ME 562 Presentation 3
Quadrotor Attitude Control

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Attitude Dynamics

$$\ddot{\phi} = \dot{\theta}\dot{\psi}(\frac{I_y - I_z}{I_x}) + \frac{L}{I_x}U_2$$

$$\ddot{\theta} = \dot{\phi}\dot{\psi}(\frac{I_z - I_x}{I_y}) + \frac{L}{I_y}U_3$$

$$\ddot{\psi} = \dot{\phi}\dot{\theta}(\frac{I_x - I_y}{I_z}) + \frac{1}{I_z}U_4$$

L: Arm length of the drone
I_x, I_y, I_z: Moments of inertia for roll, pitch, yaw coordinate
ϕ: Roll angle, θ: Pitch angle, ψ: Yaw angle (w.r.t. inertial frame)
U_2, U_3, U_4: Roll, pitch, yaw motor command

Attitude Dynamics

Consider the continuous-time dynamics,

$$\dot{x}(t) = Ax(t) + B_m\left(u(t) + \sigma_m(t)\right) + B_{um}\sigma_{um}(t), \quad x(0) = x_0$$

- State: $x(t) = [\phi, \theta, \psi, \dot{\phi}, \dot{\theta}, \dot{\psi}]^\top \in \mathbb{R}^6$
- Input: $u(t) \in \mathbb{R}^3$ (Roll Force, Pitch Force, Yaw Torque)

$$B_m = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{L}{I_x} & 0 & 0 \\ 0 & \frac{L}{I_y} & 0 \\ 0 & 0 & \frac{1}{I_z} \end{bmatrix} \qquad B_{um} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\mathcal{L}_1$ Adaptive Control Architecture

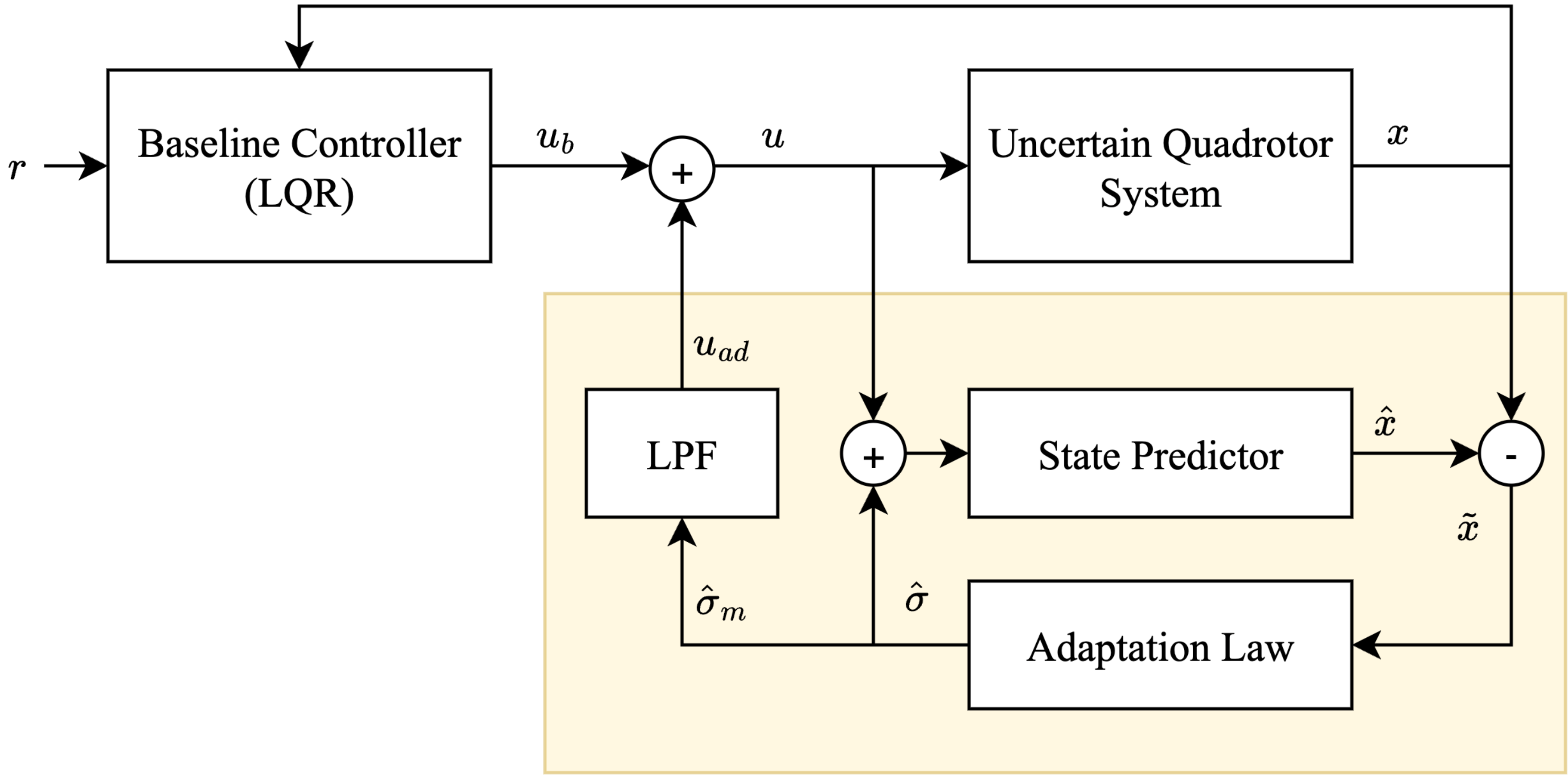


Figure: $\mathcal{L}_1$ Adaptive Control Architecture

$\mathcal{L}_1$ Adaptive Control

State Predictor We consider the state predictor given by,

$$\dot{\hat{x}}(t) = Ax(t) + B_m(u(t) + \hat{\sigma}_m(t)) + B_{um}\hat{\sigma}_{um}(t) + A_e\tilde{x}(t), \quad \hat{x}(0) = x_0$$

- $\hat{x}(t) \in \mathbb{R}^n$ is the predictor state.
- $\hat{\sigma}_m(t)$ and $\hat{\sigma}_{um}(t)$ are the adaptive estimates.
- $\tilde{x}(t) \triangleq \hat{x}(t) - x(t)$ is the state prediction error.
- $A_e \in \mathbb{R}^{n \times n}$ is a Hurwitz matrix determining the error dynamics.

The error dynamics governing $\tilde{x}(t)$ are thus,

$$\dot{\tilde{x}}(t) = A_e\tilde{x}(t) + \bar{B}(\hat{\sigma}(t) - \sigma(t)), \quad \tilde{x}(0) = 0$$

where $\bar{B} = [B_m, B_{um}]$ and $\sigma(t) = [\sigma_m(t)^\top, \sigma_{um}(t)^\top]^\top$.

$\mathcal{L}_1$ Adaptive Control

Piecewise Constant Adaptive Law

The uncertainty estimate $\hat{\sigma}(t)$ is updated at discrete sampling intervals T_s.
For $t \in [iT_s, (i+1)T_s), i \in \mathbb{N} \cup \{0\}$, the estimate is held constant,

$$\hat{\sigma}(t) = \hat{\sigma}(iT_s)$$

The update law is,

$$\hat{\sigma}(iT_s) = -\Phi^{-1}(T_s)\mu(iT_s)$$

where,

$$\Phi(T_s) = A_e^{-1}(e^{A_e T_s} - \mathbb{I}_6)\bar{B}$$
$$\mu(iT_s) = e^{A_e T_s}\tilde{x}(iT_s)$$

$\mathcal{L}_1$ Adaptive Control

Control Law
The control input $u(t)$ comprises the baseline LQR controller and the $\mathcal{L}_1$ adaptive augmentation.

$$u(t) = u_{base}(t) + u_{\mathcal{L}_1}(t) \tag{1}$$

Baseline Controller

$$u_{base}(t) = -K_{lqr}(x(t) - r(t)) \tag{2}$$

$\mathcal{L}_1$ Adaptive Augmentation The matched uncertainty estimate $\hat{\sigma}_m(t)$ is low-pass filtered to ensure robustness:

$$u_{\mathcal{L}_1}(s) = -C(s)\mathcal{L}[\hat{\sigma}_m(t)], \quad C(s) = \frac{\omega_c}{s + \omega_c}\mathbb{I}_m \tag{3}$$

where ω_c is the filter cutoff frequency satisfying the small-gain stability condition.

Performance Metric

To evaluate the tracking performance of the proposed architecture, we utilize the **Root Mean Square Error (RMSE)**. For a given trajectory of N samples, the RMSE for the roll angle ϕ is defined as:

$$\text{RMSE}_{\phi} = \sqrt{\frac{1}{N}\sum_{k=1}^{N}\left(\phi(t_k) - \phi_{ref}(t_k)\right)^2}$$

where:

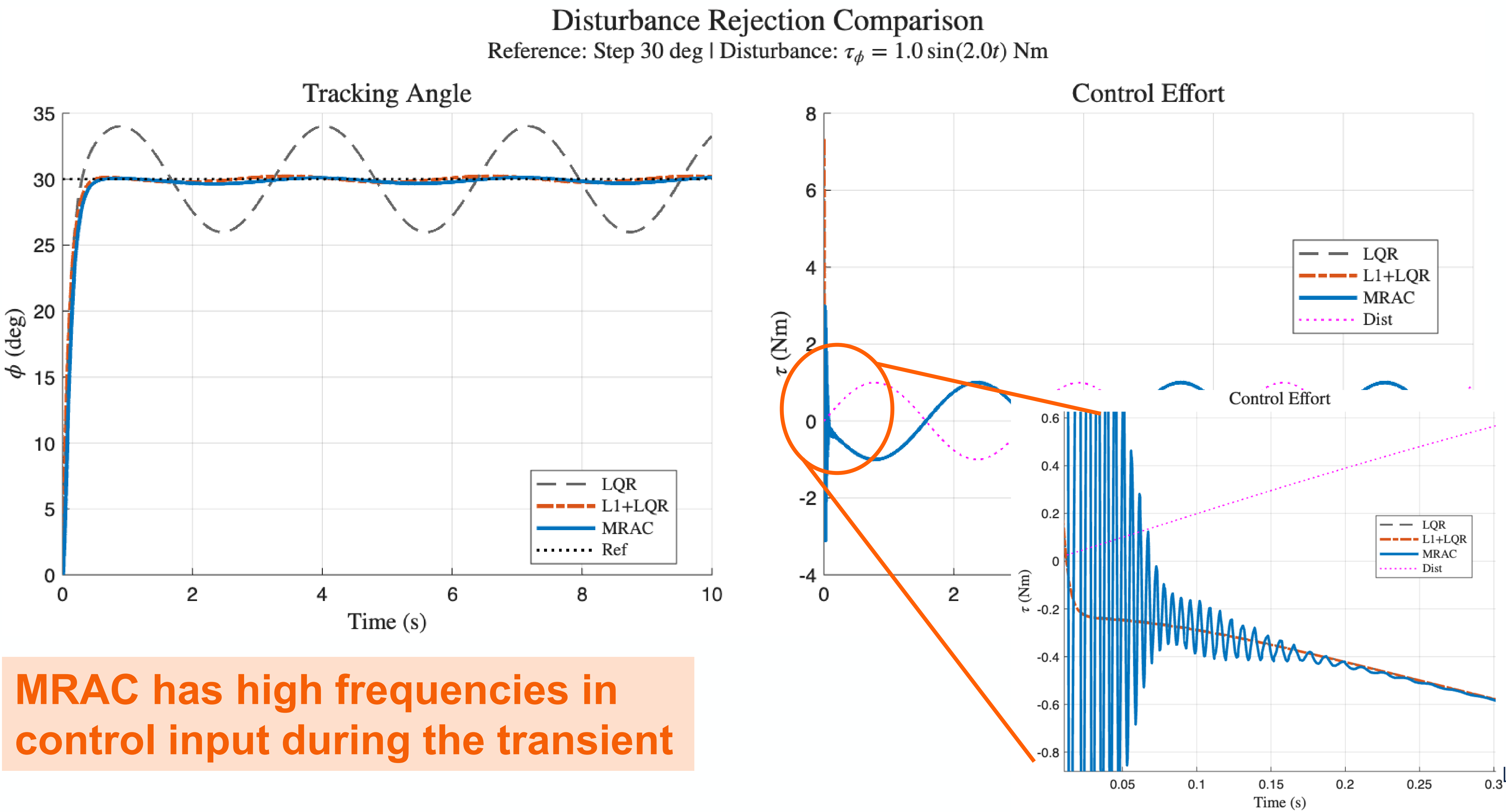
- $\phi(t_k)$ is the measured roll angle at time step k.
- $\phi_{ref}(t_k)$ is the reference command (e.g., step or sine) at time step k.

This metric quantifies the aggregate tracking deviation in the presence of disturbances.

Simulations

Tracking: MRAC vs $\mathcal{L}_1$

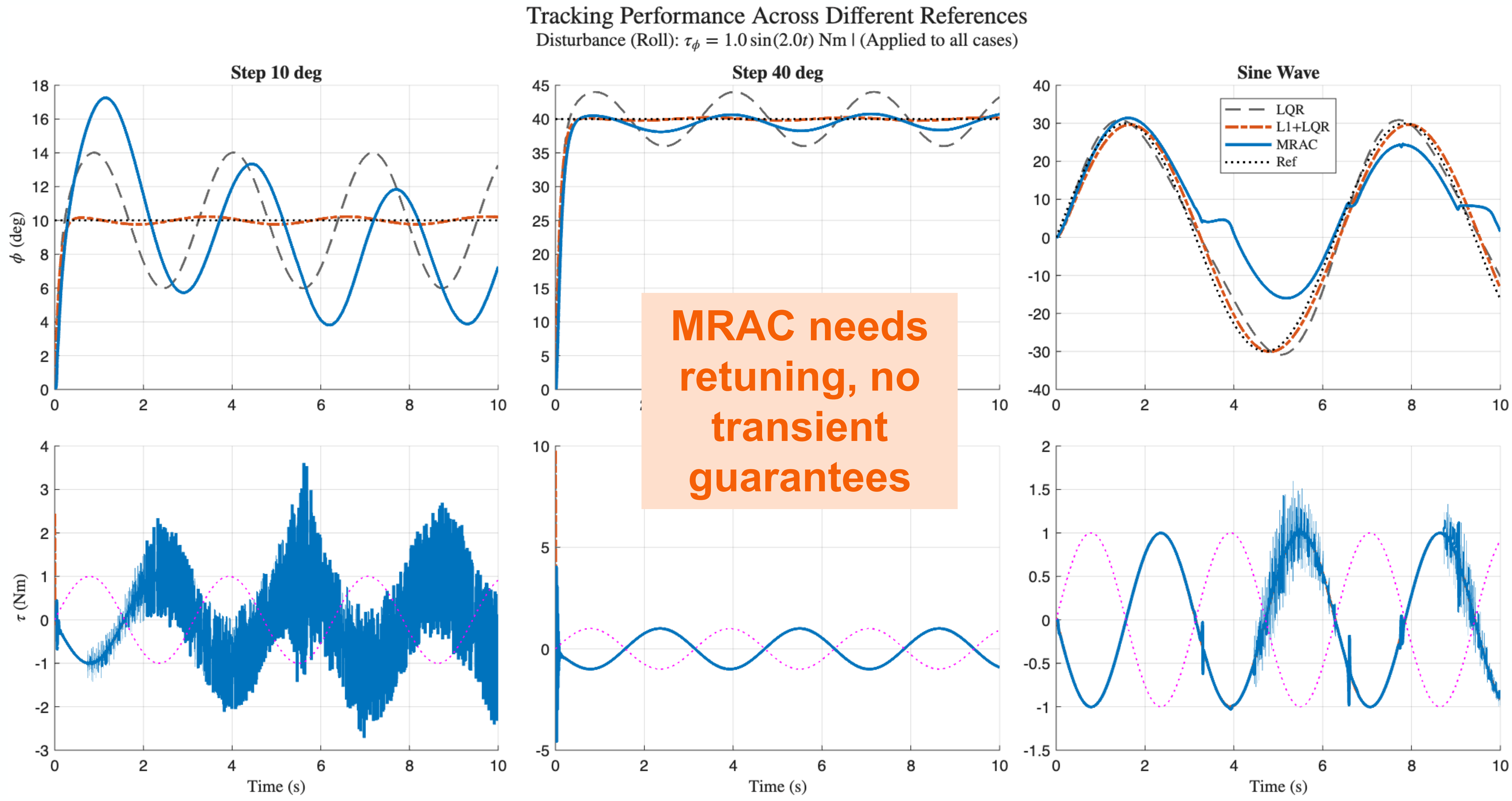
$$\Gamma_x = \Gamma_v = \Gamma_W = 5$$
$$\theta_x = \theta_r = \theta_W = 100$$
$$A_e = -10\, I_6,\ w_c = 40,\ T_s = 2e-3$$



MRAC has high frequencies in control input during the transient

Tracking Different References

$\Gamma_x = \Gamma_v = \Gamma_W = 5$
$\theta_x = \theta_r = \theta_W = 100$
$\mathrm{A_e} = -10\,\mathrm{I_6}, \mathrm{w_c} = 40, \mathrm{T_s} = 2e-3$

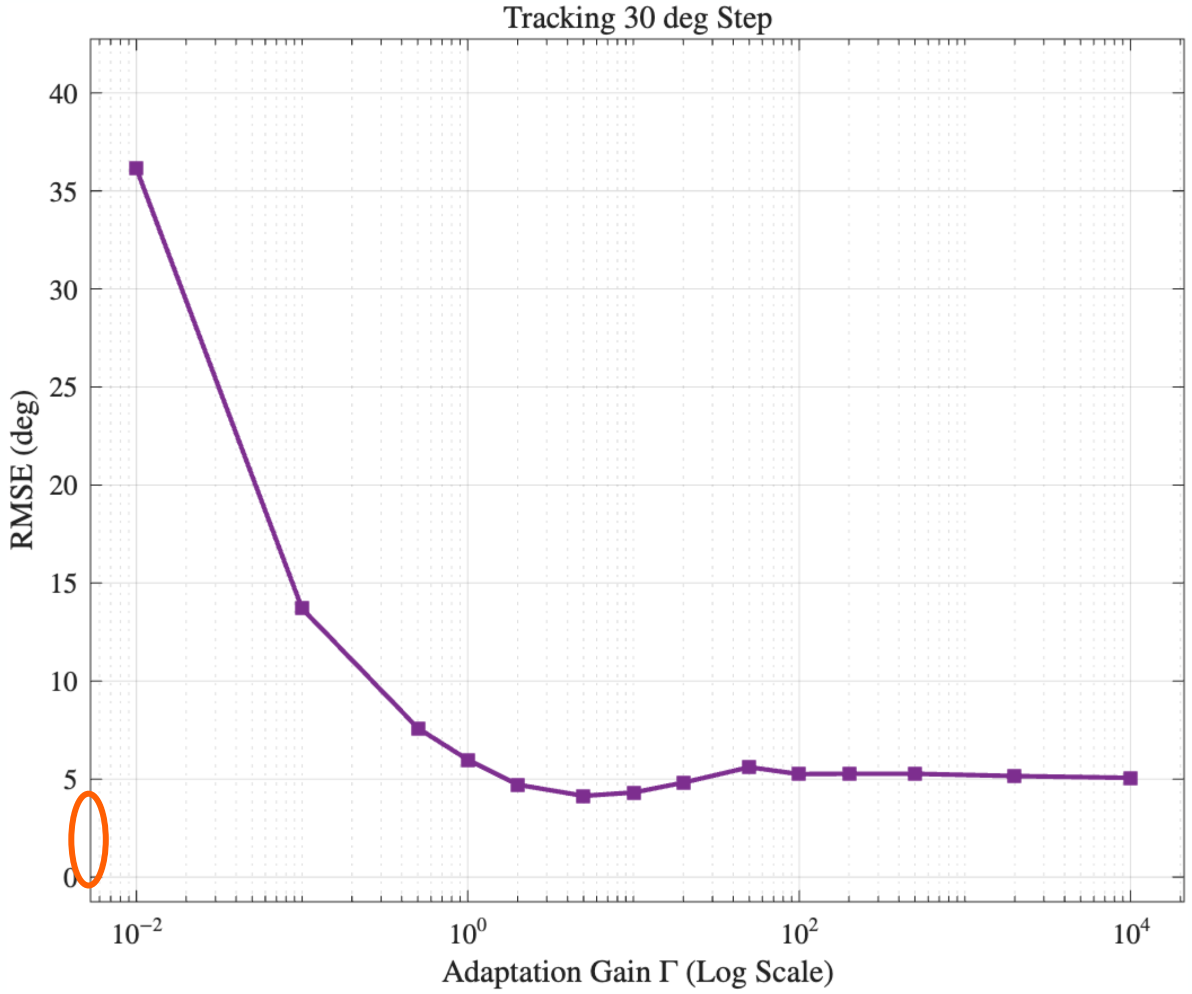


MRAC needs retuning, no transient guarantees

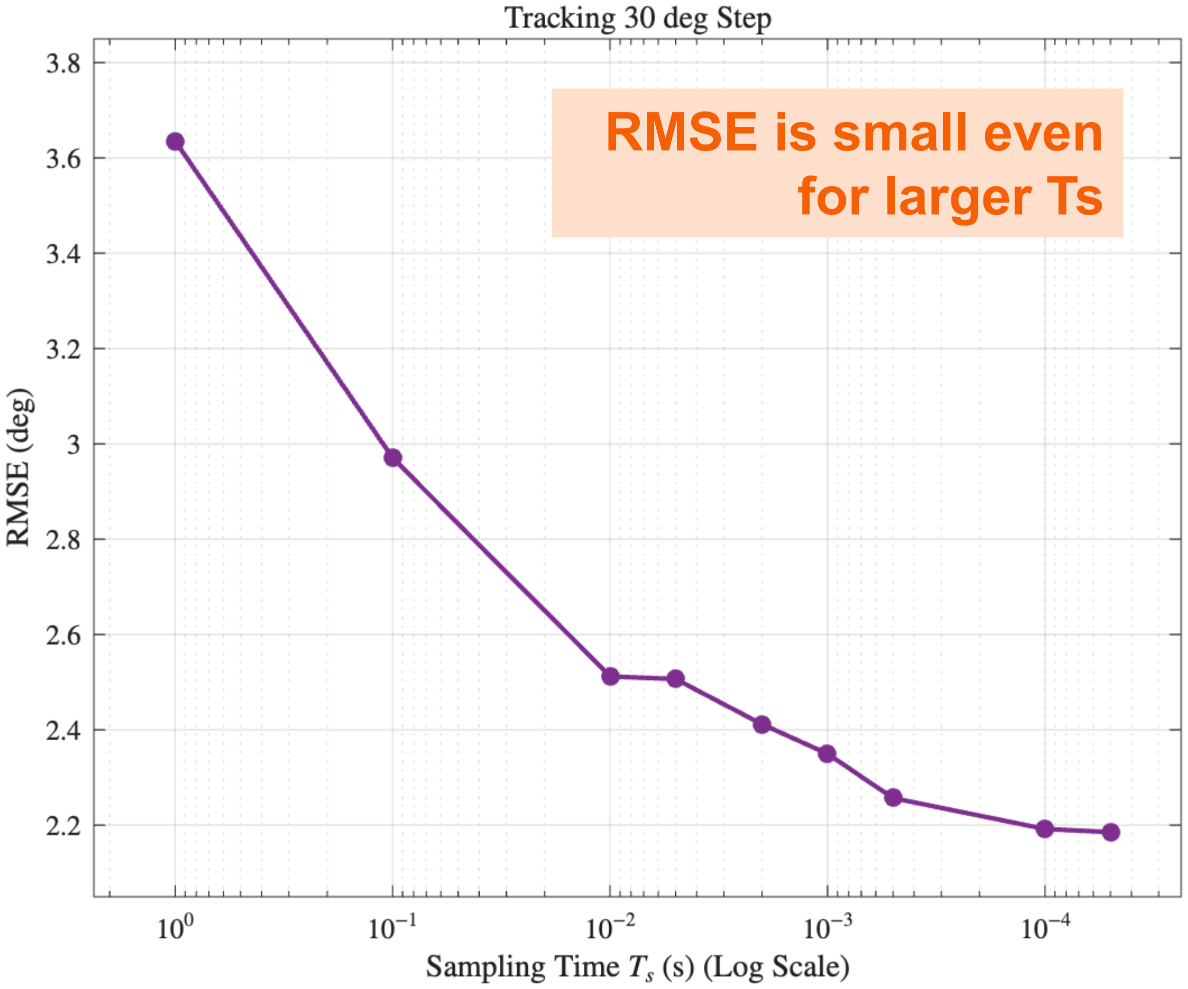
Effect of Adaptation Gain on RMSE

$\Gamma_x = \Gamma_v = \Gamma_W = \Gamma$
$\theta_x = \theta_r = \theta_W = 1000$
$A_e = -10\, I_6,\ w_c = 40$

MRAC



$\mathcal{L}_1$

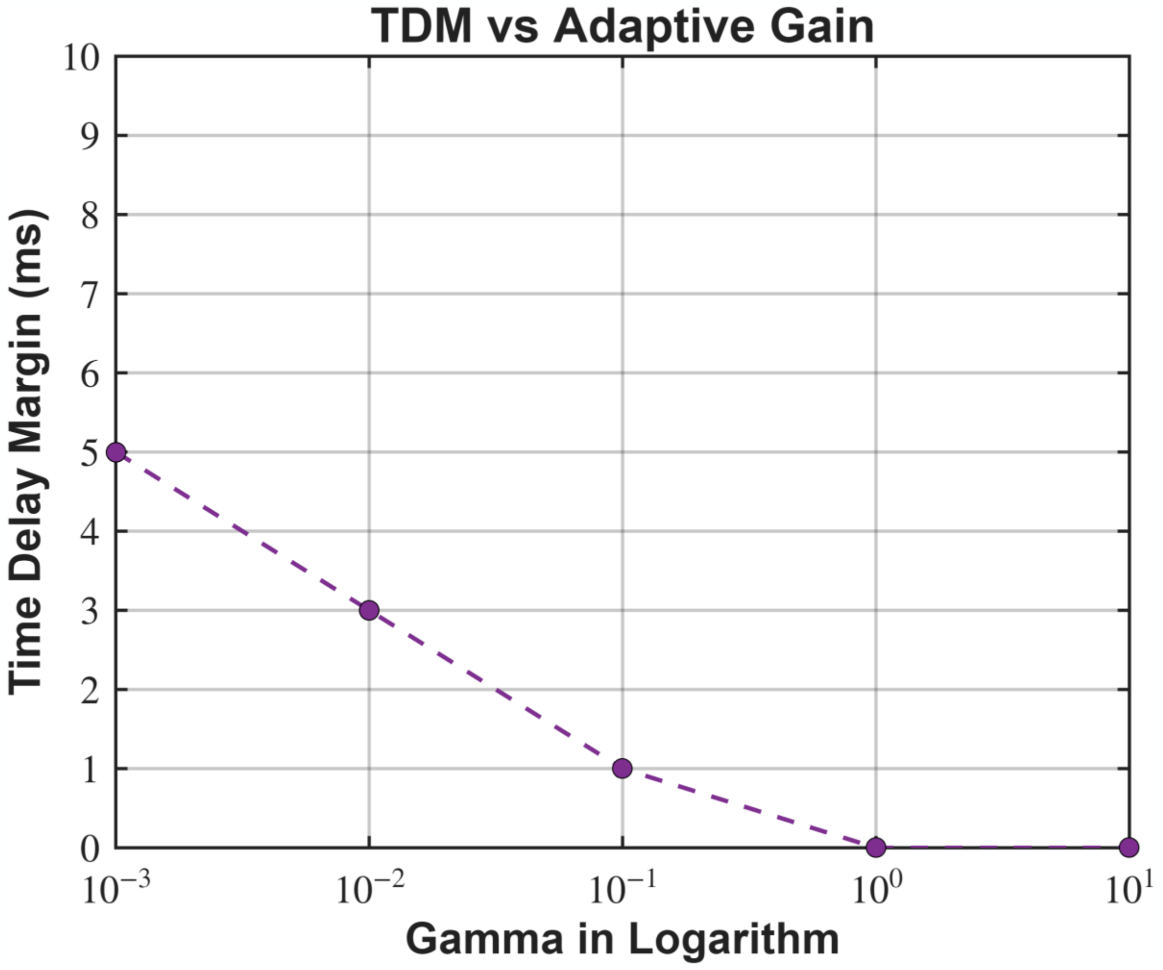


Higher gain

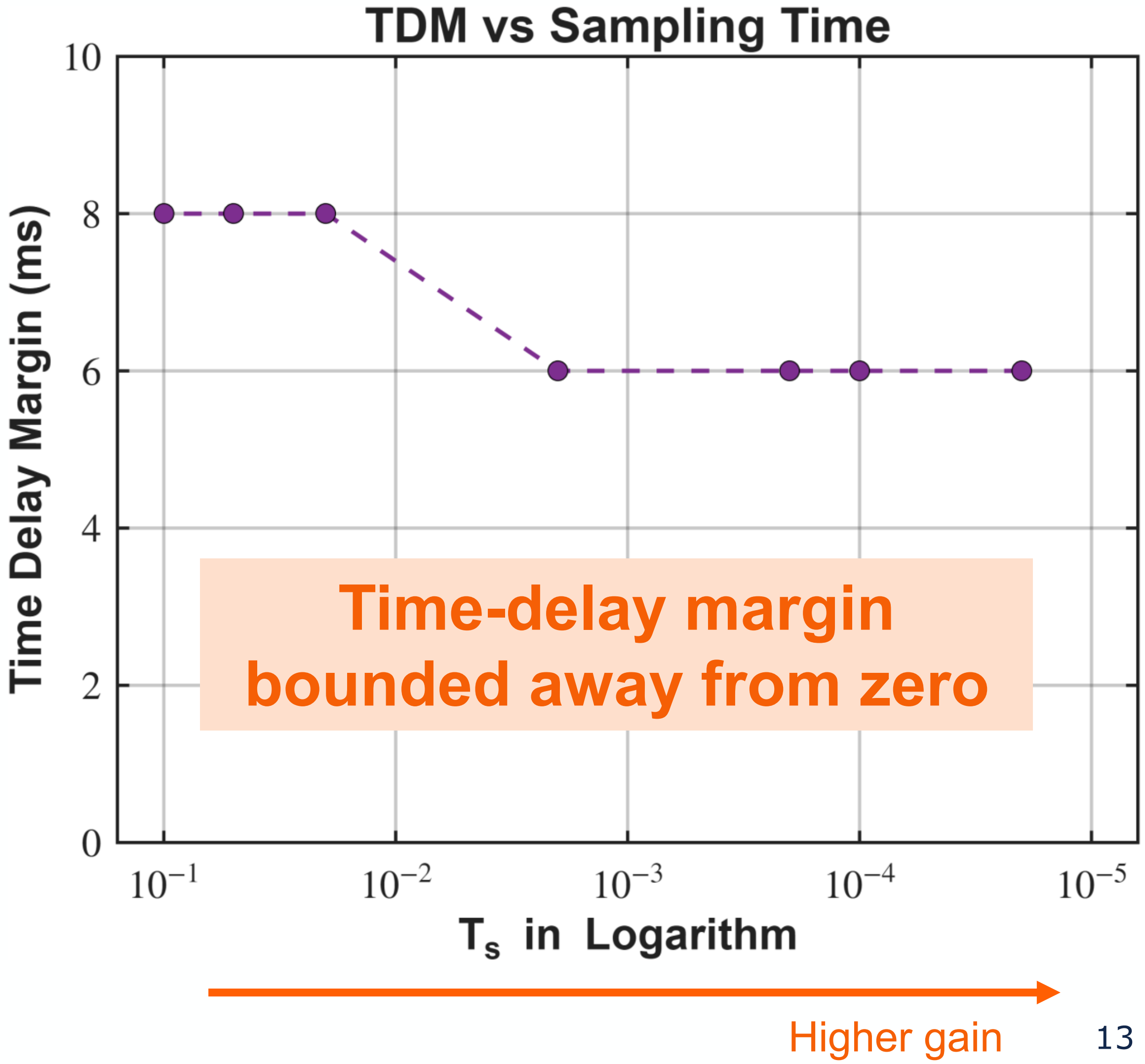
Effect of Adaptation Gain on Time-delay Margin

$\Gamma_x = \Gamma_v = \Gamma_W = 5$
$\theta_x = \theta_r = \theta_W = 100$
$A_e = -10\, I_6, w_c = 40$
$disturbance = \sin(2t)\ Nm$

MRAC

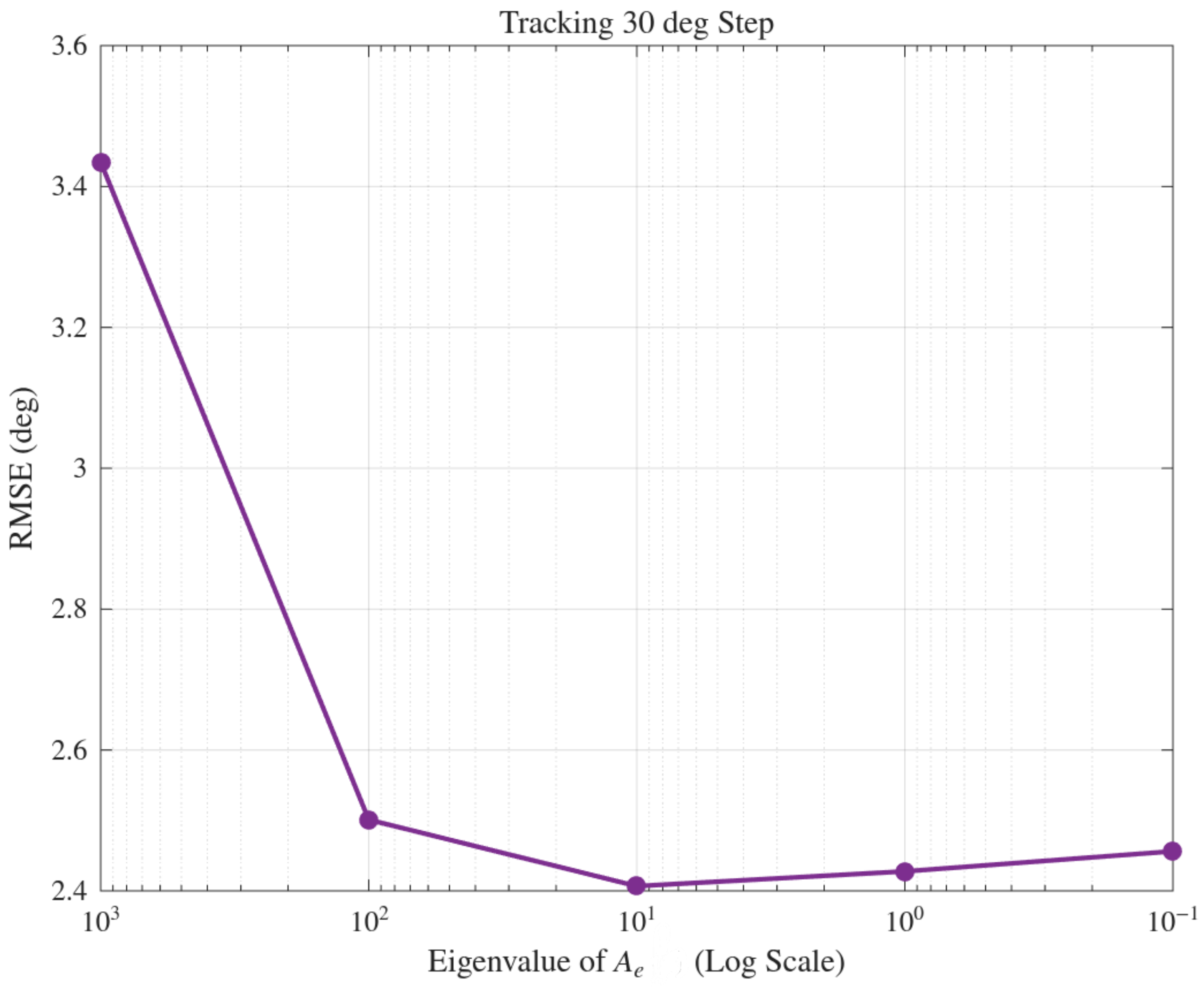


$\mathcal{L}_1$



$\mathrm{Ts} = 0.002, \omega_c = 40$

Effect of Choice of Ae

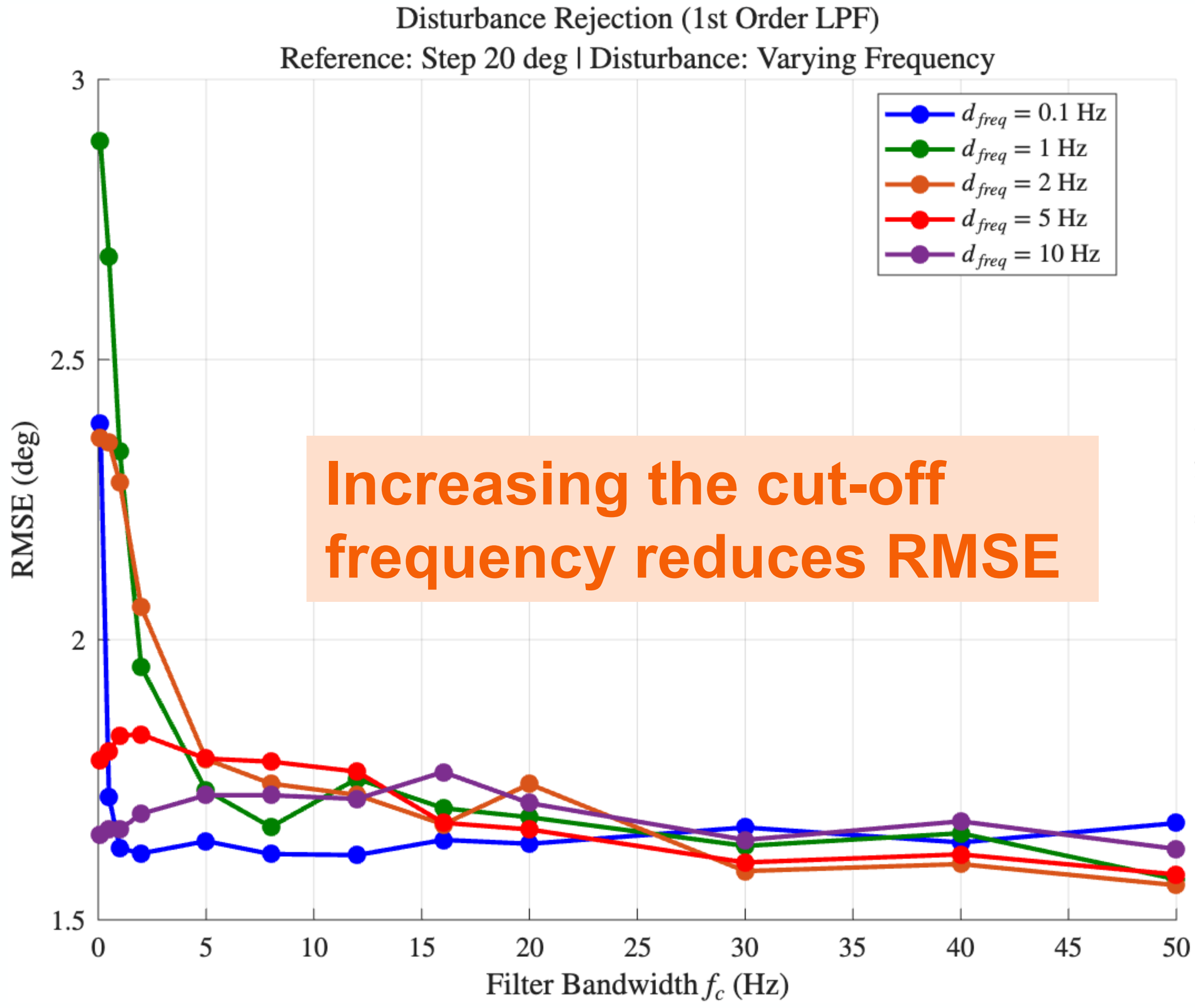


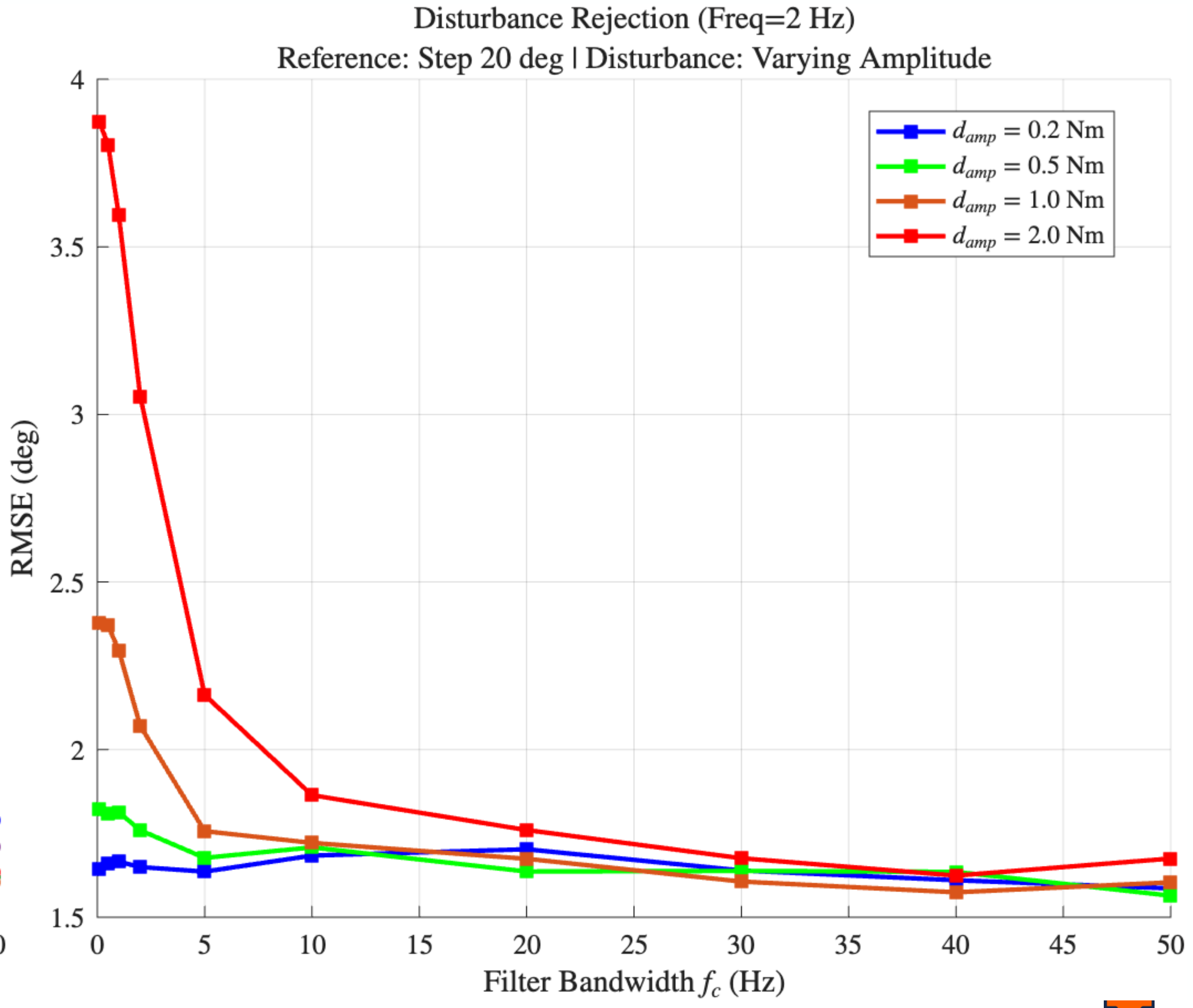
- **Ae is tuned to minimize state prediction error.**
- **If eigenvalue magnitude is too small, slow error convergence.**
- **If the eigenvalue magnitude is too large, numerical issue.**

$Ts = 0.002, A_e = -10\, I_6$

Effect of Choice of Filter Bandwidth

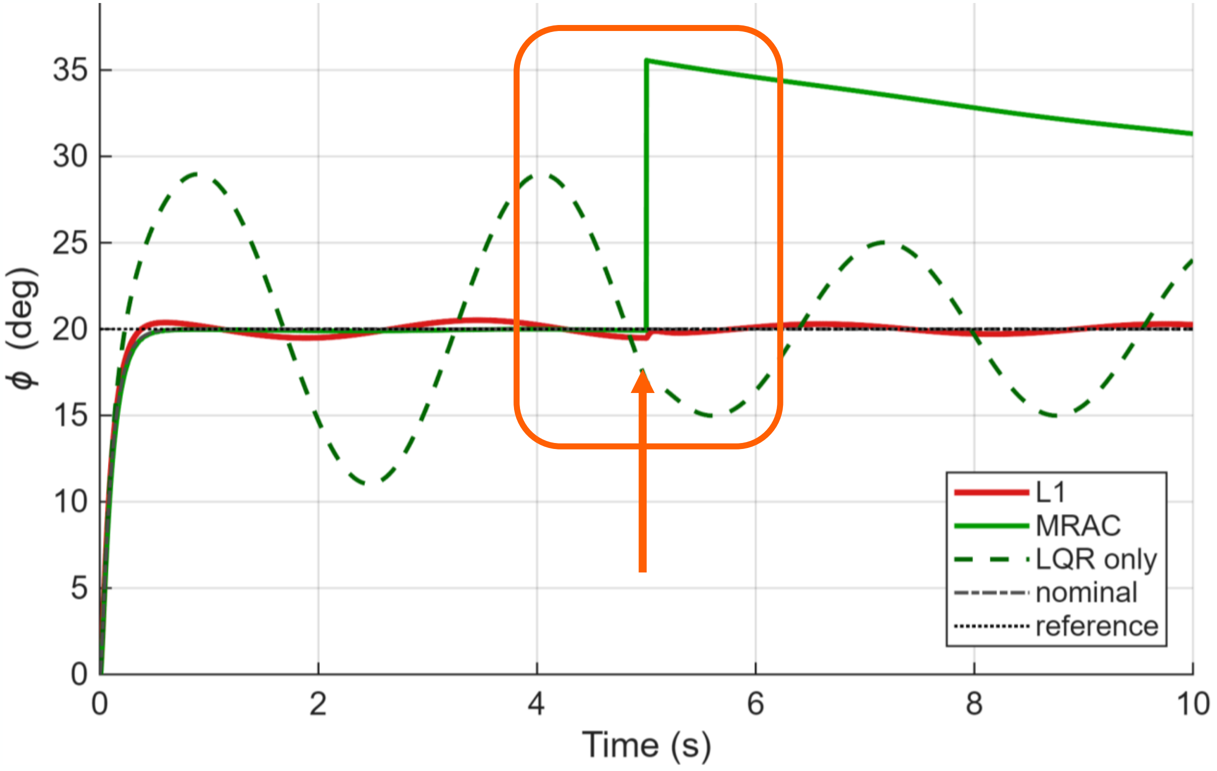
Ablations with properties of the sinusoidal disturbance

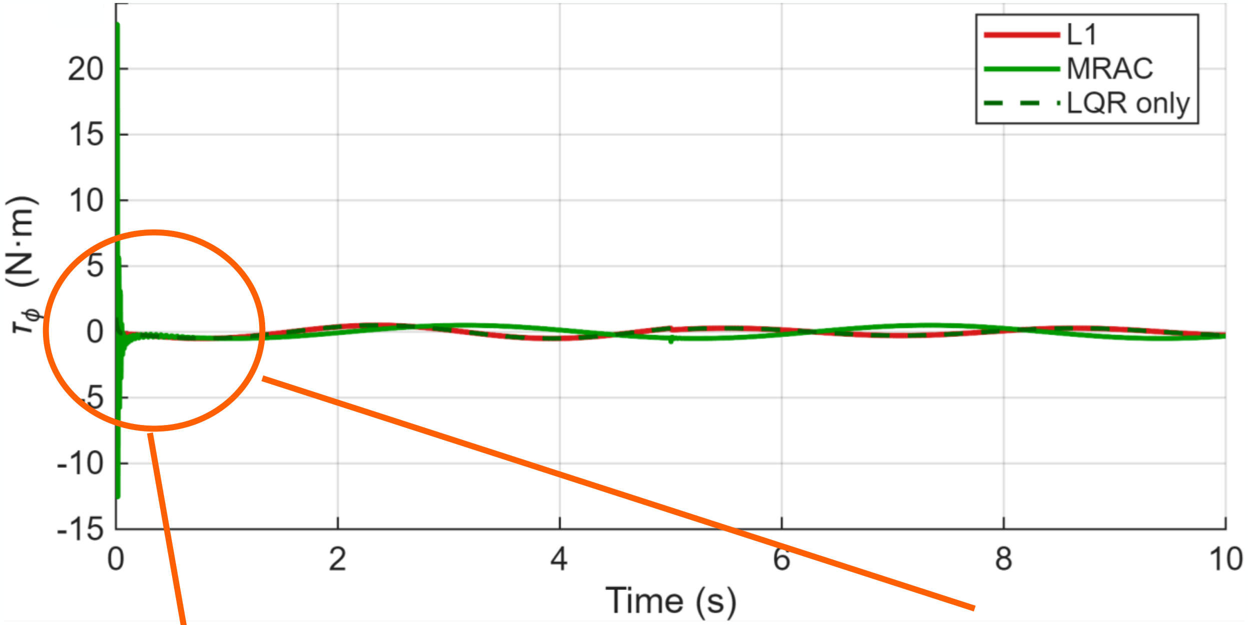


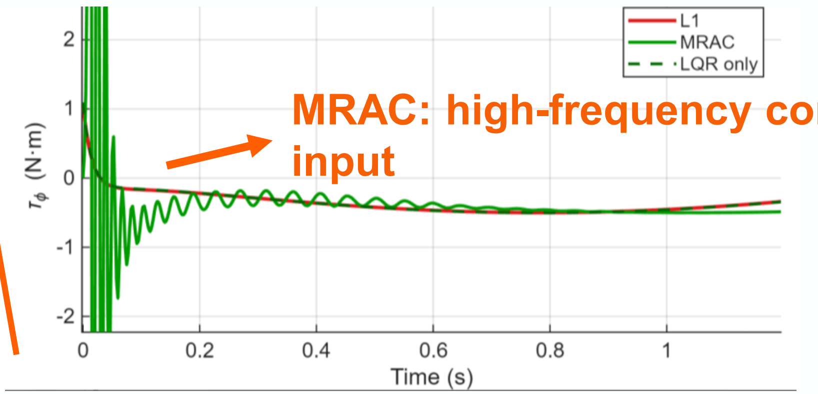


Impact of Sudden Change in Plant Parameters

Sudden parameter change at t = 5s: $L = 0.14 \rightarrow L = 0.25$







$\mathcal{L}_1$ demonstrates bounding tracking even on large parameter changes

MRAC diverges on significant parameter modification

Conclusion

- **Transient guarantees and superior tracking:** Demonstrated better tracking of different reference signals ($10^\circ, 30^\circ$ steps, sine waves) compared to Baseline LQR and MRAC in the presence of disturbances (without the need of re-tuning).

- **Robustness-performance trade-off:** Unlike MRAC, which exhibited control-input oscillations and low TDM at high gain ($\Gamma = 1000$), $\mathcal{L}_1$ maintains low-frequency control signals.

- **Robustness to uncertain plant parameters:** Unlike MRAC, which exhibited high tracking error under sudden change in plant parameters, $\mathcal{L}_1$ maintained bounded tracking error without retuning.

References

[1] Hovakimyan, Naira, and Chengyu Cao. $\mathcal{L}1$ adaptive control theory: Guaranteed robustness with fast adaptation. Society for Industrial and Applied Mathematics, 2010.

[2] Quadrotor Control Using Model Reference Adaptive Control

https://www.mathworks.com/help/slcontrol/ug/quadrotor-control-using-model-reference-adaptive-control.html

Thank you!