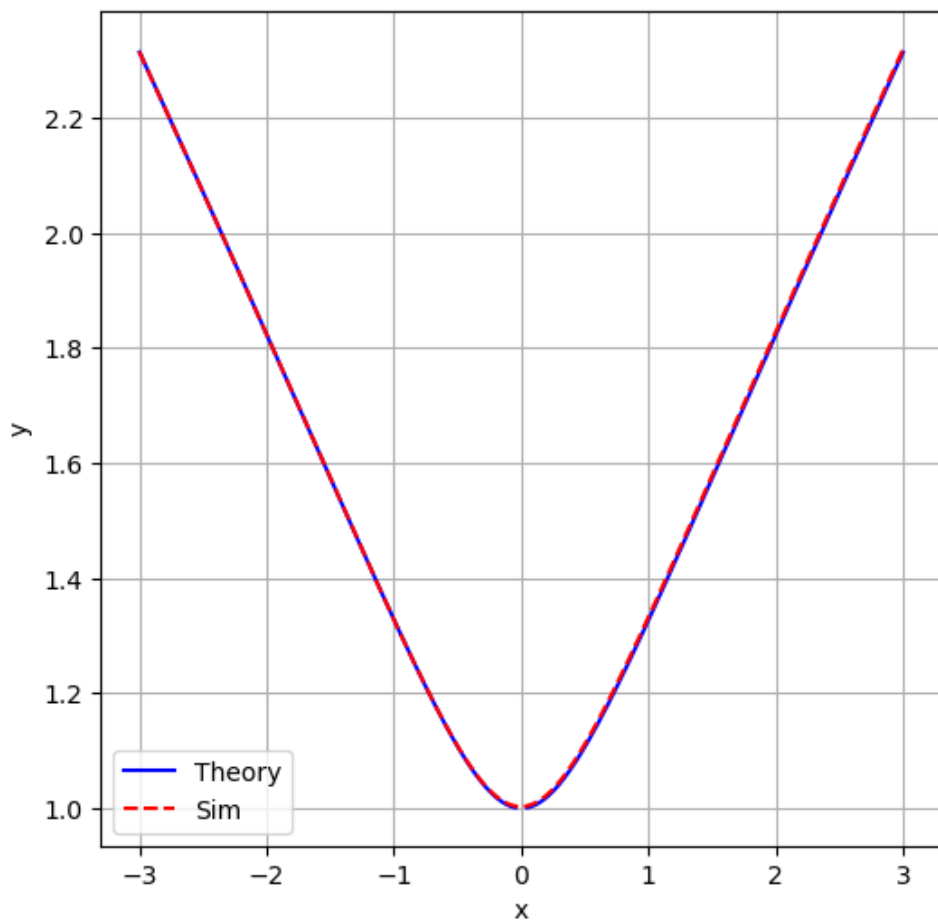


Fig. 0: Plot of the differential equation when $h = 0.01$