# Quota-Critical Witnesses

## Experimental Signatures of the Action Limit

*Part III of the Beyond the Ledger Series*

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**Abstract**

In Parts I and II, we established that a finite action ledger implies operational incompleteness and necessitates an unmeasurable substrate ($\mathcal{U}$) populated by topological Folds. In this final paper, we translate these theoretical claims into falsifiable experimental protocols. We design three *Quota-Critical Witnesses*: (1) the *Simulation Complexity Witness*, which predicts increased compressibility and min-entropy degradation that tracks the resource cap when a QRNG is driven beyond its action capacity; (2) *Ramsey Fisher Benchmarks*, which verify that Fisher information plateaus when the accumulated measurement record exceeds the ledger capacity $N_{\max}$; and (3) *Ultra-Dense Fermi Gases*, which constrain the fundamental non-locality scale $\ell_0$ via deviations in the equation of state. A positive result in any of these experiments would provide direct evidence for the resource-bounded nature of physical reality and constrain the fundamental parameters of the unmeasurable substrate.

## Contents

# 1 Introduction: Auditing the Pilot

The central claim of Action Realism is that *truth is expensive.* Specifically, we have argued that the measurable world (the **Pilot**) is a finite ledger of facts carved from an infinite potential (the **Dreamer**) at a cost of one quantum of action ($h$) per bit. This implies that physical reality is resource-bounded.

In Part I, we proved this logically: a finite ledger cannot account for its own expansion. In Part II, we modeled this physically: the expansion is driven by **Folds**—topological defects in the unmeasurable substrate $\mathcal{U}$.

But logic and ontology are not enough. If the Action Quota is a real physical boundary—a "wall" in the phase space of reality—we should be able to hit it. We should be able to design experiments that do not merely measure a value, but which force the universe to make a choice it cannot afford within its current budget.

We call such experiments **Quota-Critical Witnesses**.

A Quota-Critical Witness is a protocol where the theoretically predicted outcome is contingent on the system hitting an **action saturation boundary** ($\Delta E\,\tau \approx h$) or a **logical resource limit** ($N$ bits). The signature is a deviation from standard quantum-mechanical predictions (or a specific resource-scaling behavior) at that boundary.

Each witness is designed so that a null result does not merely fail to support the framework, but translates into a quantitative bound on its new scale parameters (e.g. $\ell_0$, ledger capacity $N$, or fold rate $\Gamma$).

In this paper, we detail three such protocols, moving from the single-qubit scale to the macroscopic many-body limit.

# 2 Operational Definitions

We distinguish (i) *persistent* records, which remain available throughout the protocol without further stabilizing action, from (ii) ephemeral working memory.

**Definition 1** (Ledger Capacity)**.** The ledger capacity $N$ is operationally defined as the maximum number of persistent, recoverable classical bits that can be maintained within the experimental apparatus during the protocol's duration, given a fixed energy/action budget. This is the effective ledger capacity of the apparatus, which can be varied by changing the hardware (e.g. memory buffer size).

A witness is *quota-critical* if its predicted scaling changes when the protocol is tuned so that the required persistent transcript would exceed $N$ under Action Realism. The ledger boundary is the apparatus+controller subsystem during the protocol interval; exporting records outside this boundary (e.g. writing to an external server) constitutes an increase in ledger capacity and is treated as changing $N$. Accordingly, witnesses are specified as online protocols or protocols with explicitly budgeted export channels. Operationally, $N$ is enforced by running the predictor $P$ on a microcontroller/FPGA with a measured RAM+non-volatile budget and disabling external writes during acquisition.

**Detection criterion.** A positive witness requires (i) a statistically significant regime change in the specified observable, (ii) reproducibility under repeated runs, and (iii) the regime boundary tracking a controlled ledger/resource parameter (e.g. $N_{\max}$, power/thermal budget), not merely uncontrolled technical drift.

# 3 Witness I: Simulation Complexity Witness

This protocol tests the **Description–Action Link** (Paper I) in hardware. It frames the generation of quantum randomness not as a given, but as a process with a finite bit-production capacity.

## 3.1 Derivation

If each new stable classical bit requires a minimum action expenditure $h$, then a physical random-bit source is not an oracle: it has a *bit-production capacity* under fixed resources. When driven beyond this capacity—e.g. by increasing the output rate while holding the device's energetic and memory budget fixed—the source must either (i) pay additional action (raising the ledger cost) or (ii) exhibit departures from ideal i.i.d. behavior (bias/correlations/compressibility).

## 3.2 The Protocol

We use a **Quantum Random Number Generator (QRNG)** based on vacuum shot noise or photon arrival times.

1. **Fixed Power Budget.** Operate the QRNG with a fixed input power $P_{\text{in}}$.
2. **Throughput Sweep.** Demand output bit rate $f$ while holding fixed all device resources (input power, bandwidth, temperature control, and on-device persistent memory $N_{\max}$). We hold the analog chain constant (same photodiode/ADC gain, same sampling rate, same filtering, same post-processing) to ensure the change-point is not an engineering artifact.
3. **Operational Threshold.** Action Realism predicts a device-dependent critical regime beyond a measured threshold $f_c$, above which the source cannot supply statistically independent, stably recorded bits without either increasing its resource footprint or exhibiting detectable departures from i.i.d. behavior. The value of $f_c$ is treated as an experimentally measured parameter (not assumed *a priori*).
4. **Analysis.** After acquisition, analyze $R_{1:m}$ with (i) offline randomness tests (e.g. NIST STS) and (ii) an online resource-bounded predictor $P$ (persistent memory $N_{\max}$) for change-point detection via compression/statistics.
5. **Control Experiment.** The PRNG control is rate-matched and passed through the same acquisition/ADC pipeline.

## 3.3 Predicted Signature

**Standard QM.** After correcting for known device imperfections, the QRNG output remains statistically indistinguishable from i.i.d. bits as throughput is increased, until ordinary engineering limits (bandwidth, thermal drift) dominate.

**Action Realism.** Holding device resources fixed, there exists a *quota-limited* regime in which the source cannot supply fresh independent bits at the demanded rate. The prediction is a **complexity wall** expressed as a regime change in the bitstream itself: beyond a critical operating point, the stream becomes detectably more compressible and fails i.i.d. tests. Operationally, we expect:

1. a rise in compression ratio for $R_{1:m}$ under fixed compressors,
2. systematic nonzero autocorrelation / min-entropy degradation,
3. reproducible dependence of these effects on the controlled resource cap.

The distinguishing feature is that the change-point tracks the controlled resource cap (e.g. persistent memory/power/thermal budget) rather than generic bandwidth limits alone.

**Metric.** We define normalized compression length $\hat{H}_{\mathsf{Comp}} := |\mathsf{Comp}(R)|/m$, and separately measure Rényi min-entropy $H_\infty$, testing for a change-point in $\hat{H}_{\mathsf{Comp}}(f)$ as $f$ is swept. Here $\mathsf{Comp}$ is a fixed, publicly specified compressor (e.g. LZ77 with fixed window). A witness requires that the detected change-point persists after conditioning on diagnostics (no correlation with saturation/drift markers). We declare a change-point if (i) $\hat{H}_{\mathsf{Comp}}$ shifts by $\geq 5\sigma$ relative to the low-$f$ baseline and (ii) at least one independent statistic (min-entropy or autocorrelation) shifts consistently.

# 4 Witness II: Ramsey Fisher Benchmarks

While Witness I tests bit generation, the Ramsey experiment tests **metrological limits**. It verifies that information extraction saturates when the ledger is full.

## 4.1 Derivation

We separate (i) *online* estimators with constant memory from (ii) *transcript-based* estimators that require a growing persistent record to reach nominal scaling under drift/nuisance parameters. Concretely, under unknown phase drift modeled as a latent stochastic process (e.g. Ornstein–Uhlenbeck drift), maximum-likelihood inference requires retaining a windowed transcript; FIFO truncation then imposes an information ceiling. Action Realism predicts a plateau only in the transcript-required mode when the transcript is capped at $N_{\max}$.

## 4.2 The Protocol

1. **Prepare Probe.** Use a sequential non-adaptive Ramsey protocol (repeated single-qubit measurements) on a spin ensemble or single ion.
2. **Variable Ledger.** Cap the persistent transcript with a FIFO buffer of $k$ measurement outcomes, so $N_{\max} = k \cdot b$ bits (where $b$ is bits per outcome). Run an online constant-memory estimator as baseline.
3. **Expend Action.** Vary the cumulative total action $S_{\text{total}} = M \cdot \Delta E \cdot \tau$ expended across the interrogation sequence.
4. **Measure.** Compute $I_{\text{FIFO}}(\phi)$ from the FIFO-limited record and $I_{\text{online}}(\phi)$ from the constant-memory estimator.
5. **Decoherence Control.** Independently measure the coherence time $T_2$. Ensure the experiment operates in the regime $S_{\text{total}}/h \ll T_2 \Delta E/h$ to distinguish ledger limits from decoherence.

## 4.3 Predicted Signature

**Standard QM.** In ideal conditions, an online estimator can continue to improve with $M$ without needing a growing transcript; any saturation is explained by decoherence, drift, or estimator choice.

**Action Realism.** In the transcript-required regime, $I_{\text{FIFO}}(\phi)$ exhibits a plateau that tracks $N_{\max}$, while $I_{\text{online}}(\phi)$ does not necessarily plateau at the same point. The smoking gun is that the plateau in $I_{\text{FIFO}}$ shifts proportionally with the physical transcript cap $N_{\max}$ under otherwise fixed conditions. We test tracking by regressing the plateau location $\widehat{(S/h)}_c$ against $N_{\max}$ and requiring slope $\approx 1$ within error.

# 5 Witness III: Ultra-Dense Fermi Gases

Finally, we look for a macroscopic signature in **condensed matter**. We test the **DFT kernel** (Paper II) directly via the equation of state (EOS).

## 5.1 Derivation

In Paper II, we introduced the DFT kernel $K(\Box) = e^{-\ell_0^2 \Box}$ to regulate the field. This non-local kernel modifies short-range interactions between fermions. Dimensional analysis suggests the non-local correction to the energy density scales as

$$\epsilon_{\text{nl}} \sim n \cdot (\ell_0^3 n) \cdot \epsilon_F,$$

which gives a pressure correction

$$\Delta P_{\text{nonlocal}} \sim \ell_0^3 n^2 \epsilon_F.$$

We treat $\ell_0$ as a free microscopic length to be constrained experimentally.

## 5.2 The Protocol (Constraint Setting)

1. **Platform.** Choose a fermionic system with tunable short-range interactions (via Feshbach resonances), such as $^6$Li or $^{40}$K, in a uniform box trap.
2. **Sweep.** Measure the equation of state $P(n)$ at zero temperature across a wide range of densities and interaction strengths.
3. **Analysis.** Perform a Bayesian model comparison between the standard local effective theory (contact + range corrections) and a model including the non-local term $\Delta P_{\text{nonlocal}}(n; \ell_0)$.

## 5.3 Predicted Signature

**Standard QM.** Residuals are consistent with known local corrections within error bars.

**Action Realism.** A non-local kernel induces a systematic EOS residual with a characteristic density dependence controlled by $\ell_0$; the sign and detailed shape are model-dependent. The experiment therefore yields either a detection or a bound on $\ell_0$ by fitting $\Delta P_{\text{nonlocal}}(n; \ell_0)$. A null result gives

$$|\Delta P| \leq \delta P \quad \Rightarrow \quad \ell_0 \lesssim \left( \frac{\delta P}{n^2 \epsilon_F} \right)^{1/3} \lambda_F,$$

where $\lambda_F$ is the Fermi wavelength, with consistent units understood.

# 6 Feasibility and Falsification

## 6.1 Quantitative Falsification Criteria

Action Realism is falsified (for the tested parameter ranges) if:

1. **Witness I.** No reproducible change-point in $\hat{H}_{\mathsf{Comp}}(f)$ or min-entropy metrics that tracks a controlled resource cap (power/memory), beyond independently measured engineering noise.
2. **Witness II.** No plateau in $I_{\text{FIFO}}(\phi)$ that shifts with the physical transcript cap $N_{\max}$ under fixed decoherence/drift.
3. **Witness III.** EOS residuals are fully absorbed by local effective parameters, yielding only upper bounds consistent with $\ell_0 = 0$.

### 6.2 Timescales and Alternatives

Witness I requires sufficient trials ($m \sim 10^4$–$10^6$) to establish complexity statistics. Witness II requires high-stability clocks (hours of averaging). Witness III relies on precision static measurements.

We distinguish Action Realism from:

- **Modified QM (CSL).** CSL predicts stochastic localization (decoherence rate) but does not predict resource-indexed transitions in bit-production capacity.
- **Computational Complexity.** Computational complexity theory limits the time required for computation but not the storage capacity of physical states under a fixed action budget.

## 7 Conclusion: The Final Compact

These three witnesses probe the Action Limit from three directions: simulation complexity, quantum metrology, and condensed matter physics. By reframing the logical paradoxes of Paper I and the topological mechanisms of Paper II into concrete physical benchmarks—**complexity walls**, **saturation plateaus**, and **EOS deviations**—we transform Action Realism from a philosophical framework into a testable physical theory.

Positive results would not only vindicate Action Realism but would establish fundamental limits on information processing, metrology, and many-body physics—requiring revision of quantum information theory at its foundations. If the Action Quota is real, the Pilot's ledger is not just a metaphor; it is a physical constraint that shapes the behavior of qubits, clocks, and fluids alike.

## A Resource Accounting for Witnesses

- **Witness I.** The persistent record is the generated bitstream $R_{1:m}$ and the internal state of the compressor. For $m = 10^6$ bits, this is $\sim 1$ MB of active ledger.
- **Witness II.** The persistent record is the FIFO buffer of the last $k$ measurement outcomes, where $k = N_{\max}$. For $k = 10^3$ outcomes at 4 bytes/outcome, this is $\sim 4$ kB.
- **Witness III.** The persistent record is the set of calibrated $(n, P)$ data points, each requiring $\sim 32$ bits of precision. For 100 points, this is $\sim 800$ bytes.