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Observability based Control for Cooperative Localization

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Abstract— **In this paper, we focus on localizing a stationary target in a GPS denied environment using a team of aerial unmanned aerial vehicles equipped with bearing-only sensors. The vehicles use cooperative localization to localize themselves and the target. We develop an observability-based controller that improves the overall localization accuracy of the team as well as of the target. We use receding horizon control to plan vehicle paths such that the nonlinear observability conditions are satisfied while improving the localization accuracy and maintaining minimum safe distance from each other and the landmarks. The controller performance is verified using simulation results.**

I. INTRODUCTION

Over the last decade, the use of Unmanned Aerial Vehicles (UAVs) have increased tremendously in both civil and military applications. The size of UAVs has also decreased because of smaller autopilots and miniature sensor technologies. Smaller UAVs offer several advantages over bigger UAVs. One such advantage is that the smaller UAVs can be operated in the tighter constraints of an urban environment. Another advantage is that large number of small UAVs can be operated together in a joint mission [1], [2]. However, urban environments offer several challenges in the operation of UAVs such as no or partial availability of GPS signals for localization. Therefore, there is a need of alternative methods to solve the localization problem. One such method is known as Simultaneous Localization and Mapping (SLAM) where the vehicle uses its external sensors like camera, laser range finder, or Sonar with data from Inertial Measurement Unit (IMU) to build a map and localize itself at the same time [3]. Over the last decade, SLAM has been extensively investigated in various forms and has been successfully implemented on various platforms. However SLAM process is computationally expensive, the estimation accuracy deteriorate with time, and require frequent loop closures, therefore, it is not suitable for multi-vehicle coordination missions where precise relative position is required.

An alternative method is to use abundantly available sources which can provide localization information known as Signals of Opportunity(SoOp). SoOp are those signals that are not originally intended (designed) for positioning but they are freely available all the time and everywhere (within a certain range). Examples of SoOp include broadcast analog and digital such as AM/FM radio and analog/digital TV (transmission towers), wireless local area network (WLAN) signals such as WiFi and WiMax (access points), cellular and mobile phone network base stations, radar sites, RFID tags, RF beacons/transponders, and partially available GPS. In a single vehicle localization problem, the vehicle uses IMU data to predict its position when any SoOP is not in its sensor range, and updates the position when a SoOp is in its sensor range. However, due to limited sensor range and occlusions the position update will not be frequent, resulting less accurate position estimates.

As mentioned earlier multiple small UAVs can be used in a cooperative setting to various control mission. Similar to cooperative control mission multiple UAVs can also be used for localization. In cooperative localization a group of robots exchange relative position measurements (between robots and between a robot and a SoOp) from their exteroceptive sensors (e.g.,camera, laser, etc.) and their motion information (velocity and turn rate) from interoceptive sensors (e.g., IMU, encoders, etc.) to collectively estimate their states. Cooperative localization has been an active area of research (e.g., [4], [5], [6], [7], [8], [9], [10], [11], [12], [13]) because it provides several potential advantages, including increased localization accuracy, sensor coverage, robustness, efficiency, and flexibility.

Cooperative localization problem has been solved using Extended Kalman Filter (EKF) [14], Minimum Mean Square Estimator (MMSE)[5], Maximum Likelihood Estimation (MLE) [15], Particle Filter [16], and Maximum A Posteriori (MAP) [17]. Furthermore the cooperative localization problem have been solved either in a centralized [8] or distributed manner [14], [5], [17]. The cooperative localization algorithm, irrespective of type of filter (centralized or distributed) will provide meaningful localization estimates (consistent and bounded), if and only if the sensors provide enough information for localization or in other words if the system is observable.

Several authors have carried out observability analysis of the cooperative localization problem [14], [18]. It has been shown that two landmarks are needed for the observability of a single vehicle [19], [20], [21] when using bearing-only measurements. Sharma *et al.* [22] extends the observability analysis from 2 to n robots, with bearing-only measurements. They have shown that in n-robot cooperative localization problem with bearing only measurements if in a given relative position measurement graph (RPMG) each vehicle node has path to two different landmarks then the system is completely observable. These results were verified through experimental results using a three robot platform equipped with omni-directional cameras [23].

From these results it is clear that a group of vehicles

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can perform cooperative localization to navigate in a GPS denied environment. The UAV paths and sensor orientation should be determined such that the observability conditions are satisfied and safe distance between vehicle is maintained. Furthermore, the relative measurements like bearing-only and range-only measurements are nonlinear therefore the amount of observability also depend on the vehicle path and the RPMG topology. Therefore, the vehicle paths can be used not only to maintain observability conditions but also improve the observability resulting in improved localization accuracy. This is demonstrated by Yu *et al.* in [24], [25], where they design a controller to improve the observability of obstacles in the field of view of the vehicle. They also showed that the obstacle avoidance and improving observability are complementary tasks. Zhang *et al.* [26], developed a controller to improve the localization accuracy of a team of robots performing cooperative localization using bearing only measurements. The controller is designed for a fixed sensing topology without considering the observability. Furthermore, the cooperative localization was performed without any source of absolute information (SoOp). In this paper, we extend the work by Zhang *et al.* [26] by designing a controller to improve the localization accuracy of team of UAVs performing cooperative localization using bearing-only measurements for a time time-varying sensing topology. The main contributions of the paper are:

- We use cooperative localization to localize a target and team of unmanned vehicles using bearing-only measuremnts in presence of known SoOp (or landmarks).
- We develop an observability-based controller for the vehicles performing cooperative localization and localizing a target. The observability based controller generates vehicle paths such that the following objectives are achieved.
  - Vehicles maintain the overall observability by satisfying the nonlinear conditions.
  - Improves the overall localization accuracy of all the vehicles and the target.
  - Vehicles maintain the minimum distance between each other, SoOps (landmarks), and the target.

II. Problem Formulation

In this paper we focus on localizing a stationary obstacle/target in a GPS denied environment using multiple aerial robots flying at a constant altitude and constant speed. To localize the obstacle successfully the UAVs should estimate their own position and heading and at least one vehicle should have the target in its field-of-view. We use bearing-only cooperative localization [23] to estimate position and heading of all the vehicles and to track the target. Cooperative Localization is briefly discussed in next subsection.

Note 1: In rest of the paper we will use 'landmark' instead of 'SoOp' and will use 'robot' or 'vehicle' instead of a UAV.

A. Bearing-only Cooperative Localization

Consider n vehicles moving in a horizontal plane performing cooperative localization. We can write the equations of motion for the i^{th} vehicle as,

$$\dot{X}_i = g_i(X_i, u_i) \triangleq \begin{pmatrix} V_i \cos\psi_i \\ V_i \sin\psi_i \\ \omega_i \end{pmatrix}, \tag{1}$$

where $X_i = [x_i \ y_i \ \psi_i]^\top \in \mathbb{R}^3$ is the robot state, including robot location $(x_i, \ y_i)$ and robot heading ψ_i, and $u_i = [V_i, w_i]^\top$ is the control input vector. We assume that onboard introspective sensor IMU measure the linear speed V_i and angular speed ω_i of the robot. Without loss of generality, we assume that robots cannot move backward ($V_i \geq 0, \ i = 1 \cdots n$). Each vehicle has an exteroceptive sensor to measure relative bearing to other vehicles and landmark that are in the field-of-view of the sensor. Relative bearing from the i^{th} robot to the j^{th} robot or landmark can be written as,

$$\eta_{ij} = \tan^{-1}\left(\frac{y_j - y_i}{x_j - x_i}\right) - \psi_i. \tag{2}$$

For cooperative localization, each vehicle exchanges their local sensor measurements (velocity, turn rate, and bearing to landmarks and other robots) with their neighbors. Let N_i^M be the set of neighbors for which robot i can obtain bearing measurements, and let N_i^C be the set of neighbors with which robots i can communicate. In this paper, we assume that $N_i^M = N_i^C$ and we will therefore denote the set of neighbors as N_i. To represent the connection topology of the robots we use a relative position measurement graph (RPMG)[27] which is defined as follows.

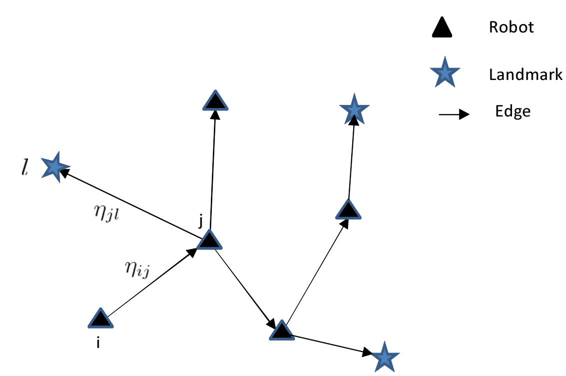

Fig. 1. Relative position measurement graph (RPMG). The nodes of an RPMG represent vehicle states and the edges represent bearing measurements between nodes.

Definition 1: An RPMG for n robots performing cooperative localization with l different known landmarks is a directed graph $G_n^l \triangleq \{\mathcal{V}_{n,l}, \mathcal{E}_{n,l}\}$, where $\mathcal{V}_{n,l} = \{1, \cdots, n, n+1, \cdots, n+l\}$ is the node set consisting of n robot nodes and l landmark nodes and $\mathcal{E}_{n,l}(t) \subset \{\mathcal{V}_{n,0} \times \mathcal{V}_{n,l}\} = \{\eta_{ij}\}, \ i \in \{1, \cdots, n\}, \ j \in \{1, \cdots, n, n+1, \cdots, n+l\}$ is the edge set representing the availability of a relative bearing measurement. We use m to denote the number of edges in the RPMG. An example RPMG (G_5^3 with $m = 7$) is shown in Fig. 1.

We assume that the robot sensors have limited sensor range R_{sensor} and limited field-of-view. Therefore, agents can only measure the bearing of those robots and landmarks that are located within the footprint of the sensor. Therefore, the graph G_n^l will likely have a time varying topology.

B. Cooperative localization implementation

The objective of the cooperative localization is to estimate the combined state $\hat{X}(k) = [\hat{X}_1(k), \cdots, \hat{X}_n(k)]^\top$. We use an extended information filter (EIF) to implement the bearing-only cooperative localization. In the information filter instead of state $\hat{X}$ and covariance $P(k)$ the information vector $\hat{y}(k)$ and information matrix $Y(k)$ is updated. The information matrix and information vector can be computed as

$$\begin{aligned} Y(k) &= P(k)^{-1}, \\ \hat{y}(k) &= Y(k)\hat{X}(k). \end{aligned}$$

Similar to an extended kalman filter (EKF) the EIF has two steps. The first is the prediction step, which is given below.

$$\begin{aligned} &Y(k+1|k) = (F(k)Y(k|k)^{-1}F(k)^\top + B(k)Q(k)B(k)^\top)^{-1}, \\ &\hat{y}(k+1|k) = Y(k+1|k)\hat{X}(k+1|k), \\ &\hat{X}(k+1|k) = X(k|k) + T_s f(\hat{X}(k|k), u(k)), \end{aligned}$$

where T_s is the sampling time, $F_k = \begin{pmatrix} F_1 & 0 & \cdots & 0 \\ 0 & F_2 & \cdots & 0 \\ \vdots & \cdots & \ddots & 0 \\ 0 & 0 & \cdots & F_n \end{pmatrix}$, $B(k) = \begin{pmatrix} B_1 \\ \vdots \\ B_n \end{pmatrix}$, and $Q(k) = \begin{pmatrix} Q_i(k) & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & Q_n(k) \end{pmatrix}$ is covariance of noise in the control input. The matrix F_i and B_i are the system jacobian with respect to state X_i and control u_i, which are given below

$$F_i = I_3 + T_s \frac{\partial f_i}{\partial X_i}|_{X_i = X_i(k)} = \begin{bmatrix} 1 & 0 & -V_i T_s \sin\psi(k) \\ 0 & 1 & V_i T_s \cos\psi(k) \\ 0 & 0 & 1 \end{bmatrix},$$

$$B_i = T_s \frac{\partial f_i}{\partial u_i}|_{u_i = u_i(k)} \begin{bmatrix} T_s \cos\psi_k & 0 \\ T_s \sin\psi_k & 0 \\ 0 & T_s \end{bmatrix},$$

and $Q_i(k) = \begin{pmatrix} \sigma_{v_i}^2 & 0 \\ 0 & \sigma_{\omega_i}^2 \end{pmatrix}$, where σ_{v_i} and σ_{ω_i} are the standard deviation in velocity input and turn rate input respectively.

The measurement update step is given as

$$\begin{aligned} Y(k+1|k+1) &= Y(k+1|k) + \sum H_{ij}(k)^\top \sigma_{\eta_{ij}}^{-2} H_{ij}(k), \\ \hat{y}(k+1|k+1) &= \hat{y}(k+1|k) \\ &\quad + \sum H_{ij}(k)^\top R_{ij}^{-1}(\mu_{ij} + H_{ij}\hat{X}(k+1|k)). \end{aligned}$$

The scalar μ_{ij} represents the innovation

$$\mu_{ij} = \eta_{ij} - h_{ij}(x(k+1|k)), \tag{3}$$

and $\sigma_{\eta_{ij}}$ is standard deviation of the noise in the bearing measurement. The row vector H_{ij} is the measurement jacobian

$$H_{ij}(k) = \frac{\partial h_{ij}}{\partial X}|_{X=X(k)}. \tag{4}$$

The EIF is dual of the EKF and the EKF is a quasi-local asymptotic observer for nonlinear systems and its convergence and boundedness are achieved when the system is fully observable [28]. In our previous work [22], we performed nonlinear analysis of the bearing-only cooperative localization problem. The following theorem summarizes the conditions required for the complete observability of the system.

Theorem 1: Given a proper RPMG G_n^l, if for each robot there exists a path to at least two landmarks and $\eta_{i1} \neq \eta_{i2}, \forall\ i = 1, \cdots, n$, then the system is completely observable, i.e., the rank of the observability matrix is $3n$.

These observability conditions are very important from a robot's path planning perspective. These conditions dictate how each vehicle's path should be planned such that the required sensing topology with required number of landmarks is maintained.

C. Control Objective

In this subsection we discuss the control objectives based on the conditions obtained from the observability analysis of the cooperative localization. The main objective is plan path for robots such that the system remains observable and the accuracy of target tracking as well the accuracy of position and heading estimate of all of the robots increases with time. The path planner should achieve following objectives.

1) To maintain the observability, the RPMG including two landmarks and the target should be connected. The connectivity of a graph is directly connected to the algebraic connectivity $\lambda_2(L)$ of the graph. The algebraic connectivity $\lambda_2(L)$ is the second smallest eigen-value of the Laplacian of the graph [29].
2) The path should be planned such that there is an increase in the information available from sensors. The information matrix is given by
$$I(t) = \sum_{i,j} w_{ij} H_{ij}^\top(t) R^{-1} H_{ij}(t) \tag{5}$$
where $H_{ij}(t) = \frac{\partial \eta_{ij}}{\partial X}|_{X=X(t)} = [0\ 0\ -\frac{y_{ij}}{\rho_{ij}^2}\ \frac{x_{ij}}{\rho_{ij}^2}\ 0\ 0\ 0]$, where w_{ij} denote the probability of detection j^{th} vehicle in the field-of-view of i^{th} vehicle, which is 1 if in field-of-view and 0 otherwise.
3) The robots should maintain safe distance from each other, from landmarks, and from the target.

These control objectives can be summarized as following receding horizon control problem.

$$u^*(\tau : t) = \max_{u^p(\tau:t)} det(\sum_{i,j} w_{ij} H_{ij}^\top(t) R^{-1} H_{ij}(t)) \tag{6}$$

subject to

$$\lambda_2(L(t)) > 0, \ \forall t$$
$$\rho_{ij} > \rho_{min}, \ \forall i, j \in [1, n]$$
$$y_{ij} \cos \psi_i - x_{ij} \sin \psi_i \neq 0, \forall \ i, j \in [1, n]$$

where $x_{ij} = x_i - x_j$, $y_{ij} = y_i - y_j$, $\rho_{ij} = \sqrt{x_{ij}^2 + y_{ij}^2}$, and ρ_{min} is minimum allowable distance between two vehicles or a vehicle and a landmark.

Note 2: The third condition ensures that vehicles do not move on the line between two vehicles, vehicle and landmark, and vehicle and the target to avoid singularity in the observability matrix.

We use a distributed receding horizon control [30] to maximize the information gain, each vehicle optimizes only for its own control at each update, and exchanges information only with neighboring subsystems. We assume that neighboring subsystems can directly communicate with one another.

Every vehicle solves its own optimal control using its current state and that of its neighbors. Each i^{th} vehicle predicts some trajectories for over each prediction horizon and prior to each update, each i^{th} vehicle receives an assumed control trajectory from each neighbor to compute the cost function.

III. Results

In this section, we present initial simulation results for the observability-based cooperative control approach discussed in this paper. For the simulation, we consider five robots performing cooperative localization in presence of two known landmarks while localizing a single stationary obstacle. Following are the different simulation parameters of a robot used in the simulation results presented in this paper.

- Sensing range of the bearing-only sensor ($R_{sensor} = 120m$).
- Linear velocity of the robot ($V = 5 \ m/s$).
- Sampling time period $T_s = 0.1 \ s$.
- Initial pose uncertainty $[\sigma_{x0} \ \ \sigma_{y0} \ \ \sigma_{\psi_0}] = [5m \ 5m \ 0.2rad]$).
- Standard deviations of process noise in velocity and turn rate $[\sigma_v \ \sigma_\omega]^T = [0.2m/s \ 0.2rad/s]^T$.
- Standard deviation of measurement noise $\sigma_{\eta_{ij}} = 0.1rad$.
- Minimum distance between two vehicles or a vehicle and a landmark $d_{min} = 20m$.

Actual and estimated trajectories generated by the observability-based control are shown in Fig. 2. It can be seen that the actual and estimated trajectories are very close to each other and the RPMG, including two landmarks and a stationary target, is connected meaning that the system is observable. It can also be seen that vehicles maintain the minimum distance of $20m$ between each other and with the landmarks as shown in Fig. 3. The Fig. 4 shows how the determinant of the overall combined covariance matrix decreases with time. This shows that how the observability-based controller improves the overall localization accuracy

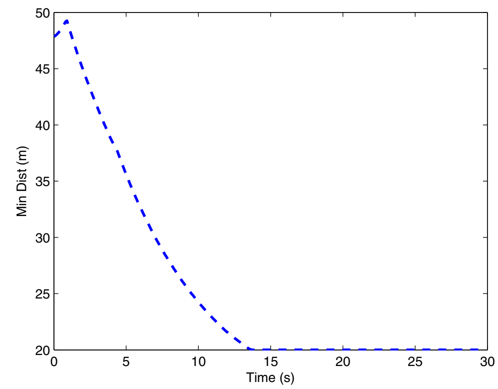

Fig. 3. This figure shows the minimum distance from the set of all the relative distance between vehicles, vehicle and landmark, vehicle and vehicle and the target.

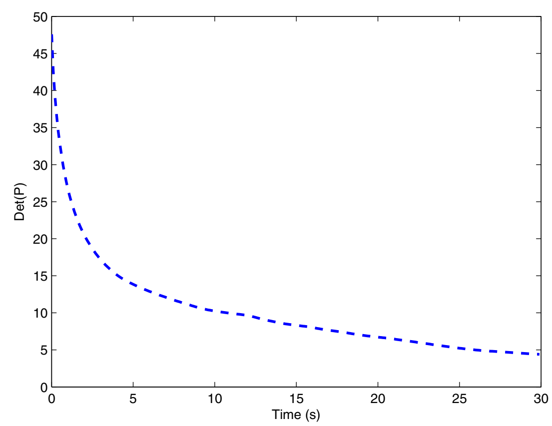

Fig. 4. Determinant of the overall covariance matrix

while maintaining the overall observability and minimum distance between vehicles and landmarks. This is also visible in Fig. 5 and Fig. 6. Fig. 5 shows the error plots (position and attitude) for vehicle 5. Similarly Fig. 6 shows the position error plots for the stationary target. Both the figures show that the observability-based controller reduces the uncertainty in the position and heading estimates.

IV. Conclusion

In this paper, we present initial results for a observability-based controller intended to improve the localization accuracy of a team of unmanned vehicles performing cooperative localization. We have used a receding horizon control to chose vehicle paths such that the observability conditions obtained in [22] are satisfied while improving the localization accuracy and maintaining minimum safe distance from each other and the landmarks. The simulation results show that using the observability based control for a time varying relative position measurement graph, the vehicles are able

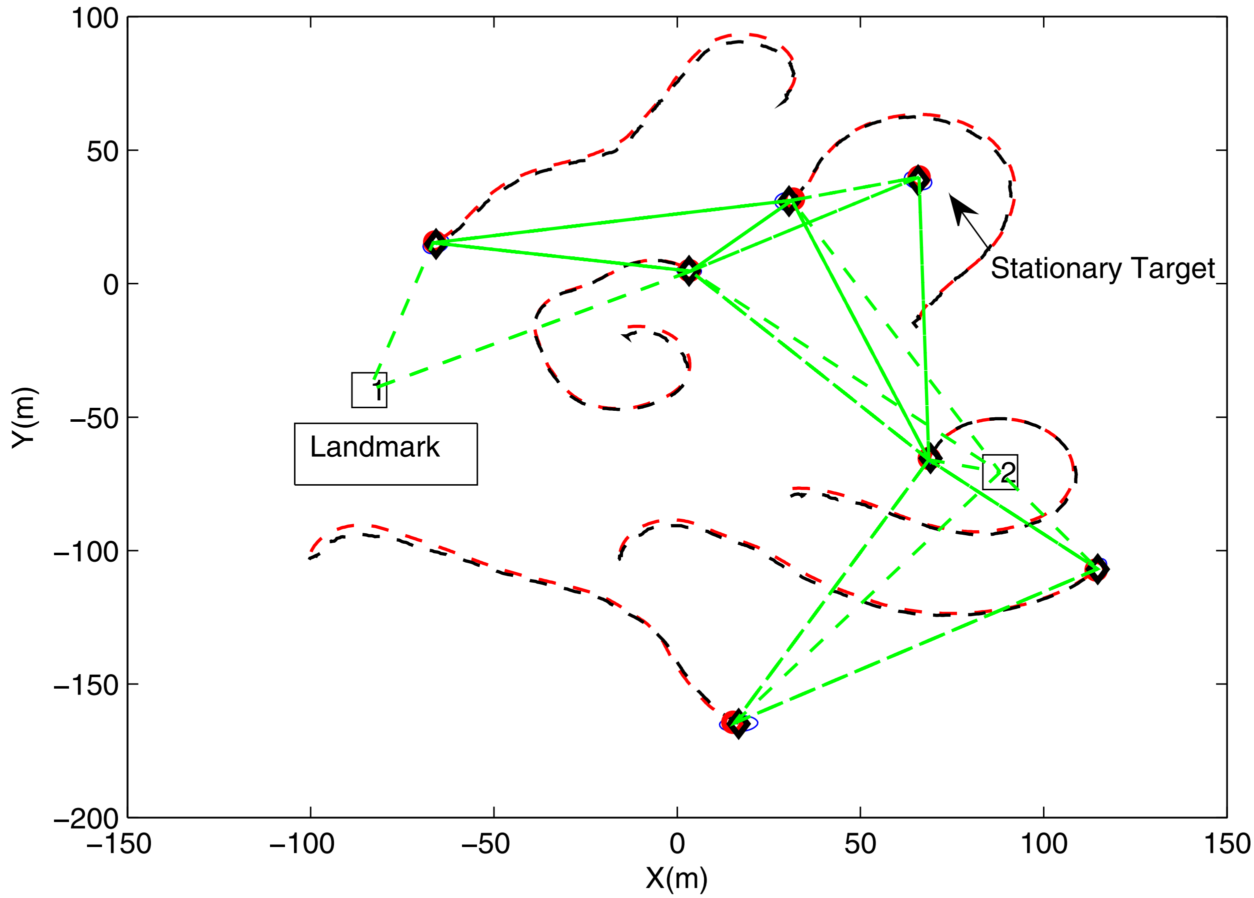

Fig. 2. Actual and estimated trajectories: The black circles represent the initial position uncertainty (3σ). The black diamonds represent the initial position estimates of robots. The red circles are the true initial positions of the robots. The dashed green line represents an edge (bearing measurement) between two nodes. Two numbered squares are the two known landmarks.

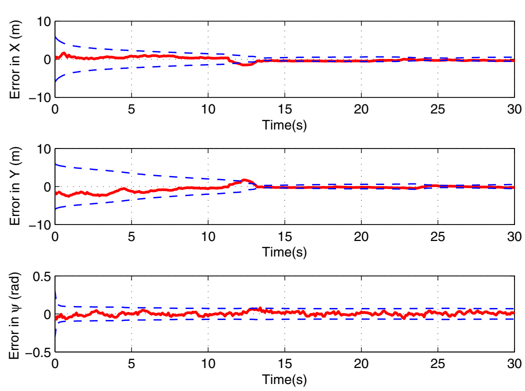

Fig. 5. Error in localization (position and heading) for the vehicle 5:The dashed blue curve represents the $\pm 3\sigma$ uncertainty. The estimation error is represented by the red solid curve.

to maintain the connected RPMG, improve their localization accuracy, and maintain the minimum safe distance.

In future, we will derive conditions that guarantee observability and minimum severation. These conditions should dictate the initial sensing topology and separation between landmarks. We will extend these results for tracking multiple moving targets in a GPS denied area and remove the constraint of constant altitude and velocity.

REFERENCES

[1] R. Sharma and D. Ghose, "Collision avoidance between uav clusters using swarm intelligence techniques," *International Journal of Systems Science*, vol. 40, no. 5, pp. 521–538, 2009.

[2] R. Sharma, M. Kothari, C. N. Taylor, and I. Postlethwaite, "Cooperative target-capturing with inaccurate target information," in *American Control Conference (ACC), 2010*, 2010, pp. 5520–5525. [Online]. Available: http://ieeexplore.ieee.org/stamp/stamp.jsp?arnumber=5530992

[3] R. Sharma, C. Taylor, D. W. Casbeer, and R. W. Beard, "Distributed cooperative slam using an information conseneus filter," in *AIAA Guidance Navigation and Control Conference. Toronto.. AIAA*, 2010, pp. 8334–8342.

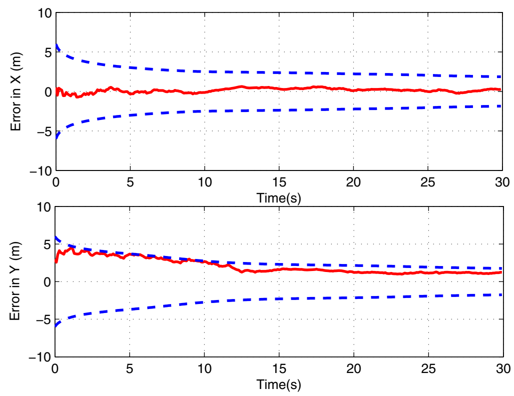

Fig. 6. Error in localization (position) for the target:The dashed blue curve represents the $\pm 3\sigma$ uncertainty. The estimation error is represented by the red solid curve.

[4] R. Kurazume and S. Hirose, "Study on cooperative positioning system: optimum moving strategies for cps-iii," in *Proc. IEEE International Conference on Robotics and Automation*, vol. 4, 16–20 May 1998, pp. 2896–2903.

[5] A. C. Sanderson, "A distributed algorithm for cooperative navigation among multiple mobile robots," *Advanced Robotics*, vol. 12, pp. 335–349, 1998.

[6] S. I. Roumeliotis and G. A. Bekey, "Collective localization: a distributed kalman filter approach to localization of groups of mobile robots," in *Proc. IEEE International Conference on Robotics and Automation ICRA '00*, vol. 3, 24–28 April 2000, pp. 2958–2965.

[7] J. Spletzer, A. Das, R. Fierro, C. Taylor, V. Kumar, and J. Ostrowski, "Cooperative localization and control for multi-robot manipulation," in *Proc. IEEE/RSJ International Conference on Intelligent Robots and Systems*, vol. 2, 2001, pp. 631–636 vol.2.

[8] I. M. Rekleitis, G. Dudek, and E. E. Milios, "Multi-robot cooperative localization: a study of trade-offs between efficiency and accuracy," in *Proc. IEEE/RSJ International Conference on Intelligent Robots and System*, vol. 3, 30 Sept.–5 Oct. 2002, pp. 2690–2695.

[9] P. Nebot, D. Gomez, and E. Cervera, "Agents for cooperative heterogeneous mobile robotics: a case study," in *Proc. IEEE International Conference on Systems, Man and Cybernetics*, vol. 1, 5–8 Oct. 2003, pp. 557–562.

[10] A. Bahr, J. J. Leonard, and M. F. Fallon, "Cooperative localization for autonomous underwater vehicles," *International Journal of Robotics Research*, vol. Volume 28 , Issue 6, pp. 714–728, 2009.

[11] A. Mourikis and S. Roumeliotis, "Analysis of positioning uncertainty in reconfigurable networks of heterogeneous mobile robots," in *Proc. IEEE International Conference on Robotics and Automation ICRA '04*, vol. 1, 2004, pp. 572–579 Vol.1.

[12] R. Sharma and C. N. Taylor, "Cooperative navigation of MAVs in GPS denied areas," in *Proc. IEEE International Conference on Multisensor Fusion and Integration for Intelligent Systems MFI 2008*, 20–22 Aug. 2008, pp. 481–486.

[13] ——, "Vision based distributed cooperative navigation for mavs in gps denied areas," in *Proceedings of the AIAA Infotech@ Aerospace Conference*, 2009. [Online]. Available: http://arc.aiaa.org/doi/pdf/10.2514/6.2009-1932

[14] S. I. Roumeliotis and G. A. Bekey, "Distributed multirobot localization," *IEEE Transaction for Robotics and Autonomus Systems*, vol. 18, no. 5, pp. 781–795, 2002.

[15] A. Howard, M. J. Matark, and G. S. Sukhatme, "Localization for mobile robot teams using maximum likelihood estimation," in *Proc. IEEE/RSJ Int Intelligent Robots and Systems Conf*, vol. 1, 2002, pp. 434–439.

[16] D. Fox, W. Burgard, H. Kruppa, and T. S, "A probabilistic approach to collaborative multi-robot localization," *Autonomous Robots*, vol. 8, pp. 325–344, 2000.

[17] E. D. Nerurkar, S. I. Roumeliotis, and A. Martinelli, "Distributed maximum a posteriori estimation for multi-robot cooperative localization," in *Proc. IEEE Int. Conf. Robotics and Automation ICRA '09*, 2009, pp. 1402–1409.

[18] A. Martinelli and R. Siegwart, "Observability analysis for mobile robot localization," in *Proc. IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS 2005)*, 2–6 Aug. 2005, pp. 1471–1476.

[19] M. Betke and L. Gurvits, "Mobile robot localization using landmarks," *IEEE Transaction for Robotics and Autonomus Systems*, vol. 13, no. 2, pp. 251–263, 1997.

[20] L. D. L. Perera, A. Melkumyan, and E. Nettleton, "On the linear and nonlinear observability analysis of the slam problem," in *Proc. IEEE Int. Conf. Mechatronics ICM 2009*, 2009, pp. 1–6.

[21] K. W. Lee, W. S. Wijesoma, and I. G. Javier, "On the observability and observability analysis of slam," in *Proc. IEEE/RSJ Int Intelligent Robots and Systems Conf*, 2006, pp. 3569–3574.

[22] R. Sharma, R. W. Beard, C. N. Taylor, and S. Quebe, "Graph-based observability analysis of bearing-only cooperative localization," *Robotics, IEEE Transactions on*, vol. 28, no. 2, pp. 522–529, 2012.

[23] R. Sharma, S. Quebe, R. W. Beard, and C. N. Taylor, "Bearing-only cooperative localization:simulation and experimental results," *Journal of Intelligent & Robotic Systems*, pp. 1–12, 2013. [Online]. Available: http://link.springer.com/article/10.1007/s10846-012-9809-z

[24] H. Yu, R. Sharma, R. W. Beard, and C. N. Taylor, "Observability-based local path planning and collision avoidance for micro air vehicles using bearing-only measurements," in *American Control Conference (ACC), 2011*, 2011, pp. 4649–4654. [Online]. Available: http://ieeexplore.ieee.org/stamp/stamp.jsp?arnumber=5991095

[25] ——, "Observability-based local path planning and obstacle avoidance using bearing-only measurements," *Robotics and Autonomous Systems*, vol. 61, no. 12, pp. 1392–1405, 2013.

[26] F. Zhang, B. Grocholsky, V. Kumar, and M. Mintz, "Cooperative control for localization of mobile sensor networks," in *Cooperative Control*. Springer, 2005, pp. 241–255.

[27] A. Mourikis and S. Roumeliotis, "Performance analysis of multirobot cooperative localization," *IEEE Transaction for Robotics and Autonomus Systems*, vol. 22, no. 4, pp. 666–681, 2006.

[28] Y. Song and J. W. Grizzle, "The extended kalman filter as a local asymptotic observer for descrete -time nonlinear systems," *Journal of Mathemetical Systems, Estimation, and Control*, vol. 5, pp. 59–78, 1995.

[29] M. C. DeGennaro and A. Jadbabaie, "Decentralized control of connectivity for multi-agent systems," in *IEEE Conf. on Decision and Control*, 2006, pp. 3628–3633.

[30] W. B. Dunbar and R. M. Murray, "Distributed receding horizon control for multi-vehicle formation stabilization," *Automatica*, vol. 42, no. 4, pp. 549–558, 2006.