Signals and Systems Analysis

Chapter 1:- Introduction

OUTLINE

- **Classifications of Signals**
- **Classifications of Systems**
- **Useful Signal Operations**
- **Elementary Signals**

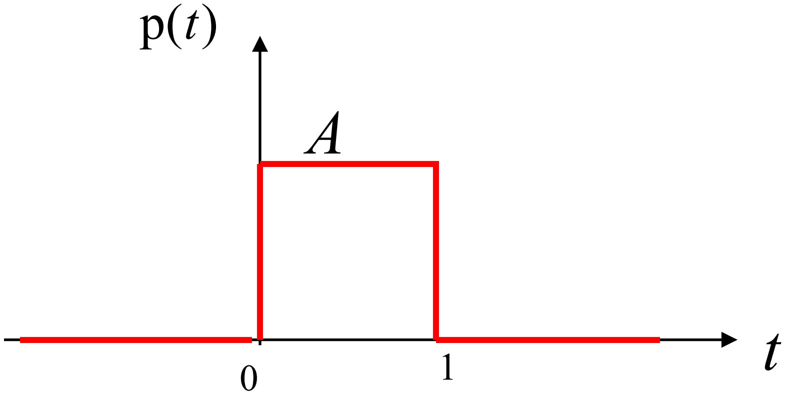
CLASSIFICATIONS OF SIGNALS

- **Classification of signals: signals can be classified as**
  - Continuous-time signal v.s. discrete-time signal
  - Analog signal v.s. digital signal
  - Even signal v.s. odd signal
  - Periodic signal v.s. Aperiodic signal
  - Power signal v.s. Energy signal
  - Deterministic and Random signals
  - ……

CLASSIFICATION: CONTINUOUS-TIME V.S. DISCRETE-TIME

- **Continuous-time (CT) signal**
  - a signal is defined over continuous time
    - E.g. sinusoidal signal $s(t) = \sin(4t)$
    - E.g. voice signal
    - E.g. Rectangular pulse function

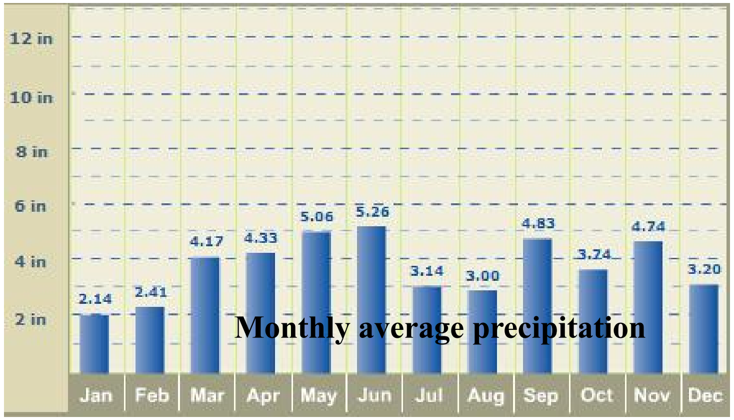
$$\mathrm{p}(t) = \begin{cases} A, & 0 \le t \le 1 \\ 0, & \text{otherwise} \end{cases}$$



Rectangular pulse function

CLASSIFICATION: CONTINUOUS-TIME V.S. DISCRETE-TIME

- **Discrete-time signal**
  - A signal that is defined at discrete values of time
  - E.g. the monthly average precipitation



$T_s = 1\text{month}$

$k = 1, 2, \ldots, 12$

CLASSIFICATION : ANALOG V.S. DIGITAL

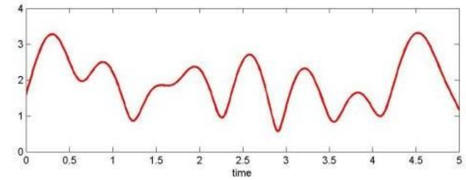
Analog:

- A signal whose amplitude can take on any value in a continuous range

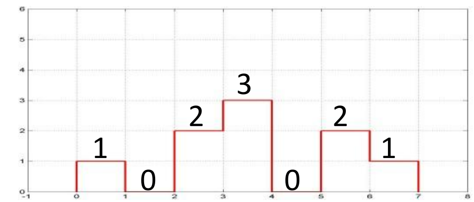
Digital:

- A signal whose amplitude can take on only a finite number of values

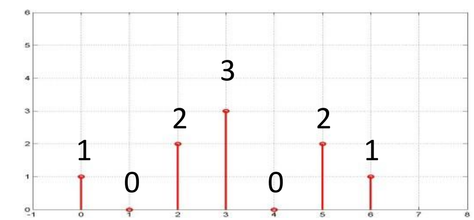
continuous-time, Analog



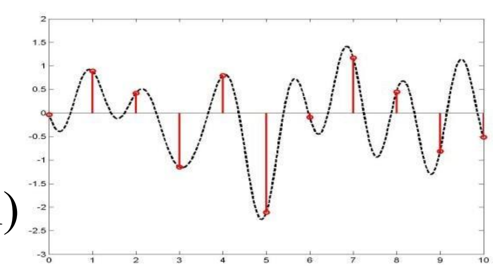
Continuous-time, Digital



Discrete-time, Digital



- Discrete-time, Analog (samples of analog signal)



Different types of signals

CLASSIFICATION: EVEN V.S. ODD

- **Even v.s. odd**
  - x(t) is an even signal if: $x(t) = x(-t)$
    - E.g. $x(t) = \cos(2t)$
  - x(t) is an odd signal if: $x(-t) = -x(t)$
    - E.g. $x(t) = \sin(2t)$
  - Some signals are neither even, nor odd
    - E.g. $x(t) = e^t$ $\quad x(t) = \cos(2t), t > 0$
  - Any signal can be decomposed as the sum of an even signal and an odd signal

$$y(t) = \underbrace{y_e(t)}_{\text{even}} + \underbrace{y_o(t)}_{\text{odd}}$$

$$\begin{aligned} y_e(t) &= 0.5\,[y(t) + y(-t)] \\ y_o(t) &= 0.5\,[y(t) - y(-t)] \end{aligned}$$

CLASSIFICATION : PERIODIC V.S. APERIODIC

- **Periodic signal v.s. aperiodic signal**
  - A signal is periodic if
    - There is a positive real value T such that $f(t) = f(t + nT)$
    - It is defined for all possible values of t, $-\infty \le t \le \infty$ (why?)
  - Period T_0: the smallest positive integer T_0 that satisfies

$$f(t) = f(t + nT_0)$$

$$T_1 = 2T_0$$

$$f(t + T_1) = \mathrm{f}(t + 2T_0) = f(t)$$

CLASSIFICATION: ENERGY V.S. POWER

- **Energy of signal *x*(*t*) over** $t \in [-\infty, +\infty]$

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

A signal with finite energy is called an energy signal.

- **Average power of signal *x*(*t*)**

$$P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$$

A signal with finite and non-zero power is called a power signal.
Example: *sin(t), cos(t)*

- A signal with finite energy has zero power
- A signal with finite power has infinite energy
- A signal can be an energy sign or a power signal, or neither NENP(Ramp signal), but not both.

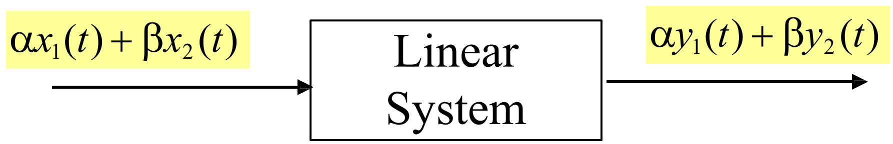
OUTLINE

- Classifications of Signals
- Classifications of Systems
- Useful Signal Operations
- Elementary Signals

CLASSIFICATIONS OF Systems

- **Classifications**
  - Linear v.s. non-linear
  - Time-invariant v.s. time-varying
  - Dynamic v.s. static (memory v.s. memoryless)
  - Causal v.s. non-causal
  - Stable v.s. non-stable (Reading Assignment)

- **Linear system**
  - Let $y_1(t)$ be the response of a system to an input $x_1(t)$
  - Let $y_2(t)$ be the response of a system to an input $x_2(t)$
  - The system is linear if the superposition principle is satisfied:
    1. the response to $x_1(t)+x_2(t)$ is $y_1(t)+y_2(t)$
    2. the response to $\alpha x_1(t)$ is $\alpha y_1(t)$

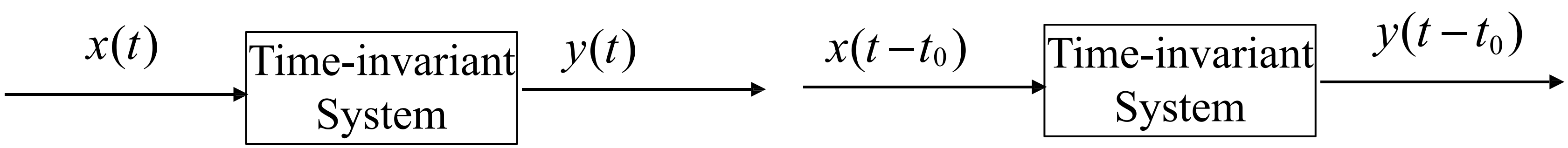


Linear system

- **Non-linear system**
  - If the superposition principle is not satisfied, then the system is a non-linear system

CLASSIFICATIONS: TIME-VARYING V.S. TIME-INVARIANT

- **Time-invariant**
  - A system is time-invariant if a time shift in the input signal causes an identical time shift in the output signal



- **Examples**
  - $y(t) = \cos(x(t))$

CLASSIFICATIONS: MEMORY V.S. MEMORYLESS

Memoryless system (Instantaneous)

- the present value of the output depends only on the present value of the input
  - Example: input x(t): the current passing through a resistor output y(t): the voltage across the resistor

$$y(t) = Rx(t)$$

  - The output value at time *t* depends only on the input value at time *t*.

System with memory

- the present value of the output depends on not only the present value of input, but also previous input values.
  - Example: capacitor, current: x(t), output voltage: y(t)

$$y(t) = \frac{1}{C}\int_0^t x(\tau)\,d\tau$$

  - the output value at *t* depends on all input values before *t*

CLASSIFICATIONS: CAUSAL V.S. NON-CAUSAL

- **Causal system (Physical/non-anticipated)**
  - The output depends on only input from the past and present
  - Example 4

$$y(t) = x(t-3) + x(t)$$

- **Non-causal system**
  - A system is non-causal if the output depends on the input from the future (prediction).
  - Example 5: $y(t) = x(t-2) + x(t+2)$

  - All practical systems are causal.

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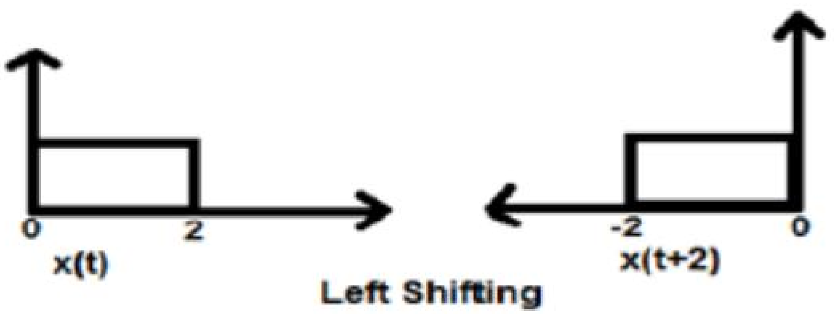
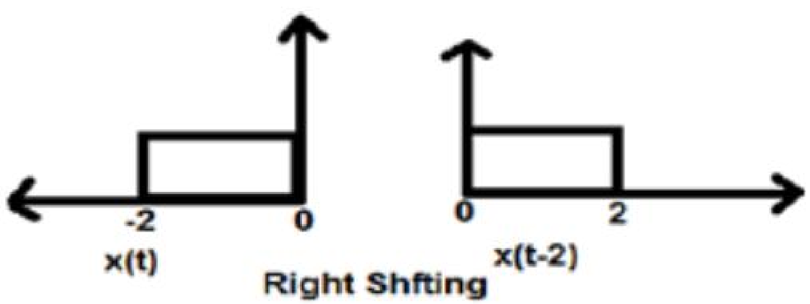
- Elementary Signals

OPERATIONS: TIME SHIFTING

- **Time Shifting operation:** shifting a signal with respect to time
  - $x(t-T)$: represents $x(t)$ shifted by T seconds

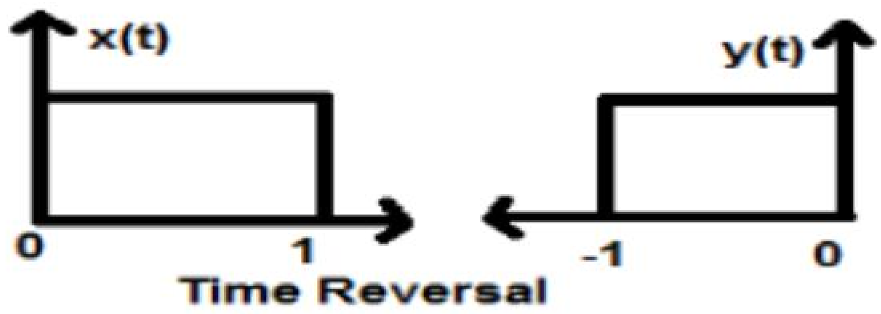
- If T is positive, the shift is to the right (delay)
- If T is negative, the shift is to the left (Advance)





OPERATIONS: TIME REVERSAL/REFLECTION

- **Time Reversal:** a signal's time is multiplied by -1
- In this case, the signal produces its mirror image about Y-axis.
- Mathematically, this can be written as $x(-t)$

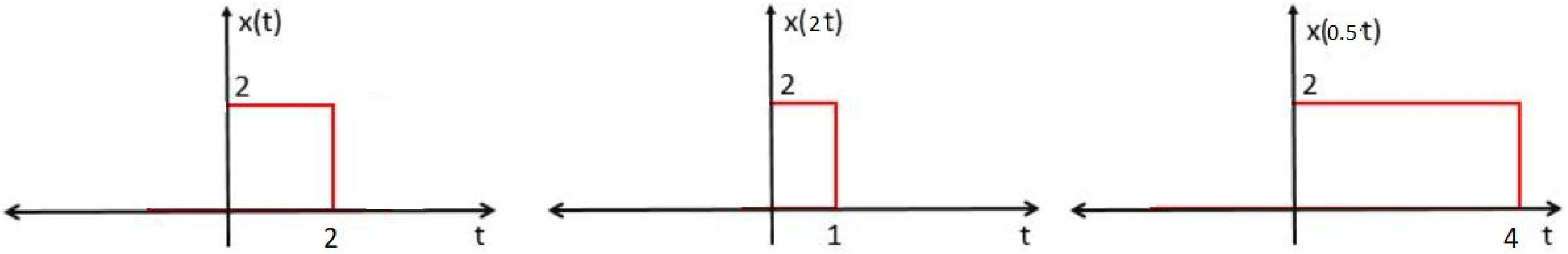


OPERATIONS: TIME-SCALING

- **Time-scaling:** compression or expansion of a signal in time $x(at)$

In time scaling the time t is multiplied by a constant which is not equal to zero

- $|a|>1$, Time compression
- $|a|<1$, Time expansion



OPERATIONS: TIME-SCALING

Combined operation $x(at+b)$

Method 1:

1. Time scale the signal by a: $x(at)$
2. Time shift the scaled signal by b/a: $x(a(t+b/a))=x(at+b)$

Method 2:

1. Time shift the signal by b: $x(t+b)$
2. Time scale the shifted signal by a: $x(at+b)$

- The operations are always performed w.r.t. the time variable t

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ELEMENTARY SIGNALS

- are also called singularity functions/switching functions

- They are functions that are either discontinuous or have discontinuous derivatives. (that is when the function jumps from one value to another without taking on any intermediate value)

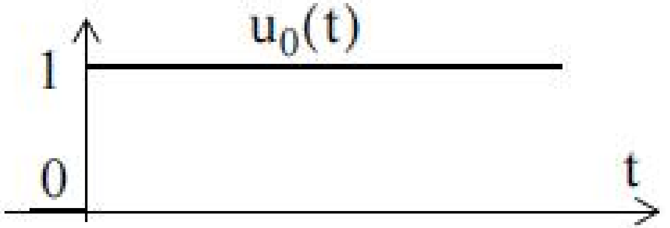
- They are very useful in representing switching operations.

- The three most widely used singularity functions are: unit step, unit impulse and, unit ramp functions

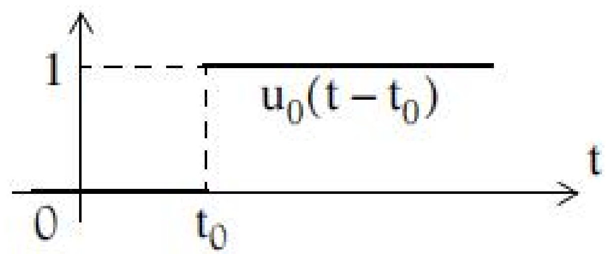
ELEMENTARY SIGNALS: UNIT STEP FUNCTION

- **Unit step function**

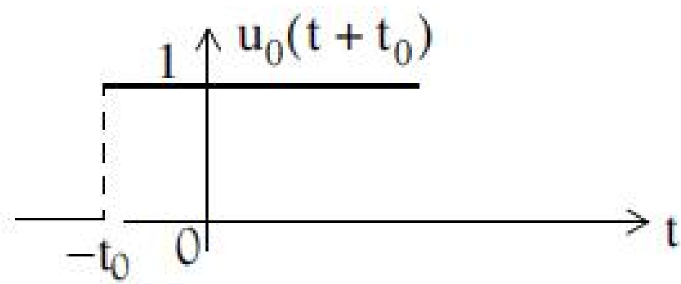
$$\boxed{u_0(t) = \begin{cases} 0 & t < 0 \\ 1 & t > 0 \end{cases}}$$



$$u_0(t - t_0) = \begin{cases} 0 & t < t_0 \\ 1 & t > t_0 \end{cases}$$

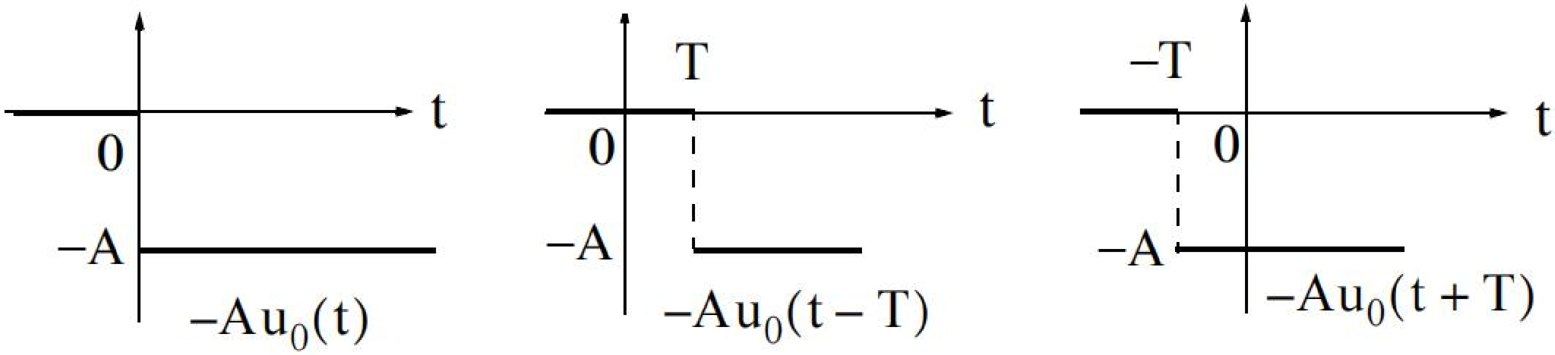


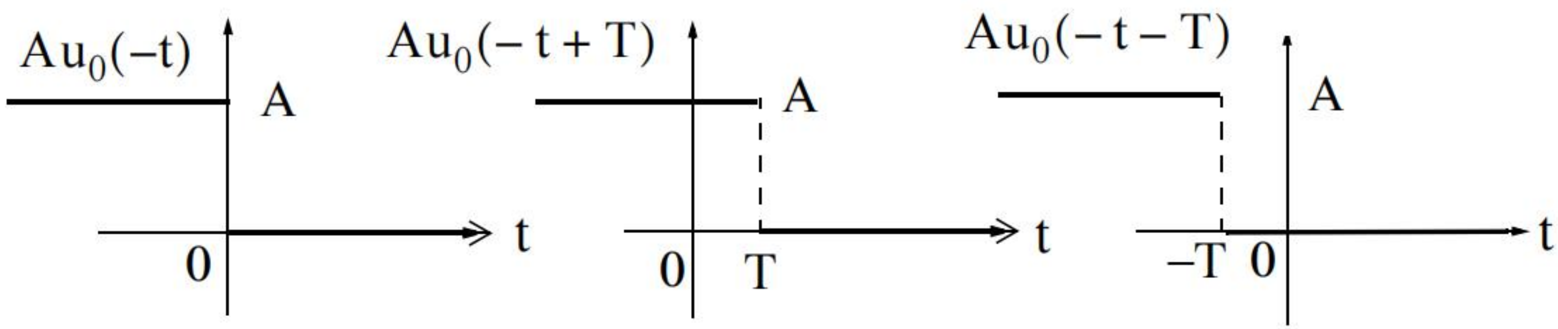
$$u_0(t + t_0) = \begin{cases} 0 & t < -t_0 \\ 1 & t > -t_0 \end{cases}$$



ELEMENTARY SIGNALS: UNIT STEP FUNCTION

Other forms of the unit step function

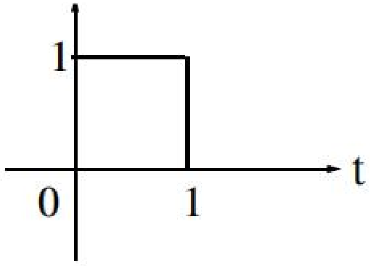




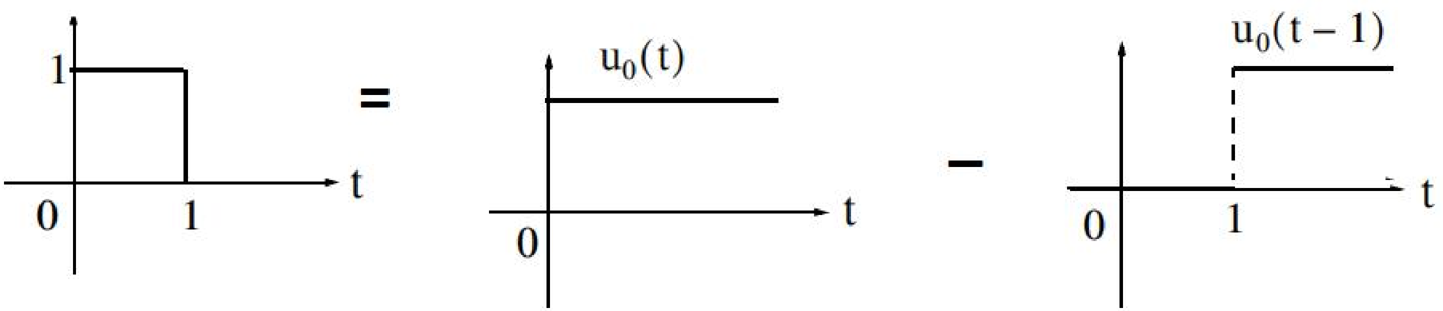
ELEMENTARY SIGNALS: UNIT STEP FUNCTION

- Unit step functions can be used to represent other time−varying functions such as the rectangular pulse shown below

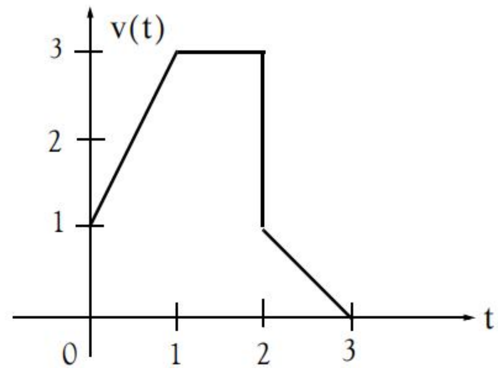
Example : Express the square pulse a s a sum of the unit step function



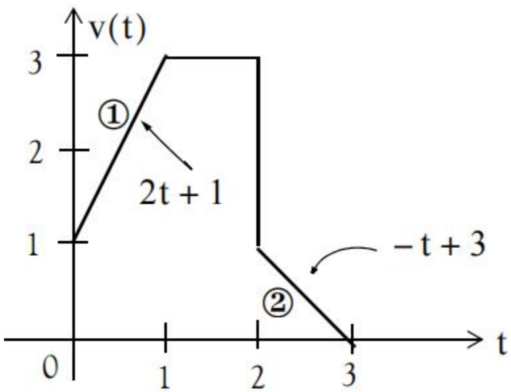
Solution:



Example : Express the waveform as a sum of unit step functions



Solution: First find the equations of the linear segments



$$v(t) = (2t+1)[u_0(t) - u_0(t-1)] + 3[u_0(t-1) - u_0(t-2)] + (-t+3)[u_0(t-2) - u_0(t-3)]$$

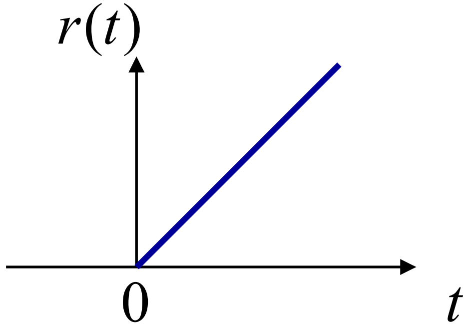
Simplify:

$$v(t) = (2t+1)u_0(t) - 2(t-1)u_0(t-1) - tu_0(t-2) + (t-3)u_0(t-3)$$

ELEMENTARY SIGNALS: RAMP FUNCTION

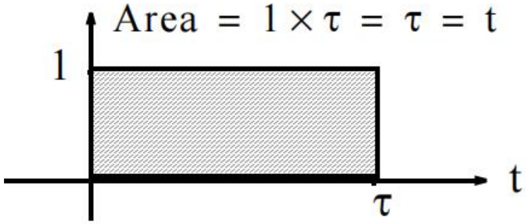
The Ramp function $\boldsymbol{u_1(t) = r(t)}$

$$u_1(t) = \int_{-\infty}^{t} u_0(\tau)d\tau$$



Unit ramp function

We can evaluate the integral by considering the area under the unit step function $u_0(t)$from 0 to ∞



Therefore the ramp function is defined as

$$u_1(t) = \begin{cases} 0 & t < 0 \\ t & t \geq 0 \end{cases}$$

Since is the integral of $u_0(t)$ is $u_1(t)$, then $u_0(t)$must be the derivative of $u_1(t)$

$$\frac{d}{dt}u_1(t) = u_0(t)$$

ELEMENTARY SIGNALS: RAMP FUNCTION

Higher-order functions of t can be generated by repeated integration of the unit step function.

$$u_2(t) = \begin{cases} 0 & t < 0 \\ t^2 & t \geq 0 \end{cases} \qquad \text{or} \qquad u_2(t) = 2\int_{-\infty}^{t} u_1(\tau)d\tau$$

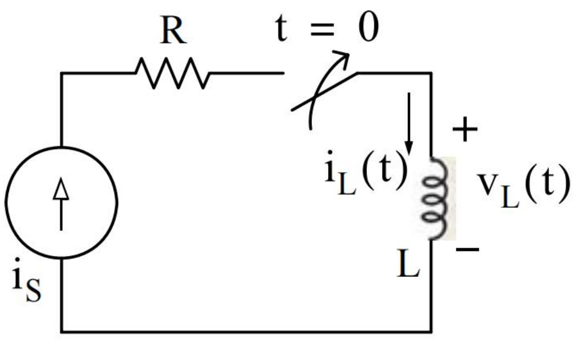
$$u_3(t) = \begin{cases} 0 & t < 0 \\ t^3 & t \geq 0 \end{cases} \qquad \text{or} \qquad u_3(t) = 3\int_{-\infty}^{t} u_2(\tau)d\tau$$

$$u_n(t) = \begin{cases} 0 & t < 0 \\ t^n & t \geq 0 \end{cases} \qquad \text{or} \qquad u_n(t) = n\int_{-\infty}^{t} u_{n-1}(\tau)d\tau$$

ELEMENTARY SIGNALS: UNIT IMPULSE FUNCTION

- There are frequent occasions in which we are interested in the derivative of the unit step function $u_0(t)$

$$v_L(t) = L\frac{di_L}{dt}$$



$$i_L(t) = i_S\, u_0(t)$$

$$\boxed{v_L(t) = L i_S \frac{d}{dt} u_0(t)}$$

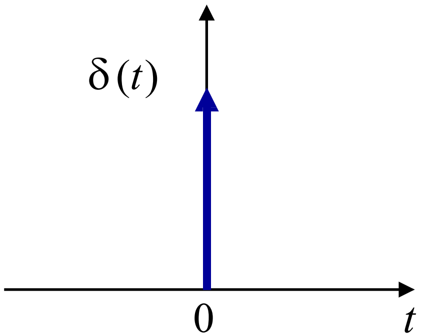
- The derivative of the unit step has a non-zero value only at t=0.
- The derivative of the unit step function is defined in the this section

Unit impulse function (delta function)

- The *unit impulse* or *delta function*, denoted as $\delta(t)$, is the derivative of the unit step $u_0(t)$.

$$\int_{-\infty}^{t} \delta(t)dt = u(t)$$

$$\delta(t) = \frac{du(t)}{dt}$$



Unit impulse function

$$\delta(0) = \infty$$
$$\delta(t) = 0, t \neq 0$$
$$\int_{-\infty}^{t} \delta(t)dt = \begin{cases} 1, & t > 0 \\ 0, & t < 0 \end{cases}$$

Properties of the delta function

- The *sampling property* of the delta function states that

$$\boxed{f(t)\delta(t-a) = f(a)\delta(t-a)}$$

$$\boxed{f(t)\delta(t) = f(0)\delta(t)}$$

The *sifting property* of the delta function states that

$$\boxed{\int_{-\infty}^{\infty} f(t)\delta(t-\alpha)dt = f(\alpha)}$$

Higher order delta functions

- An *nth-order delta function* is defined as the nth derivative of the unit step function

$$\boxed{\delta^n(t) = \frac{d^n}{dt}[u_0(t)]}$$

- The derivation of the sampling property of the delta function can be extended to show that

$$\boxed{f(t)\delta'(t-a) = f(a)\delta'(t-a) - f'(a)\delta(t-a)}$$

- The derivation of the sifting property of the delta function can be extended to show that

$$\boxed{\int_{-\infty}^{\infty} f(t)\delta^n(t-\alpha)dt = (-1)^n \frac{d^n}{dt^n}[f(t)]\bigg|_{t=\alpha}}$$

ELEMENTARY SIGNALS: UNIT IMPULSE FUNCTION

Example : Evaluate the following expressions:

a. $3t^4\delta(t-1)$ b. $\int_{-\infty}^{\infty} t\delta(t-2)dt$ c. $t^2\delta'(t-3)$

Solutions:

a) The sampling property states that

$$\boxed{f(t)\delta(t-a) = f(a)\delta(t-a)}$$

$$f(t) = 3t^4 \qquad a = 1$$

$$\boxed{f(t)\delta(t) = f(0)\delta(t)}$$

$$3t^4\delta(t-1) = \{3t^4\big|_{t=1}\}\delta(t-1) = 3\delta(t)$$

b) The sifting property states that

$$\boxed{\int_{-\infty}^{\infty} f(t)\delta(t-\alpha)dt = f(\alpha)}$$

$$\int_{-\infty}^{\infty} t\delta(t-2)dt = f(2) = t\big|_{t=2} = 2$$

c) From $f(t)\delta'(t-a) = f(a)\delta'(t-a) - f'(a)\delta(t-a)$

$$t^2\delta'(t-3) = t^2\big|_{t=3}\delta'(t-3) - \frac{d}{dt}t^2\big|_{t=3}\delta(t-3) = 9\delta'(t-3) - 6\delta(t-3)$$

SUMMARY

- **Signals and Classifications**
  - Continuous-time v.s. discrete-time
  - Analog v.s. digital
  - Odd v.s. even
  - Periodic v.s. aperiodic
  - Power v.s. energy
- **Systems and Classifications**
  - Continuous-time signal v.s. discrete-time signal
  - Analog signal v.s. digital signal
  - Even signal v.s. odd signal
  - Periodic signal v.s. Aperiodic signal
  - Power signal v.s. Energy signal
- **Useful Signal Operations**
  - Time shifting
  - reflection
  - Time scaling
- **Elementary Signals**
  - Unit step, unit impulse, ramp