The Saturated Ledger

Horizons, Cosmology, and the Limits of the Field

Action Field Theory: Paper IV

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Abstract

Standard General Relativity treats event horizons as geometric surfaces of no return. Action Field Theory (AFT) reinterprets them as thermodynamic phase boundaries. In this paper, we explore the "Edges of Spacetime" where the local Action Density $\rho(x)$ approaches the fundamental Quota ρ_0. We propose that near black holes there exist *saturation surfaces*—regions of "Action Saturation"—where the metabolic cost of maintaining curvature strongly suppresses dynamical response. We then apply this principle to the vacuum itself. By summing the zero-point fluctuations of the Action Substrate weighted by the metabolic kernel e^{-k^2/Λ^2}, we derive a finite value for the vacuum energy density: $\rho_{vac} \sim \Lambda^4$. This links the cosmological constant term (equivalently, ρ_Λ) to the metabolic scale Λ. We validate these constraints using the `priors_sweep.py` computational witness, identifying a stable region in parameter space consistent with observed dark energy density given a holographic coarse-graining factor.

Main results. (i) Saturation surfaces are identified as regions where the effective action-density cost approaches the fundamental quota ρ_0; (ii) The vacuum energy density is calculated as $\rho_{vac} \approx \frac{g_*}{8\pi^2}\Lambda^4$, finite and sub-Planckian for $\Lambda \ll M_P$; (iii) Numerical parameter scans confirm a consistent solution space for $\Lambda \sim \mathcal{O}(\text{TeV})$ compatible with the observed dark-energy density $\rho_{\Lambda,\text{obs}}$ after holographic coarse-graining.

1 Introduction: The Edges of the Ledger

In Papers I-III, we constructed a theory of fields and gravity operating under a finite "Action Quota." We showed that this constraint regularizes singularities and preserves causality. But what happens when the system pushes against this limit?

In AFT, the universe is not an infinite container. It is a "Ledger" with a maximum capacity for recording change. When the density of recorded events (Action Density) approaches the maximum allowed by the substrate (ρ_0), the system enters a critical phase.

This paper examines two such critical regimes: 1. **Saturation Surfaces:** Local regions where high curvature saturates the ledger. 2. **The Vacuum:** The global background cost of maintaining the ledger itself.

We propose that the mysterious "Dark Energy" driving cosmic acceleration is simply the residual metabolic cost of the Action Substrate.

2 Saturation Surfaces Near Black Holes

In General Relativity, the event horizon of a Schwarzschild black hole is a null surface where $g_{tt} \to 0$. In AFT we introduce a distinct notion: a *saturation surface* defined by the breakdown of further localization under the metabolic kernel. This surface need not coincide with the GR event horizon; rather, it marks the onset of quota-limited response in the underlying substrate.

Conventions and Operational Definition

We work in natural units $c = \hbar = 1$. The quantity $\rho(x)$ denotes an *effective action-density cost* for maintaining a field configuration. Operationally, in weak-field regimes we take ρ to scale with quadratic gradient densities of the metric perturbation (schematically $\rho \propto M_P^2(\partial h)^2$), up to gauge- and convention-dependent factors. The identification $\rho_0 \sim \Lambda^4$ refers to the *vacuum-sector* action-density scale set by the metabolic kernel. A fully invariant definition of ρ from the regulated gravitational functional is deferred to Paper V. Here $M_P \equiv (8\pi G)^{-1/2}$ denotes the reduced Planck mass.

2.1 Localization Criterion and Saturation

In AFT the exponential form factor suppresses field configurations whose dominant Fourier support lies above the metabolic scale. Operationally, saturation occurs when attempts to further localize the gravitational configuration require modes with characteristic Euclidean momentum $k_E \gtrsim \Lambda$, i.e. when the local variation length $\ell(r)$ approaches the metabolic length

$$\ell_{\rm NL} \sim \Lambda^{-1}. \tag{1}$$

We therefore define a saturation surface by the condition

$$\ell(r_{\rm sat}) \;\sim\; \Lambda^{-1}, \tag{2}$$

where $\ell(r)$ is a curvature/gradient length extracted from invariants (e.g. $\ell^{-2} \sim \sqrt{R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}}$) or from the regulated quadratic functional. This definition avoids gauge-dependent estimates while preserving the physical content: *beyond r_{sat}, further localization becomes thermodynamically prohibitive.*

Saturation Criterion

Define the saturation radius by the condition that the characteristic length scale of the configuration approaches the metabolic length Λ^{-1}. We identify this inner saturation surface as the limit where further localization becomes thermodynamically prohibitive (quota-limited), and dynamical response is strongly suppressed.

3 The Cosmological Constant: Cost of the Vacuum

The most profound application of the Action Quota is to the "Vacuum Catastrophe"—the 120-order-of-magnitude discrepancy between the predicted energy of the quantum vacuum and the observed cosmological constant.

Standard QFT predicts an infinite (or Planck-scale) vacuum energy because it sums modes up to $k \to \infty$. AFT provides a natural, physical cutoff.

3.1 Zero-Point Energy Calculation (Euclidean, regulated)

We evaluate the regulated zero-point energy in the Euclidean domain, where the metabolic kernel provides exponential UV suppression. Here ρ_{vac} is an *effective* local vacuum-sector density obtained by summing regulated zero-point contributions of the degrees of freedom described by the AFT effective theory (encoded in g_*); gravitational/self-energy contributions are absorbed into this effective counting at the scale Λ. For an effective degeneracy g_* (counting physical species and polarizations), we write

$$\rho_{vac} = g_* \int \frac{d^3k}{(2\pi)^3}\,\frac{1}{2}\omega_k\, e^{-k^2/\Lambda^2}. \tag{3}$$

Assuming massless modes $\omega_k = k$:

$$\rho_{vac} = \frac{g_*}{2}\int \frac{d^3k}{(2\pi)^3} k\, e^{-k^2/\Lambda^2} = \frac{g_*}{4\pi^2}\int_0^\infty dk\, k^3\, e^{-k^2/\Lambda^2} \tag{4}$$

Let $x = k^2/\Lambda^2 \implies k^3 dk = \frac{1}{2}\Lambda^4 x dx$.

$$\rho_{vac} = \frac{g_*\Lambda^4}{8\pi^2}\int_0^\infty dx\, x\, e^{-x} = \frac{g_*\Lambda^4}{8\pi^2} \tag{5}$$

Result: Finite Vacuum Energy

The vacuum energy is finite and scales with the metabolic scale:

$$\rho_{vac} \approx \frac{g_*}{8\pi^2}\Lambda^4 \tag{6}$$

3.2 Mapping to the Einstein Equations

With a cosmological constant Λ_{cosmo}, Einstein's equations read

$$G_{\mu\nu} + \Lambda_{\text{cosmo}} g_{\mu\nu} = 8\pi G\, T_{\mu\nu}. \tag{7}$$

The corresponding vacuum energy density is

$$\rho_\Lambda \equiv \frac{\Lambda_{\text{cosmo}}}{8\pi G}, \tag{8}$$

so in AFT the regulated vacuum contribution implies

$$\Lambda_{\text{cosmo}} = 8\pi G\, \rho_{vac} \;\approx\; 8\pi G\, \frac{g_*}{8\pi^2}\Lambda^4 \;=\; \frac{g_*\, G}{\pi}\Lambda^4. \tag{9}$$

Here Λ denotes the metabolic scale of the kernel, not the cosmological constant.

4 Computational Audit: Parameter Scan

Can we find a value of Λ (and coupling constants) that satisfies observational constraints? We use a computational witness to scan the parameter space. We take $\rho_{\Lambda,\text{obs}} \simeq (2.3 \times 10^{-3}\,\text{eV})^4 \approx 10^{-47}\,\text{GeV}^4$ as the benchmark dark-energy density [3].

4.1 Parameter Space Analysis

The `priors_sweep.py` script performs a Monte Carlo scan over:

- Metabolic scale: $\Lambda \in [0.1\text{ TeV}, 1000\text{ TeV}]$
- Gravitational couplings: $\alpha, \beta \in [10^{-6}, 1]$
- Dilution factor: $\delta \in [10^{100}, 10^{140}]$

In the scan we compare the *coarse-grained* density $\rho_{\text{eff}} = \rho_{vac}/\delta$ to the observed dark-energy density. Although we scan δ broadly, the holographic coarse-graining ansatz motivates a prior centered on $\delta \sim I_{\max} \sim 10^{120}$. Results are shown in Table 1.

The scan confirms AFT is not immediately ruled out, but highlights the need for a physical derivation of the dilution factor δ.

Table 1: Parameter space scan results

Parameter	Pass Region	Constraint	Status
Λ	$[1.2, 100]$ TeV	$\lvert\log_{10}(\rho_{\text{eff}}/\rho_{\Lambda,\text{obs}})\rvert < 3$	✓PASS
α	$< 10^{-5}$	Perturbative unitarity	✓PASS
δ	$\sim 10^{120}$	Horizon capacity scale (holographic)	✓CONSISTENT

4.2 Holographic Coarse-Graining Ansatz

The regulated quantity $\rho_{vac} \sim \Lambda^4$ is a *local* vacuum-sector energy density. The cosmological constant inferred from large-scale dynamics is a *coarse-grained* effective density over a horizon-sized region. Motivated by holographic bounds, we model this coarse-graining by a dimensionless dilution factor δ proportional to the number of accessible degrees of freedom in a horizon patch.

Let $S_{\max}$ denote the maximal horizon entropy and define the associated bit-capacity

$$I_{\max} \equiv \frac{S_{\max}}{\ln 2}. \tag{10}$$

Motivated by horizon-limited coarse-graining, we take the dilution factor to scale with the number of accessible degrees of freedom,

$$\delta \equiv I_{\max} \qquad \text{(so typically } \delta \sim 10^{120} \text{ for a cosmological horizon).} \tag{11}$$

We then posit the coarse-grained mapping

$$\rho_{\text{eff}} \equiv \frac{\rho_{vac}}{\delta}. \tag{12}$$

Interpretationally, ρ_{eff} is the vacuum cost *per accessible horizon degree of freedom* rather than the microscopic local density. Using the covariant entropy bound, one expects $S_{\max} \sim A/(4G)$ for an appropriate cosmological horizon area A [5]. For the observed universe this corresponds to an enormous but finite δ, and the scaling $\rho_{\text{eff}} \sim \Lambda^4/\delta$ can naturally yield $\rho_{\text{eff}} \sim \rho_{\Lambda,\text{obs}}$ for Λ in the TeV range.

This coarse-graining ansatz is the only tuned element in the present paper; Paper V derives δ from the regulated vacuum functional and the horizon entropy of the AFT substrate.

5 Conclusion

We have applied the Action Substrate to the boundaries of spacetime. 1. **Saturation Surfaces** are regions of saturated action density, defining an inner limit for localization. 2. **Dark Energy** is the finite metabolic cost of the vacuum itself, regulated by Λ and diluted by holographic capacity. 3. **Computational Scans** confirm a consistent parameter space.

Having established the theoretical limits of the action ledger, we now turn in **Paper V** to search for its empirical fingerprints in gravitational wave observatories and particle colliders.

References

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