CFD HOMEWORK-5

HAZIRLAYAN

ADI SOYADI : ENES ŞAHINER

ÖĞRENCI NUMARASI : 30200018

TESLIM TARIHI : 28.5.24

DERS ADI : COMPUTIONAL FLUID DYNAMICS

DERS YÜRÜTÜCÜSÜ : SERTAÇ ÇADIRCI

Table of contents

Steady Burger Equation

It can be said that steady burger equations are the similar to Navier Stokes equations, the only difference is that there is no pressure gradient. Additionally, since there are two equations, iteration is necessary. Equations are shown below.

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{1}{Re}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = \frac{1}{Re}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)$$

Standard Central Difference

Using standard central difference approximation is used to solve the equations. Steps are shown below. (for u and v same approximation was applied)

If the standard central difference values of the parameters are written,

$$\frac{\partial u}{\partial x} = \frac{u_{i+1}^{j} - u_{i-1}^{j}}{2\Delta x}$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{u_{i+1}^{j} - 2u_i^j + u_{i-1}^{j}}{\Delta x^2}$$

$$\frac{\partial v}{\partial y} = \frac{v_i^{j+1} - v_i^{j-1}}{2\Delta y}$$

$$\frac{\partial^2 v}{\partial y^2} = \frac{v_i^{j+1} - 2v_i^j + v_i^{j-1}}{\Delta y^2}$$

If these values are put in place of the parameters,

$$u * \frac{u_{i+1}^{j} - u_{i-1}^{j}}{2\Delta x} + v * \frac{u_i^{j+1} - u_i^{j-1}}{2\Delta y} = \frac{1}{Re} * \left(\frac{u_{i+1}^{j} - 2u_i^j + u_{i-1}^{j}}{\Delta x^2} + \frac{u_i^{j+1} - 2u_i^j + u_i^{j-1}}{\Delta y^2}\right)$$

$$u * \frac{v_{i+1}^{j} - v_{i-1}^{j}}{2\Delta x} + v * \frac{v_i^{j+1} - v_i^{j-1}}{2\Delta y} = \frac{1}{Re} * \left(\frac{v_{i+1}^{j} - 2v_i^j + v_{i-1}^{j}}{\Delta x^2} + \frac{v_i^{j+1} - 2v_i^j + v_i^{j-1}}{\Delta y^2}\right)$$

Here,

$$\Delta x^2 = h^2 \; and \; \Delta y^2 = k^2 \rightarrow \gamma = \frac{h^2}{k^2}$$

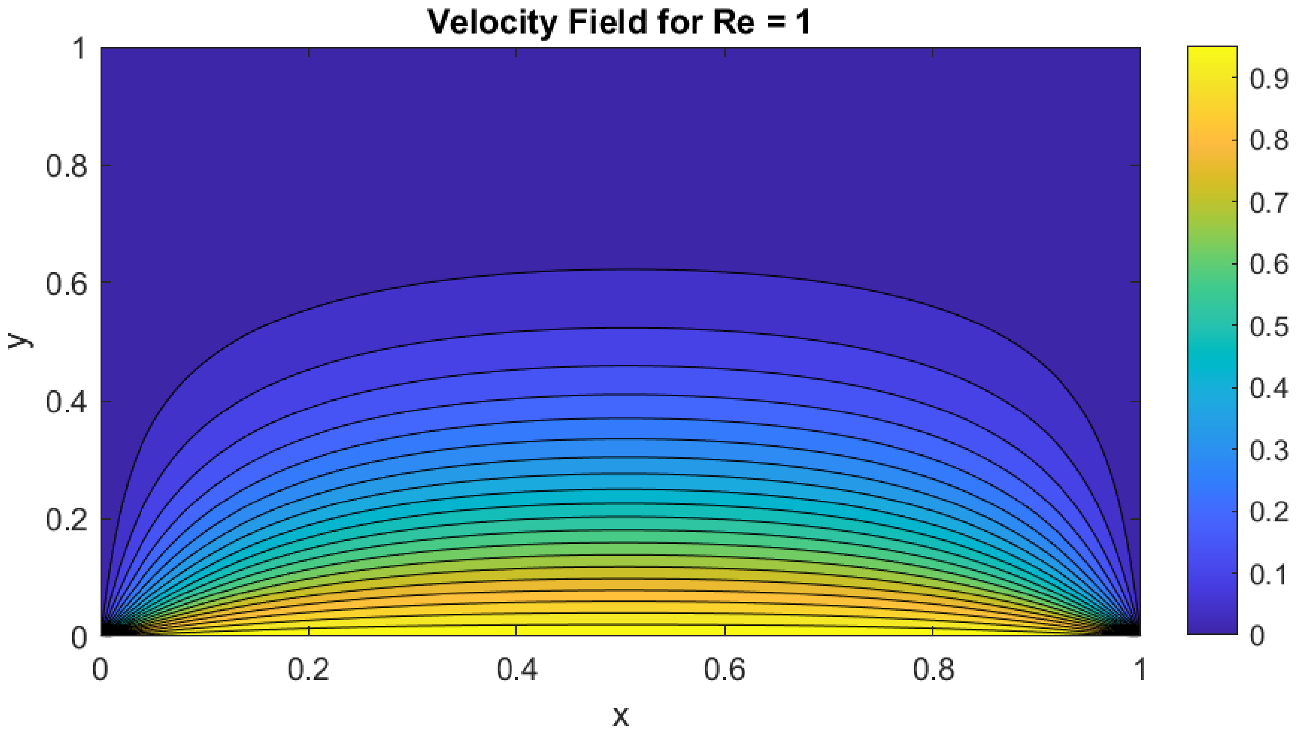
The discrete expression for u is as follows

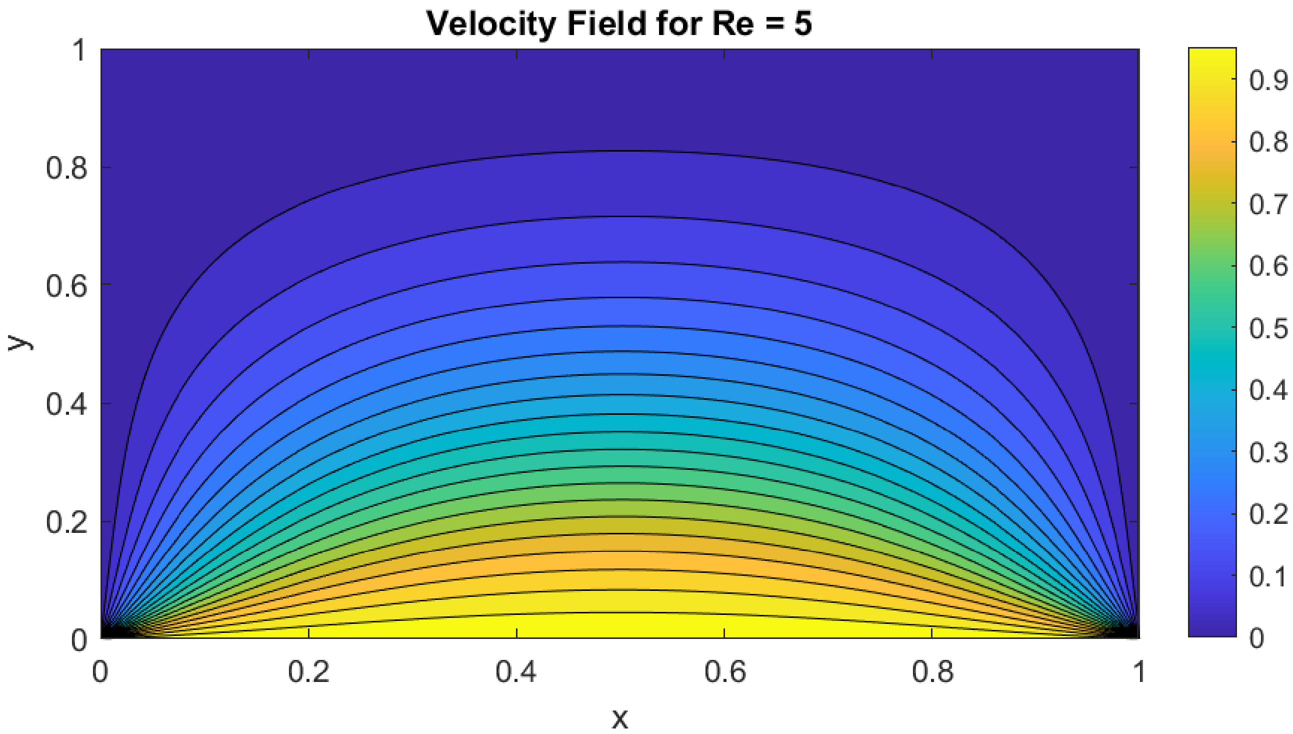
$$u_{i+1,j} + u_{i-1,j} + \gamma * u_{i,j-1} - 2 * (1+\gamma) * u_{i,j} - Re * \frac{h}{2} * \left[u_{i,j} * \left(u_{i+1,j} - u_{i-1,j} \right) + v_{i,j} * \gamma^{\frac{1}{2}} * \left(u_{i,j+1} - u_{i,j-1} \right) \right] = 0$$

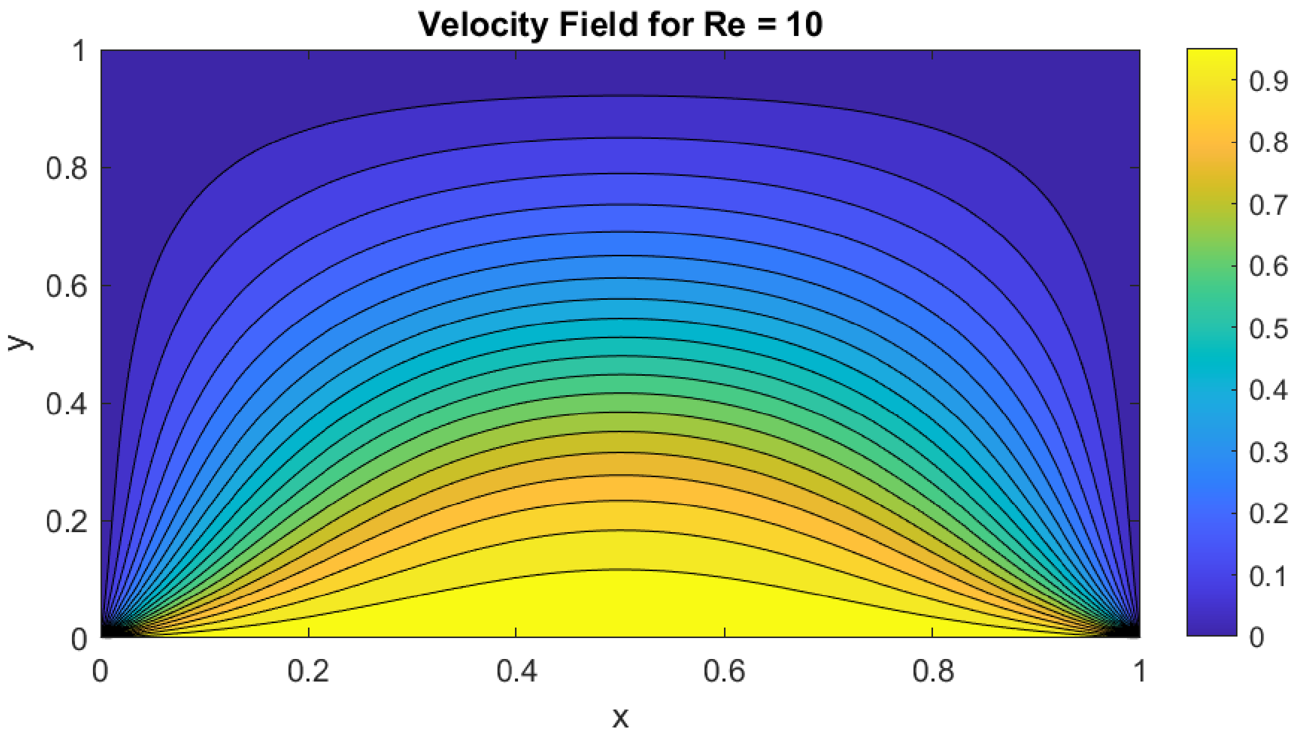
The final equation becomes:

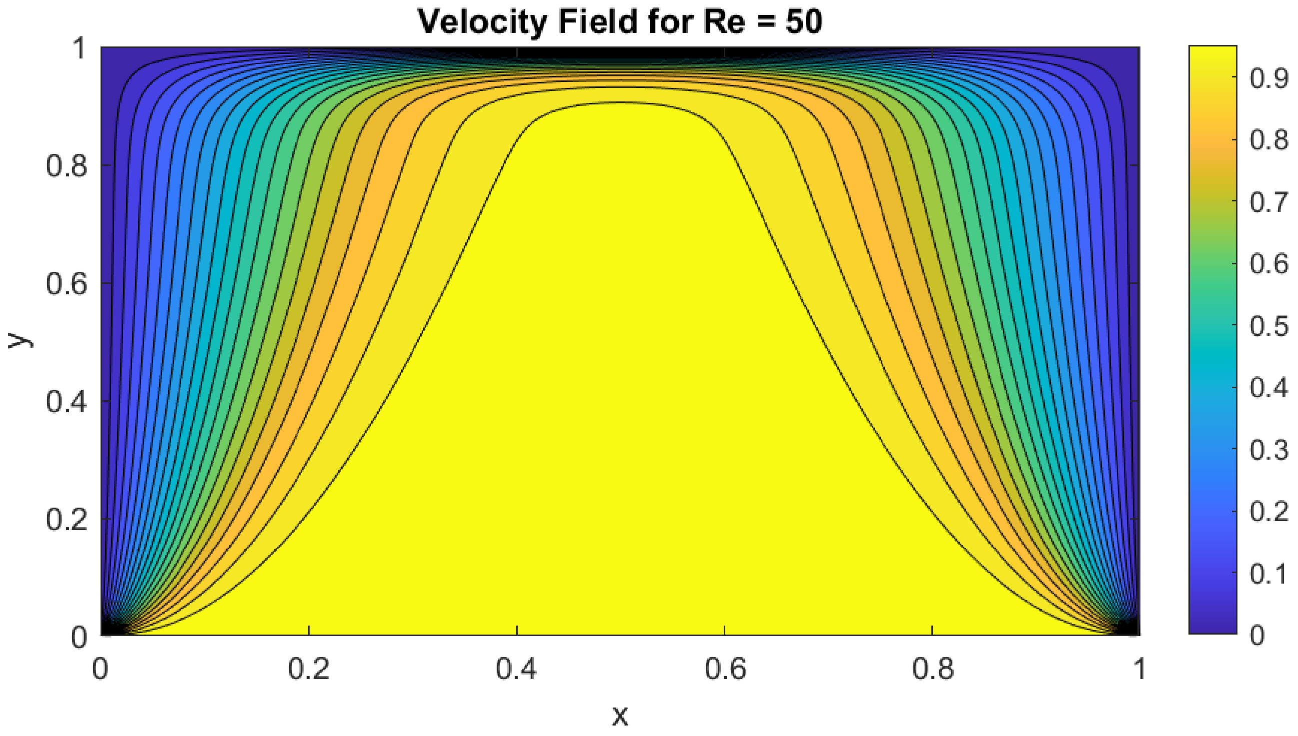
$$u_{i+1,j}^{(n)} \left\{ 1 - \frac{h \mathrm{Re}}{2} u_{i,j}^{(n-1)} \right\} + u_{i-1,j}^{(n)} \left\{ 1 + \frac{h \mathrm{Re}}{2} u_{i,j}^{(n-1)} \right\} + u_{i,j+1}^{(n)} \left\{ \gamma - \frac{\mathrm{Re}}{2} \gamma^{1/2} h v_{i,j}^{(n-1)} \right\} + u_{i,j-1}^{(n)} \left\{ \gamma + \frac{\mathrm{Re}}{2} \gamma^{1/2} h v_{i,j}^{(n-1)} \right\} - 2(1+\gamma) u_{i,j}^{(n)} = 0 \quad (53)$$

Graphics




Velocity Field for Re = 5
y
x
0
0.2
0.4
0.6
0.8
1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9


Velocity Field for Re = 10
y
x
0
0.2
0.4
0.6
0.8
1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9

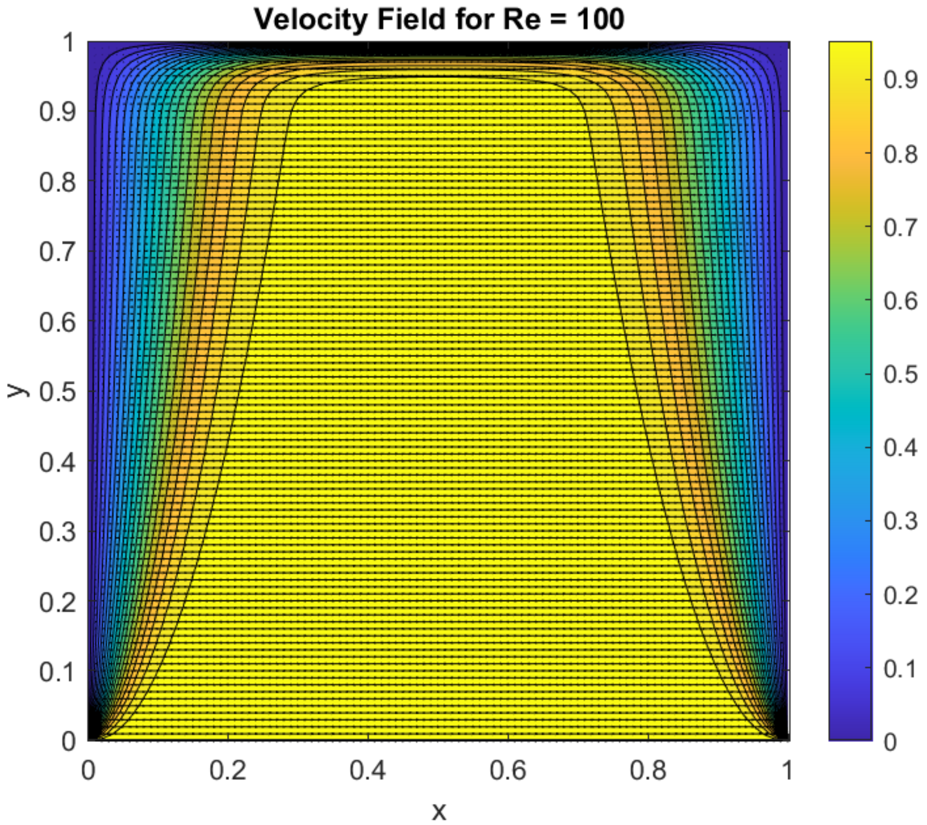


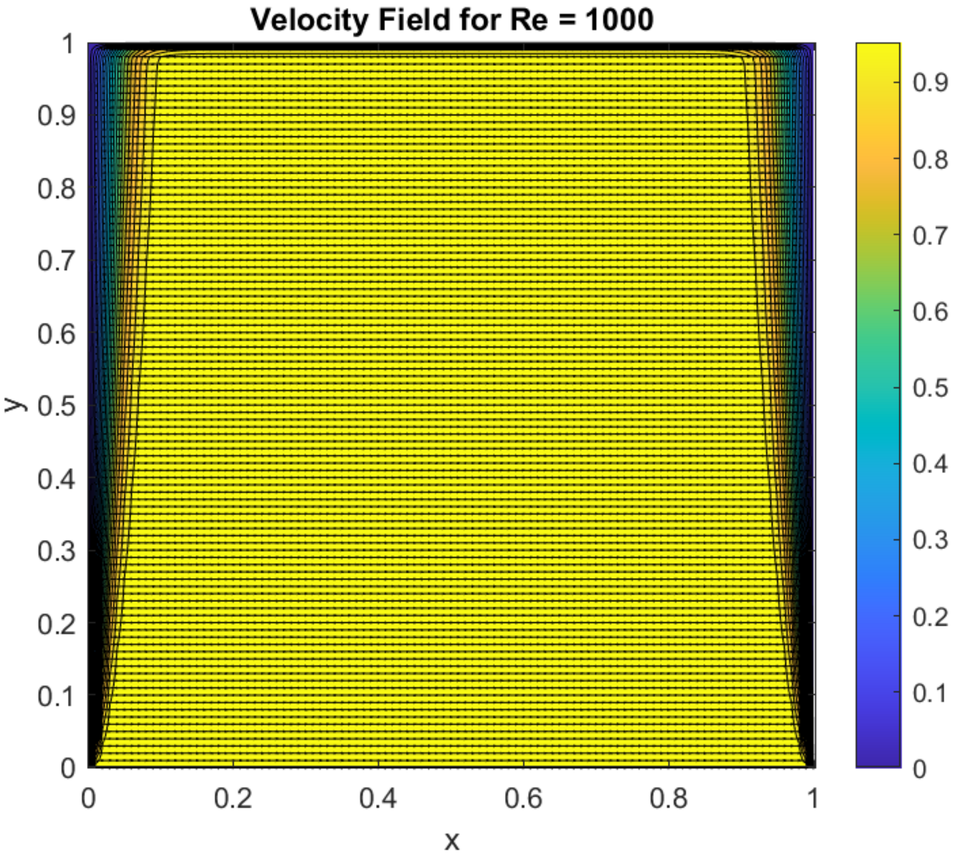
As can be seen from the graphs, as Re increases, the velocity also increases. Because increasing Re is associated with increasing velocities.

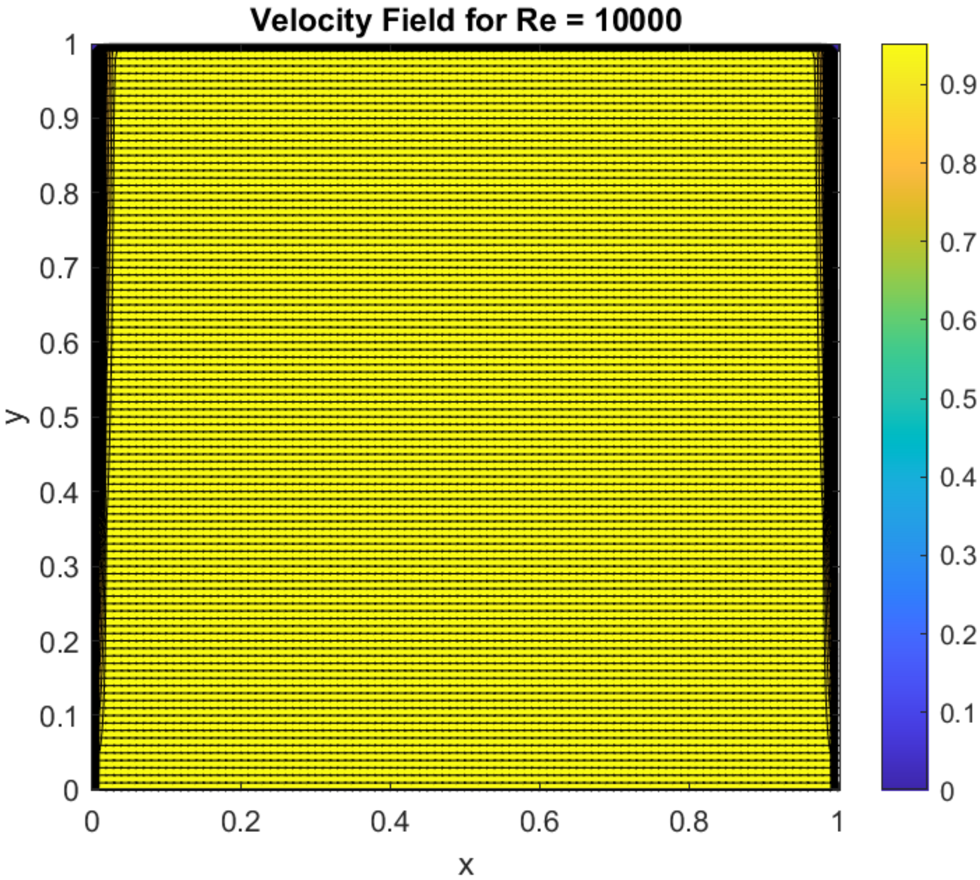
Upwind-downwind algorithm

As Re increases, it becomes difficult to reduce the mesh size while maintaining diagonal dominance. It is necessary to consider differences in local flow direction. So this algorithm was used to show higher Re values.

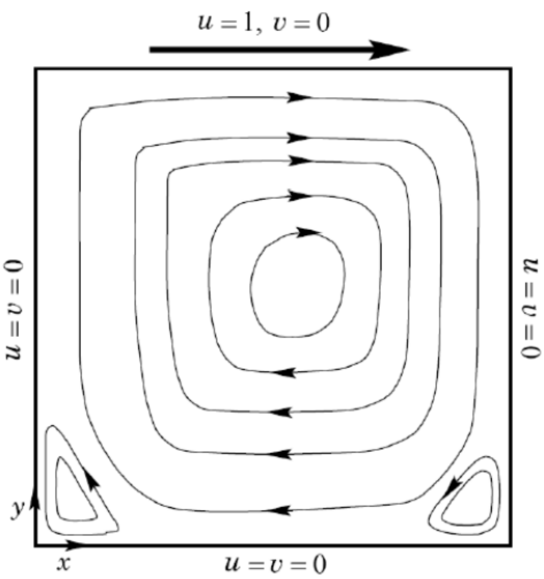
Graphics


Velocity Field for Re = 100
y
x
0
0.2
0.4
0.6
0.8
1
0.1
0.3
0.5
0.7
0.9


Velocity Field for Re = 1000
y
x
0
0.2
0.4
0.6
0.8
1
0.1
0.3
0.5
0.7
0.9



As the Re value increases, the velocity value begins to equal the value of the external flow everywhere. The physical state of the flow can be analyzed better with streamlines, but unfortunately there is not enough time for this. As an example, streamlines can be shown below.



Lid-Driven Cavity Problem

The 2-D vorticity transport equation shown below.

$$\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}=v\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial x^2}\right)$$

FTCS Scheme

If the equation is rewritten in accordance with the FTCS scheme.

$$\frac{w_{i,j}^{n+1}-w_{i,j}^{n}}{\Delta t}+u_{i,j}\,\frac{w_{i+1,j}^{n}\;-w_{i-1,j}^{n}}{2\Delta x}+u_{i,j}\,\frac{w_{i,j+1}^{n}\;-w_{i,j-1}^{n}}{2\Delta y}$$
$$=v\left(\frac{w_{i+1,j}^{n}\;-2w_{i-1,j}^{n}\;+w_{i-1,j}^{n}}{\Delta x^2}+\frac{w_{i,j+1}^{n}\;-2w_{i,j}^{n}\;+w_{i,j-1}^{n}}{\Delta y^2}\right)$$

The final equation,

$$w_{i,j}^{n+1}\;=\;w\_(i,j\,)\text{^}n+\Delta t[-u_{i,j}\frac{w_{i+1,j}^{n}\;-w_{i-1,j}^{n}}{2\Delta x}-\;u_{i,j}\;\frac{w_{i,j+1}^{n}\;-w_{i,j-1}^{n}}{2\Delta y}$$
$$+v\left(\frac{w_{i+1,j}^{n}\;-2w_{i-1,j}^{n}\;+w_{i-1,j}^{n}}{\Delta x^2}+\frac{w_{i,j+1}^{n}\;-2w_{i,j}^{n}\;+w_{i,j-1}^{n}}{\Delta y^2}\right)$$

Gauss-Seidel SOR

Here the main equation is that

$$\nabla^2\psi=-\Omega$$

If the equation discretizes using central difference method,

$$\frac{\psi_{i+1,j}-2\psi_{i,j}+\psi_{i-1\,,j}}{\Delta x^2}+\frac{\psi_{i,j-1}-2\psi_{i,j}+\psi_{i\,,j+1}}{\Delta y^2}=\;-w_{i,j}$$

$$\psi_{i,j}=\frac{1}{2\left(\frac{1}{\Delta x^2}+\frac{1}{\Delta y^2}\right)}\left[\frac{\psi_{i+1,j}+\psi_{i-1\,,j}}{\Delta x^2}+\frac{\psi_{i,j-1}+\psi_{i\,,j+1}}{\Delta y^2}-w_{i,j}\right]$$

Applying SOR method.

$$\psi^{new}{}_{i,j} = (1-w)\,\psi^{old}{}_{i,j} + \frac{w}{2\left(\frac{1}{\Delta x^2}+\frac{1}{\Delta y^2}\right)}\left[\frac{\psi_{i+1,j}+\psi_{i-1\,,j}}{\Delta x^2}+\frac{\psi_{i,j-1}+\psi_{i\,,j+1}}{\Delta y^2}-w_{i,j}\right]$$

The Poisson Equation

$$\nabla^2 p = 2p\left(\frac{\partial u}{\partial x}\frac{\partial v}{\partial y}-\frac{\partial u}{\partial y}\frac{\partial v}{\partial x}\right)$$

u and v equation are needed, here is another main function about streamlines.

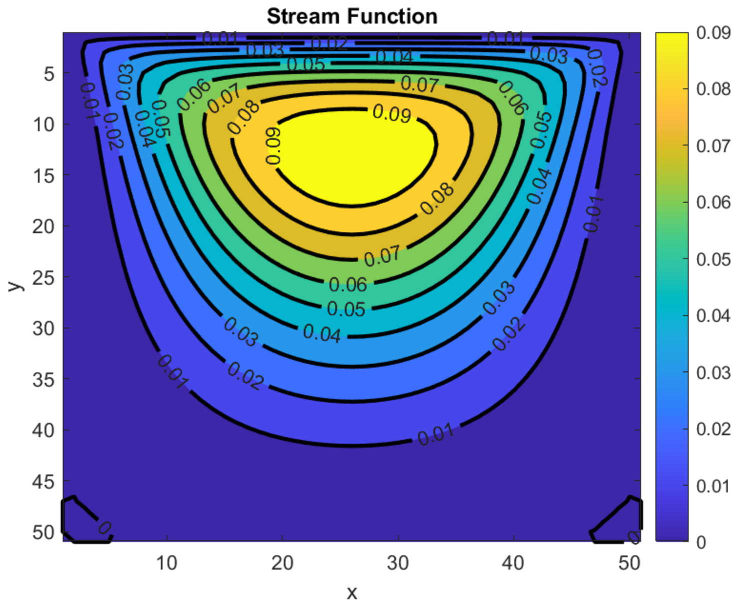
$$u = \frac{\partial \psi}{\partial y} = 0$$

$$v = -\frac{\partial \psi}{\partial x} = 0$$

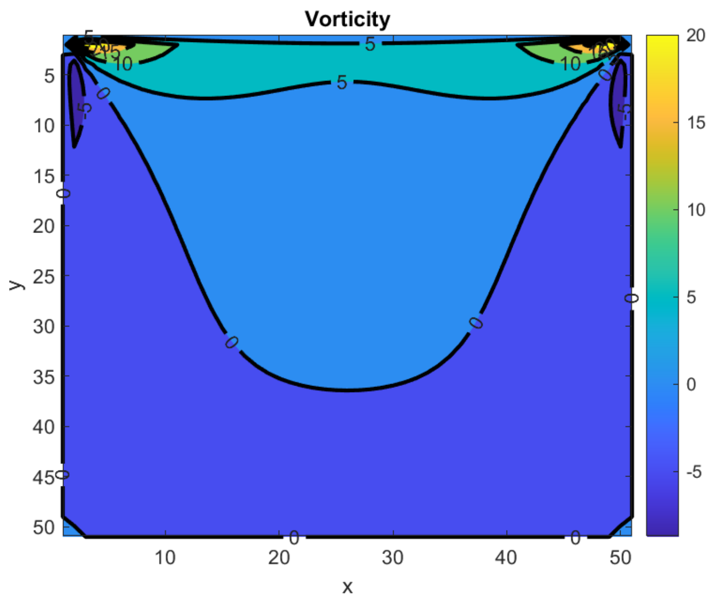
If u and v are discriminated with central difference and replaced in the Poisson equation, the final equation is:

$$\frac{p_{i+1,j}-2p_{i,j}+p_{i-1,j}}{\Delta x^2}+\frac{p_{i,j-1}-2p_{i,j}+p_{i,j+1}}{\Delta y^2} = 2p\,(\frac{u_{i+1,j}-u_{i-1,j}}{2\Delta x}\frac{v_{i,j+1}-v_{i,j-1}}{2\Delta y}-\frac{u_{i,j+1}-u_{i,j-1}}{2\Delta y}\frac{v_{i+1,j}-v_{i-1,j}}{2\Delta x}$$

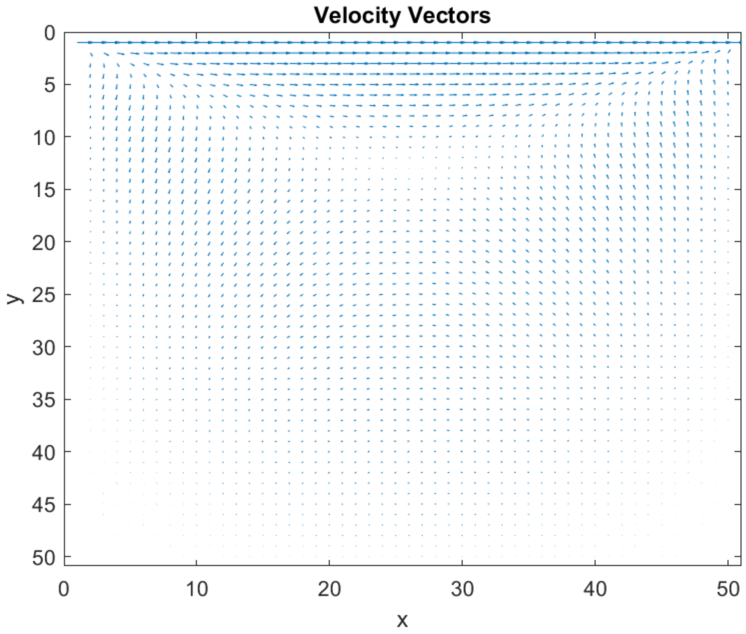
Graphics



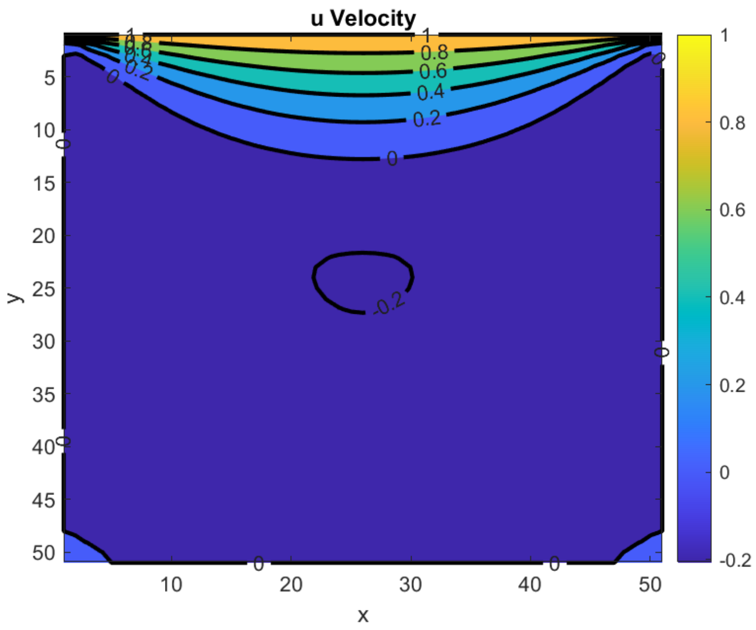
The contour plot of the stream function reveals intricate flow patterns, showcasing how fluid elements navigate through the domain. The dense concentration of contour lines in certain areas hints at regions of intense flow, while sparse lines suggest calmer regions or areas of stagnation.



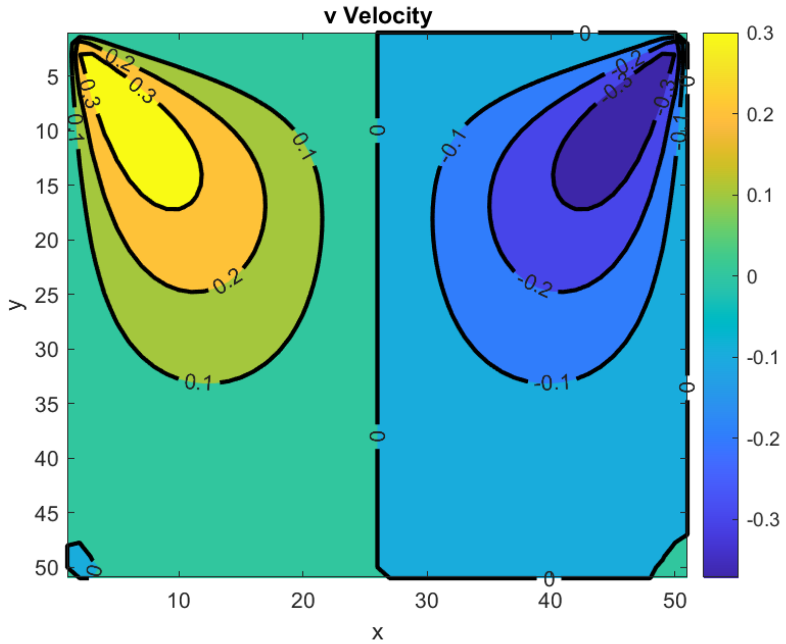
Through the vorticity contour plot, one can discern areas where fluid rotation is prominent, unveiling the dynamic behavior of the fluid. The swirling patterns depicted by the contours unveil the presence and locations of vortices, shedding light on the complexity of fluid motion.



The velocity vector plot offers a visual narrative of how fluid velocity varies across the domain, painting a picture of the fluid's journey. The arrangement and length of arrows depict the strength and direction of flow, offering insights into the flow's behavior at different points.



Examining the contour plot of the u-velocity component, one can discern regions of predominant horizontal flow, providing clues about how momentum is distributed within the fluid. Variations in contour density expose regions of acceleration or deceleration along the horizontal axis.



The contour plot of the v-velocity component uncovers the vertical flow dynamics within the domain, offering a glimpse into how fluid parcels ascend or descend. Changes in contour intensity hint at areas of upward thrust or downward pull, enriching our understanding of vertical fluid motion.

Reference List

Lecture notes, Çadırcı, S.