Gravity and Orbits

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Hello, we hope you all had a great break. Since F=ma is coming up soon, we're going to rush through gravity and orbits in the same week. This paper will be basically a cheat sheet of formulas, but we will try to talk about them in depth during the meeting. If you are at all confused by any section, please ask a captain for clarification.

1 Newton's Law of Universal Gravitation

Newton postulated that the force of gravity between any two objects, of masses m_1 and m_2, separated by distance r was equal to:

$$F_g = \frac{Gm_1m_2}{r^2}$$

where G is roughly equal to $6.67 * 10^{-11} \frac{\text{N m}^2}{\text{kg}^2}$.

2 Shell Theorem

Because we want to eventually to talk about things on a large scale (like planets and orbits), we will introduce this extremely useful concept called the shell theorem. It comes in two parts and allows one to simplify the effects of the force of gravity quickly.

1. When an object is inside a shell of uniform density, the net force of gravity on the object from to the shell is 0. Note that this means *anywhere* inside the shell, not just the center.

2. When an object is outside a shell of uniform density, the net force of gravity on the object is the same as if the shell were just a point mass at the center of mass of the shell. Therefore, we can pretend a sphere of uniform mass is just a point mass located at the center of the sphere. This is useful when dealing with stars or planets.

3 Energy of Orbits

When two objects are locked in orbit, they conserve energy between them. Kinetic energy is the conventional kinetic energy; but potential energy is in a form you may not have seen before.

$$T = \frac{1}{2}mv^2 \qquad\qquad U = \frac{-Gm_1m_2}{r}$$

4 Kepler's Laws

Kepler came up with three laws that explain the motion of planets trapped in orbit. They are derived using energy of orbits, conservation of momentum, etc. They cannot be derived using gravity because gravity does not affect objects in space – if they did, all the planets would just fall down.

1. The orbit of a planet is an ellipse with the sun as one of the foci.
2. A line segment joining a planet and the sun sweeps out equal areas in equal amounts of time. This law is a result of the conservation of angular momentum.
3. The square of a planet's orbital period is directly proportional to the cube of the semi-major axis of the orbit. This means that if two orbits have the same orbital distance and combined mass, they orbit with the same period (part of derivation).

5 Problems

1. A 5kg object is placed on the surface of a planet with uniform density and feels a force due to gravity F. If a 10 kg object is placed half way to the center of that planet, and the planet is of uniform density, how much force will that object feel in terms of F?

2. Arnold and Jeremy get into an argument over the path of objects thrown off the Earth upwards. Arnold says that in freshman physics, he learned that when you throw objects up at 45 degrees, they follow a parabolic path; Jeremy says that when you throw objects up at 45 degrees, they should follow an elliptical orbit, by Kepler's First Law. Who's right? Who's wrong? Could they both be wrong?

3. Verify Kepler's third law for circular orbits. Try finding the period of a circular orbit with radius a, then finding the period of a circular orbit with radius $2a$, then repeat with ka.