Theorem 2.3.9. *The running time of* EUCLID(a, b) *is* $O(\lg b)$.

<u>Proof</u>: We may assume $a > b \geq 1$

To determine ~~#~~ upper bound on number of rec calls, need to find smallest k s.t. $b < F_{k+1}$

Use $F_{k+1} \approx \frac{\phi^{k+1}}{\sqrt{5}}$ $\phi = \frac{1+\sqrt{5}}{2}$

(HW: $F_n = \frac{\phi^n + \hat{\phi}^n}{\sqrt{5}}$)

Want to find k s.t.

$b = \frac{\phi^{k+1}}{\sqrt{5}}$ solve for k:

$\sqrt{5}\, b = \phi^{k+1}$

$\log_\phi \sqrt{5}\, b = k+1$

$k = \log_\phi \sqrt{5}\, b - 1 = \log_\phi b + \text{const.}$

# of rec calls is bounded above by $\approx \log_\phi b + \text{const}$.

Since everything else in algorithm takes constant time, running time is $O(\#\text{ rec calls})$ so

running time $T(b) = O(\log_\phi b) = O(\lg b)$

$\blacksquare$