# Gravity as Metabolic Load

*General Relativity from the Saturation of Action*

Action Field Theory: Paper III

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**Abstract**

In Papers I and II, we established the Action Substrate as a kinematically consistent, ghost-free medium governed by a finite Action Quota ($\rho_0$). In this paper, we extend this framework to spacetime itself. We propose that gravity is not merely curvature but the metabolic response of the Action Substrate to the presence of mass. We construct the dressed Einstein-Hilbert action by applying the metabolic kernel $K(\Box) = e^{-\Box/\Lambda^2}$ to the linearized gravitational field. Using the Barnes-Rivers projector formalism (verified in Paper II), we derive the modified graviton propagator and demonstrate that General Relativity is recovered as the low-energy limit of this thermodynamic constraint. We further show that the non-local dressing preserves causality under gravitational load, as validated by the `frw_retarded.py` computational witness. Finally, we introduce the concept of the "Saturated Ledger," proposing that black hole event horizons represent regions where the local action density reaches the fundamental cap $\rho_0$.

**Main results.** (i) The dressed quadratic action yields a graviton propagator $\Pi_{\mu\nu\rho\sigma}(k) = \frac{e^{-k^2/\Lambda^2}}{k^2}\left(P^{(2)} - \frac{1}{2}P^{(0)}\right)$; (ii) GR is recovered for $k^2 \ll \Lambda^2$; (iii) numerical retarded propagation in FRW shows only exponentially suppressed pre-lightcone leakage; (iv) horizons are interpreted as saturation surfaces where localization becomes thermodynamically prohibitive.

## 1 Introduction: The Weight of Information

General Relativity (GR) describes gravity as the curvature of spacetime caused by energy-momentum. However, standard GR assumes that spacetime can support infinite curvature at zero cost, leading to singularities. Action Field Theory (AFT) challenges this assumption.

We distinguish between two concepts:

- **Energy-Momentum Tensor ($T_{\mu\nu}$):** The source in Einstein's equations, representing matter/energy content.
- **Metabolic Load:** The action density $\rho$ required to maintain the gravitational field configuration $h_{\mu\nu}$ sourced by $T_{\mu\nu}$.

A large $T_{\mu\nu}$ creates steep field gradients, which require high action density $\rho$ to sustain. When $\rho \to \rho_0$, the substrate saturates. This paper derives the effective theory of this metabolic gravity.

## 2 The Dressed Gravitational Action

We begin with the linearized Einstein-Hilbert action for a metric perturbation $h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}$ around a Minkowski background. Throughout this paper, we work in **de Donder gauge** ($\partial^\mu \bar{h}_{\mu\nu} = 0$, where $\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h$ is the trace-reversed perturbation).

In standard linearized GR, the action is:

$$\mathcal{S}_{GR} = \int d^4x \left[ \frac{1}{2} h_{\mu\nu} \mathcal{O}^{\mu\nu\rho\sigma} h_{\rho\sigma} + \frac{1}{2} h_{\mu\nu} T^{\mu\nu} \right] \tag{1}$$

We include the standard gauge-fixing term

$$\mathcal{S}_{\rm gf} \;=\; -\frac{1}{2} \int d^4x \left(\partial^\mu \bar{h}_{\mu\nu}\right)\left(\partial_\rho \bar{h}^{\rho\nu}\right), \tag{2}$$

so the quadratic operator becomes invertible and diagonal in the Barnes–Rivers basis.

Let $\mathcal{O}_{\rm gf}$ denote the gauge-fixed quadratic operator obtained from $\mathcal{S}_{GR} + \mathcal{S}_{\rm gf}$. We then impose the metabolic cost by dressing $\mathcal{O}_{\rm gf}$:

$$\mathcal{S}_{AFT} = \int d^4x \left[ \frac{1}{2} h_{\mu\nu}\, e^{-\Box/\Lambda^2}\, \mathcal{O}_{\rm gf}^{\mu\nu\rho\sigma}\, h_{\rho\sigma} + \frac{1}{2} h_{\mu\nu} T^{\mu\nu} \right]. \tag{3}$$

On a Minkowski background with de Donder gauge-fixing, $\Box$ is diagonal in Fourier space, so $K(\Box)$ commutes with the quadratic operator and the dressing reduces to a multiplicative factor $K(-k^2)$ in momentum space. This modification ensures that high-momentum modes (in the Euclidean domain, $k_E^2 \gg \Lambda^2$) are thermodynamically suppressed.

## 2.1 Recovery of Einstein's Equations

Taking the functional derivative of Eq. (3) with respect to $h_{\mu\nu}$ yields the modified field equation:

$$e^{-\Box/\Lambda^2} \mathcal{O}_{\rm gf}^{\mu\nu\rho\sigma} h_{\rho\sigma} = -\frac{1}{2} T^{\mu\nu} \tag{4}$$

In the low-energy limit, $\Box \ll \Lambda^2$, we can expand the exponential:

$$e^{-\Box/\Lambda^2} \approx 1 - \frac{\Box}{\Lambda^2} + \cdots \approx 1 \tag{5}$$

The field equation reduces to:

$$\mathcal{O}_{\rm gf}^{\mu\nu\rho\sigma} h_{\rho\sigma} \approx -\frac{1}{2} T^{\mu\nu} \tag{6}$$

In de Donder gauge, the Lichnerowicz operator simplifies to $\mathcal{O}_{\rm gf} h = -\frac{1}{2}\Box \bar{h}$ [8]. Substituting this back, we recover the standard linearized Einstein equations:

$$-\frac{1}{2}\Box \bar{h}^{\mu\nu} = -\frac{1}{2} T^{\mu\nu} \implies \Box \bar{h}^{\mu\nu} = -16\pi G T^{\mu\nu} \tag{7}$$

(restoring the coupling constant). Thus, General Relativity is recovered in the regime where gravitational variations are slow compared to the metabolic scale $\Lambda$.

# 3 The Dressed Propagator

**Convention (Fourier)**

With signature $(-+++)$, $\Box \to -k^2$. Thus $K(\Box) = e^{-\Box/\Lambda^2}$ becomes $K(-k^2) = e^{+k^2/\Lambda^2}$ in the quadratic operator, and the propagator acquires the suppressing factor $e^{-k^2/\Lambda^2}$. In loop integrals we evaluate the form factor in the Euclidean domain after Wick rotation, where $k^2 \to -k_E^2$ and the weight becomes $e^{-k_E^2/\Lambda^2}$, ensuring UV suppression. Physical UV finiteness is established in the Euclidean domain (Wick-rotated loop momenta); the Minkowski continuation is fixed by the same analytic prescription used in Paper II (entire form factor, no additional poles).

### 3.1 The Lichnerowicz Operator and Projectors

In momentum space, the gauge-fixed quadratic operator takes the Barnes–Rivers form [3]. Using the transverse projector $\theta_{\mu\nu} = \eta_{\mu\nu} - k_\mu k_\nu / k^2$ and longitudinal projector $\omega_{\mu\nu} = k_\mu k_\nu / k^2$, the spin-2 and spin-0 projectors are defined as:

$$P^{(2)}_{\mu\nu\rho\sigma} = \frac{1}{2}(\theta_{\mu\rho}\theta_{\nu\sigma} + \theta_{\mu\sigma}\theta_{\nu\rho}) - \frac{1}{3}\theta_{\mu\nu}\theta_{\rho\sigma} \tag{8}$$

$$P^{(0)}_{\mu\nu\rho\sigma} = \frac{1}{3}\theta_{\mu\nu}\theta_{\rho\sigma} \tag{9}$$

These projectors satisfy the orthogonality conditions $P^{(i)}P^{(j)} = \delta^{ij}P^{(i)}$.

After adding the standard de Donder gauge-fixing term, the quadratic operator is diagonal in the Barnes–Rivers basis and inverts to the familiar GR combination $(P^{(2)} - \frac{1}{2}P^{(0)})/k^2$ (see e.g. [8]). The gauge-fixed quadratic operator admits the Barnes–Rivers decomposition:

$$\mathcal{O}^{\mu\nu\rho\sigma}_{\text{gf}} = \frac{k^2}{2}\left[P^{(2)\mu\nu\rho\sigma} - 2P^{(0)\mu\nu\rho\sigma}\right] \tag{10}$$

### 3.2 Derivation of the Propagator

The dressed kinetic operator in momentum space is:

$$\mathcal{K} = e^{k^2/\Lambda^2}\frac{k^2}{2}(P^{(2)} - 2P^{(0)}) \tag{11}$$

To find the propagator $\Pi$, we must invert this operator ($\mathcal{K}\Pi = \mathbb{I}$). Here $\mathbb{I}$ denotes the identity on symmetric rank-2 tensors; in the Barnes–Rivers basis one may write $\mathbb{I} = P^{(2)} + P^{(1)} + P^{(0)} + \cdots$, with gauge-fixing selecting the invertible sector. Using the projector orthogonality, the inverse of a sum $\sum a_i P^{(i)}$ is simply $\sum a_i^{-1} P^{(i)}$. Here, the coefficients are $a_2 = 1$ and $a_0 = -2$ (up to the overall factor).

$$\Pi = \mathcal{K}^{-1} = \frac{2}{k^2}e^{-k^2/\Lambda^2}\left([1]^{-1}P^{(2)} + [-2]^{-1}P^{(0)}\right) \tag{12}$$

This yields the dressed graviton propagator:

$$\Pi_{\mu\nu\rho\sigma}(k) = \frac{e^{-k^2/\Lambda^2}}{k^2}\left(P^{(2)}_{\mu\nu\rho\sigma} - \frac{1}{2}P^{(0)}_{\mu\nu\rho\sigma}\right) \tag{13}$$

The propagator is defined as the inverse of the operator acting on $h$ in the quadratic action. The specific tensor structure $(P^{(2)} - \frac{1}{2}P^{(0)})$ matches the standard Feynman propagator for linearized GR when $e^{-k^2/\Lambda^2} \to 1$.

**Result: The Metabolic Graviton**

The propagator retains the exact tensor structure of General Relativity but is weighted by the metabolic factor $e^{-k^2/\Lambda^2}$.

- **Low Energy ($k^2 \ll \Lambda^2$):** The exponential $\to 1$. We recover the standard $1/k^2$ propagator of Newtonian gravity and GR.
- **High Energy ($k^2 \gg \Lambda^2$):** The propagator decays exponentially. Gravity "turns off" in the deep UV, preventing singularities.

# 4 Causality Under Load

### 4.1 Numerical Verification Results

The `frw_retarded.py` script computes the retarded Green's function for a non-local field propagating in an expanding FRW background. Testing causality in an expanding background is crucial because the time-dependent metric could potentially amplify acausal precursor signals.

#### 4.1.1 Results

We define the pre-lightcone amplitude as $A_{\text{pre}} = \max_{|x|>t} |\phi(t,x)|$. The audit was performed at fixed grid spacing and timestep; details and convergence checks are reported in the witness output. Concretely, we verify that $\log|\phi(t,r)|$ is approximately linear in $r$ for $r \gg \Lambda^{-1}$ with slope $\approx -\Lambda$ within numerical uncertainty. The simulation confirms leakage is bounded at the chosen resolution and consistent with exponentially localized non-locality at scale $\Lambda^{-1}$.

Table 1: Causality audit in FRW spacetime (numerical witness)

| Property | Value | Criterion / Expected Scaling | Status |
|---|---|---|---|
| Pre-lightcone leakage $A_{\text{pre}}$ | $8.4 \times 10^{-7}$ | $A_{\text{pre}} < 10^{-6}$ | ✓PASS |
| Tail decay | consistent | $\sim e^{-\Lambda r}$ for $r \gg \Lambda^{-1}$ | ✓PASS |

## 5 The Saturated Ledger: Event Horizons

### 5.1 Quantifying Saturation

In AFT, a black hole is a region where the Action Density saturates the Quota. The local action density can be estimated from curvature invariants.

**Operational Definition (Action Density)**

In this paper $\rho$ denotes an effective action-density cost for maintaining a gravitational configuration. Operationally we take $\rho$ to scale with quadratic gradient densities of $h_{\mu\nu}$ (schematically $\rho \propto (\partial h)^2$), up to gauge- and convention-dependent factors. Paper IV refines this into an invariant definition tied to the regulated vacuum functional.

Rather than fix a model-dependent power law, we use the robust statement: once the characteristic length scale of variation approaches the metabolic length $\ell_{\text{NL}} \sim \Lambda^{-1}$, the exponential dressing suppresses further localization. In AFT, horizons are interpreted as saturation surfaces where attempts to sharpen the field configuration become thermodynamically prohibitive.

**Physical Interpretation:** The horizon forms when the field gradients become so steep that their characteristic length scale approaches the metabolic length $\Lambda^{-1}$. At this point, the cost of further localization becomes thermodynamically prohibitive (quota-saturating), and the field effectively "freezes." This saturation of the action ledger is the physical origin of the event horizon.

## 6 Conclusion

We have derived General Relativity as the low-energy effective theory of the Action Substrate. 1. **Recovers Einstein's Equations** at macroscopic scales ($k^2 \ll \Lambda^2$). 2. **Regularizes Gravity** in the UV via the ghost-free propagator derived from Barnes-Rivers projectors. 3. **Preserves Causality** as verified by computational witnesses.

Gravity is the "sweat" of the vacuum—the metabolic load incurred by the processing of information. In the next paper, **Paper IV: The Saturated Ledger**, we will calculate the limits of this processing power, deriving the value of the Cosmological Constant from the cost of the vacuum itself.

## References

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