Name: _____

Due Date: Thursday, May 28, 11:59 PM

Exercise 1

(a) The following table was generated using the function $f(x) = e^{2x}$. Use the best possible finite-difference schemes from your notes to determine the missing entry in the following table.

x	$f(x)$	$f'(x)$
1.1	9.025013	
1.2	11.02318	
1.3	13.46374	
1.4	16.44465	

(b) Find the error bounds and compute the actual errors.

Exercise 2 Consider Poisson's equation

$$-\frac{\partial^2 u}{\partial x^2} = f(x), \quad 0 \leq x \leq 1,$$

for $f(x) = 32\pi^2 \sin(2\pi(2x-1)) + 40$.

(a) Verify that the true solution to the differential equation is given by

$$u(x) = 2\sin(2\pi(2x-1)) - 20x(x-1)$$

(b) Use the centered difference scheme to approximate the solution using 4,16,32, and 64 nodes. Plot your solutions along with the true solution. Do your approximations approach the true solution as you increase the number of nodes?

Exercise 3 Approximate the integral

$$\int_1^{1.5} x^2 \ln x dx$$

using the Midpoint, Trapezoidal, and Simpson's 1/3 rule. Compare each approximation with the true integral value (this you can compute using integration by parts).

Exercise 4 The quadrature rule

$$\int_{-1}^{1} f(x)dx = c_0 f(-1) + c_1 f(0) + c_2 f(1)$$

is exact for all polynomials of degree less than or equal to 2. Determine c_0, c_1, and c_2.