Gravity as a Macroscopic Caustic

Paper IV of Series III: The Geometry of the Limit

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Abstract

We extend the Action Quota framework to the macroscopic scale, treating General Relativity as the optics of a capacity-limited field. We show how the fold normal form in Fermat optics yields an Airy-profile universal shape, modeling the Event Horizon as a physical **Macroscopic Fold** where the action density approaches the fundamental ceiling ρ_0. Using the Fermat potential formalism, we derive the characteristic scaling length from local derivatives. We propose ρ_0 as the physical parameter controlling the effective semiclassical scale, leading to a universal **Airy Law** intensity profile. We outline observational tests via wave-optics lensing in gravitational wave (GW) and electromagnetic (EM) regimes, where the collapsed profile is operationally robust under Doppler rescaling.

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1 Introduction: The Limit of Spacetime

In Papers I-III, we explored the consequences of the Action Quota ρ_0 for microscopic systems. We found that the cost of a turn creates the Uncertainty Principle (Paper I), the stability of a fold creates discrete Energy Levels (Paper II), and the topology of a twisted fold creates Spin and Exclusion (Paper III).

Now we turn to the macroscopic limit. General Relativity describes gravity as the curvature of spacetime. In our framework, we reinterpret this curvature as the modulation of the **Action Capacity** of the manifold. Gravity is the "refractive index" experienced by the flow of Action.

We propose that the **Event Horizon**—and more generally, any gravitational caustic—is the macroscopic manifestation of the same folding mechanism that creates the electron. It is a region where the action flow saturates the capacity of the spacetime manifold. The field buckles, creating a singular boundary (a fold) that separates the accessible universe from the forbidden region.

In this paper, we derive the universal wave-optics signature of this limit: the Airy collapse of gravitational lensing.

Proposal: Macroscopic Saturation

We propose that horizons behave as **macroscopic fold boundaries for action flow** in an appropriate optical (eikonal) reduction: null congruences focus and form a boundary where the semiclassical map becomes singular. In this paper we test only the caustic optics consequence. We do not claim the thin-lens fold is literally the horizon; we claim the same normal-form optics controls wave propagation near any fold boundary.

2 The Macroscopic Fold

We work in the thin-lens approximation of General Relativity, which captures the essential topological features of the fold without the full complexity of the Einstein field equations. Validity requires the source, lens, and observer to be well-separated and the deflection angles to be small.

2.1 Fermat Potential and Action

The time delay for a signal traversing a gravitational lens is determined by the dimensionless **Fermat Potential** $\Phi(\theta;\beta)$, where θ is the image plane coordinate and β is the source position. We take Φ to be the dimensionless lensing time-delay potential, so that:

$$\Delta t(\theta;\beta) = \frac{D}{c}\Phi(\theta;\beta), \qquad S(\theta;\beta) = E\Delta t = \frac{ED}{c}\Phi(\theta;\beta), \tag{1}$$

where D is the usual lensing distance combination. The action along the path is $S = E\Delta t$, so the phase is $\phi = S/\hbar$, where we identify the reduced Action Quota $\hbar$ with Planck's constant as established in the series. The wave-optics integral near a fold thus depends on ρ_0 exactly as standard optics depends on $\hbar$.

2.2 Local Normal Form Near a Fold

A **Fold Caustic** occurs where the map from image to source space is singular. Mathematically, this corresponds to the vanishing of the determinant of the Hessian matrix $A(\theta) = I - \nabla\nabla\psi(\theta)$. Near a fold, we expand the Fermat potential in adapted coordinates (u, v) (normal, tangent) around the singular point:

$$\Phi(u, v;\beta) \approx \Phi_0 + \frac{\lambda_t}{2}v^2 + \frac{1}{6}\Phi_{uuu}u^3 - \beta u. \tag{2}$$

Here Φ_{uuu} is the third derivative along the fold normal.

2.3 The Airy Length

Define the dimensionless wave-optics frequency parameter

$$\Omega := \omega \frac{D}{c}. \tag{3}$$

The wave field is locally:

$$\Psi(s) \propto \iint \exp\left(i\Omega\Phi(u, v; s)\right) du\, dv. \tag{4}$$

Integrating out the quadratic tangent direction yields a prefactor $\propto \lambda_t^{-1/2}$. For the normal direction, the phase is:

$$\phi(u) \sim \Omega \left[\frac{\Phi_{uuu}}{6} u^3 - su\right]. \tag{5}$$

Rescaling $u = \ell_{\text{img}}\tilde{u}$ to put the phase into the Airy normal form requires:

$$\Omega \frac{|\Phi_{uuu}|}{6} \ell_{\text{img}}^3 = \frac{1}{3} \qquad \Longrightarrow \qquad \ell_{\text{img}} = \left(\frac{2}{\Omega|\Phi_{uuu}|}\right)^{1/3} \tag{6}$$

reduces the exponent to the Airy normal form $x^3/3 - zx$, where the control parameter is

$$z \;=\; s\,\Omega\,\ell_{\text{img}} \;\propto\; s\,\Omega^{2/3}. \tag{7}$$

Equivalently, writing $z := -s/\ell_A(\Omega)$ defines the *source-plane* (observable) Airy scale

$$\ell_A(\Omega) \;\propto\; \Omega^{-2/3}. \tag{8}$$

This length scale represents the "thickness" of the caustic surface as perceived by the wave.

3 The Universal Airy Law

3.1 Connection to Standard Wave Optics

The Airy function profile at gravitational caustics is a known result of catastrophe optics (Ref. 2). Our contribution is identifying the Action Quota ρ_0 as the physical parameter that sets the characteristic length scale ℓ_A, providing an interpretation of why this particular scale governs the diffraction pattern.

3.2 Collapse of the Wave Field

We propose that the intensity I near any gravitational fold, when plotted against the scaled coordinate $z = -s/\ell_A$ (where s is the source separation), must collapse onto a single universal curve.

Result: The Universal Airy Law

$$I(s;\omega) = I_0(\omega)\ \mathrm{Ai}^2\left(-\frac{s}{\ell_A(\omega)}\right), \qquad \ell_A \propto \Omega^{-2/3}. \tag{9}$$

The overall amplitude $I_0(\omega)$ scales with frequency due to the diffraction integral prefactor, but the *shape* of the profile is universal. This states that all gravitational folds collapse onto the universal Airy curve.

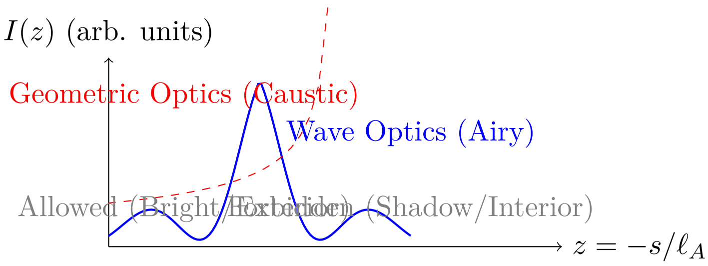

Figure 1: Schematic intensity profile (qualitative approximation of the Airy function). The intensity profile near a gravitational fold regularizes the geometric divergence into a smooth, oscillatory pattern. The horizontal axis is the scaled coordinate $z = -s/\ell_A$.

3.3 Observer Robustness (Practical Invariance)

Because $\ell_A \propto \omega^{-2/3}$, the predicted profile is most cleanly tested using the observed frequency in the scaling variable $z = -s/\ell_A(\omega_{\rm obs})$. This is not a claim of covariant geometry, but of operational universality: observers at different velocities measure different frequencies $\omega_{\rm obs}$ and different fringe scales $\ell_A(\omega_{\rm obs})$, but when they plot their data using their own measured $\omega_{\rm obs}$, all recover the same functional form $\mathrm{Ai}^2(z)$.

4 Implications for the Horizon

4.1 Entropy and the Saturated Ledger (Heuristic Argument)

We sketch a heuristic connection between horizon entropy and the Action Quota. If the horizon is a surface saturated at ρ_0, and if we count one bit per cell of size $\approx \ell_P^2$ (derived from ρ_0), the Bekenstein-Hawking formula $S = A/(4\ell_P^2)$ follows (Ref. 5). The horizon is a "Saturated Ledger." However, a rigorous derivation of this connection—particularly the factor of 4 and the transition from phase-space cells to horizon area—requires a full quantum gravity treatment and remains an open problem. The standard derivation counts quantum microstates of horizon geometry itself, not phase-space cells of propagating modes.

4.2 High-Frequency Cutoff (Conjecture)

A potential consequence of the Action Capacity is that linear waves might overshoot the ceiling that static curvature respects. Dimensional analysis suggests that linear wave propagation would break down when the action density $\sim \hbar\omega/\bar{\lambda}^3$ approaches the Planck density $\sim \hbar/\ell_P^4$, implying a cutoff frequency $\omega_{max} \sim c/\ell_P$. This suggests that the Action Quota acts as a natural UV cutoff for gravity.

4.3 Cusp Catastrophes (Pearcey Function)

While folds are the most common singularity (codimension 1), higher-order singularities like **Cusps** (codimension 2) also occur. These correspond to the meeting of two folds. Our framework predicts that these regions are governed by the **Pearcey Integral**. These frequency scalings ($\ell_C \propto \omega^{-1/2}$) follow from the Pearcey normal form of the cusp integral (Ref. 2). The intensity near a cusp scales as $I \propto \omega^{1/2}$ (compared to $\omega^{1/3}$ for folds), offering a powerful observational discriminator.

5 Conclusion: The Geometry of the Limit

This paper completes the "Geometry of the Limit" series. We have traced the consequences of the Action Quota from the microscopic spin of an electron to the macroscopic event horizon of a black hole.

We find that reality is not a featureless continuum. It is textured by the cost of **action**.

- **Locally:** The cost of a turn creates the Quantum (Paper I).
- **Globally:** The stability of a fold creates the Atom (Paper II).
- **Topologically:** The twist of a fold creates Matter (Paper III).
- **Macroscopically:** The saturation of the field creates Gravity (Paper IV).

The architecture of physical law, from the uncertainty principle to black hole thermodynamics, emerges as the geometry of a continuum operating under a finite action budget. The "singularity" at the center of a black hole—and the "singularity" of the Big Bang—may not be breakdowns of physics, but simply the points where the Action Budget of the universe is fully invested. Beyond this limit, the classical concepts of space and time may no longer apply, pointing toward a domain of pure, unmeasurable potential—the semantic reservoir from which the physical universe emerges.

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