Name: ______________________________

Due Date: Thursday, April 23

Exercise 1 Use algebraic manipulations to show that each of the following functions has a fixed point at p precisely when $f(p) = 0$, where $f(x) = x^4 + 2x^2 - x - 3$.

(a) $g_1(x) = (3 + x - 2x^2)^{1/4}$

(b) $g_2(x) = \left(\dfrac{x + 3 - x^4}{2}\right)^{1/2}$

Exercise 2 Given the following sequence $p_{n+1} = \dfrac{p_n^2 + 3}{2p_n}$

(a) Calculate p_1 and p_2 with $p_0 = 3$.

(b) Show by definition that the given sequence is actually a sequence generated by Newton's method to find a solution of the equation $x^2 - 3 = 0$.

Exercise 3 Consider the following non-linear equation: $f(x) = x^2 - 3 = 0$ on $[0, 4]$

(a) Write an expression for p_n using the secant method with the f provided above. Compute p_2 and p_3 using starting points $p_0 = 1$ and $p_1 = 3$.

(b) Compute p_2 and p_3 using the method of false position with f given above, and with starting points $p_0 = 1$ and $p_1 = 3$.

Exercise 4

(a) Use the Fixed Point Theorems from Section 2.2 to show that $g(x) = \pi + 0.5\sin(x/2)$ has a unique fixed point on $[0, 2\pi]$.

(b) Use the theoretical result to estimate the number of iterations required to achieve 10^{-4} accuracy.

(c) (Programming) Use fixed-point iteration to find an approximation to the fixed point that is accurate to within 10^{-4} using stopping criteria based on $|g(p_n) - p_n|$ and $|p_n - p_{n-1}|$. Create three figures for the following convergence histories: $|g(p_n) - p_n|$, $|p_n - p_{n-1}|$, and $|p_n - p|$. You must submit your code along with your homework.