MATH182 FINAL
DUE July 31, 2020

Question 1. *(5pts) Write an algorithm which implements depth-first search without recursion. You might find a* stack *data structure to be useful. Your algorithm should take as input a graph* $G = (V, E)$ *and upon termination should successfully assign the* predecessor, discovery time, *and* finishing time *attributes to all vertices. You can also use the* color *attribute.*

Question 2. *(5pts) Give the optimal parenthesization for a matrix chain product* $\langle A_1, A_2, A_3, A_4, A_5, A_6 \rangle$, *where*

- A_1 *has size* 5×10
- A_2 *has size* 10×3
- A_3 *has size* 3×12
- A_4 *has size* 12×5
- A_5 *has size* 5×50
- A_6 *has size* 50×6

Question 3. *(5pts) Suppose* $G = (V, E)$ *is a connected, undirected graph. Show that if an edge* (u, v) *is contained in some minimum spanning tree of* G, *then it is a light edge crossing some cut of the graph.*

Question 4. *(5pts) Describe an efficient algorithm that, given a set* $\{x_1, x_2, \ldots, x_n\}$ *of points on the real line, determines the smallest set of unit-length closed intervals that contains all of the given points. Argue that your algorithm is correct.*

Question 5. *(True/False) For each of the following statements indicate whether they are* ***true*** *or* ***false****. Each question is worth 2pts, a blank answer will receive 1pt. Recall that "true" means "always true" and "false" means "there exists a counterexample".*

(1) Since MAX-HEAPIFY *runs in* $O(\lg n)$ *time, the process* BUILD-MAX-HEAP *runs in* $\Theta(n \lg n)$ *time.*
(2) In the rod-cutting problem, the naive recursive solution CUT-ROD *runs in polynomial time.*
(3) If a problem has a greedy algorithm solution, then it necessarily does not have a dynamic programming solution; and vice versa.
(4) The algorithm BFS *visits every vertex of the graph.*
(5) During DFS, if a vertex v *has its color changed from white to gray while* u *is gray, then* $u.f < v.f$.