6.

a) To eliminate s.s. position tracking error for step input, controller must have the form $\left(K_p + \frac{K_1}{s}\right)$ in it

Thus, we have a P.I. controller.

Through testing in MATLAB, we found

$$C_{p1} = 10^{-7} + \frac{10^{-2}}{s}$$

To track a 3,000 rad/s input, yet reject 10^5 rad/s noise, we set gain cross over frequency between those two values. A $W_g \approx 10^4$ should accomplish this. We chose $W_g = 1.67 \times 10^4$ to be a better median point between the input frequency and noise frequency.

For the compensator, we chose a lead-lag controller.

Lead

Set zeros a decade before W_g and poles a decade after

$$C_{lead} = \left(\frac{1 + \frac{s}{6 \times 10^3}}{1 + \frac{s}{6 \times 10^4}}\right)^3$$

With this, the loop needs 92 dB of Gain to have the cross over frequency at 1.67×10^4 rad/s.

$$92 = 20 \log(K)$$

$$K = 10^{92/20}$$

so $$C_{lead} = 10^{92/20} \left(\frac{1 + \frac{s}{6 \times 10^3}}{1 + \frac{s}{6 \times 10^4}}\right)^3$$

Lag

$$K_p = 100{,}000$$

This is set to ensure low s.s. tracking error for the step input.

$$K_{p_{old}} = \lim_{s \to 0} s \cdot C_{lead} \cdot L(s) = 10^{42/20} \cdot -5$$

$$K_{lag} = \frac{K_p}{K_{p_{old}}} = \frac{100{,}000}{K_{p_{old}}} = -0.502377$$

C_{lag} takes the form $K_{lag} \dfrac{(1+\tau s)}{(1+\alpha\tau s)}$

We keep the high frequency gain of the lag compensator 1 so as to not change the cross over frequency.

So, $\dfrac{|K_{lag}|}{\alpha} = 1 \Rightarrow \alpha = 0.502377$

Similarly, the zero of the lag compensator must be AT LEAST a decade below ω_g.

So, $\tau = \dfrac{1}{100}$

so, $C_{lag} = -5.02377 \times 10^{-1} \cdot \dfrac{\left(1 + \frac{s}{100}\right)}{\left(1 + 5.02377 \times 10^{-3} s\right)}$

Thus

$$C_{mech}(s) = C_{PI} \cdot C_{lead} \cdot C_{lag}$$

$$= \left(10^{-7} + 10^{-2}/s\right) \cdot 10^{42/20} \cdot \left(\frac{1 + \frac{s}{6\times10^{3}}}{1 + \frac{s}{6\times10^{4}}}\right)^{3} \cdot -5.02377\times10^{-1} \frac{\left(1 + \frac{s}{100}\right)}{\left(1 + 5.02377\times10^{-3} s\right.}$$

This compensator design focuses heavily on steady-state performance. We achieved a sinusoidal tracking error under 10% while nearly eliminating noise error. Unfortunately, the noise attenuation came at the cost of the phase margin and overshoot (23%).

As mentioned earlier, the PI compensator is used to eliminate step tracking error. Then, the Lead compensator is used to increase input frequency gain and phase margin at the cross over frequency. The lag compensator lowers the gain at the noise frequency.