ECE 141 - Principles of Feedback Control
Winter 2019
Final Design Project:
Controller Design for a Fast Tool Servo

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4.

a) $P_{elec}(s) = \frac{V_s(s)}{V_c(s)} = \frac{V_a}{V_c} \cdot \frac{I_c}{V_a} \cdot \frac{V_s}{I_c}$

$$\frac{V_a}{V_c} = 5 \qquad \frac{V_s}{I_c} = 5 \cdot R_s = 1$$

$$I_c = \frac{V_a - V_{emf}}{L_c s + R_c}$$

$V_{emf} = K_f X s$

$$F = K_f I_c = bXs + k_1 x + mXs^2$$

$$X = \frac{K_f I_c}{ms^2 + bs + k_1} \qquad \text{so, } V_{emf} = \frac{K_f^2 I_c s}{ms^2 + bs + k_1}$$

Thus, $\frac{I_c}{V_a} = \frac{1}{(L_c s + R_c)\left(1 + \frac{K_f^2 s}{(ms^2 + bs + k_1)(L_c s + R_c)}\right)}$

$$\boxed{\frac{V_s}{V_c} = \frac{5}{(L_c s + R_c)\left(1 + \frac{K_f^2 s}{(ms^2 + bs + k_1)(L_c s + R_c)}\right)}}$$

b) $P_{elec} = \frac{V_s}{V_c} = \frac{V_a}{V_c} \cdot \frac{I_c}{V_a} \cdot \frac{V_s}{I_c}$

$$= 5 \cdot \frac{1}{L_c s + R_c} \cdot 1$$

$$= \boxed{\frac{5}{L_c s + R_c}}$$

c) At high frequencies, both transfer functions will be the same because $\lim_{s \to \infty} \frac{K_f^2 s}{(ms^2 + bs + k_1)(L_c s + R_c)} = 0$

Thus, any affect $V_{back\,emf}$ has disappears

d) To obtain DC gain of $G_u = -0.5$ A/V, set $s=0$ for C.L. TF

so, $\frac{-R_2}{R_1} \frac{1}{1+L} \cdot 5R_s \Rightarrow \frac{R_2}{10{,}000} \cdot 0.2 = \frac{1}{10}$

$$\boxed{R_2 = 5{,}000\,\Omega}$$

To obtain a gain cross over frequency of $w_c = 6\times10^5$ rad/sec, we set the zero of C_{elec} a decade before desired w_c and the pole of C_{elec} a decade after desired w_c.

Therefore,

$$\boxed{\begin{aligned} R_3 &= 121{,}000\,\Omega \\ C_1 &= 1.3\times10^{-9}\ F \\ C_2 &= 1.3\times10^{-12}\ F \end{aligned}}$$

e)

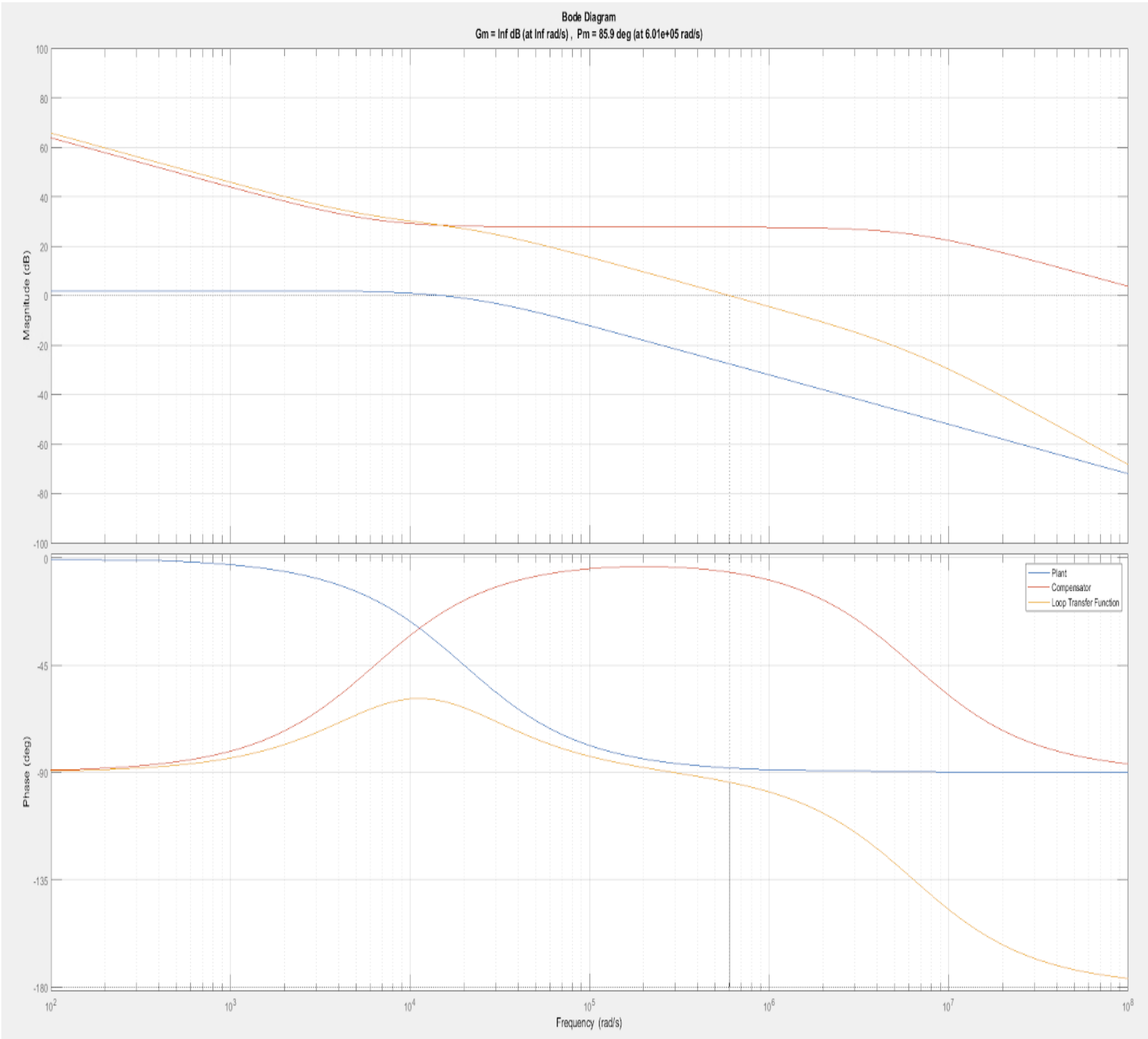

Figure 1: Bode plots of Plant, Compensator, and Loop Transfer Function

5.

a) $\cancel{A} \; K_f I_c - bXs - k_1 X = mXs^2$

$$F = K_f I_c = mXs^2 + bXs + k_1 X$$

$$\frac{X}{F} = \frac{1}{ms^2 + bs + k_1} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Resonant Freq seems to lead to a 20dB jump.

so $20 = 20 \log(M)$

$M = 10$

to get ζ, $M = \dfrac{1}{2\zeta\sqrt{1-\zeta^2}}$

so, $\zeta = 0.05$

From the given Bode plot, ω_n is at $\omega_n = 10^3$

Then, we can solve for m_1, b_1, and k_1.

$$\boxed{\begin{aligned} m_1 &= \tfrac{1}{100} \\ b_1 &= 1 \\ k_1 &= 10^4 \end{aligned}}$$

b) From part a), we can see that

$$G_p(s) = \frac{1}{10^4} \cdot \frac{10^6}{s^2 + 100s + 10^6} = \boxed{\frac{100}{s^2 + 100s + 10^6}}$$

with the time delay accounted for,

$$\boxed{G_p(s) = \frac{100 \cdot e^{-1.2 \times 10^{-3} s}}{s^2 + 100s + 10^6}}$$

b)

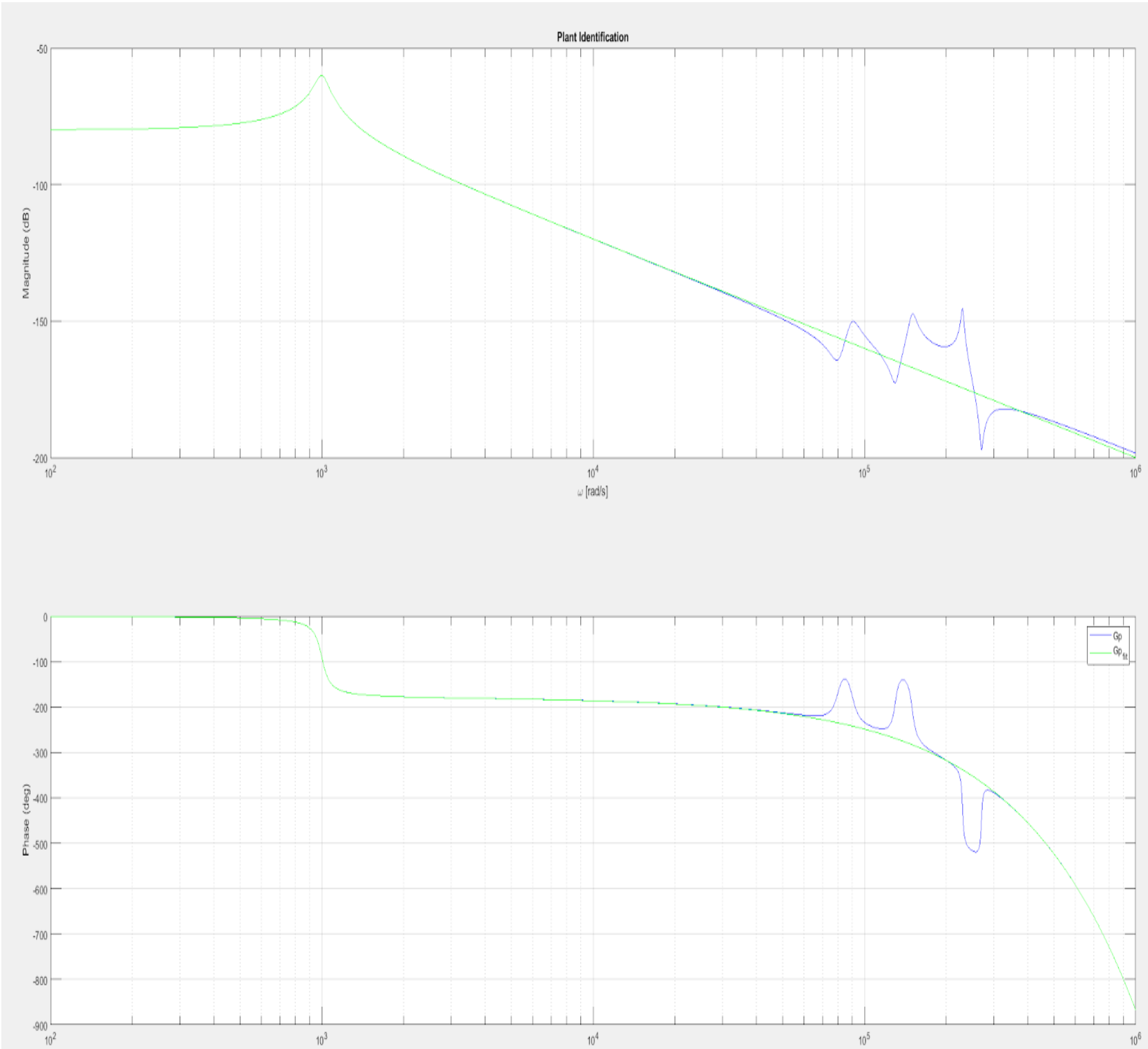

Figure 2: Comparison of actual G_P with the estimation we calculated

6.

a) To eliminate ~~[illegible]~~ s.s. position tracking error for step input, controller must have the form $\left(K_p + \frac{K_1}{s}\right)$ in it

Thus, we have a P.I. controller.

Through testing in MATLAB, we found

$$C_{P1} = 10^{-7} + \frac{10^{-2}}{s}$$

To track a 3,000 rad/s input, yet reject 10^5 rad/s noise, we set gain cross over frequency between those two values. A $W_g \approx 10^4$ should accomplish this. ~~[illegible]~~

~~[illegible]~~ We chose $W_g = 1.67 \times 10^4$ to be a better median point between the input frequency and noise frequency.

For the compensator, we chose a lead-lag controller.

Lead

Set zeros a decade before W_g and poles a decade after

$$C_{lead} = \left(\frac{1 + \frac{s}{6 \times 10^3}}{1 + \frac{s}{6 \times 10^4}}\right)^3$$

With this, the loop needs 92 dB of Gain to have the cross over frequency at 1.67×10^4 rad/s.

$$92 = 20 \log (K)$$

$$K = 10^{92/20}$$

so $$C_{lead} = 10^{92/20} \left(\frac{1 + \frac{s}{6 \times 10^3}}{1 + \frac{s}{6 \times 10^4}}\right)^3$$

Lag

$K_p = 100{,}000$

This is set to ensure low s.s. tracking error for the step input.

$$K_{p_{old}} = \lim_{s \to 0} s \cdot C_{lead} \cdot L(s) = 10^{92/20} \cdot -5$$

$$K_{lag} = \frac{K_p}{K_{p_{old}}} = \frac{100{,}000}{K_{p_{old}}} = -0.502377$$

C_{lag} takes the form $K_{lag} \dfrac{(1+\tau s)}{(1+\alpha \tau s)}$

We keep the high frequency gain of the lag compensator 1 so as to not change the cross over frequency.

so, $\dfrac{|K_{lag}|}{\alpha} = 1 \Rightarrow \alpha = 0.502377$

Similarly, the zero of the lag compensator must be AT LEAST a decade below w_g.

so, $\tau = \frac{1}{100}$

so, $$C_{lag} = -5.02377 \times 10^{-1} \cdot \frac{\left(1 + \frac{s}{100}\right)}{\left(1 + 5.02377 \times 10^{-3} s\right)}$$

Thus

$$C_{mech}(s) = C_{PI} \cdot C_{lead} \cdot C_{lag}$$

$$= \left(10^{-7} + 10^{-2}/s\right) \cdot 10^{92/20} \cdot \left(\frac{1 + \frac{s}{6 \times 10^{3}}}{1 + \frac{s}{6 \times 10^{4}}}\right)^{3} \cdot -5.02377 \times 10^{-1} \frac{\left(1 + \frac{s}{100}\right)}{\left(1 + 5.02377 \times 10^{-3} s\right)}$$

This compensator design focuses heavily on steady-state performance. We achieved a sinusoidal tracking error under 10% while nearly eliminating noise error. Unfortunately, the noise attenuation came at the cost of the phase margin and overshoot (23%).

As mentioned earlier, the PI compensator is used to eliminate step tracking error. Then, the Lead compensator is used to increase input frequency gain and phase margin at the cross over frequency. The lag compensator lowers the gain at the noise frequency.

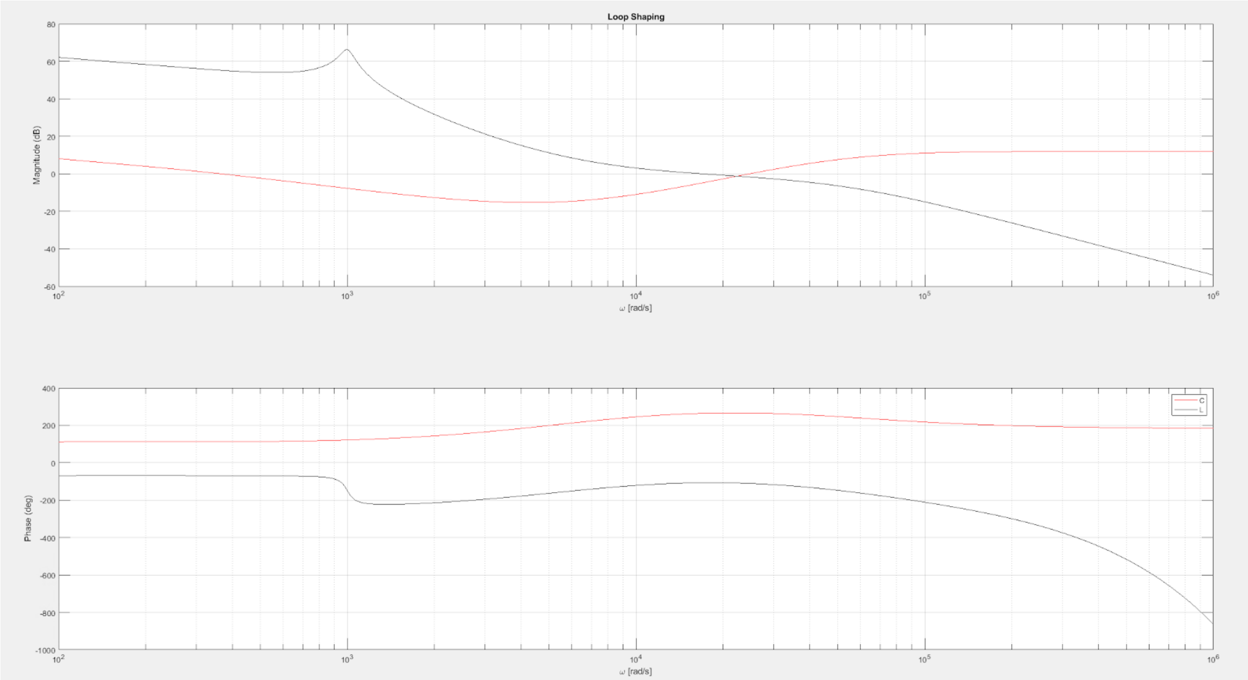

Figure 3: Bode plots of the loop shaping

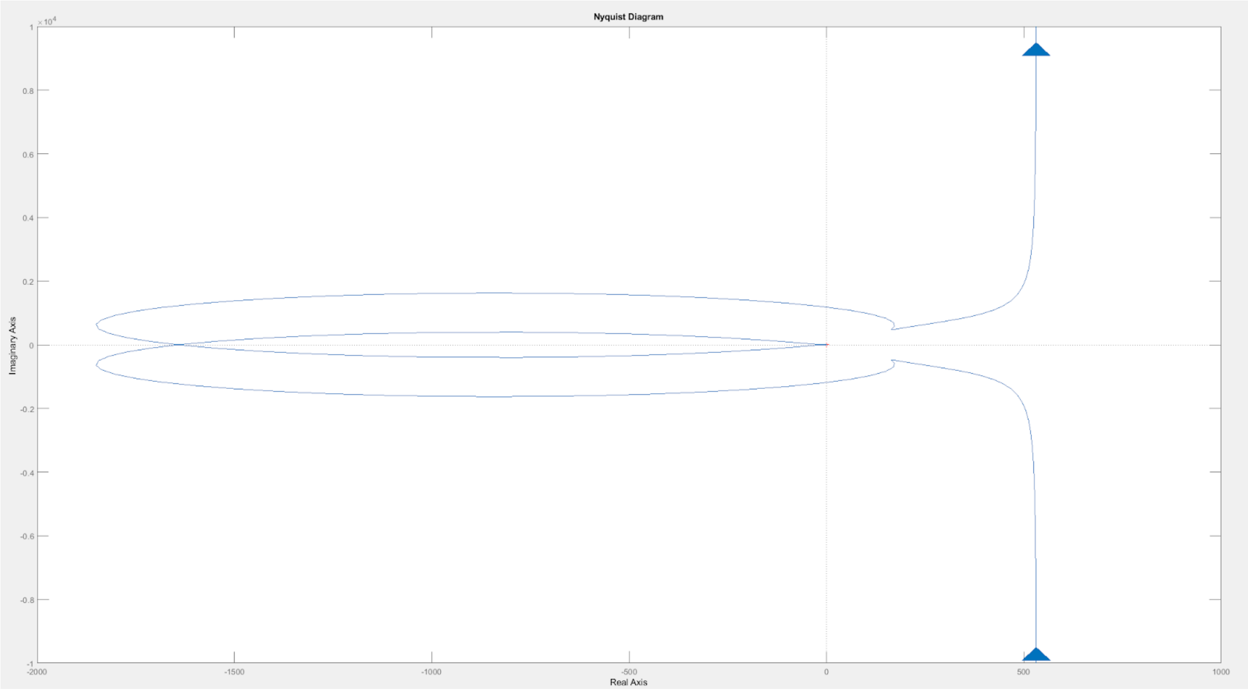

Figure 4: Nyquist Diagram of Loop

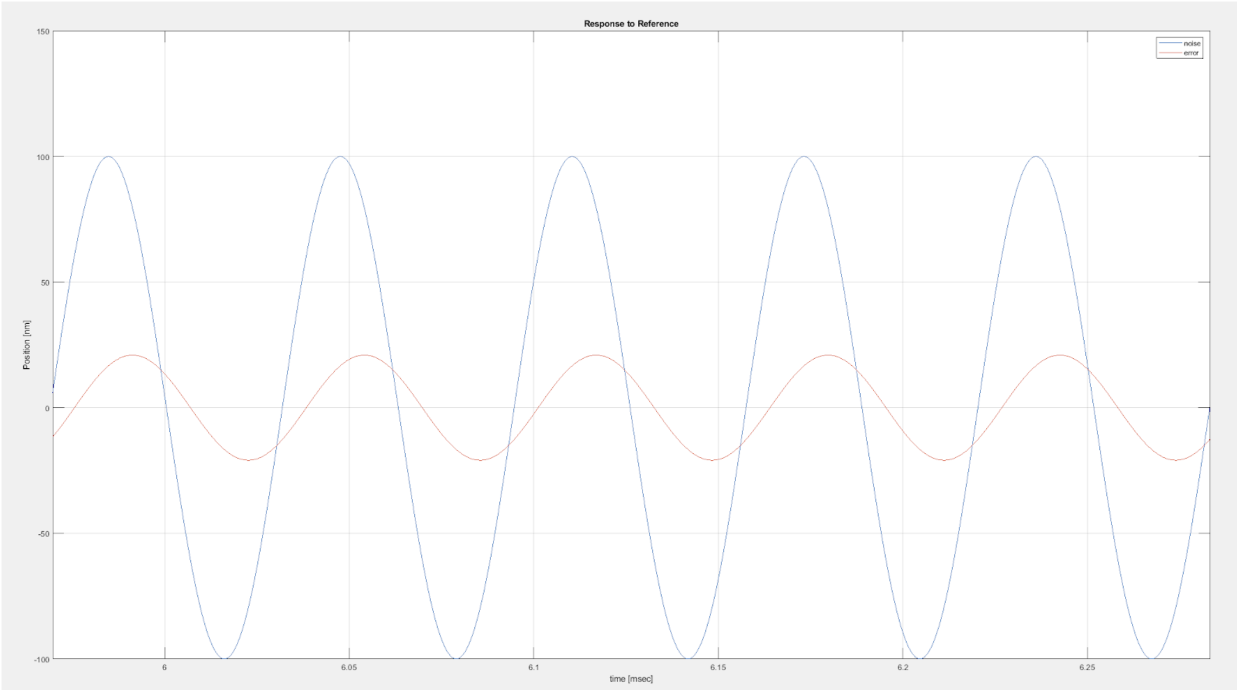

Figure 5: Response to Reference

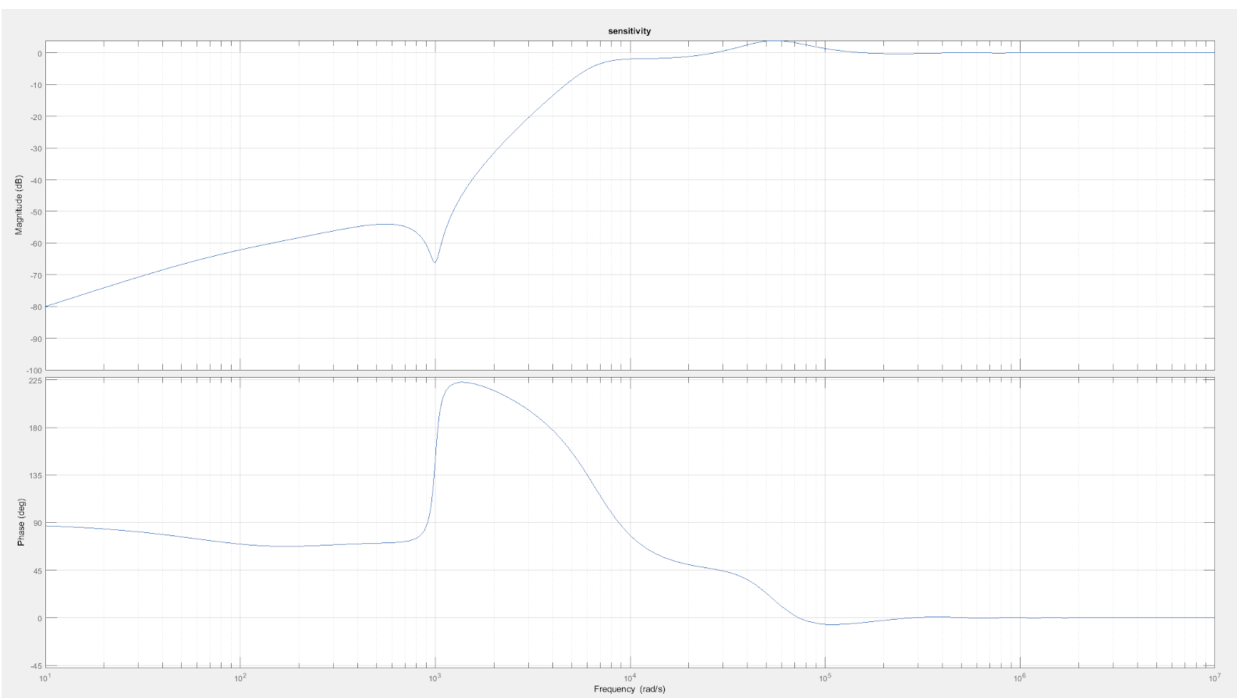

Figure 6: Sensitivity

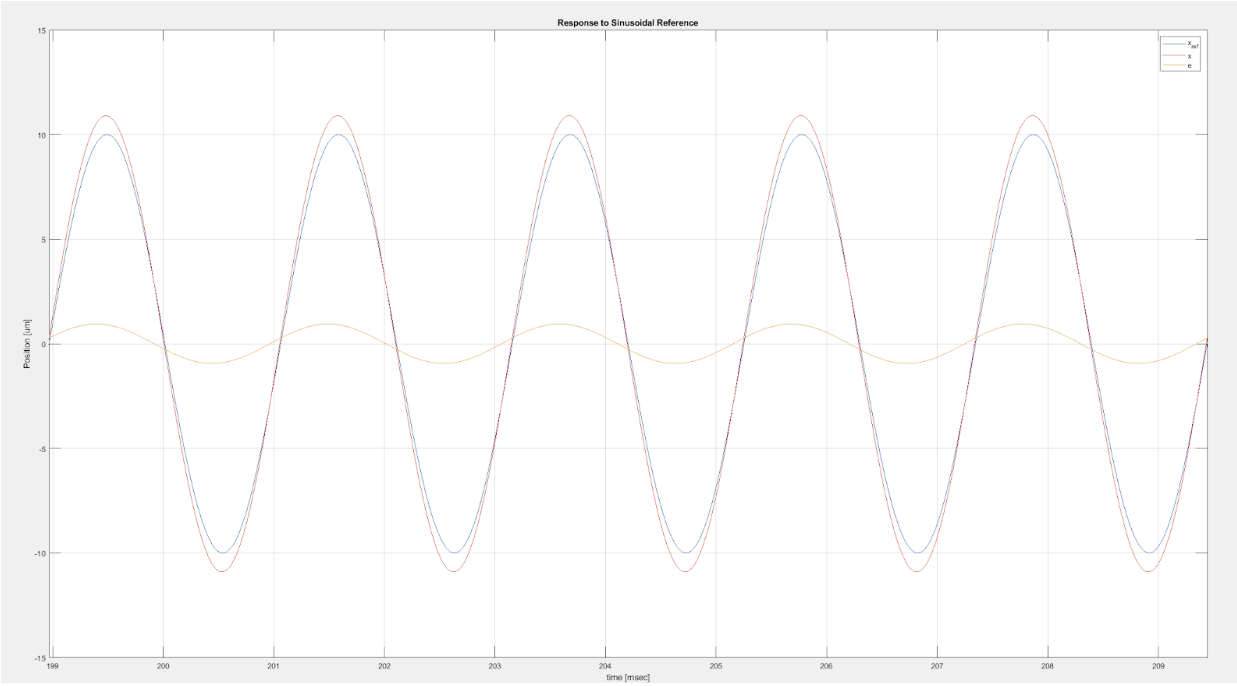

Figure 7: Sinusoidal Response

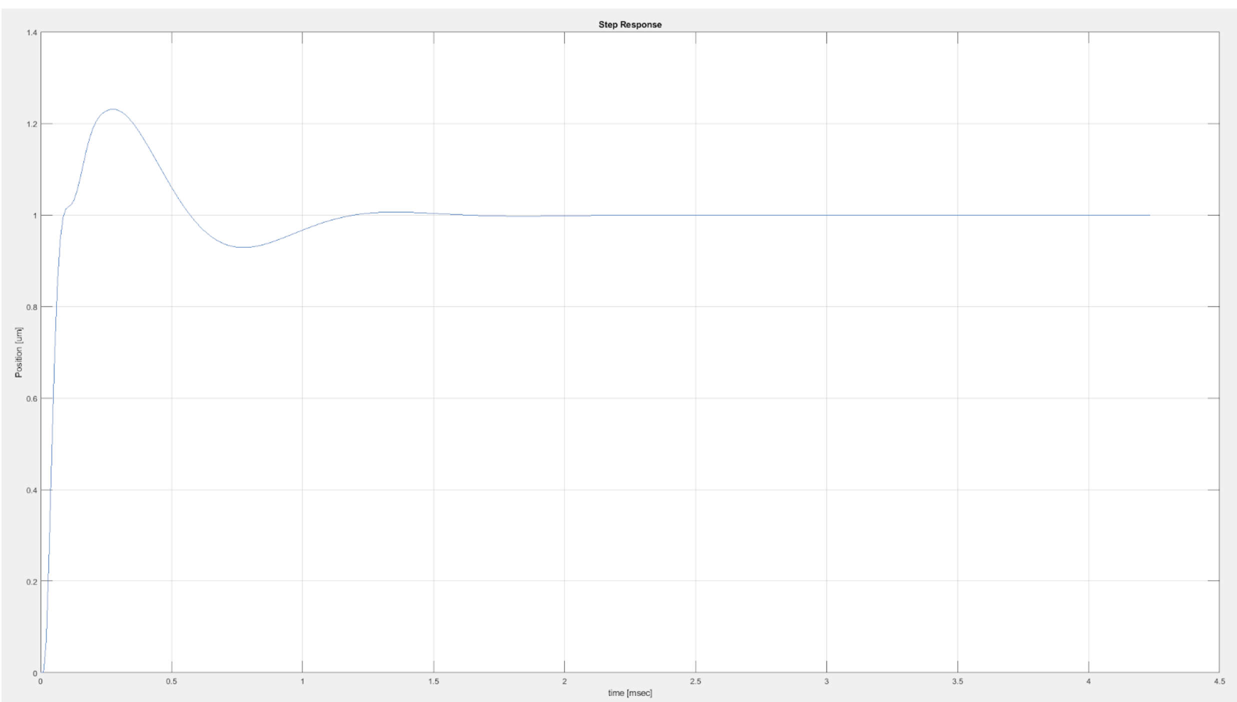

Figure 3: Step Response