Information-Optimizing Control of Multi-Agent Systems

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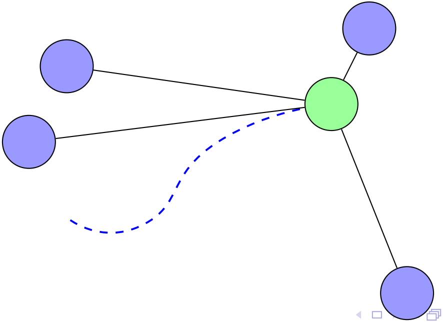
Problem Motivation

- For tractability; oftentimes, controls and estimation are decoupled.
- Estimation is generally used to improve the performance of a controller, in the presence of noise.
- How can a controller be used to improve the performance of an estimator?

Problem of Interest (Autonomous Convoy)

- Consider a multi-agent system equipped with two types of sensors: a global sensor, a relative sensor, and communicator.
- Global sensor provides position, relative sensor provides ranges between agents.
- Assume that the *global sensor fails on one of the agents* (green, primary agent). How should the rest of the fleet (secondary agents) navigate to 'help' the primary agent?
- We define 'help' as maximizing the position information that can be derived from the relative sensor measuremnts with the secondary agents.



Geometric Dilution of Precision

- Conveniently, as we have set up the problem here, Geometric Dilution of Precision (GDOP) is a convenient proxy for "information" we are interested.
- GDOP is a measure of the sensitivity of measurement error on the output position, as a function of the geometries of the agents. A lower GDOP value is prefered.
- In our 2D example, GDOP is calculated as follows:

$$A = \begin{bmatrix} \frac{x_1 - x}{R_1} & \frac{y_1 - x}{R_1} & -1 \\ & \vdots & \\ \frac{x_n - x}{R_n} & \frac{y_n - x}{R_n} & -1 \end{bmatrix}$$

$$\mathrm{GDOP} = tr\{(A^T A)^{-1}\}$$

- This metric is commonly used in sattelite positioning, which also relies on ranging.

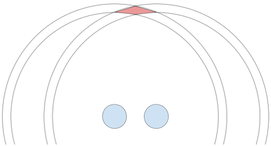


Figure: Example of poor spatial diversity, high dilution of precision.

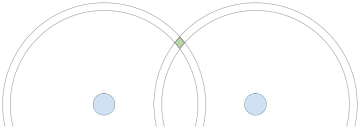


Figure: Example of good spatial diversity, low dilution of precision.

Problem Formulation

- The goal is to minimize the objective function

$$J = \phi(x(t), u(t), t),$$

subject to dynamic constraints

$$\dot{x} = Ax + u$$

and path constraints

$$c(x(t), u(t), t) \leq 0$$

- The state, x, contains the states of all the agents and is jointly optimized.

$$x = [x_1, y_1, \dot{x}_1, \dot{y}_1, \ \ldots, x_n, y_n, \dot{x}_n, \dot{y}_n]^T$$

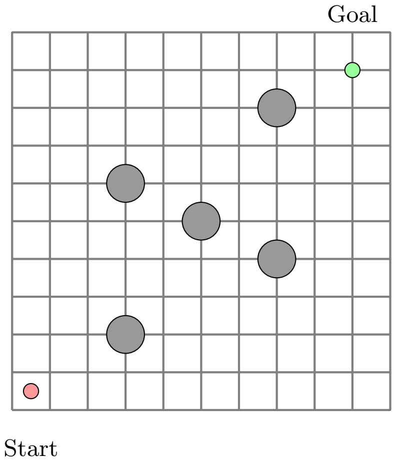


Figure: Example of the map. The red and green points denote the start and goal, respectively. The gray points indicate obstacles that the final paths may not intersect.

State Dynamics and Constraints

- Agents have full control authority in each dimension of their state.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{y}_1 \\ \ddot{x}_1 \\ \ddot{y}_1 \\ \vdots \\ \dot{x}_n \\ \dot{y}_n \\ \ddot{x}_n \\ \ddot{y}_n \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 & & & & & \\ 0 & 0 & 0 & 1 & \dots & & & & \\ 0 & 0 & 0 & 0 & & & & & \\ 0 & 0 & 0 & 0 & & & & & \\ \vdots & & & & \ddots & & & & \\ & & & & & 0 & 0 & 1 & 0 \\ & & & & & 0 & 0 & 0 & 1 \\ & & & & & 0 & 0 & 0 & 0 \\ & & & & & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ \dot{x}_1 \\ \dot{y}_1 \\ \vdots \\ x_n \\ y_n \\ \dot{x}_n \\ \dot{y}_n \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ u_{x_1} \\ u_{y_1} \\ \vdots \\ 0 \\ 0 \\ u_{x_n} \\ u_{y_n} \end{bmatrix}$$

- The control is constrained.

$$u_{\min} \leq u \leq u_{\max}$$

- The secondary agents must be within the communication range of the primary agent.

$$d(t) \leq r_{\max}$$

- The final paths may not intersect with any obstacles, O.

$$x(t) \cap O \leq 0$$

Results

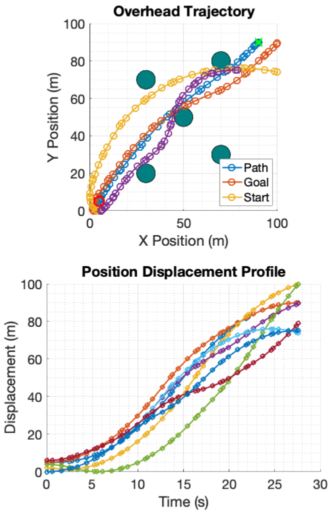


Figure: GPOPS-II results for optimal paths.

Conclusion

- This projects uses GPOPS-II to optimize the paths of a multi-agent system, in a manner that attempts to maximize information.
- We can jointly solve for the optimal paths under the provided constraints. And we find that GDOP is a useful 'information' metric, for the solver.
- Future Work:
  - How could this be implemented in a decentralized manner?
  - Are there methods to implemt this type of control in real-time, operating in dynamic environments?
  - How could we extend the 'information' metric more generally to other types of nonlinear sensors? (i.e. bearing)
  - Investigate complex agent dynamics.

References

- https://en.wikipedia.org/wiki/Dilution_of_precision_(navigation)
- F. Meyer, H. Wymeersch, M. Fröhle and F. Hlawatsch, "Distributed Estimation With Information-Seeking Control in Agent Networks," in IEEE Journal on Selected Areas in Communications, vol. 33, no. 11, pp. 2439-2456, Nov. 2015, doi: 10.1109/JSAC.2015.2430519.
- J. Spletzer, A. K. Das, R. Fierro, C. J. Taylor, V. Kumar and J. P. Ostrowski, "Cooperative localization and control for multi-robot manipulation," Proceedings 2001 IEEE/RSJ International Conference on Intelligent Robots and Systems. Expanding the Societal Role of Robotics in the the Next Millennium (Cat. No.01CH37180), Maui, HI, USA, 2001, pp. 631-636 vol.2, doi: 10.1109/IROS.2001.976240.