Math 151A - Spring 2020

Midterm 1 : Instructions

- Name: ____
- Student ID: ____

- This exam is open notes/book, however, you may not use the internet. The only exception is for specific searches about the programming language, e.g., you may search for a command about plotting, printing, debugging, etc.

- Write legibly. No points will be given if we cannot understand your work. Submit all your codes and **show all** work needed to obtain your answers

Problem	**Points**
1	20
2	20
3	20
4	20
5	20

1. Consider computing the quantity:

$$y = \frac{1}{x} - \frac{1}{x+1}, \quad x > 0$$

(a) For what values of x do you expect cancellation of significant digits? Explain.

(b) Rewrite the expression for computing y so that it avoids cancellation for those values of x identified in part (a).

2. Consider the function $f(x) = e^x + 2^{-x} + 2\cos(x) - 6$ for $1 \leq x \leq 2$. Use Newton's method, secant method, and the modified Newton's method to find a root of f to accuracy within 10^{-5} for the following two criteria: $|f(p_n)|$ *and* $|p_n - p_{n-1}|$. Plot your solutions and explain your findings.

3. (a) By a theorem from class, show that the function $g(x) = 1 + e^{-x}$ has a unique fixed point on $[1, 2]$ (given values: $e^{-1} = 0.3679$, $e^{-2} = 0.1353$).

   (b) Using $p_0 = 1$, how many iterations does the theory predict it will take to achieve 10^{-5} accuracy, to approximate the fixed point, starting with $p_0 = 1$?

   (c) Program a standard fixed point iteration, Aitken's method, and Steffensen's method to find a fixed point with accuracy 10^{-5} using $p_0 = 1$. Use the stopping criteria: $|g(p_n) - p_n|$. Plot your solutions and explain your findings.

4. Suppose that a function f has m continuous derivatives on the interval $[a, b]$ containing p. Show: f has a zero of multiplicity m at p if and only if

$$0 = f(p) = f'(p) = \cdots = f^{(m-1)}(p) \text{ but } f^{(m)}(p) \neq 0.$$

5. (a) State two equivalent definitions for a zero of multiplicity m for a function $f \in C^m[a,b]$.

(b) Suppose p is a zero of multiplicity m of f, where $f^{(m)}$ is continuous on an open interval containing p. Show that the following fixed point method has $g'(p) = 0$:

$$g(x) = x - \frac{mf(x)}{f'(x)}.$$