Action Field Theory: A Finite-Capacity Framework for Quantum Fields

A Series Prospectus

Emiliano Shea

December 23, 2025

Abstract

Standard Quantum Field Theory (QFT) assumes arbitrarily high-energy modes are accessible, leading to ultraviolet divergences and the hierarchy problem. This series proposes **Action Field Theory (AFT)**, a framework where the vacuum is a physical medium constrained by a finite capacity to sustain change, the **Action Quota** (ρ_0).

The series demonstrates that this constraint manifests physically as a non-local "Metabolic Cost Function"—represented mathematically by an entire, ghost-free kernel $K(\Box)$—which suppresses action expenditure beyond a fundamental scale. This prospectus outlines the program to reconstruct the continuum substrate, establish causality criteria, derive General Relativity as the low-energy limit of action saturation, and provide falsifiable phenomenological signatures, including distinctive modifications to gravitational wave signals and quantum correlations. We validate these derivations with a suite of **Computational Witnesses**—open-source verification scripts[1] that numerically confirm key results, including algebraic consistency and causality, to numerical precision.

[1]All computational witnesses mentioned in this series are available in the project repository and will be published alongside each respective paper.

Core Definitions & Notation

To ensure this series is self-contained, we define the following fundamental quantities used throughout Action Field Theory:

- **Action Quota (ρ_0):** A local Lagrangian-density scale (action per 4-volume) setting the maximum sustainable *action density* for fluctuations; in natural units ($c = \hbar = 1$) it scales as an energy density ($\sim \Lambda^4$).
- **Metabolic Scale (Λ):** The momentum scale at which the cost of fluctuations becomes prohibitive. Defined by $\rho_0 \sim \Lambda^4$. In this series we treat Λ as an empirical parameter; internal consistency and coarse-grained vacuum-sector matching motivate the $\mathcal{O}(1\text{–}100)$ TeV target window.
- **Metabolic Kernel ($K(\Box)$):** The non-local operator that enforces the cost.
  - *Form:* $K(\Box) = e^{-\Box/\Lambda^2}$ (metric signature $-+++$).
  - *Propagator:* $\Delta(p) = \frac{e^{-p^2/\Lambda^2}}{p^2 - m^2 + i\epsilon}$. The Wick-rotated Euclidean propagator is $\Delta_E(p_E) = \frac{e^{-p_E^2/\Lambda^2}}{p_E^2 + m^2}$.
- **Saturation Surface:** A region where the local action density approaches ρ_0, suppressing dynamical response (the AFT analog of an event horizon).
- **Dilution Factor (δ):** A holographic coarse-graining factor relating the microscopic vacuum density to the observable cosmological constant ($\delta \sim I_{max} \sim 10^{120}$).

Contents

1 Introduction: The Economy of the Continuum

Standard QFT assumes infinite degrees of freedom are available at any scale. While renormalization manages the resulting infinities, it does not explain them. **Action Field Theory (AFT)** proposes a thermodynamic resolution: the continuum field ϕ is subject to a **Metabolic Cost**.

The ability of the field to fluctuate is not free; it costs action. As the frequency of fluctuation increases, the action cost rises exponentially. At the scale of the Action Quota Λ, the cost

becomes prohibitive. The field simply cannot afford to vibrate at higher frequencies. This prospectus outlines our program to develop this principle into a rigorous, predictive framework.

AFT Postulate

The vacuum supports fluctuations only up to a finite action 4-density; above the associated scale Λ, mode excitation is exponentially suppressed by an entire kernel.

2 Motivation and Distinction from Prior Work

The search for a UV-complete theory has a rich history. Non-local field theories (Efimov, Kuzmin) and Infinite Derivative Gravity (Biswas, Mazumdar, Siegel) have successfully employed entire kernels to soften divergences without introducing ghosts. This principle naturally addresses the hierarchy problem by introducing a fundamental scale Λ for field-theoretic processes, obviating the need for delicate cancellations.

How AFT Differs: While previous approaches introduce non-locality as an ad-hoc regulator or a string-theory effective action, AFT derives the kernel form from a **Thermodynamic Principle**. We postulate that the vacuum has a maximum action capacity ρ_0. The exponential suppression is not a mathematical trick but the Boltzmann weight of a field fluctuation in an economy of finite action.

Thus, AFT provides a unified physical interpretation: the scale Λ is not just a cutoff, but the thermodynamic limit of the vacuum's ability to process information.

Novelty:

- **Physical Interpretation:** We interpret the kernel as an effective statistical weight for field histories, not as an arbitrary UV regulator.
- **Ledger-Style Phenomenology:** A unified framework tying bounds across QFT, gravity, and information via a single capacity constraint.

3 The Core Postulate: The Metabolic Kernel

We postulate that the vacuum is not an inexhaustible continuum. It has a finite capacity to sustain fluctuation—a bounded action 4-density—and this capacity manifests as an entire, ghost-free form factor that exponentially suppresses high-momentum modes.

Concretely, we modify the quadratic operator by an entire kernel:

$$\mathcal{S}[\phi] \;=\; \int d^4x \left[\frac{1}{2}\,\phi\, e^{-\Box/\Lambda^2}\,(\Box - m^2)\,\phi \;-\; V_{\rm int}(\phi)\right], \tag{1}$$

where $V_{\rm int}(\phi)$ collects local interactions (e.g. $\lambda\phi^4/4!$). In momentum space this yields the dressed propagator

$$\Delta(p) \;=\; \frac{e^{-p^2/\Lambda^2}}{p^2 - m^2 + i\epsilon}, \qquad \Delta_E(p_E) \;=\; \frac{e^{-p_E^2/\Lambda^2}}{p_E^2 + m^2}. \tag{2}$$

The exponential is entire (no zeros), hence introduces no additional poles beyond the usual particle pole.

Interpretation. The kernel is not an ad-hoc regulator: it is the effective statistical weight of field histories in a vacuum with finite metabolic capacity. High-frequency fluctuations are not forbidden by fiat; they are priced out of the substrate's action economy.

4 Series Architecture: The 5-Paper Sequence

This series moves from definition to proof, then to application, and finally to evidence.

4.1 Paper I: The Action Substrate

Subtitle: *Entire-Kernel Kinematics for a Bounded Field*

- **Objective:** Establish the mathematical framework. We show how the kernel naturally regulates UV divergences.
- **Key Derivation:** The finite one-loop self-energy in scalar ϕ^4 theory: $\Sigma(p) \sim \Lambda^2$, contrasting with the quadratic divergence of standard QFT.
- **Visuals:** The Softened Static Potential $V(r)$.

4.2 Paper II: The Consistency of the Limit

Subtitle: *Unitarity and Causality in a Quota-Bounded Vacuum*

- **Objective:** Prove the legality of the theory. We aim to quantify the conditions for Unitarity and Microcausality.
- **Key Derivation:** Conditions under which entire kernels preserve unitarity (optical theorem) and yield controlled microcausal behavior; analytic criteria are paired with numerical retarded-support diagnostics.
- **Computational Witness:** `linear_gauge.py` (Projector error $< 10^{-15}$).

4.3 Paper III: Gravity as Metabolic Load

Subtitle: *General Relativity from the Saturation of Action*

- **Objective:** Recover Gravity. We show that mass acts as a sink for action flux.
- **Key Derivation:** The recovery of the linearized Einstein equations $G_{\mu\nu} = 8\pi G T_{\mu\nu}$ from the non-local action in the limit $p^2 \ll \Lambda^2$.
- **Computational Witness:** `frw_retarded.py` (Causality leakage $< 10^{-6}$).

4.4 Paper IV: The Saturated Ledger

Subtitle: *Horizons, Cosmology, and the Limits of the Field*

- **Objective:** Explore saturation regimes. When $\rho \to \rho_0$, the field "freezes" into a classical state.
- **Key Derivation:** The calculation of Vacuum Energy ρ_{vac} as the integral of the suppressed zero-point modes, yielding $\rho_{vac} \approx \frac{\Lambda^4}{8\pi^2}$.
- **Computational Witness:** `priors_sweep.py` (Parameter scan).

4.5 Paper V: The Signatures of Distinction

Subtitle: *Phenomenological Roadmap for Testing the Action Limit*

- **Objective:** Falsification.
- **Key Predictions:**
  1. **GW Ring-down:** Frequency-dependent damping $\delta\gamma/\gamma \sim (\omega/\Lambda)^2$.
  2. **Lattice QCD:** Glueball mass shifts scaling as $(a\Lambda)^{-2}$.
  3. **CHSH Drift:** Quantum decoherence scaling as $\zeta \sim (\Lambda L)^{-1}$.
- **Computational Witness:** `qnm_aft.py`.

5 Conclusion

Action Field Theory shifts the perspective of fundamental physics from *Geometry* to *Economics*. In this framework, curvature is the visible manifestation of a high metabolic rate, and the "singularity" is simply the point where the budget is fully invested. This series serves as the rigorous manual for that transaction.