1D Kinematics Solutions

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1. To solve this problem, one has to realize the problem essentially is composed of two parts. One, where the coke is falling freely through air because of gravity up until just before it hits the car, and two, the instant the coke hits the car and comes to a complete stop.
Solve these parts individually. So we want to solve for the speed of the coke right before it hits the car. Use the equation $V_f = V_o + at$. That gives us $V_f = -4.9m/s^2$) This means that the speed of the can when it hits the car is $-4.9m/s^2$.
Onto the second part of the question. Use the equation $V_f^2 = V_o^2 + 2ad$. Solving for acceleration gives us $12.005m/s^2$. Note that the acceleration is positive, as the velocity of the coke was negative.

2. First order of business, change $150km/hr$ into nicer units. We typically use m/s in physics. So there are 1000m in 1k, and 3600s in 1hr. $150km/hr * 1hr/3600s * 1000m/1km$ gives us a velocity of $41.66m/s$ for the speed of the train. Because Eshan very much wants to stay alive in this problem, he wants the velocity of the train to become 0, which should be our final velocity. Now we just have to find how long it would take for a train traveling at $41.66m/s$ to reach $0m/s$.
Use the equation $V_f = V_o + at$. Solving for t gives us $47.39s$. So if the train is 2 minutes away (120 seconds), Eshan therefore has $120s - 47.39s$ to warn the train, or 72.6s.

3. Remember that velocity is equal to distance divided by time. So for part a, Siddharth drives at two speeds, $60km/hr$, and $100km/hr$. Keep in mind that he drives for the same time at these speeds. So we'll call the time he drives at each speed t, meaning the total time driven this journey is 2t. This means that he travels different distances at these two speeds. Lets call the distance he travels while at $60km/hr$ d_1, and the distance he travels while at $100km/hr$ d_2. These give the equations $60 = d_1/t$ and $100 = d_2/t$. Rearranging these equations gives us $d_1 = 60t$ and $d_2 = 100t$. So, the total distance traveled on the way to Cincinnati is $d_1 + d_2 = 60t + 100t = 160t$. Now lets bring back that equation $v = d/t$So we have $v = 160t/2t$, giving us $80km/hr$ for the average speed to Cincinnati.
Phew. Now let's do part 2 of this question. We'll call each distance he drives for some time d, meaning the returning journey he drives a total distance of 2d. Now let's call the time it takes him driving at $60km/hr$ t_1, and the time it takes him driving at $100km/hr$ t_2. That gives us equations $60 = d/t_1$ and

$100 = d/t_2$. Rearranging gives us $t_1 = d/60$ and $t_2 = d/100$. Bringing back our old friend $v = d/t$, we can solve for v, with the total distance being 2d, and the total time being $d/60 + d/100$. This gives us an average velocity of $75km/hr$ back to Silver Spring.
Don't worry, this part is easy. Ignoring the fact that I asked for the average velocity of the entire trip (which would be 0 because we return to where we came from), we can solve this essentially the same as the other two parts of the question. We have the velocities of both legs of the trip, $80km/hr$, and $75km/hr$. We also have the total distance of the trip, 2d, assuming one leg of the trip is d. We'll call the time at $80km/hr$ t_1 and $75km/hr$ t_2. That gives us equations $60 = d/t_1$ and $100 = d/t_2$. Rearranging, $t_1 = d/80$ and $t_2 = d/75$. So the total distance is 2d, and the total time is $d/80 + d/75$. Average speed is then $77.42km/hr$.

4. Similar to question 1, this problem forces you to divide the situation into a lot of smaller problems. For both Richard and Siddharth, there are two parts of their motion, the motion while accelerating, and the motion while at top speed. We also have to split the problem into the section before Richard start and the section after. We first consider Siddharth during his acceleration period. We know $V_0 = 0$, $a = 1.67m/s^2$, and $V_f = 4$, since he stops accelerating once he gets to his top speed. We then use the formula $V_f = V_0 + at$, but rearrange to solve for the time, which gives us $t = \frac{V_f = V_0}{a}$. Plugging our numbers in, we find $t = \frac{4-0}{1.67} = 2.4s$. We also want to know the distance that Siddharth travels while accelerating. We then need to use formula $V_f^2 = V_0^2 + 2ad$, which we can rearrange to get $d = \frac{V_f^2 - V_0^2}{2a}$. Plugging in our numbers, we get $d = \frac{16-0}{2\times1.67} = 4.79m$. Next, we want to analyze the time Richard is accelerating. In this case, $V_0 = 0$, $a = 1.67m/s^2$, and $V_f = 6$, which gives us as before, $t = 3.6s$ and $d = 10.78m$. We have to remember now, that Richard starts moving at $t = 5$, so he really stops accelerating at $t = 5 + 3.6 = 8.6s$. This means we have the following events in order.

- At $t = 0s$ Siddharth starts accelerating
- At $t = 2.4s$ Siddharth reaches top speed and stops accelerating
- At $t = 5s$ Richard starts accelerating
- At $t = 8.6s$ Richard reaches top speed and stops accelerating
- After $t = 8.6s$ both Richard and Siddharth are at top speed

We now want to find the distance between Richard and Siddharth at $t = 8.6$, when they are both at top speed. We know Siddharth moves $4.79m$ while he is accelerating and moves $d = V\Delta t = 4(8.6-2.4) = 24.8m$. This means Siddharth is at $x = 4.79+24.8 = 29.59m$. We also know that Richard moves $10.78m$ while he is accelerating from $t = 5$ to $t = 8.6$, which means Richard is at $x = 10.78$. The difference in their distances is $29.59-10.78 = 18.81m$. We now want to find how long it takes them to reach each other. Since Richard is moving at $6m/s$ and Siddharth is moving at $4m/s$, Richard is catching up to Siddharth by $2m/s$. This means the time it takes to close the $18.81m$ between them is $t = \frac{d}{v} = 9.41s$. Since we are already at $t = 8.6$, the total time is $t = 8.6 + 9.41 = 18.01s$.

5. This question is easier than it seems. The easiest way to do it is to ignore gravity until the very last step of the problem. Because the cannonballs are going towards each other, they are approaching each other at a rate of $25m/s + 55m/s = 80m/s$. So we can think of this as a single particle traveling at $80m/s$. The distance between the 2 cannons is 200 m, so we do $200/80 = 2.5 seconds$ to find the time it takes for the two cannonballs to hit each other. Nowe we re-introduce gravity back into the problem. How much would the cannonballs "fall" in 2.5 seconds? We use the equation $d = V_o t + 1/2at^2$, to find $d = 0(2.5) + 1/2(-9.8)(2.5)^2 = -31.25m$. So the cannonballs "fall" 31.25 meters in this time. Using common sense, because the top cannon is traveling faster, it covers more distance, and the place where the two cannonballs meet must be closer to the bottom cannon than the top one. So we can subtract 31.25 meters from 200meters, and get 168.75 meters below the top cannon as our answer.