1

$$g = \frac{GM}{R^2}$$

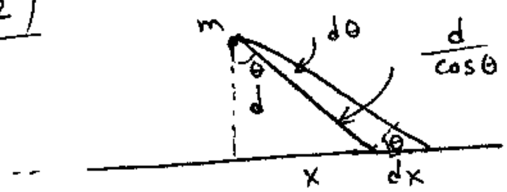
$$m_r = \frac{4}{3}\pi r^3 \rho$$

$$M = \frac{4}{3}\pi R^3 \rho.$$

$$g_r = \frac{G m_r}{r^2} \qquad \frac{m_r}{M} = \left(\frac{r}{R}\right)^3$$

$$\frac{g_r}{g} = \frac{\left(\frac{G m_r}{r^2}\right)}{\frac{GM}{R^2}} = \frac{m_r}{M}\left(\frac{R^2}{r^2}\right) = \frac{r}{R} \qquad \therefore \boxed{g_r = \frac{g r}{R}}$$

2

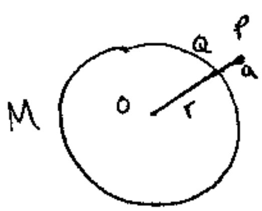


$$\left(\frac{d}{\cos\theta}\, d\theta\right)\frac{1}{\cos\theta} = dx$$

$$dM = \lambda\, dx = \frac{\lambda d}{\cos^2\theta}\, d\theta$$

$$F = \int_{-\frac{\pi}{2}}^{\pi/2} \frac{Gm\left(\frac{\lambda d}{\cos^2\theta}\right)d\theta}{\left(\frac{d}{\cos\theta}\right)^2} = \boxed{\frac{2G\lambda m}{d}}$$

3



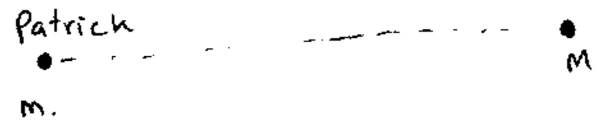
Let the planet have radius r. Suppose we are a height 'a' above the planet then the potential at P, minus potential at Q is what we want.

$$\frac{-GMm}{r+a} + \frac{GMm}{r} = \frac{GMma}{r(r+a)} \qquad (3)$$

since $r+a \simeq r$ (3) becomes $\simeq \boxed{\underbrace{\frac{GM}{r^2}}_{g}\, ma} = \boxed{mga}$

what we wanted.

4



Jnitially, both E and p are zero, so they must be zero at all times.

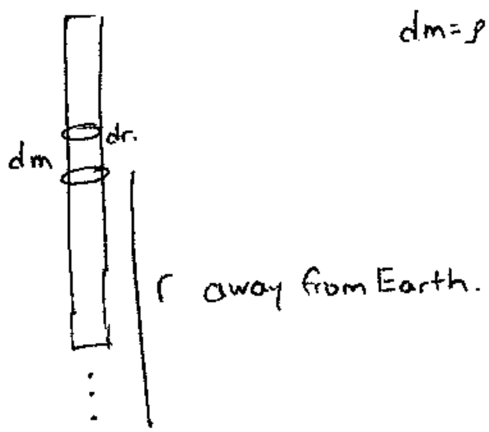
At some later time, $mv_1 = MV_2$ (4)

$$\frac{1}{2}mv_1^2 + \frac{1}{2}mV_2^2 - \frac{GMm}{r} = 0$$

$$\frac{1}{2}m\left(\frac{MV_2}{m}\right)^2 + \frac{1}{2}mV_2^2 - \frac{GMm}{r} = 0$$

$$V_2 = m\sqrt{\frac{2G}{M+m}\left(\frac{1}{r}\right)} \quad , \text{ then } V_1 = M\sqrt{\frac{2G}{M+m}\left(\frac{1}{r}\right)}$$

5



$dm = \rho A dr$

$$\frac{GM\rho A dr}{r^2} - dm\omega^2 r = dTA$$

$$\frac{GM\rho A dr}{r^2} - \rho A dr\,\omega^2 r = dTA$$

$$\rho A\left(\frac{GM}{r^2} - \omega^2 r\right) = \frac{dTA}{dr}$$

$$\rho r\left(\frac{GM}{r^3} - \omega^2\right) = \frac{dT}{dr}$$

if $\frac{dT}{dr} = 0$, then that is the max tension.

$$\boxed{r = \sqrt[3]{\frac{GM}{\omega^2}}}$$