Name: _____

Due Date: Thursday, April 16

Exercise 1 Consider the function $f(x) = \dfrac{1}{1+2x} - \dfrac{1-x}{1+x}$, with $x > 0$.

(a) For what values of x do you expect cancellation of significant digits? Explain.

(b) Rewrite the expression for computing $f(x)$ so that it avoids cancellation for those values of x identified in part (a).

Exercise 2 Suppose $f(x)$ is continuous on $[a, b]$, and $f(x) \in [a, b]$ for any $x \in [a, b]$. Show that f has at least one fixed point on $[a, b]$.

Exercise 3 Consider the following non-linear equation: $f(x) = x^2 - 0.7x = 0$ on $[0.5, 1]$

(a) Show that $f(x)$ has exactly one root on $[0.5, 1]$ without solving the equation.

(b) Consider the bisection algorithm starting with the interval $[0.5, 1]$, i.e. consider $[a_1, b_1] = [0.5, 1]$ and $p_1 = 0.75$. Find the minimum number of iterations required to approximate the solution with an absolute error of less than 10^{-5}.

(c) (Programming) Now program a bisection algorithm to verify this. In particular, create three figures.

- In the first figure, plot the values $|p - p_n|$ on the y-axis, and the iteration number in the x-axis.
- In the second figure, plot $|p_n - p_{n-1}|$ in the y-axis and the iteration number in the x-axis.
- In the third figure, plot the values for $|f(p_n)|$ on the y-axis and the iteration number in the x-axis.

Do your experiments coincide with 3b)?

Exercise 4 Repeat Exercise 3 for the function $f(x) = \sqrt{x} - \cos x$ in the interval $[0, 1]$.