Torque/More Rotation

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1 Review

For reference, here are the tables provided from last meeting.

Linear	Rotational	Relationship
$\vec{d}$	$\vec{\theta}$	$\vec{d} = \vec{r} \times \vec{\theta}$
$\vec{v}$	$\vec{\omega}$	$\vec{v} = \vec{r} \times \vec{\omega}$
$\vec{a}$	$\vec{\alpha}$	$\vec{a} = \vec{r} \times \vec{\alpha}$
m	I	

Linear Formula	Rotational Formula
$V_f = V_o + at$	$\omega_f = \omega_o + \alpha t$
$d = V_o t + \frac{1}{2}at^2$	$\theta = \omega_o t + \frac{1}{2}\alpha t^2$
${V_f}^2 = {V_o}^2 + 2ad$	${\omega_f}^2 = {\omega_o}^2 + 2\alpha\theta$
$F = ma$	$\tau = I\alpha$
$p = mv$	$L = I\omega$
$KE_{lin} = \frac{1}{2}mv^2$	$KE_{rot} = \frac{1}{2}I\omega^2$
$a_c = \frac{v^2}{r}$	$a_c = \omega^2 r$

Shape/Object	Moment of Inertia
Point	mr^2
Thin rod, through center	$\frac{1}{12}mr^2$
Thin rod, through end	$\frac{1}{3}mr^2$
Hoop	mr^2
Solid disk	$\frac{1}{2}mr^2$
Solid sphere	$\frac{2}{5}mr^2$
Hollow sphere	$\frac{2}{3}mr^2$

2 Torque

Torque is the rotational analog of force. It measures how much a force will rotate an object. It is defined as $\vec{\tau} = \vec{r} \times \vec{F}$, where $\vec{r}$ is the displacement vector between the pivot point and where the force is applied. Imagine that you're trying to push open your bedroom door with your fingers. The closer you are to the hinges (the pivot point), the harder it is to push, because the decrease in $\vec{r}$ results in a decrease in torque. For many problems, $\vec{r} \times \vec{F}$ is often equal to rF, as the distance and force vectors are perpendicular.

3 New Insights

Now that we know about rotation and moment of inertia, it allows us to have more realistic problems. For example, let's look at some toilet paper, and then an Atwood machine.

1. Arnold has a big roll of toilet paper, and is also standing on a tall cliff. Then Arnold holds onto one end of the roll, and lets the other end fall down and unroll. Assume changes in the mass and radius of the roll are negligible. Find the position of the center of the toilet paper after time t.

2. Arnold has a pulley (solid disk) with two masses, m_1 and m_2. Previously, we assumed the pulley was ideal and massless, and the string was massless as well.

   (a) If $m_2 > m_1$, find the acceleration of the system. Hint: draw a diagram, then $F_{net} = ma$.

   (b) Now, assume the pulley has mass m_p, but the string is still massless. Find the new acceleration of the system. Hint: $\vec{\tau} = \vec{r} \times \vec{F}$.

4 Problems from last week

1. A 2 kilogram solid ball with radius 1 meter is sitting on top of a 10 meter tall ramp with a 30 degree incline. The coefficient of friction is high enough that the ball rolls down the ramp without slipping.

   (a) What is the velocity of the ball when it reaches the bottom of the ramp?

   (b) What if the sphere was hollow?

   (c) What if it was a M kilogram hollow ball with radius R meters?

2. A hamster in space is inside of a hamster ball of radius R and moment of inertia MR^2. On the outside of it, there are two rockets that are set up to rotate the ball and can each provide a thrust, T. How long do the rockets need to be turned on for the hamster to feel an acceleration due to gravity, g? The answer should be in terms of g, R, M, and T.