F=ma Solutions

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Note: In these solutions, we assume $g = 10$ m/s^2.

1 Collision Problems

1. Arnold is originally to the left of Jeremy. Arnold (mass: 8 kg) is moving at 5 m/s to the right, while Jeremy (mass: 4 kg) is moving at a speed of 2 m/s to the left. After they collide elastically, what are Arnold and Jeremy's velocities?

   Solution: Let moving to the right be positive, and moving to the left as negative. The total momentum in our problem is: (8 kg) $\times$ (5 m/s) + (4 kg) $\times$ (-2 m/s) = 32 kg m/s.

   After shifting our frame by x m/s to the left, the total momentum is now:

   8 kg$\times(5-x)$ m/s+4 kg$\times(-2-x)$ m/s. Equating this expression to 0, we get that $x = \frac{8}{3}$ m. Applying this shift, we now have: Arnold is moving at $\frac{7}{3}$ m/s to the right, while Jeremy is moving at $\frac{14}{3}$ m/s to the left. After reversing the direction of these velocities and reversing our original switch, we get our final answer: $\boxed{\text{Arnold is moving } \dfrac{1}{3} \text{ m/s to the right, while Jeremy is moving } \dfrac{22}{3} \text{ m/s to the right.}}$ We can check this with the original conditions to confirm that momentum and energy are conserved.

2. Arnold is moving at a speed of 5 m/s. He travels 3 m, when he then collides elastically with Jeremy, who is at rest. If the coefficient of kinetic friction between both Arnold and Jeremy and the surface is 0.2, how far will Jeremy travel before stopping? Use the same masses from the previous problem.

   Solution: Originally, Arnold's kinetic energy is equal to $\frac{1}{2}mv^2 = \frac{1}{2}(8)(5)^2 = 100$ J. Before he collides with Jeremy, he loses energy due to friction; the amount of energy is lost is: $F_f d = (8)(10)(0.2)(3) = 48$ J. Thus, right before Arnold collides with Jeremy, he has 52 J of kinetic energy. Equating this with $\frac{1}{2}mv^2$, we get that Arnold is traveling at a speed of $\sqrt{13}$ m/s.

   After the collision, Arnold is moving at $\frac{\sqrt{13}}{3}$ m/s to the right, while Jeremy is moving at $\frac{4\sqrt{13}}{3}$ m/s to the right (Work not shown for these calculations). Thus, Jeremy's kinetic energy is equal to $\frac{1}{2}mv^2 = \frac{1}{2}(4)(\frac{4\sqrt{13}}{3})^2 = \frac{416}{9}$ J. When Jeremy stops, he will have lost all his energy due to friction. We can equate: $\frac{416}{9} = F_f d = (4)(10)(0.2)(d)$. Solving for d, we get our final answer of $\boxed{\dfrac{52}{9} \text{ m}}$.

2 Motion

1. Arnold (5 m tall) is on an elevator accelerating upwards at 3 m/s^2. If Arnold drops his phone when the elevator is 10 meters above the ground, what will the phone's velocity be when it hits the bottom of the elevator?

   Remark. *This question is slightly altered from the problem on the original handout.*

Solution: Arnold's phone is accelerating at 10 m/s^2 down. Meanwhile, the elevator floor is accelerating upwards at 3 m/s^2 up. Thus, we want to find when the two objects, combined, cover a total distance of 5 m. We can write the following equations:

$$d_{phone} = V_o t + \frac{1}{2}at^2 = 5t^2 \tag{1}$$

$$d_{elevator} = V_o t + \frac{1}{2}at^2 = \frac{3}{2}t^2 \tag{2}$$

$$d_{phone} + d_{elevator} = 5. \tag{3}$$

After substituting equations (1) and (2) into equation (3), we get: $\frac{13}{2}t^2 = 5$. Solving for this, we find that $t = \sqrt{\frac{10}{13}}$ s. Then, the phone's final velocity is $V_f = V_o + at = at = 10\sqrt{\frac{10}{13}} = \boxed{\dfrac{10\sqrt{130}}{13}\ \mathrm{m/s}}$.

2. Arnold shoots a rock with his rock gun at an angle θ with respect to the horizontal. Find the horizontal distance D the rock travels, and the maximum height H it reaches during the flight. What is the value of H/D?
   If we assume that $g = 10\ \mathrm{m}/s^2$ throughout the rock's flight, the rock will travel in a parabola. If the rock is originally shot at a speed of V_o, then $V_{ox} = V_o \cos\theta$ and $V_{oy} = V_o \sin\theta$. At the maximum height of the flight, the y-velocity is equal to 0. Thus, we can write, $V_{fy} = V_{oy} + at$. Solving for t, we get that $t = \frac{V_o \sin\theta}{g}$. Since a parabola is symmetric, we know that the total time of the rock's flight is $2t = \frac{2V_o \sin\theta}{g}$.
   Since x-velocity is constant, we can easily solve for the horizontal distance: $D = V_{ox}t = V_o \cos\theta \times \frac{2V_o \sin\theta}{g} = \frac{2(V_o)^2 \sin\theta\cos\theta}{g}$. Then, we can find the max height with the formula $d_y = V_{oy}t + \frac{1}{2}at^2 = V_o \sin\theta \times \frac{V_o \sin\theta}{g} - \frac{1}{2}g \times (\frac{V_o \sin\theta}{g})^2$. Simplifying this, we get: $H = \frac{(V_o)^2(\sin\theta)^2}{2g}$. After a bunch of fraction skills, we see that when dividing D by H, V_o and g cancel, and we are finally left with: $\boxed{\dfrac{H}{D} = \dfrac{1}{4}\tan\theta}$.

3 Rotation

Note: See Arnold's handwritten solutions below.

1. Calculate the moment of inertia of a disk of uniform density, mass $4M$, and radius $2R$ about its center.
2. Calculate the moment of inertia of the same disk about a point on its circumference.
3. Calculate the new moment of inertia of the resultant disk after a circle of radius R is cut out from the middle of it.
4. Calculate the moment of inertia of the resultant disk after a circle of radius R is cut out in a Pac-man like shape (the cutout is between the circumference and the center).
5. Order the last 4 objects from easiest to rotate to hardest to rotate.

4 Energy

1. What are the units of energy in terms of the base SI units? (kg, m, s, etc.)
   Answer: kg $\mathrm{m}^2/\mathrm{s}^2$

2. A block of mass m is moving on a horizontal table surface at v_o. It then moves smoothly onto a sloped big block of mass M. The big block can also move on the table surface. Assume that everything moves without friction. How high is the small block before it falls back down the ramp? What is the speed of the small block after it leaves the slope?
   Solution: See Arnold's handwritten solutions below.

5 Orbits and Gravity

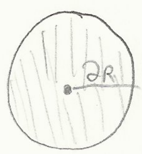
1. Four masses m are arranged at the vertices of a tetrahedron of side length a. What is the gravitational potential energy of this arrangement in terms of G, m, and a?
   Solution: See Arnold's handwritten solutions below.

2. A satellite of small mass, m, orbits a planet of large mass M in an elliptical orbit with closest point R and farthest point $2R$ away. At the farthest point away in the orbit, the satellite is moving with velocity v_o. Then, the satellite is instructed to begin orbiting in a circle of radius $2R$. What is the new speed of the rocket in terms of v_o?
   Solution: See Arnold's handwritten solutions below.

Section 5: Rotation

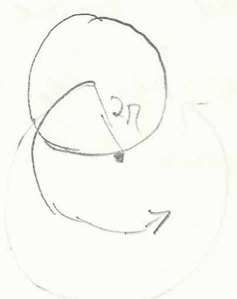
$I_{disk} = \frac{1}{2}MR^2$ $\qquad$ $I_{parallel} = I_{com} + mr^2$

1.



$I_{com} = \frac{1}{2}(4M)(2R)^2 = 8MR^2$

2.

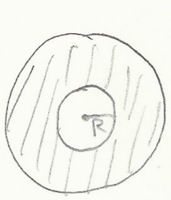


$I_p = 8MR^2 + (4M)(2R)^2$

$= 8MR^2 + 16MR^2$

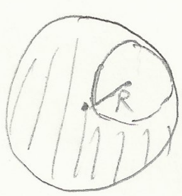
$= 24MR^2$

3.



$I = 8MR^2 + \frac{1}{2}\left(-\frac{4M}{4}\right)(R)^2 = 7.5MR^2$

4.



Empty about its own COM: $\frac{1}{2}\left(\frac{4M}{4}\right)R^2 = \frac{1}{2}MR^2$

Empty about whole disk COM: $\frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$

Whole disk's $I = 8MR^2 - \frac{3}{2}MR^2 = 6.5MR^2$.

5. easiest $\longrightarrow$ hardest

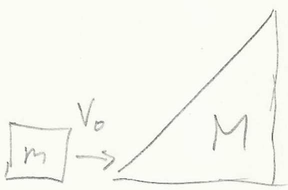
4, 3, 1, 2

Section 6: Energy

1. Energy has units like potential energy:

$$E = mgh = kg \cdot \frac{m}{s^2} \cdot m = kg\frac{m^2}{s^2}$$

2.



The ramp and block move at same velocity togeter at m's max height, find that velocity by conserving momentum:

$$mv_0 + M(0) = (m+M)v_f$$

$$v_f = \frac{mv_0}{(m+M)}$$

Then, conserve energy:

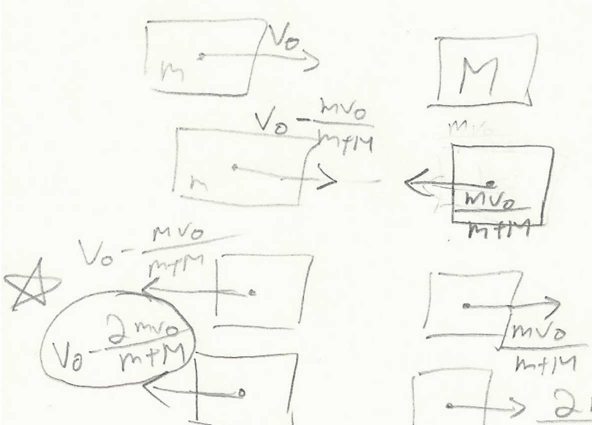
$$\frac{1}{2}mv_0^2 = \frac{1}{2}(m+M)(v_f)^2 + mgh$$

$$mgh = \frac{1}{2}mv_0^2 - \frac{1}{2}\frac{mv_0^2}{m+M}$$

$$h = \frac{1}{2g}\left[v_0^2 - \frac{mv_0^2}{m+M}\right] = \frac{1}{2g}\left[\frac{Mv_0^2}{m+M}\right]$$

Notice question 2 is an elastic collision end product.

$$V_{COM} = \frac{mv_0}{m+M}$$



original frame

COM frame

after collision

original frame

☆ $V_0 - \frac{2mv_0}{m+M}$

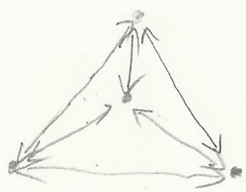
$$= \left(\frac{M-m}{m+M}\right)V_0$$

7. Remember that

$$PE = -\frac{Gm_1m_2}{r} \quad \text{and not} \quad -\frac{2Gm_1m_2}{r} \text{ for a system of 2 particles.}$$

m_1 $\quad$ m_2

Don't double-count potentials!



6 arrows of $\frac{-Gm^2}{a}$ potential so

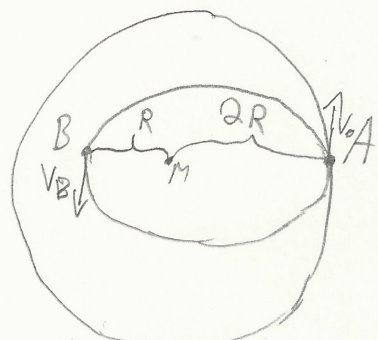
$\frac{-6Gm^2}{a}$ total gravitational potential energy

b. In circular orbit of $2R$:

$$F_g = F_c$$

$$\frac{GMm}{(2r)^2} = \frac{mv_f^2}{(2r)}$$

$$v_f = \sqrt{\frac{GM}{2R}}$$



To find velocity of elliptical orbit, Conserve energy and angular momentum.

Momentum: $2\cancel{R}(v_0) = \cancel{R}(v_B) \rightarrow v_B = 2v_0$

Energy: $$\underbrace{\frac{1}{2}mv_0^2 - \frac{GMm}{2R}}_{\text{Energy at A}} = \underbrace{\frac{1}{2}mv_B^2 - \frac{GMm}{R}}_{\text{energy at B}}$$

$$\frac{3}{2}mv_0^2 = \frac{GMm}{2R}$$

$$v_0 = \sqrt{\frac{GM}{3R}}$$

$$v_f = \sqrt{\frac{3}{2}}\, v_0$$ This is a hard question.