```
GeneratePermutations(A)
perm_list = []
n = A.length
for i = 0 to n-1
    B = A[0,...,i-1, i+1...n-1]
    for perm in GeneratePermutations(B)
        append perm ^ A[i] to perm_list
return perm_list
```

$$T(n) = c_1 + n \cdot T(n-1) + c_4\, n \cdot (n-1)!$$

$$\Rightarrow T(n) = \cancel{\Theta(n!)}$$

$$\cancel{\Theta(c_2^n \cdot n!)}$$

$$= \Theta(n!)$$

```
IS_sorted(A)
for i = 1 to A.length - 1
    if A[i] ≱ A[i-1]
        return false
return true
```

$$T(n) = \Theta(n)$$

```
BogoSort(A)
for perm in GeneratePermutations(A)
    if IS_sorted(perm)
        return perm
```

up to $n!$ times

n

$$O(n \cdot n!)$$

Suppose f is asymptotically positive, $\alpha > 0$. Then $O(f)^{\alpha} = O(f^{\alpha})$, $O(f)^{-\alpha} = \Omega(f^{-\alpha})$, $o(f)^{\alpha} = o(f)^{\alpha}$, and $o(f)^{-\alpha} = \omega(f^{-\alpha})$.

Pf: Let $g = O(f)$, g asymptotically positive. Let $c > 0$, n_0 be such that for all $n \geq n_0$, $0 < g(n) \leq c f(n)$. Then for any $n \geq n_0$, $0 < g(n)^{\alpha} \leq c^{\alpha} f(n)^{\alpha}$, i.e. $g^{\alpha} = O(f^{\alpha})$. Also, $0 < c^{-\alpha} f(n)^{-\alpha} \leq g(n)^{-\alpha}$, ~~i.e. $0 < f(n)^{-\alpha} \leq c^{\alpha} g(n)^{\alpha}$~~, so $g^{-\alpha} = \Omega(f^{-\alpha})$. For o and ω, we need to quantify differently: replace "Let $c > 0$, n_0 be ..." with "let $c > 0$, and let n_0 be ...". The rest of the proof is identical, except that we must replace c by $c^{1/\alpha}$ to ensure that any constant works.

Suppose f is increasing asymptotically positive, $c > 1$, ~~and~~

$k \in \mathbb{N}$, and for sufficiently large n,

$f(n+k) \geq c \cdot f(n)$. Then $\sum_{n=1}^{m} f(n) = \Theta(f(m))$.

Pf: ~~f~~ Assume m large. Then

n_0 large

$$\sum_{n=1}^{m} f(n) = \sum_{n=1}^{n_0+k-1} f(n) + \sum_{n=n_0+k}^{m} f(n)$$

$$\leq \sum_{n=1}^{n_0+k-1} f(n) + \sum_{n=n_0+k}^{m} f(n_0+k) \cdot c^{\left\lfloor \frac{n-n_0}{k} \right\rfloor}$$

$$\leq A + f(n_0+k) \cdot \sum_{j=0}^{m-n_0-k} c^{j}$$

$$f(n) \leq c \cdot f(n-k) \leq c^2 \cdot f(n-2k) \leq \cdots \leq c^{\left\lfloor \frac{n-n_0}{k} \right\rfloor} f(n_0+r)$$

$0 \leq r < k$

$$\leq c^{\left\lfloor \frac{n-n_0}{k} \right\rfloor} f(n_0+k)$$

$$\leq A + f(n_0+k) \cdot \Theta\left(c^{\frac{m-n_0-k}{k}}\right)$$

$$\leq A + f(m) \cdot c$$

$$= O(f(m)).$$

Also, $f(m) \leq \sum_{n=1}^{m} f(n)$. Thus $\sum_{n=1}^{m} f(n) = \Theta(f(m))$.

$n^{(\sqrt{n})!}$ $\qquad 2^{2^n}$

$(\lg n)\cdot \cancel{\log}(\sqrt{n})!$ $\qquad 2^n$

n

$\lg\lg n + \sqrt{n}\lg\sqrt{n} - \sqrt{n} + \lg(\sqrt{2\pi}) + \lg\sqrt{n} + \Theta\left(\frac{1}{\sqrt{n}}\right)$

$\sqrt{n}! = \left(\frac{\sqrt{n}}{e}\right)^{\sqrt{n}} \cdot \sqrt{2\pi\sqrt{n}} \cdot \left(1 + \Theta\left(\frac{1}{\sqrt{n}}\right)\right)$