Reconstructing the Periodic Table
A Geometric Guide for Students

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Why You Should Care

Understanding *why* electrons arrange themselves the way they do gives you predictive power over matter. Instead of just memorizing rules, you will learn the geometric logic that determines:

- Why iron rusts but gold remains shiny.

- Why metals conduct electricity while wood does not.

- How to design new materials with specific properties.

- The deep connection between geometry and the elements.

Abstract

This guide provides a step-by-step reconstruction of the entire Periodic Table using the principles of **Action Realism**. Instead of memorizing the Madelung rule $(n + \ell)$, we calculate the "packing cost" for each electron shell based on geometric constraints. We compare these results side-by-side with standard quantum mechanics, showing how simple cost functions can predict the structure of matter, including the famous anomalies like Chromium and Copper.

Contents

1 How to Get the Most From This Guide

1. Read through once without doing calculations.

2. Go back and do all "Predict First" boxes.

3. Try the practice problems without peeking.

4. Teach the concept to a friend (best way to learn!).

5. Use the quick reference card for exams.

2 Prerequisites: Quantum Numbers Quick Review

Before we begin, let's review the language of the atom.

- **Principal Quantum Number (n):** The Shell/Size (1, 2, 3...). Think of this as the distance from the nucleus.

- **Azimuthal Quantum Number (ℓ):** The Shape (0=s, 1=p, 2=d, 3=f). Think of this as the complexity of the fold.

- **Magnetic Quantum Number (m):** Orientation ($-\ell$ to $+\ell$).

- **Spin Quantum Number (s):** Up or down ($\pm 1/2$).

Note: For determining the filling order, we only need n and ℓ.

3 The Historical Puzzle: Order from Chaos

When Dmitri Mendeleev first laid out the Periodic Table in 1869, it was a map without a legend. He knew that elements with similar properties recurred at regular intervals, but he didn't know *why*.

1869	**Mendeleev**: Pattern recognized (Periodicity).
1920s	**Bohr**: Shell model proposed (n).
1936	**Madelung**: The $(n + \ell)$ rule observed empirically.
1960s	**Standard QM**: Calculated via Hartree-Fock approximations.
2020s	**Action Realism**: Geometric explanation via "Packing Costs."

4 The Rules of the Game

To build an atom, we need to pack electrons into the available space around the nucleus.

Two Ways to Pack

■ Standard QM (The Empirical Rule)	♦ Action Realism (The Cost Function)
Rule: Orbitals fill in order of increasing value of $(n + \ell)$. If values are equal, the lower n fills first.	**Concept:** Nature minimizes the "Action Cost" $\mathcal{C}$ to stabilize a fold.
Justification: Penetration and shielding effects. s-orbitals ($\ell = 0$) penetrate closer to the nucleus than d-orbitals ($\ell = 2$), lowering their energy.	**Calculation:** $\mathcal{C} \approx n + \ell$. Twisting a fold ($\ell$) incurs a comparable cost to expanding it (n). The tie-breaker is volume (n).

> **The Cost Formula**
>
> For any orbital defined by size n and twist ℓ:
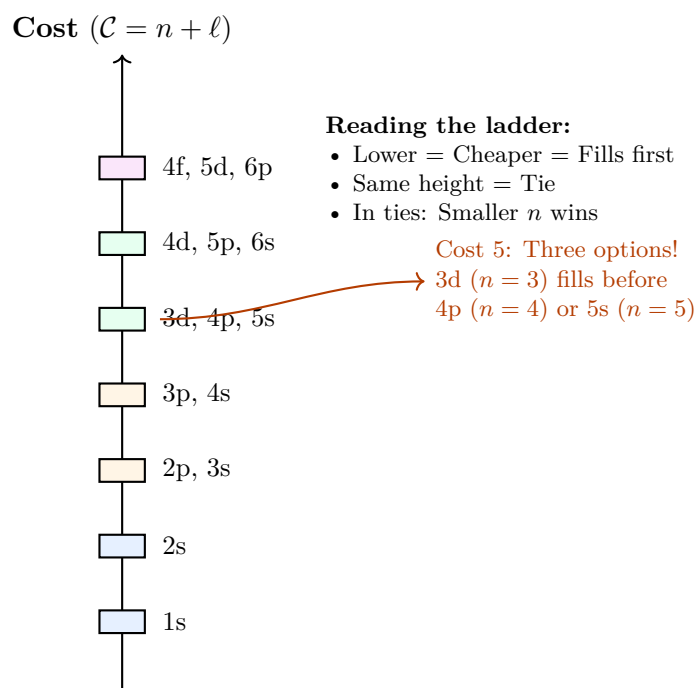> $$\text{Cost } \mathcal{C} = n + \lambda\ell, \quad \lambda \approx 1$$

> **Simplest Model:** $\mathcal{C} \approx n + \ell$.
> **Tie-Breaker:** If two orbitals have the same cost, pick the one with the smaller n (smaller volume).
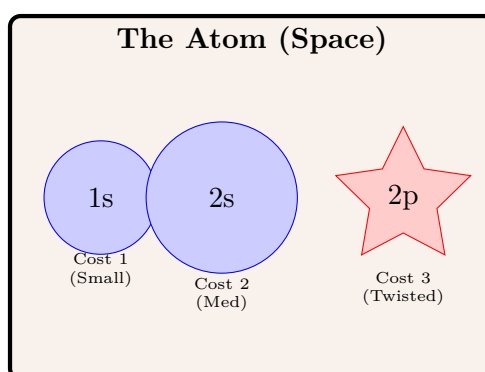
Watch Out!

Many students get confused here! Remember:

- n is the Principal Quantum Number (Size/Shell).

- ℓ is the Angular Momentum Quantum Number (Shape/Twist: s=0, p=1, d=2, f=3).

- We are comparing the **sum** $n + \ell$, not just n!

Cost ($\mathcal{C} = n + \ell$)

4f, 5d, 6p	**Reading the ladder:**
4d, 5p, 6s	• Lower = Cheaper = Fills first
	• Same height = Tie
	• In ties: Smaller n wins
3d, 4p, 5s	Cost 5: Three options!
	3d ($n = 3$) fills before
	4p ($n = 4$) or 5s ($n = 5$)
3p, 4s	
2p, 3s	
2s	
1s	

The Suitcase Analogy

The Atom (Space)

1s — Cost 1 (Small)
2s — Cost 2 (Med)
2p — Cost 3 (Twisted)

Packing Rule: We fill the suitcase with the "cheapest" (lowest Cost) items first.
Simple shapes pack easier than twisted ones.

Reading Electron Configurations

The notation $[Ar]4s^2 3d^5$ means:

- $[Ar]$: "Start with Argon's 18 electrons" (shorthand)

- $4s^2$: 2 electrons in the 4s orbital

- $3d^5$: 5 electrons in the 3d orbital

- **Total**: $18 + 2 + 5 = 25$ electrons (this is Manganese)

Why $n + \ell$?

The Physics Behind the Formula:

The n term (size cost): Larger orbitals ($n = 2, 3, 4...$) require more "volume" in phase space. Cost grows linearly with n.

The ℓ term (twist cost): Adding angular momentum means creating curvature/twist in the fold. Each unit of twist ($\ell = 0, 1, 2, 3$) requires bending the geometry, which costs action.

Why equal weight? In the Action Realism framework, we model:

$$\mathcal{C} \approx \alpha n + \beta \ell \tag{1}$$

where α and β are geometric coefficients. Empirically, $\alpha \approx \beta \approx 1$ (both costs are comparable). This is the **key geometric insight**: twisting and expanding cost roughly the same amount of action per unit.

The result: The simplest cost function that works is $\mathcal{C} = n + \ell$.

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5 Period 1: The Simple Start (H, He)

- **Available Seats:** $1s$ ($n = 1, \ell = 0$).

- **Cost Calculation:** $\mathcal{C}_{1s} = 1 + 0 = \mathbf{1}$.

Element	Z	Choice	AR Cost	Standard Config	Match?
Hydrogen	1	$1s$	1	$1s^1$	Yes
Helium	2	$1s$	1	$1s^2$	Yes

6 Periods 2 & 3: The Pattern Emerges

We now have two options to fill:

- **$2s$ Orbital:** $n = 2, \ell = 0$. Cost $\mathcal{C} = 2 + 0 = \mathbf{2}$.

- **$2p$ Orbital:** $n = 2, \ell = 1$. Cost $\mathcal{C} = 2 + 1 = \mathbf{3}$.

Predict First

Quick calculation:

- $\mathcal{C}_{2s} = \underline{\quad}$, $\mathcal{C}_{2p} = \underline{\quad}$

- Which fills first? $\underline{\qquad}$

Check your answers after calculating, then read on to verify.

Prediction: The $2s$ shell (Cost 2) must fill before the $2p$ shell (Cost 3).

Period 2 (Lithium – Neon)

Z	Element	Filling Step	Winner	Configuration	Match?
3	Li	$2s$ (Cost 2) vs $2p$ (Cost 3)	$2s$	$[He]2s^1$	Yes
4	Be	Finish $2s$	$2s$	$[He]2s^2$	Yes
5	B	$2s$ Full. Start $2p$ (Cost 3)	$2p$	$[He]2s^22p^1$	Yes
6	C	Continue $2p$	$2p$	$[He]2s^22p^2$	Yes
7	N	Continue $2p$	$2p$	$[He]2s^22p^3$	Yes
8	O	Continue $2p$	$2p$	$[He]2s^22p^4$	Yes
9	F	Continue $2p$	$2p$	$[He]2s^22p^5$	Yes
10	Ne	Finish $2p$ (Shell Closed)	$2p$	$[He]2s^22p^6$	Yes

Period 3 (Sodium – Argon)

Now we compare the next available shells.

- **$3s$ Orbital:** $n = 3, \ell = 0$. Cost $\mathcal{C} = 3 + 0 = \mathbf{3}$.

- **$3p$ Orbital:** $n = 3, \ell = 1$. Cost $\mathcal{C} = 3 + 1 = \mathbf{4}$.

> **Note**
>
> $2p$ and $3s$ both have Cost 3. The tie-breaker (lower n) favors $2p$ ($n = 2$) over $3s$ ($n = 3$), which is why Period 2 finishes before Period 3 starts.

Z	Element	Filling Step	Winner	Configuration	Match?
11	Na	$3s$ (Cost 3) vs $3p$ (Cost 4)	$3s$	$[Ne]3s^1$	Yes
12	Mg	Finish $3s$	$3s$	$[Ne]3s^2$	Yes
13	Al	$3s$ Full. Start $3p$ (Cost 4)	$3p$	$[Ne]3s^23p^1$	Yes
14	Si	Continue $3p$	$3p$	$[Ne]3s^23p^2$	Yes
15	P	Continue $3p$	$3p$	$[Ne]3s^23p^3$	Yes
16	S	Continue $3p$	$3p$	$[Ne]3s^23p^4$	Yes
17	Cl	Continue $3p$	$3p$	$[Ne]3s^23p^5$	Yes
18	Ar	Finish $3p$ (Shell Closed)	$3p$	$[Ne]3s^23p^6$	Yes

Progress: You've now predicted 18 elements correctly! Only 100 to go...

7 Period 4: The First Crossover (K - Kr)

> **Predict First**
>
> Before looking at the table, predict:
>
> - Which orbital fills first in Period 4: 3d or 4s?
>
> - Calculate: $\mathcal{C}_{3d} = $ ___ and $\mathcal{C}_{4s} = $ ___
>
> - Winner: _____
>
> *Check your answers after calculating, then read on to verify.*

> **✓ Answer Check**
>
> Did you predict correctly?
>
> - $\mathcal{C}_{3d} = 3 + 2 = \mathbf{5}$

- $\mathcal{C}_{4s} = 4 + 0 = \mathbf{4}$

- Winner: **4s** (lower cost)

Here is where standard intuition fails. Why fill $4s$ before $3d$?

- **3d Orbital:** $n = 3, \ell = 2$. Cost $\mathcal{C} = 3 + 2 = \mathbf{5}$.

- **4s Orbital:** $n = 4, \ell = 0$. Cost $\mathcal{C} = 4 + 0 = \mathbf{4}$.

Result: $4 < 5$. It is cheaper to build a big simple shell ($4s$) than a medium twisted shell ($3d$).

Z	Sym	Filling Step	Winner	Configuration	Match?
19	K	$4s$ (Cost 4) vs $3d$ (Cost 5)	$4s$	$[Ar]4s^1$	Yes
20	Ca	Finish $4s$	$4s$	$[Ar]4s^2$	Yes
21	Sc	$4s$ Full. Start $3d$ (Cost 5)	$3d$	$[Ar]4s^23d^1$	Yes
22	Ti	Continue $3d$	$3d$	$[Ar]4s^23d^2$	Yes
23	V	Continue $3d$	$3d$	$[Ar]4s^23d^3$	Yes
24	Cr	**Anomaly:** 1/2-filled lock	$3d$	$[Ar]4s^13d^5$	**Yes***
25	Mn	Continue $3d$	$3d$	$[Ar]4s^23d^5$	Yes
26	Fe	Continue $3d$	$3d$	$[Ar]4s^23d^6$	Yes
27	Co	Continue $3d$	$3d$	$[Ar]4s^23d^7$	Yes
28	Ni	Continue $3d$	$3d$	$[Ar]4s^23d^8$	Yes
29	Cu	**Anomaly:** Full-filled lock	$3d$	$[Ar]4s^13d^{10}$	**Yes***
30	Zn	Finish $3d$	$3d$	$[Ar]4s^23d^{10}$	Yes
31	Ga	$3d$ Full. Start $4p$ (Cost 5*)	$4p$	$[Ar]\ldots4p^1$	Yes
32	Ge	Continue $4p$	$4p$	$[Ar]\ldots4p^2$	Yes
33	As	Continue $4p$	$4p$	$[Ar]\ldots4p^3$	Yes
34	Se	Continue $4p$	$4p$	$[Ar]\ldots4p^4$	Yes
35	Br	Continue $4p$	$4p$	$[Ar]\ldots4p^5$	Yes
36	Kr	Finish $4p$ (Shell Closed)	$4p$	$[Ar]\ldots4p^6$	Yes

Wait—What About Chromium and Copper?

You may have noticed Z=24 (Cr) and Z=29 (Cu) are marked with asterisks. The simple cost rule predicts $[Ar]4s^23d^4$ for Cr, but nature chooses $[Ar]4s^13d^5$ instead. Why?

Quick Answer: Half-filled and fully-filled d-shells get a "stability bonus" (see "Topological Locking" later in the text).

Try It Yourself: Scandium (Z=21)

Given: We've filled $[Ar]4s^2$ ($18 + 2 = 20$ electrons). **Question:** Where does electron #21 go?

Step 1: Calculate costs of available orbitals:

- $\mathcal{C}_{3d} = 3 + 2 = \mathbf{5}$

- $\mathcal{C}_{4p} = 4 + 1 = \mathbf{5}$

Step 2: Apply tie-breaker (lower n wins): $3d$ ($n = 3$) beats $4p$ ($n = 4$) ✓

Result: Scandium $= [Ar]4s^23d^1$ ✓

8 Period 5: The Second Crossover (Rb - Xe)

Z	Sym	Step	Config (Outer)	Match?
37-38	Rb, Sr	$5s$	$5s^{1-2}$	Yes
39-40	Y, Zr	$4d$	$5s^2 4d^{1-2}$	Yes
41	Nb	Anomaly	$5s^1 4d^4$	Yes (Partial)*
42	Mo	Lock	$5s^1 4d^5$ (half-shell)	**Yes**
43	Tc	$4d$	$5s^2 4d^5$	Yes
44-46	Ru-Pd	Complex	Various anomalies	No*
47	Ag	Lock	$5s^1 4d^{10}$ (full shell)	**Yes**
48	Cd	$4d$	$5s^2 4d^{10}$	Yes
49-54	In-Xe	$5p$	$5s^2 4d^{10} 5p^{1-6}$	Yes

Note on Nb: AR predicts an anomaly due to locking proximity. Actual is $5s^1 4d^4$, very close to the $4d^5$ lock state. Ru-Pd involve complex correlation effects not captured by this simple model.

9 Period 6: The Lanthanide Contraction (Cs - Rn)

Now we introduce the f-orbitals ($n = 4, \ell = 3$).

- $\mathcal{C}_{6s} = 6 + 0 = \mathbf{6}$.

- $\mathcal{C}_{4f} = 4 + 3 = \mathbf{7}$.

- $\mathcal{C}_{5d} = 5 + 2 = \mathbf{7}$.

- $\mathcal{C}_{6p} = 6 + 1 = \mathbf{7}$.

  Period 6 Order:

1. **6s** (Cost 6) $\rightarrow$ Fills first.

2. **4f** (Cost 7, $n = 4$) $\rightarrow$ Lowest n in tie.

3. **5d** (Cost 7, $n = 5$) $\rightarrow$ Middle n in tie.

4. **6p** (Cost 7, $n = 6$) $\rightarrow$ Highest n in tie.

   Note: Lanthanum (Z=57) is an exception: it fills $5d^1$ before $4f$ starts. This is due to very small energy differences that our simple cost function doesn't capture. For our purposes, we'll treat 4f as starting at Ce (Z=58).

Z	Sym	Block	Configuration	Notes
55-56	Cs-Ba	s-block	$6s^{1-2}$	Fills first (Cost 6)
57	La	d-block	$6s^2 5d^1$	Anomaly (5d starts early)
58-71	Ce-Lu	f-block	$6s^2 4f^{1-14}$	4f fills (Cost 7, low n)
72-80	Hf-Hg	d-block	$6s^2 4f^{14} 5d^{2-10}$	5d finishes (Cost 7, high n)
81-86	Tl-Rn	p-block	$6s^2 \ldots 6p^{1-6}$	6p fills last (Cost 7, highest n)

The Action Realism Explanation for Anomalies

Why do Cr and Cu break the rules?

> ### Topological Locking (The "Smooth Surface" Bonus)
>
> **The Concept:** In standard QM, we call this "Exchange Energy." In Action Realism, it is **Topological Locking**.
>
> **The Geometry:**
>
> - **Domain Walls:** Electrons with opposite spins (↑↓) create "walls" or "seams" between them in phase space—like patches sewn together with visible stitches.
>
> - **Locking:** Aligning all spins (↑↑↑↑↑) removes these walls—creating one smooth surface with no seams.
>
> **Why This Matters:** A smooth, seamless surface is geometrically "cheaper" (lower action cost) than a patched-together one. For Chromium ($3d^5$), the cost savings from making 5 aligned spins outweighs the cost of "stealing" one electron from the 4s shell.
>
> **When Does This Happen?**
>
> - **Half-filled d-shells** (d^5): All 5 d-orbitals have one spin-up electron → smooth surface (maximize parallel spins).
>
> - **Fully-filled d-shells** (d^{10}): All 5 d-orbitals have both spins paired → complete pairing closes the subshell (additional stability).
>
> **Analogy:** Imagine building a tent:
>
> - **Regular filling**: Add fabric panels one at a time. Each seam costs energy (stitching).
>
> - **Half-filled**: Use 5 seamless panels, one per pole. Minimal seams!
>
> - **Fully-filled**: Complete sphere with no external seams at all.

<div align="center">

Costly: Domain Walls   **Cheaper: Aligned Spins**

</div>
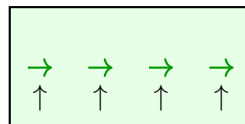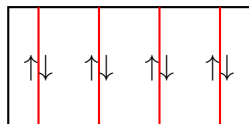
10 The Big Picture: Why Simple Math Describes Complex Atoms

From Rules to Understanding

Standard Approach: "Memorize that 4s fills before 3d." **AR Approach:** "Calculate that $4+0 < 3+2$, therefore 4s fills first."

Real-World Impact

> **Real-World Impact**
>
> **Why Chromium doesn't rust like iron:** Chromium's half-filled d-shell makes it form a protective oxide layer. This "topological locking" is why we use chromium in stainless steel!

- **Why gold is special:** The 6s orbital is so "cheap" relative to 5d that gold's chemistry is unusual (relativistic effects + cost inversion).

- **Why lanthanides are similar:** They differ only in the buried 4f shell, so their chemistry (determined by 6s/5d) is nearly identical.

- **Why transition metals conduct:** Their d-electrons are high-cost and loosely bound—easy to delocalize into metallic bonds.

Common Misconceptions

Mistake 1: "3d should fill before 4s because 3 < 4"

- **Why it's wrong:** You're only comparing n, not total cost.

- **Correct thinking:** Compare $\mathcal{C} = n + \ell$, not just n.

  Mistake 2: "The cost formula is exact"

- **Reality:** $\mathcal{C} = n + \ell$ is an approximation. Electron-electron repulsion creates small corrections (explaining some 4d anomalies).

- **But:** It reproduces the main ground-state filling order and the major crossovers (4s/3d, etc.) and helps explain several classic anomalies.

  Mistake 3: "Anomalies mean the theory is broken"

- **Reality:** Cr and Cu anomalies arise from a SECOND geometric principle (topological locking), not from the failure of the first.

Self-Check: Can You Do This?

Skill	Can Do?
Calculate the cost of any orbital given n and ℓ	☐
Predict which orbital fills first between two choices	☐
Apply the tie-breaker rule when costs are equal	☐
Explain why 4s fills before 3d using the cost function	☐
Identify which elements will have anomalies (half/full d-shells)	☐
Write the electron configuration for any element Z < 36	☐

Practice Problems

Problem 1: Predict Titanium (Z=22)

Given: Calcium (Z=20) $= [Ar]4s^2$.

(a) What orbitals are available for electrons 21 and 22?

(b) Calculate their costs.

(c) Predict Ti's configuration.

Problem 2: The Gallium Question

After filling $3d^{10}$ (Zinc, Z=30), which orbital fills next: 3f, 4p, or 4d?

Problem 3: Predict an Anomaly

Will Molybdenum (Z=42, in the 4d series) show an anomaly? Why or why not?

Problem 4: Crossing the Blocks

Explain why the p-block appears in Period 2 but the d-block doesn't appear until Period 4.

Problem 5: The Zinc Exception

Zinc (Z=30) has configuration $[Ar]4s^2 3d^{10}$. Explain why it does NOT show an anomaly like Copper, even though its d-shell is full.

Solutions

Solution 1:

(a) Available: 3d and 4p.

(b) $\mathcal{C}_{3d} = 3 + 2 = 5$; $\mathcal{C}_{4p} = 4 + 1 = 5$.

(c) Tie-breaker: 3d wins ($n = 3 < n = 4$). Ti $= [Ar]4s^2 3d^2$ ✓

Solution 2:

- 3f: doesn't exist (ℓ cannot exceed $n - 1$).

- $\mathcal{C}_{4p} = 4 + 1 = 5$. $\mathcal{C}_{4d} = 4 + 2 = 6$.

- Winner: 4p ✓

Solution 3: Yes! Mo will be $[Kr]5s^1 4d^5$ instead of $[Kr]5s^2 4d^4$ because the half-filled 4d shell (d^5) triggers topological locking, making it more stable to "steal" an electron from 5s. ✓

Solution 4: The d-block requires $\ell = 2$. The first available d-orbital is 3d with cost $\mathcal{C}_{3d} = 3 + 2 = 5$. By the time we reach cost 5, we've already filled 1s (cost 1), 2s (cost 2), 2p/3s (cost 3), and 3p/4s (cost 4). The p-block appears earlier because $\mathcal{C}_{2p} = 2 + 1 = 3$ is achievable sooner. ✓
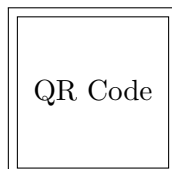
Solution 5: Zinc does have a full d-shell ($3d^{10}$), but it ALSO has a full s-shell ($4s^2$). The "locking bonus" only matters if it creates a MORE stable configuration than the default. For Zinc, keeping $4s^2$ is already stable—there's no benefit to disrupting it. Copper (Z=29) benefits because it would otherwise have $4s^2 3d^9$, which leaves the d-shell incomplete. By going to $4s^1 3d^{10}$, it achieves full d-shell locking. ✓

Appendix: Quick Reference Card

Filling Order
Cost 1:   1s
Cost 2:   2s
Cost 3:   2p, 3s
Cost 4:   3p, 4s
Cost 5:   3d, 4p, 5s
Cost 6:   4d, 5p, 6s
Cost 7:   4f, 5d, 6p, 7s
Cost 8:   5f, 6d, 7p, 8s

Scan for interactive practice:

https://actionrealism.org/periodic-table