Rotation

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1 Big Tables

In the first unit of physics, we learned about 1D and 2D motion. We used the various variables for equations to solve problems. These variables all have rotational analogs. One important thing to note is that time (t) is always the same. Here is a table:

Linear	Rotational	Relationship
$\vec{d}$	$\vec{\theta}$	$\vec{d} = \vec{r} \times \vec{\theta}$
$\vec{v}$	$\vec{\omega}$	$\vec{v} = \vec{r} \times \vec{\omega}$
$\vec{a}$	$\vec{\alpha}$	$\vec{a} = \vec{r} \times \vec{\alpha}$
m	I	

From these new variables, we can also write corresponding equations. Some are blank for you to fill in.

Linear Formula	Rotational Formula
$V_f = V_o + at$	$\omega_f = \omega_o + \alpha t$
$d = V_o t + \frac{1}{2}at^2$	$\theta = \omega_o t + \frac{1}{2}\alpha t^2$
${V_f}^2 = {V_o}^2 + 2ad$	
$F = ma$	$\tau = I\alpha$
$p = mv$	$L = I\omega$
$KE_{lin} = \frac{1}{2}mv^2$	$KE_{rot} = \frac{1}{2}I\omega^2$

Quick Check: For uniform circular motion, the formula for centripetal acceleration is $a_c = \frac{v^2}{r}$. Write this in terms of ω and r.

1.1 No-Slip Condition

In most instances, we can use the fact that things roll smoothly to find relationships between translational and rotational motion. Smooth rolling is defined to be when rolling where one full rotation leads to a linear movement of the center of mass by 1 circumference. This lets us make the following conclusions:

$$d = \theta r \tag{1}$$

$$v = \omega r \tag{2}$$

$$a = \alpha r \tag{3}$$

2 Moment of Inertia

Just like how mass is how hard it is to linearly move an object, an object's moment of inertia is how hard it is to rotate the object. For a point particle, moment of inertia is defined to be:

$$I = mr^2 \tag{4}$$

r refers to the distance between the point and the rotation axis. An important concept is that moment of inertia is additive. That is, for a system of a bunch of points, the total moment of inertia is:

$$I = \sum mr^2 \tag{5}$$

Every object has a moment of inertia. However, they are not all equal to mr^2. In order to calculate the moment of inertia of say, a solid sphere, we need calculus, so for now, you can just have the formulas.

Shape/Object	Moment of Inertia
Point	mr^2
Thin rod, through center	$\frac{1}{12}mr^2$
Thin rod, through end	$\frac{1}{3}mr^2$
Hoop	mr^2
Solid disk	$\frac{1}{2}mr^2$
Solid sphere	$\frac{2}{5}mr^2$
Hollow sphere	$\frac{2}{3}mr^2$

3 Problems

1. Arnold is traveling in a circular path. If his current angular velocity is 5 rad/s and his angular acceleration is 2 rad/s^2, how long will it take until Arnold is going 17 rad/s?

2. Ok, number 1 was probably way too easy. Now, let's say Arnold's circular path has a radius of 5 m. If the same conditions from number 1 are met, how long will Arnold accelerate until his centripetal acceleration is 500 m/s^2?

3. A 2 kilogram solid ball with radius 1 meter is sitting on top of a 10 meter tall ramp with a 30 degree incline. The coefficient of friction is high enough that the ball rolls down the ramp without slipping.

   (a) What is the velocity of the ball when it reaches the bottom of the ramp?

   (b) What if the sphere was hollow?

   (c) What if it was a M kilogram hollow ball with radius R meters?

4. A hamster in space is inside of a hamster ball of radius R and moment of inertia MR^2. On the outside of it, there are two rockets that are set up to rotate the ball and can each provide a thrust, T. How long do the rockets need to be turned on for the hamster to feel an acceleration due to gravity, g? The answer should be in terms of g, R, M, and T.