Oscillations

M. Yu

September 2015

Leave all answers in a form with only variables given and any constants needed.

1. Derive the formula for the period of a physical pendulum. Remember that torques are applied at the center of mass. Let the distance from the support to the center of mass be ℓ and the moment of inertia about the support be I. The physical pendulum has mass m.

2. Suppose that a mass m is under a force of $F(t) = mbe^{-kt}$. Given that the initial position and initial velocity are both zero, solve for $x(t)$

3. Patrick stands on a board of mass m that is floating in some water. Suppose that he depresses the board by a distance x from its equilibrium. Give the period of the board's oscillation once Patrick gets off.

   Remark. *The board is a rectangular prism of base area A and the water is of density ρ.*

4. Patrick is trapped in a hamster ball. He stands on one side of a platform which is a chord of the ball. The board is below the center of the ball. Supposed that the distance from the center of the ball to the platform is h. Patrick moves in such a way so that neither the platform or the ball moves. Describe Patrick's motion.

5. Patrick wants to travel from one end of the Earth, mass M, to the other end through a tunnel dug through the Earth. For this problems, assume the tunnel is not through the diameter. Determine Patrick's round trip time.

6. Consider a physical pendulum which starts at an initial angle to the vertical which is not small. We can not apply the approximation we did in problem 1 for this case. The energy equation for a physical pendulum is:

   $$\frac{1}{2}I\dot{\theta}^2 + mgh = E.$$

   Give the equation for the period of this pendulum as an integral with respect to θ.

   Remark. *This integral which you get can be solved with special functions known as elliptic integrals. Other special functions used in physics are but not limited to the Bessel function, Legendre Polynomials, and Laguerre Polynomials. The latter three used in quantum mechanics.*

7. ***Challenge:*** Two masses of mass m are connected to each other and to two walls via three springs. The spring constant in each spring is k. Start by giving the masses some small displacement from equilibrium. Give the general solution for the position of the masses as a function of time.