

An introduction to the mechanical properties of ceramics

David J. Green



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An Introduction to the Mechanical Properties of Ceramics

This book is a comprehensive introduction to the mechanical properties of ceramics, and is designed primarily as a textbook for undergraduate and graduate students in materials science and engineering.

Over the past 25 years ceramics have become key materials in the development of many new technologies as scientists have been able to design these materials with new structures and properties. An understanding of the factors that influence their mechanical behavior and reliability is essential. Some of these new applications are structural, and for these it is important to understand the factors that control their mechanical behavior. Non-structural applications are also being developed, but in each case it is necessary to design mechanically reliable materials. This is a particular challenge for materials that are inherently brittle. This book will introduce the reader to current concepts in the field. It contains problems and exercises to help readers develop their skills.

Although designed principally as a textbook for advanced undergraduates and graduate students, this book will also be of value as a supplementary text for more general courses and to industrial scientists and engineers involved in the development of ceramic-based products, materials selection and mechanical design.

An Introduction to the Mechanical Properties of Ceramics

Cambridge Solid State Science Series

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An Introduction to the Mechanical Properties of Ceramics

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Preface

The aim of this book is to provide a text for a senior undergraduate course on the mechanical behavior of ceramics. There are, however, some advanced sections that would allow the book to be used at the graduate level (marked ††). The format of the book owes much to the text, *Mechanical Properties of Matter*, by A. H. Cottrell, which helped me through graduate school. In teaching a course in this area, it has always been frustrating that there are so few texts aimed primarily at ceramics. There is often the concern of discerning whether ideas applied to other materials could also be used to understand ceramic materials. I have also been fortunate in being involved in the field of structural ceramics at a time it has undergone remarkable developments and I have tried to incorporate my interpretation of these recent advances into the text.

I would be amiss in not acknowledging the support I have received in undertaking this project. I owe much to Pat Nicholson, Dave Embury and Dick Hoagland, who patiently introduced me to this field of research and to Tom Wheat, who taught me about the processing of ceramics. I am particularly grateful to Fred Lange, who took a chance on me and became my mentor. His enthusiastic, intuitive advice and sense of fun encouraged me to pursue many new ideas. I also appreciate the interaction with my other colleagues at Rockwell International Science Center. The undergraduates at Penn State in Ceramic Science and Engineering should be acknowledged for suffering through the various versions of this book. I should particularly thank Fred Fitch for patiently pointing out many typographical errors in an earlier version of the book. Thanks are also owed to Brian Watts and Patty Phillips for their patient proof-reading skills. I would also thank George Scherer, David Clarke, David

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Finally, I should mention the encouragement I have received from my extended family; Mel and Vera Smith, the Knapps, Marc, Tina and Tony. The emotional support from Chris Cushing Green, Cyndi Asmus and Patty Phillips was also essential in completing this project.

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Chapter 1

Introduction

In the last 25 years there has been a strong movement to use ceramics in new technological applications and a key facet of this work has been directed at understanding the mechanical behavior of these materials. First, let us consider the various **technological functions** of ceramics as shown in Table 1.1. The diverse properties of ceramics are not always appreciated. For structural functions, adequate mechanical properties are of prime importance. Ceramic materials that are considered for these applications are termed **structural ceramics**. In some cases, such as engine parts, the choice is based on their high-temperature stability and corrosion resistance. These factors imply the engine temperature could be raised, making the overall performance more efficient. Unfortunately, ceramics can be brittle, failing in a sudden and catastrophic manner. Consequently, there has been a strong emphasis on understanding the mechanical properties of ceramics and on improving their strength, toughness and contact-damage resistance. Indeed, it is appropriate to state that there has been a revolution in the understanding of these properties and the associated research has led to the discovery of new classes of structural ceramic materials.

It is important to realize that mechanical properties can also be critical in non-structural applications. For example, in the design of the thermal protection system of the space shuttle, highly porous, fibrous silica tiles are used. The micro-structure of these materials, shown in Fig. 1.1, consists of a bonded array of fibers, usually based on silica glass. Clearly, the prime reason for using these materials was their low thermal conductivity but the resistance to thermal and structural stresses was a key item in the final design. In some non-structural applications, mechanical properties can be important in determining the lifetime

Table 1.1 Functions and technological applications of ceramics

(Adapted from Kenney and Bowen, 1983, reproduced courtesy of The American Ceramic Society.)

Function	Primary characteristic	Examples of applications
Electrical	Electrical insulation (e.g., Al_2O_3 , BeO)	Electronic substrates and packages, wiring, power-line insulators
	Ferroelectricity (e.g., BaTiO_3 , SrTiO_3)	Capacitors
	Piezoelectricity (e.g., PZT)	Vibrators, oscillators, filters, transducers, actuators, spark generators
	Semiconductivity (e.g., BaTiO_3 , SiC , $\text{ZnO-Bi}_2\text{O}_3$, CdS , V_2O_5)	NTC thermistor (temperature sensor) PTC thermistor (heater element, switch) CTR thermistor (heat sensor) Thick-film thermistor (IR sensor) Varistor (noise elimination, surge arrestors)
	Ionic conductivity (β -alumina, ZrO_2)	Solar cells, furnace elements Solid state electrolytes (batteries, fuel cells, oxygen sensors)
	Superconductivity (YBCO)	Magnets, electronic components
	Soft magnets (ferrites) Hard magnets (ferrites)	Magnetic recording heads Magnets, electric motors
Optical	Translucency (Al_2O_3 , MgO , mullite, Y_2O_3 , PLZT)	High-pressure sodium-vapor lamps, IR windows, lighting tubes and lamps, laser materials, light memory, video display and storage, light modulation and shutters.
	Transparency (silicate glasses)	Optical fibers, containers, windows
Chemical	Chemical sensors (ZnO , Fe_2O_3 , SnO_2)	Gas sensors and alarms, hydrocarbon and fluorocarbon detectors, humidity sensors

Table 1.1 (cont.)

(Adapted from Kenney and Bowen, 1983, reproduced courtesy of The American Ceramic Society.)

Function	Primary characteristic	Examples of applications
Thermal	Catalyst carriers (cordierite, Al_2O_3)	Emission control, enzyme carriers, zeolites
	Electrodes (titanates, sulfides, borides)	Electrowinning, photochemical processes
	Thermal insulation (fiberglass, aluminosilicate fibers)	IR radiators, thermal protection systems for aerospace vehicles
Structural	Thermal conduction (diamond films, AlN)	Heat sinks in electronic devices
	Thermal stability (AZS, Al_2O_3)	Refractories
	Hardness (SiC, TiC, TiN, Al_2O_3)	Cutting tools, wear-resistant materials, mechanical seals, abrasives, armor, bearings
Biological	Stiffness and thermal stability (SiC, Si_3N_4)	Ceramic engine parts, turbine parts, burner nozzles, radiant tubes, crucibles.
	Chemical stability (hydroxyapatite, Al_2O_3)	Artificial teeth, bones and joints
	Nuclear fission (UO_2 , PuO_2)	Nuclear fuels, power sources, Cladding and shielding
Nuclear	Neutron absorption (C, SiC, B_4C)	

Notes:

PZT – lead zirconium titanate NTC – negative temperature coefficient

YBCO – yttrium barium copper oxide PTC – positive temperature coefficient

PLZT – lead lanthanum zirconium titanate CTR – critical temperature resistance

AZS – Alumina zirconium silicate IR – infra-red

of the component. For example, the sodium–sulfur battery, that has high energy and power density, has been developed for transportation and energy-storage applications. This battery is based on the use of a solid ceramic electrolyte known as β -alumina, but cracking during recharging can lead to a limited lifetime. Clearly, improvements in the mechanical properties of the electrolyte could significantly impact the economic viability of the battery.

Before launching into the details of the various aspects of mechanical properties, it is worthwhile considering the overall philosophy of **materials science and engineering**. Figure 1.2 shows an overview of the way this discipline is used in

developing technological applications. An understanding of the possible properties of materials allows a particular application to be identified and a design put forward. This is usually in the realm of engineering and it is important to be able to identify and measure all of the critical properties required in the design. In materials science one is also concerned with properties, but here it is usually the optimization of properties, structure and processing that is the key item. The aim is often to adjust the processing so that a particular structure, that gives the best set of properties, is obtained. *The search for new or improved materials is an*

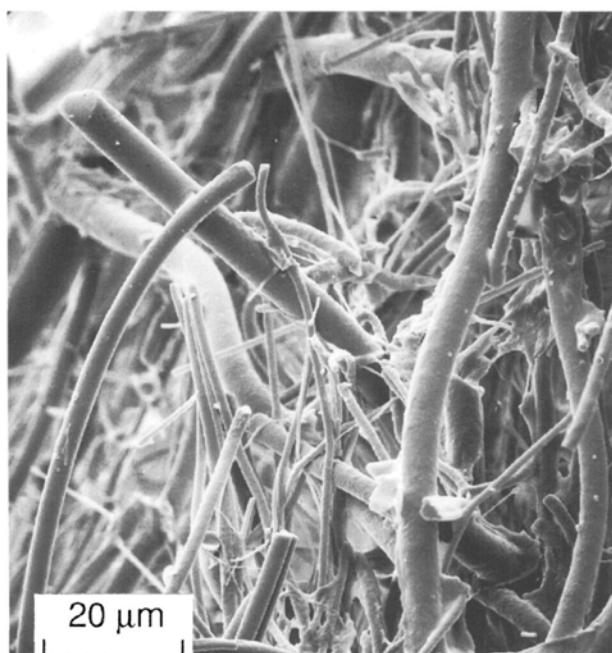


Figure 1.1 Microstructure of space shuttle tile material, a high-porosity fibrous silica; secondary electron image using the scanning electron microscope (SEM). (Reproduced courtesy of Plenum Press, New York.)

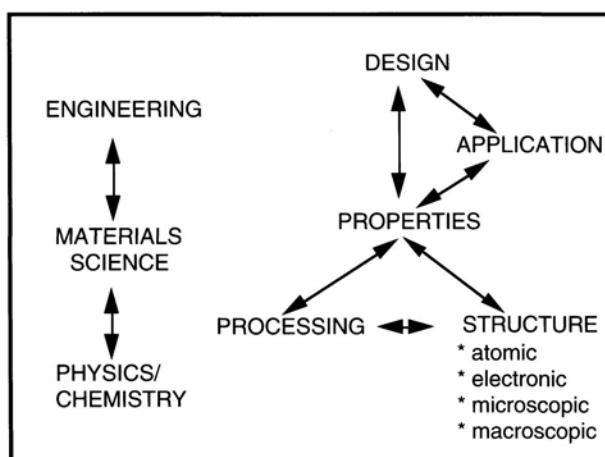


Figure 1.2 Overview of scientific approach for the development of materials for technological applications.

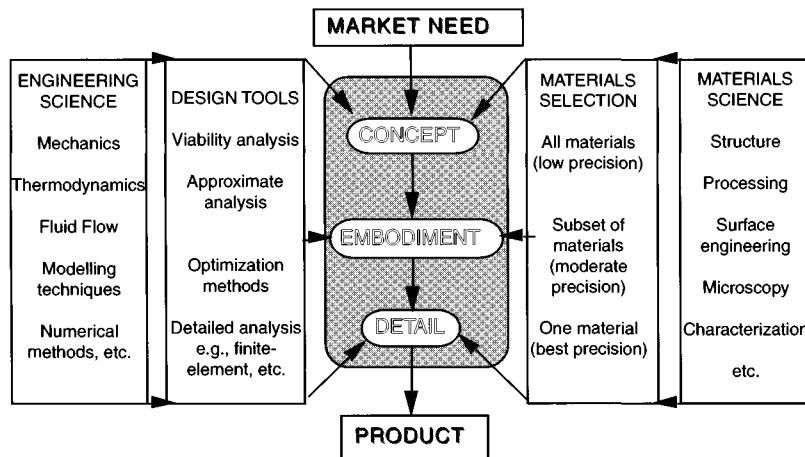


Figure 1.3 A flow chart showing how design tools and materials selection enter the design process. (Adapted from Ashby, 1992. Reproduced courtesy of Heineman Publishers, Oxford, UK.)

essential aspect of the field. The structure of the material that controls a particular property can occur at different scale levels: from the electron and atomic structure through to the macroscopic structure. For example, elastic properties are basically dependent on the chemical bonding, which determines the stiffness of the atomic bonds. Thus, one approach to changing the elastic behavior would be to adjust the composition by adding a solute. It is, however, also possible to change the elastic properties of a material by adjusting the microstructure, i.e., at a larger scale. For example, adding other phases or changing the amount of porosity at the microscopic level will also change the elastic properties. The introduction of ceramic fibers into glass, polymers or metals can substantially increase stiffness and has led to the development of **fiber composites**. At the macroscopic level, the use of different materials in particular configurations and geometries can be important. For example, in laminated structures the elastic behavior can be controlled by placing a stiffer component in the region of highest stress, such as in **sandwich structures**. As indicated in Fig. 1.2, materials science often plays an intermediary role between engineering and the basic sciences of physics and chemistry.

Figure 1.3 shows a flow chart that describes the overall design process. As the design moves from concept to product, design tools of increasing sophistication are needed from engineering science. Concurrently, materials science needs data on the properties of materials with increasing degrees of precision, in order to select the best candidate. In the early stages, approximate data are useful to identify the best group of materials but, later, standard test procedures and in-house testing are often required. The material selection process is becoming more

sophisticated as computer databases are being developed. Such databases are expected to increase in sophistication and 'intelligence'. This latter aspect is particularly important in being able to check and correct any input errors. Clearly this process will be influenced by many other factors, such as the economics and the aesthetics of the design. It is important to realize that there are often many different processing routes to produce a particular material. Shifting economic patterns and the development of new processing techniques can strongly impact the final decisions. There is also greater concern over 'greener' processing and industrial processes are now being studied from an ecological viewpoint (**industrial ecology**).

To illustrate techniques that have been introduced to improve **mechanical reliability**, it is useful to consider some examples for a particular material, say alumina (Al_2O_3). The microstructure of hot-pressed alumina, which consists of fine equiaxed grains ($\sim 5 \mu\text{m}$), is shown in Fig. 1.4. Polycrystalline alumina has found applications from electronic circuit substrates through to armor plating. It is now established, however, that the strength and toughness of alumina can be substantially improved by several techniques. For example, adding zirconia as a second phase gives microstructures similar to that shown in Fig. 1.5. In this approach, a mechanism known as **transformation toughening** can be introduced which increases toughness and strength. The toughness of alumina can also be

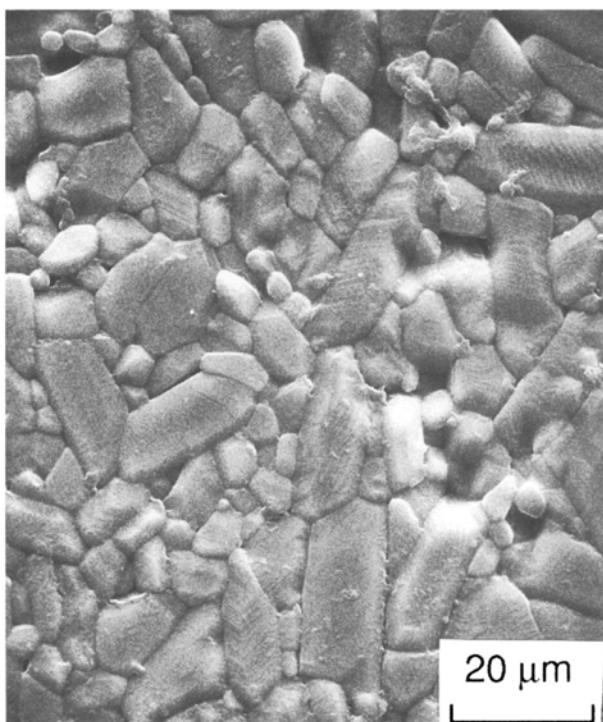


Figure 1.4 Microstructure of hot-pressed polycrystalline aluminum oxide; secondary electron image using the SEM.

increased by incorporating ceramic fibers, whiskers or platelets into the microstructure. For example, Fig. 1.6 shows the microstructure of Al_2O_3 containing SiC platelets. As shown in Chapter 8, **crack bridging** and the frictional pull-out of the reinforcements is often the source for the improved properties of these **ceramic-matrix composites**. The structure of an alumina product can also be manipulated at the macroscopic level. For example, Fig. 1.7 shows the loading of an alumina ceramic sandwich panel. The incorporation of a porous alumina core between dense alumina plates can be used to produce materials with maximum strength or stiffness at minimum weight, especially in flexural loading modes. There is also a current interest in producing ceramic hybrid laminates in which layers of different compositions are interspersed. For example, Fig. 1.8 shows a laminate consisting of alternating alumina and zirconia layers and such a structure can significantly influence the crack-propagation behavior. The above approaches emphasize the use of composites in controlling the structure. There is, however, a push to produce materials in which **self-reinforcement** is ‘grown’ into the microstructure. For example, Fig. 1.9 shows the microstructure of alumina, in which grain shape and texture are used to control the physical properties and Fig. 1.10 shows the microstructure of self-reinforced silicon nitride in which the production of ‘fibrous’ grains is found to increase fracture toughness.

The way a particular material is processed is also very important in deter-

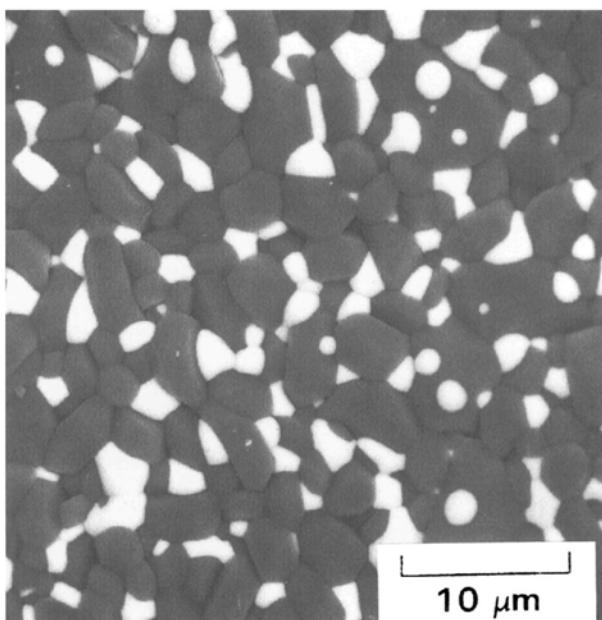


Figure 1.5 Microstructure of a zirconia-toughened alumina; back-scattered electron image using the SEM. The bright phase is zirconia.

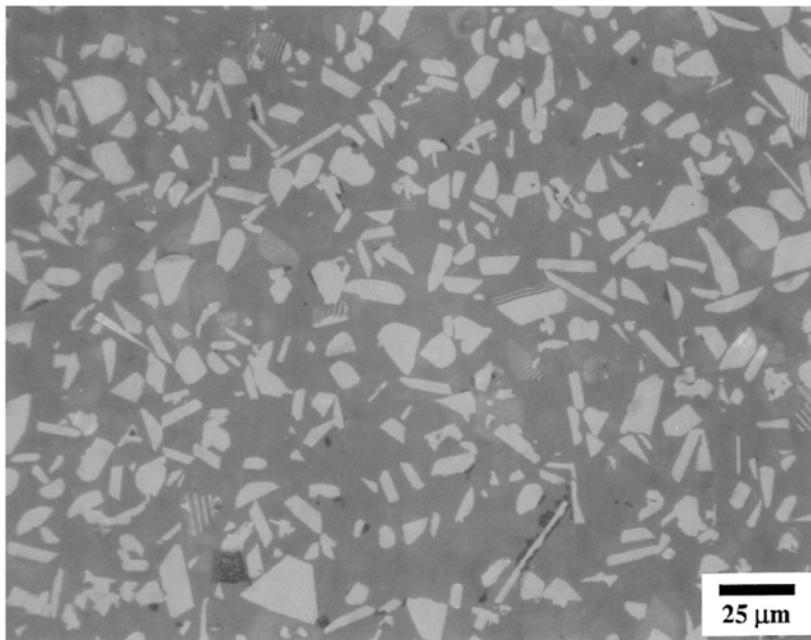


Figure 1.6 Microstructure of a SiC platelet-reinforced alumina; optical micrograph.
(Courtesy of Matt Chou.)

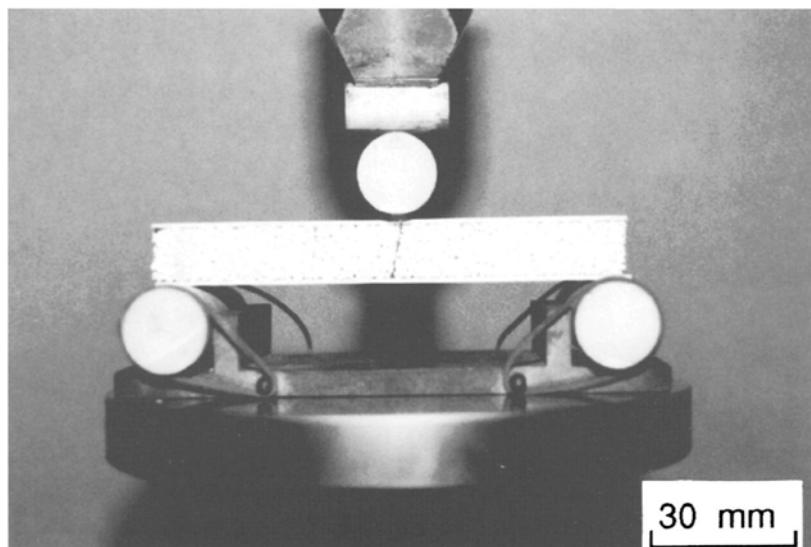


Figure 1.7 Three-point bending of an alumina sandwich panel, consisting of a porous, cellular core and dense faceplates. (Reproduced courtesy of The American Ceramic Society, Westerville OH.)

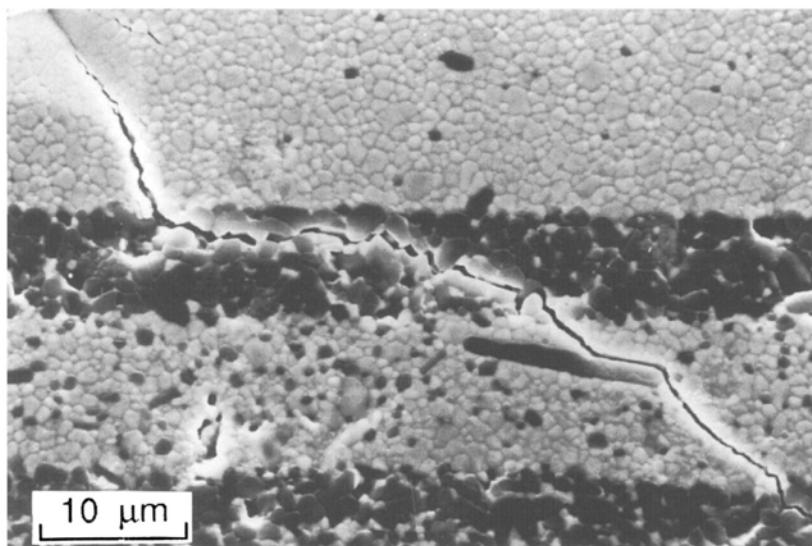


Figure 1.8 Crack deflection in an alumina–zirconia hybrid laminate. The layered structure was produced by electrophoretic deposition; scanning electron micrograph. (Courtesy of P. Sarkar and P. S. Nicholson.)

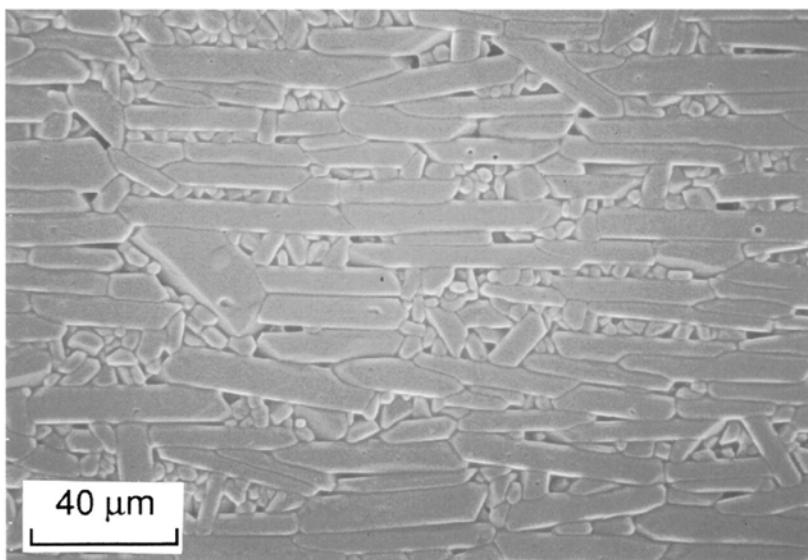


Figure 1.9 Microstructure of alumina in which grain shape and texture are used to control physical properties. (Courtesy of Matthew Seabaugh and Gary L. Messing.)

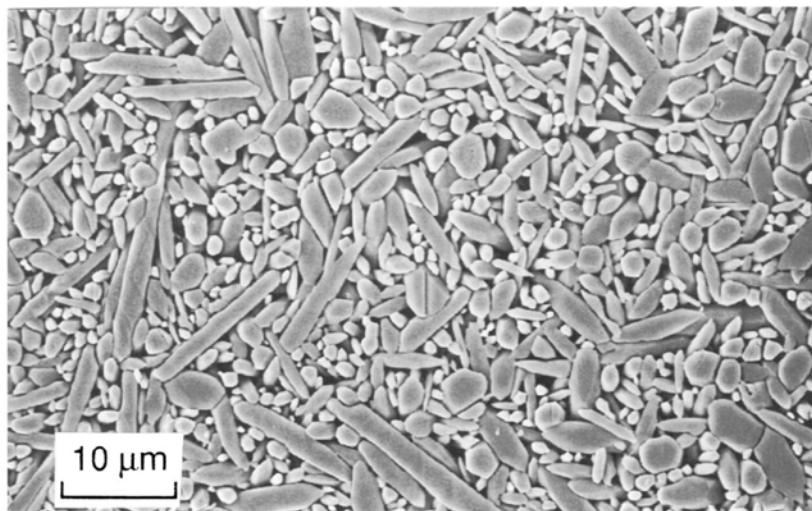


Figure 1.10 Microstructure of self-reinforced silicon nitride in which grain shape is used to control mechanical reliability. (Courtesy of Chien-Wei Li, Allied-Signal Corp.)

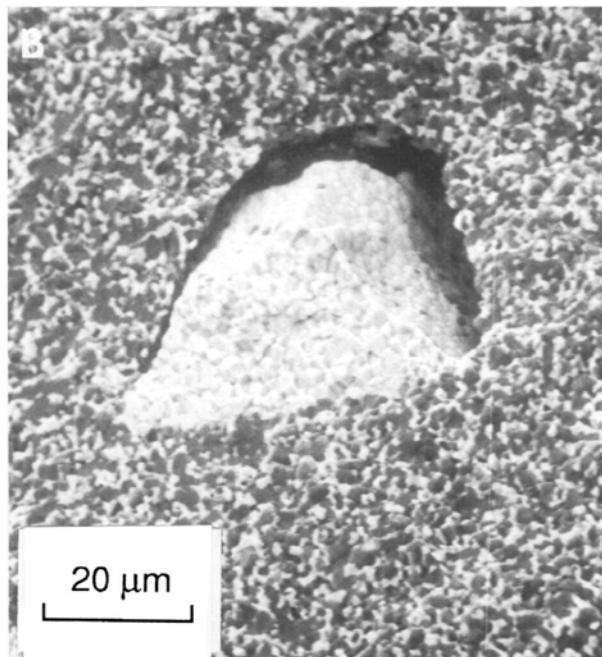


Figure 1.11 Failure origin in a zirconia-toughened alumina, caused by the presence of a zirconia aggregate in the starting powder. (Reproduced courtesy of CRC Press, Boca Raton, FL.)

mining mechanical behavior, as failure is often initiated at **microstructural imperfections**. A useful philosophy in improving the strength of ceramics is to identify the nature of the failure origins in the material. The processing is then adjusted to eliminate the particular source of failure. Figure 1.11 shows an example of a failure origin in zirconia-toughened alumina that was caused by poor mixing of the two phases. In this case, a zirconia agglomerate densified more extensively than the surrounding material and led to a ‘crack-like’ void. Clearly, more attention to the mixing of the components would eliminate this type of defect and thereby lead to an improvement in strength. Techniques are also being developed to identify microstructural imperfections in a non-destructive fashion. This approach can also be used to ‘improve’ strength by identifying and eliminating components that contain large defects.

Figure 1.12 summarizes the tremendous improvements that have been made in the strength of ceramics over the last 25 years. This is a direct result of improved processing, the detection of new toughening mechanisms, and the refined understanding of the relationships between microstructure and mechanical properties. In the remainder of this book, the various mechanical properties of ceramics will be discussed, both in terms of a formal description and the way

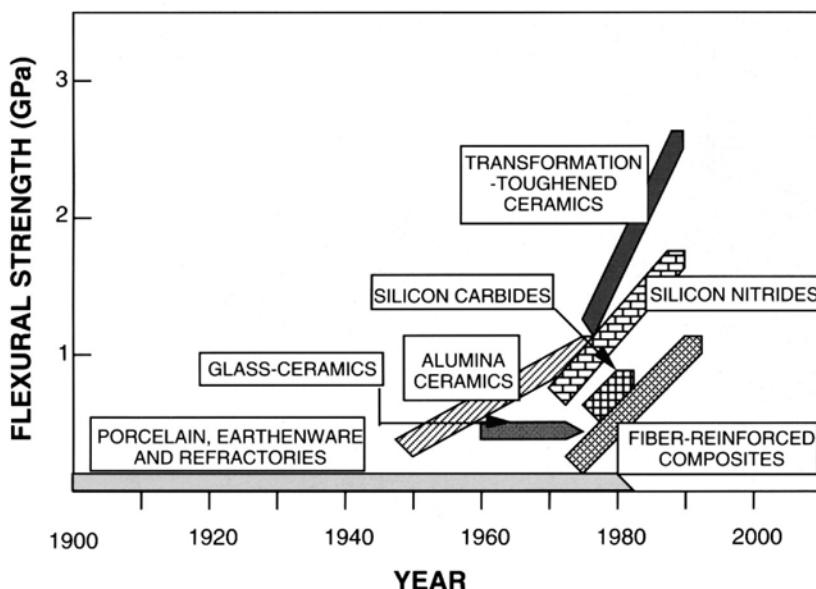


Figure 1.12 The dramatic increase in the strength of bulk ceramics in the last 25 years.

these properties are linked to the structure. The initial chapters will emphasize the elastic behavior of ceramics as this is a fundamental mode of deformation in ceramics. Indeed, engineering structural design is primarily based on the elastic properties of materials. At high temperatures, however, ceramics may undergo permanent deformation by processes such as viscous flow, plastic deformation and creep, and these are covered in Chapters 5 to 7. Inevitably, if stresses are increased further, fracture will occur, and Chapter 8 considers the fundamental basis for understanding fracture in brittle materials such as ceramics. Chapter 9 is concerned with the methodologies needed in the engineering design of ceramic components to ensure reliability under the action of mechanical or thermal stresses.

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Chapter 2

Elastic behavior

In describing the mechanical properties of materials, one is interested in understanding the response of the materials to force. For example, consider the forces that are exerted on materials as we walk around. The forces arise because our bodies are being acted on by gravity and this force is acting on each particle of our body. A force that acts on every particle of a body, animate or not, is known as a **body force**. The force produced by our bodies is then transmitted to the floor through our feet. As the force is being transmitted via a surface, it is known as a **surface force**. Now, let us consider what is happening to the floor as we transmit this force. In general, we do not notice much of a reaction but, following Newton's Third Law, we know for every action there is an equal and opposite reaction. In a way this is rather remarkable, as it indicates the floor is pressing back on our feet with exactly the same force as that caused by our weight. If the reactive force was less, we would sink and if it was too high, we would rise. To understand the mechanical properties of materials, it is important to understand how this reaction arises and as materials scientists we are interested in determining whether this reaction can be controlled.

The reaction of a material becomes clearer if one walks across a wooden plank. The force exerted by our body causes the plank to bend and the only logical explanation is that the atoms in the plank have moved to create the reactive force. Once one has completed the journey across the plank (unless fracture intervenes), the plank usually returns to its original position. If a deformation is reversible, it is termed **elastic** and this phenomenon is very common in ceramics and glasses, except at high temperatures. Clearly, many materials are not elastic (**inelastic**) and the forces can create a permanent change in size or shape. For example, if we walk through mud or wet sand we often

create, at least temporarily, a lasting impression. There are various types of inelastic responses in materials, such as **plasticity** and **viscosity** and, in some cases, time can be important in the deformation. For example, hanging a weight on a string of chewing gum can cause the **permanent deformation** to increase with time. This type of behavior is known as **creep** and this process often occurs in ceramics and glasses at high temperatures. Ultimately, if the force on a material is high enough, whether the deformation is elastic or inelastic, **fracture** will occur. In the current chapter, we will be concerned with the formal description of elasticity. Inelastic deformation and fracture will be the subject of later chapters.

2.1 Elastic deformation of atomic bonds

An indirect way of demonstrating that atoms move in a material as one applies a force is shown schematically in Fig. 2.1. A polycrystalline material is being bent by a set of forces, while x-ray diffraction is used to determine the spacing of atomic planes inclined to the surface. The interplanar spacing d of a particular set of planes can be determined from the angle θ associated with a particular diffraction peak and the wavelength of the radiation λ using Bragg's Law. i.e., $d = \lambda / (2 \sin \theta)$. As the force on the bending specimen increases, the diffraction peak changes its angular position, indicating an increase in interplanar spacing, i.e., the atoms have moved. If the atoms move back to their original positions after the stress is removed, the material is termed **elastic**. As shown in Fig. 2.2, one can plot the form of the relationship between the applied force F and the interplanar spacing d . If this relationship is linear and the material is elastic, the material or the deformation is termed **linear elastic**.

If elastic behavior is associated with the movement of atoms, consider a force

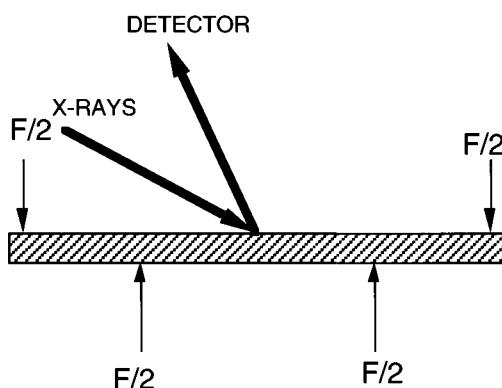


Figure 2.1 Schematic of experimental arrangement for the study of atomic displacements under the application of stress.

f being applied to a pair of atoms, as shown in Fig. 2.3. Ignoring atomic vibrations, the atoms would be at their equilibrium spacing a_0 , if $f=0$. In a simple sense, one can consider the atomic bond to be a sort of spring. As f increases, the atoms will move apart or, if the force direction is reversed, the atoms should move closer. If the force increases the interatomic spacing, it is termed **tensile** and if it decreases the spacing, **compressive**. One would expect the ease of this deformation to depend on the chemical nature of the atomic bond and it is of interest to determine how the deformation relates to the interatomic potential ϕ . This potential consists of two important components, an attractive term and a repulsive term, as shown in Fig. 2.4(a). The summation of these two terms leads to the minimum in the potential that is associated with the equilibrium atomic spacing. Clearly, the interatomic potential changes if one displaces the atoms, the displacement u being defined as $(a-a_0)$. In order to transform the potential-displacement relationship to force and displacement, one needs to use $f=d\phi(u)/du$. If one performs this differentiation, a curve such as that illustrated in Fig. 2.4(b) is obtained. In the vicinity of a_0 , the slope of the force-displacement function is expected to be approximately linear and it is in this region that one expects to obtain linear elastic behavior.

In order to move the atoms depicted in Fig. 2.3, work has to be performed on the system. For example, if one atom is moved by an amount δu , the work done is $f \delta u$, using the **principle of virtual work**. In doing this work, the bond energy has changed by an amount $\Delta\phi=\phi(u+\delta u)-\phi(u)$. As indicated earlier, the equilibrium condition for the deformation is given by $f=d\phi(u)/du$ and thus as δu approaches zero, $\Delta\phi=f\delta u$. Thus, for small displacements, the work done on the atoms is equivalent to the change in bond energy. It is now possible to define a

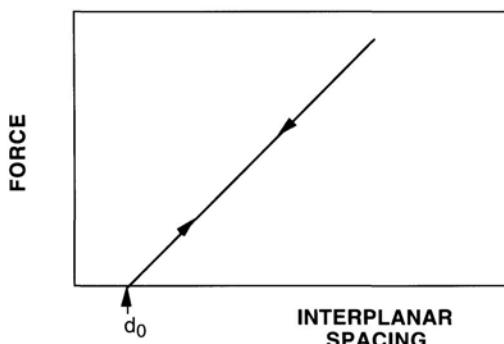


Figure 2.2 Example of possible output from experimental arrangement shown in Fig. 2.1 for a linear elastic material.

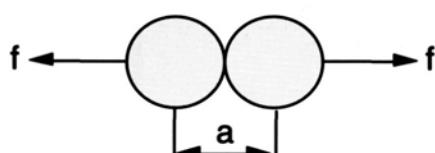


Figure 2.3 Tensile force acting on a pair of atoms.

perfect elastic deformation; it is a deformation that has the following qualities: a) the force is a function of displacement; b) for each displacement there is a corresponding force; and c) it is reversible.

Let us consider the force-displacement relationship for the two atoms in Fig. 2.3 in more detail. First, note that $\phi(u)$ is a continuous function and has a minimum value when $u=0$, implying that $d\phi(u)/du=0$ at the equilibrium spacing. As $\phi(u)$ is continuous, one can expand this function around the equilibrium position as a Taylor series, i.e.,

$$\phi(u)=\phi+\left(\frac{d\phi}{du}\right)_0 u+\left(\frac{d^2\phi}{du^2}\right)_0 \left[\frac{u^2}{2}\right]+\dots \quad (2.1)$$

where the subscript zero is used to represent the value at $u=0$. It has already been indicated that $(d\phi/du)_0=0$ so the second term on the right-hand side vanishes. In addition, for small values of u , one can neglect the term in u^3 and the higher terms. Thus, using $f=d\phi/du$, one obtains

$$f=\left(\frac{d^2\phi}{du^2}\right)_0 u=ku \quad (2.2)$$

i.e., a linear relationship between force and displacement is expected. Thus, for small displacements one predicts **linear elastic** behavior in a material. Moreover,

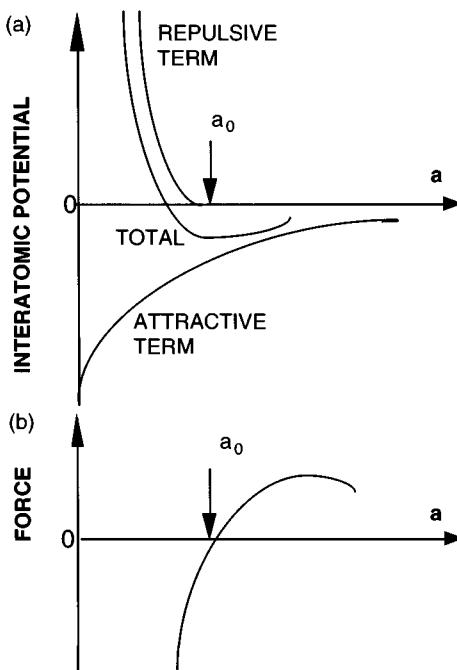


Figure 2.4 Schematic of (a) interatomic potential and (b) force relationships for a pair of atoms as a function of their separation, a . The potential is composed of attractive and repulsive terms.

one can define a proportionality constant, $k = (d^2\phi/du^2)_0$, which is an **elastic constant** for a given material. This constant represents the stiffness of the atomic bond and depends on the curvature of the interatomic potential at $u=0$. For interatomic potentials with a small radius of curvature at $u=0$, k will be large and hence the bond will exhibit a high stiffness. Equation (2.2) is an atomic version of the law known as **Hooke's Law**, which is used to describe linear elastic deformations. An important property of linear elastic deformations is that displacements produced by more than one force can be superposed, provided the forces do not strain the bonds beyond the range of constant curvature in ϕ near $u=0$. This property is known as the **principle of superposition**. The link between elastic behavior and atomic structure has now been established and these ideas will be extended further in Chapter 3.

2.2 Failure of Hooke's Law

Linear elastic behavior is common in macroscopic bodies, especially in ceramics. Indeed, one expects all materials to obey Hooke's Law over some range of displacements. When the deformation of the bonds becomes too large, however, one expects that Hooke's Law will fail to describe the behavior. Figure 2.5 shows the wide range of deformation behaviors that can be found in ceramic materials. Many ceramics, such as Al_2O_3 , are linear elastic but ceramic whiskers, such as SiC, can often be so strong that one can no longer assume the displacements are small. Higher-order terms can no longer be neglected in Eq. (2.1) and the force-displacement behavior becomes non-linear but still elastic. If one considers the slope of the force-displacement function in Fig. 2.4(b), one can see the bonds become more compliant for large tensile deformations and stiffer for compressive deformations. **Non-linear elastic** behavior is also found

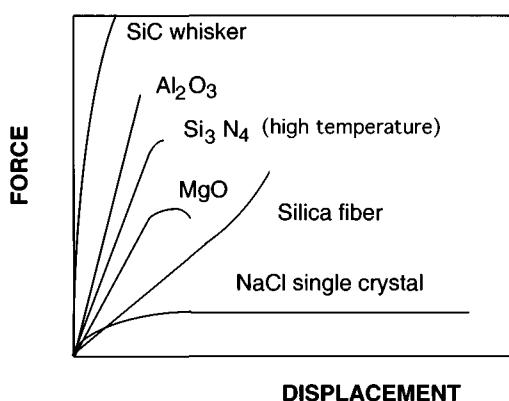


Figure 2.5 Schematic of load-displacement behavior for different ceramics, showing various types of deformation behavior.

in high-strength, silica-glass fibers but in this case tensile deformations lead to an increase in stiffness. This effect has been related to the rotation of the $(\text{SiO}_4)^{4-}$ tetrahedra that form the glass structure, giving an increase in stiffness once the rotation has occurred. An important distinction in the various types of non-linear behavior is whether the material remains entirely elastic or whether **non-elastic** or **inelastic** processes intervene. For example, in polycrystalline MgO at high temperatures (>1800 °C), the initial deformation behavior is linear elastic but once a critical force is passed, termed the **yield point**, the material becomes inelastic. The implication is that inelastic materials have the ability to ‘flow’. In the case of MgO and the alkali halides, atoms can slide past each other and this is termed **plastic deformation**. The process by which this occurs will be discussed in more detail in Chapter 6. Inelastic deformation is also found in many covalent ceramics, such as Si_3N_4 , at high temperatures. In these cases, the inelasticity is caused by **viscous flow** of a glassy phase, which is often present in the grain boundaries of these materials. Viscous flow will be discussed in Chapter 5. It is clearly important to be able to distinguish non-linear elasticity from inelasticity and this is accomplished by unloading the material. This is shown in Fig. 2.6 for an inelastic material and one finds there is no longer a single functional dependence on load. Some of the deformation is still elastic but there is also a permanent displacement that remains at zero force. If a material was non-linear elastic, the displacement would return to zero.

For the remainder of this chapter, we will return to the subject of linear elasticity and concentrate on the way it is formally described for macroscopic bodies. At first, uniform deformations in a body will be considered but then we will move to using continuum mechanics for the description of non-uniform deformations.

2.3 Engineering elastic constants

For large bodies that are linear elastic, the version of Hooke’s Law given as Eq. (2.2) is not very useful. Indeed, even the use of force and displacement to describe the deformation becomes inappropriate. Consider the atomic arrangements shown schematically in Fig. 2.7. For the four pairs of atoms in (b), one would need to apply four times the force to get the same displacement as a single pair of atoms in (a). For the string of three atoms in (c), one would apply only one-half the force to get the same displacement as the single pair. This implies Hooke’s Law would involve body dimension terms as well as k . This is somewhat awkward as it would be preferable to describe the linear elastic response of bulk materials in the simplest way possible. This problem can be overcome by normal-

izing the force to the cross-sectional area of the body and the displacement to the length of the body.

Consider a body, initial cross-sectional area A_0 and length L_0 being pulled in uniaxial tension by a force F , as shown in Fig. 2.8. The body is assumed to be an isotropic uniform continuum, (i.e., independent of direction with no internal structure). Under the action of the force, the body will increase in length by an

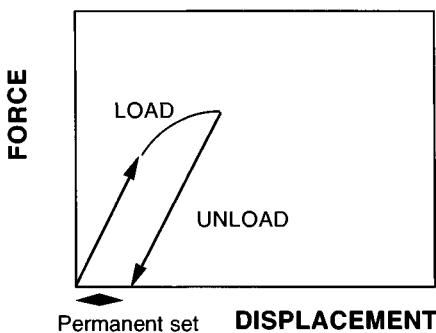


Figure 2.6 Unloading of inelastic materials gives rise to a permanent displacement at zero load.

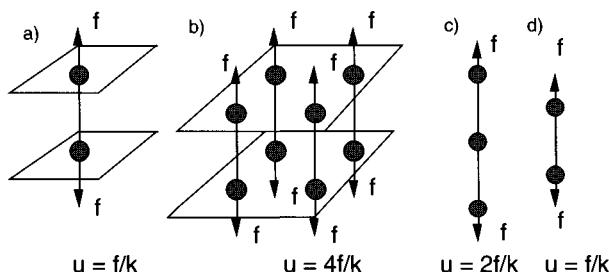


Figure 2.7 Effect of specimen size on a linear elastic deformation. To obtain the same displacement in b) as in a), one would need four times the applied force. For the same force, one would obtain twice the displacement in c) compared to d).

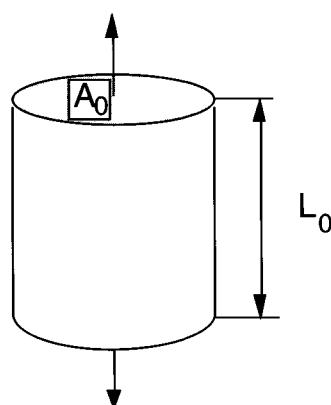


Figure 2.8 Uniaxial tensile deformation of a body.

amount ΔL . Two new parameters, **stress** $\sigma = F/A_0$ and **strain** $\varepsilon = \Delta L/L_0$, can now be defined replacing force and displacement. As a first approximation, let us consider the macroscopic body in Fig. 2.8 as ‘strings’ of atoms. This ignores interactions between the strings and other longer-range interactions. For one string of atoms, $\varepsilon = u/a_0$ and as the force acting on the string is $f = F/N$, where N is the number of atomic strings, one can rewrite Eq. (2.2) as $F = Nk\varepsilon a_0$ or $\sigma = (N/A)k\varepsilon a_0$. Recognizing that the number of bonds per unit area, $N/A = 1/a_0^2$, one obtains $\sigma = (k/a_0)\varepsilon$. More generally, this is written as

$$\sigma = E\varepsilon \quad (2.3)$$

where E is termed the **Young’s modulus**, which represents a material constant for a uniaxial tensile deformation. Although the above derivation is approximate and $E = k/a_0$ is only an estimate, this macroscopic version of Hooke’s Law is exact for a linear elastic material under a uniaxial tensile or *compressive* stress. It is often forgotten that the simple version of Hooke’s Law given in Eq. (2.3) applies only for one type of loading geometry. The application of a more complex stress state will change the form of this equation. Generalized versions of Hooke’s Law will be discussed further in Section 2.10. If a uniaxial tension test was performed on a linear elastic material, a response similar to Fig. 2.9 would be obtained, with the slope of the stress-strain curve given by E . In dense polycrystalline ceramics, the value of E is generally in the range of 100–800 GPa and for silicate glasses from 60–80 GPa. The structural factors that influence the magnitude of E will be discussed in Chapter 3. Figure 2.9 also demonstrates the type of behavior obtained if the material remains linear elastic to failure. Such behavior is common in covalently bonded ceramics (e.g., carbides, borides, oxides and nitrides) below ~ 1000 °C. The points on the two axes corresponding to the failure point are then termed the **fracture stress** (σ_f) and **fracture strain** (ε_f). The fracture stresses of dense ceramics and silicate glasses are often in the range 70–700 MPa but polycrystalline ceramics have been developed recently with fracture stresses (**tensile strengths**) exceeding 2 GPa. Moreover, in the form of single-crystal

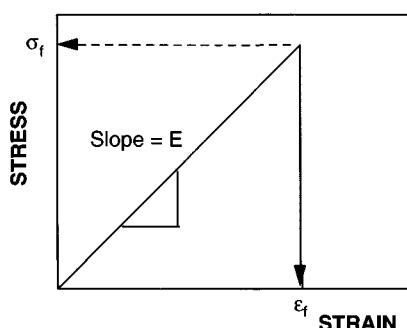


Figure 2.9 Schematic of stress–strain curve for a linear elastic material.

whiskers, the strengths can be even higher (up to ~ 25 GPa). Using typical numbers for the fracture stress and Young's modulus in Eq. (2.3), fracture strains for polycrystalline ceramics are typically <0.001 (0.1%).

In the uniaxial tension test (Fig. 2.8), there is usually a transverse strain, i.e., a strain perpendicular to the applied stress. This can be used to define a second elastic constant, **Poisson's ratio** (ν), as the negative ratio of the transverse strain (ϵ_T) to the longitudinal strain (ϵ_L), i.e., $\nu = -\epsilon_T/\epsilon_L$. For isotropic materials, it can be shown from thermodynamic arguments, that $-1 \leq \nu \leq 0.5$. For many ceramics and glasses, ν is usually in the range 0.18–0.30.

The definitions of stress and strain, given above, do pose a problem in terms of their generality, especially for large deformations. Consider the series of axial tensile deformations shown in Fig. 2.10. We have defined strain by normalizing the longitudinal displacement to L_0 ; this is termed the **engineering strain** (ϵ_E). If, however, the initial specimen size was L_3 and the specimens were under uniaxial compression, one would presumably normalize the axial displacements to L_3 . This process would imply that the absolute magnitude of the strain in going from L_0 to L_3 would be different than in going from L_3 to L_0 . Clearly, one would prefer the two strains to be the same magnitude but with opposite signs. To overcome this problem, one can define strain in terms of the changes in length normalized by the instantaneous length and this is termed the **true strain** (ϵ_t). With respect to Fig. 2.10, if the true strain is visualized as increments of engineering strain, one can write

$$\epsilon_t = \left(\frac{L_1 - L_0}{L_0} \right) + \left(\frac{L_2 - L_1}{L_1} \right) + \left(\frac{L_3 - L_2}{L_2} \right) \quad (2.4)$$

By making the increments in the displacements small in such a summation (i.e., $\Delta L = dL$), one obtains

$$\epsilon_t = \int_{L_0}^{L_F} \frac{dL}{L} = \ln\left(\frac{L_F}{L_0}\right) \quad (2.5)$$

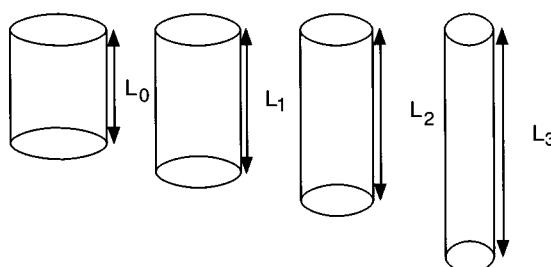


Figure 2.10 A series of deformations on a material to show that engineering strain depends on the stress direction.

where L_0 and L_F are the initial and final lengths of the specimen. From the definition of engineering strain, it is simple to show that $\varepsilon_t = \ln(1 + \varepsilon_E)$. Figure 2.11 shows the two strains in terms of $(\Delta L/L)$ and, for values $< 10\%$, the difference between true and engineering strains is found to be very small. Thus, for linear elasticity, the difference is often ignored but in inelastic processes, such as plastic deformation, the difference can become important. There is a similar problem in the definition of stress; our previous definition, normalized by A_0 , is termed the **engineering stress**, whereas **true stress** is defined by normalizing the force to the *instantaneous* cross-sectional area. In a similar fashion to strain, the difference between true stress and engineering stress is generally small for linear elastic materials.

In scientific processes one is often concerned with energy changes and thus it is worthwhile considering the energy involved in a linear elastic deformation. Consider the uniaxial tension test shown in Fig. 2.8. The force F , in causing displacements u in the body, is doing work W on the body and it was shown earlier that this work is stored as an increase in the interatomic potential. It is useful to obtain expressions for this **stored elastic strain energy**, which is equivalent to W . In uniaxial tension, the total work done on the body is given by

$$W = \int_0^u F du = \int_0^u \sigma A du \quad (2.6)$$

For a linear elastic body, using Eq. 2.3, one can write

$$W = \int_0^u EA \varepsilon du = \int_0^u EA \left(\frac{u}{L}\right) du = \frac{EAu^2}{2L} = \frac{VE\varepsilon^2}{2} \quad (2.7)$$

where V is the volume of the body. Another common energy parameter is the **elastic strain energy density** U , the elastic energy per unit volume, i.e.,

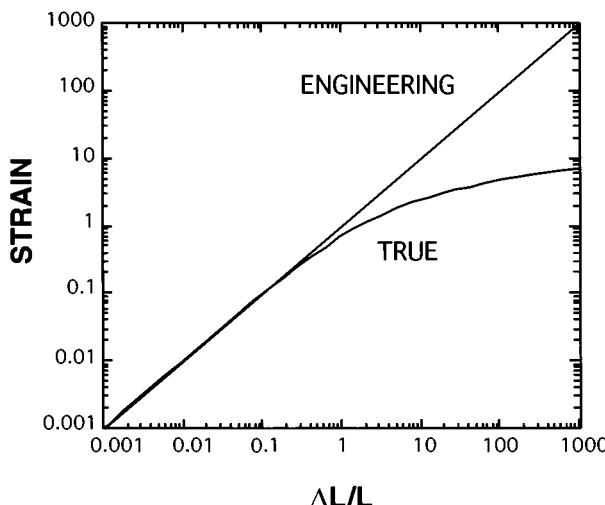


Figure 2.11 Difference between true and engineering strain for various values of fractional length change.