Name: _____

Due Date: Friday, May 15 (During TA Session)

Exercise 1 Given data $(-1, 0), (0, 1), (1, 3)$, construct the Power Series, Newton, and Lagrange interpolating polynomial of the data. Show that they are the same polynomial.

Exercise 2 (Programming) Consider the function $f(x) = \frac{1}{1+x^2}$ on the interval $[-5,\ 5]$. For $n = 5, 10,\ 25$, and 50. Plot $f(x)$ and $p_n(x)$ in one figure, and $|e_n(x)| = |f(x) - p_n(x)|$ in another figure using

(a) $n + 1$ equally spaced nodes, and

(b) $n + 1$ Chebyshev nodes.

Explain your findings. Here, you may use any of the three interpolating polynomials mentioned in Exercise 1.

Exercise 3 Suppose we want to approximate the function $f(x) = \exp(-2x) + 2x^2 + x + 1$ on the interval $[0, 1]$ using a piecewise linear polynomial $S_{1,n}$ that is constructed using the $n + 1$ equidistantly spaced nodes $x_i = ih$, where $h = \frac{1}{n}$ and $i = 0, ..., n$.

(a) Using the error bound we learned in class, determine the smallest value of n that guarantees that $|f(x) - S_{1,n}(x)| \leq 10^{-5}$, $\forall x \in [0, 1]$.

(b) (Programming) For the value of n determined in the first item, evaluate f and $S_{1,n}$ at 1000 equally spaced points between 0 and 1. Include a plot of f and $S_{1,n}$ that shows that the two curves are close to each other and report the maximum value of $|f(x) - S_{1,n}(x)|$ at those points. Is this value less than 10^{-5}?