Problem Definition

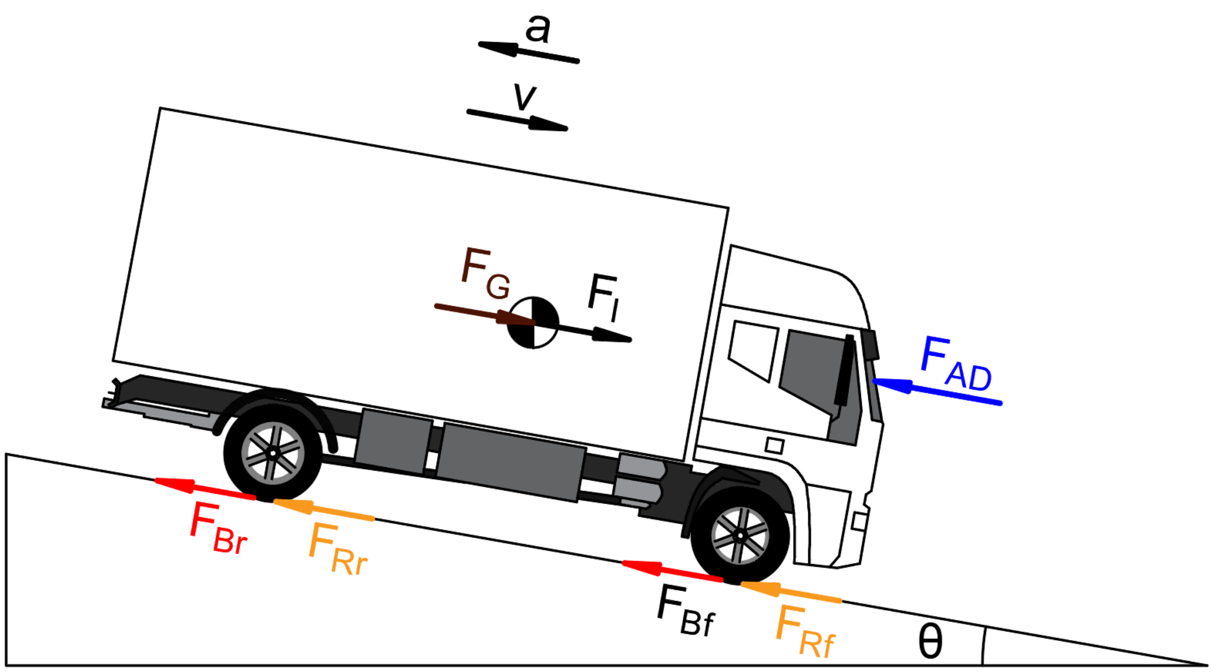

Figure 1: FBD of a decelerating truck on a downhill road.

Figure 1 shows the FBD (Free body diagram) of a decelerating truck, whose deceleration is **a** and speed is **v,** on a downhill road. The forces acting on the truck on its **longitudinal** axis are named, described and formulated in **Table 1**.

Table 1: Description of the forces in FBD.

Abbreviations	Description	Formula	
F_I	Inertial force	$F_I = \lambda ma$	(1)
F_G	Gravitational force	$F_G = mgsin\theta$	(2)
F_{AD}	Aerodynamic drag force	$F_A = \frac{1}{2}\rho C_D A_p v^2$	(3)
*F_{Bx}	Braking force	Not necessary for this problem.	(4)
*F_{Rx}	Rolling resistance force	$F_R = F_{Rf} + F_{Rr} = f_R\, mgcos\theta$	(5)

* x:f for front, x:r for rear axles.

Fort his truck, you are supposed to make thermal analysis of the installed braking system for the scenarios described below. The heat analysis will be made using the so called "Lumped Capacitance Model" described in [1] detailly. This model, treats a body as a point mass which can have a heat generation inside of it, and it can have heat transfer via convection/radiation with its surroundings.

A vehicle brake system utilizes the braking pedal force applied by the driver via a mechanism (which can include mechanical, electrical, hydraulic or pneumatic components). This force is converted to a frictional force on the braking system (between brake pads and discs for a disc brake system and between drums and shoes on a drum brake system) and finally between the tires and the ground to decelerate or stop the vehicle. You can see [2] to better understand vehicle brake system's working principle visually. There is also a course named "MAK 4057E : Vehicle Systems Design" lectured in Automotive Laboratory which covers braking systems in detail.

The friction between the brake system components creates heat which increases their temperature. The heat rate (power) generated on a braking system can be calculated with equation (5) as following:

$$P_B = F_B v \tag{5}$$

Where $\mathbf{F_B}$ is the total braking force, **v** is the truck's speed and $\mathbf{P_B}$ is the total heat generation occurring on the brake discs.

Then, using the lumped capacitance model and the "Newton's Law of Cooling/Convection", the heat balance equation for a disc brake system can be written as in equation (6):

$$m_b c \frac{dT}{dt} + hA(T - T_a) = P_B \tag{6}$$

Where **T** is the temperature of the braking discs, **A** is the cooling surface area of the braking discs, $\mathbf{T_a}$ is the ambient temperature, $\mathbf{m_b}$ is the mass of the braking discs, **c** is the specific heat value of the braking discs and **h** is the coefficient of heat convection. You can use this equation separately for front and rear axle systems by considering the brake force distribution mentioned below.

All values for the necessary parameters are given in **Table 2**.

1) (80 pts.) Scenario 1 : Constant speed on downhill road.

Assumptions & Simplifications:

- Heat convection occurs only between the brake discs and the ambient.
- **h** is assumed to be constant. (not affected by the disc rotational speed)
- **Neglect** aerodynamic drag/lift forces and pitch moments.
- There is no engine/retarder brake applied.
- **Initial** temperature of the brake discs is equal to **ambient temperature**.
- ABS system is working, there is no slip/lock on any tire of the truck. The normal force between the tires and the ground carries all the force applied on the braking system.

In this scenario the truck driver is applying brake to keep its speed constant at speed $\mathbf{v_0}$ on a downhill road for a distance of 6000 m.

a) (10 pts.) Using the FBD in Figure 1, calculate the necessary braking force to keep the truck's speed constant at $\mathbf{v_0}$ during the motion.

b) (5 pts.) Calculate the total heat energy rate (power) generated on the braking system. Assume that the installed braking system on truck applies 60% of the braking force on the front axle and 40% of the braking force on the rear axle. Calculate the power dissipated on both axles.

c) (25 pts.) Solve the heat equation given in (6) **analytically** for T and plot brake disc temperatures of the front and rear axles with respect to time on the same graph.

d) (30 pts.) Solve the heat equation given in (6) using the classical (4^{th} order) **Runge-Kutta** method. Plot brake disc temperatures of the front and rear axles with respect to time on the same graph.

e) (10 pts.) Compare the analytical and numerical method results an elaborate on them.

2) (20 pts.) Scenario 2 : Emergency brake (full stop) on a **flat** road ($\theta = 0$ deg)

Assumptions & Simplifications:

- Heat convection occurs only between the brake discs and the ambient.
- **h** is assumed to be constant. (not affected by the disc rotational speed)
- **Neglect** aerodynamic lift force and pitch moments.
- Aerodynamic **drag** force **is not** neglected.
- There is no engine/retarder brake applied.

- Initial temperature of the brake discs is equal to ambient temperature.

In this scenario, while cruising at a speed of $\mathbf{v_{flat}}$, driver applies an emergency brake that decelerates the truck with a **constant** deceleration of $\mathbf{a_{target}}$ until the truck stops.

Be aware of those:

- You will have inertial force in your longitudinal dynamic equation because of the deceleration!
- Aerodynamic force is a function of vehicle speed, you need to recalculate its value on every time step!
- The power dissipation on the discs $\mathbf{P_B}$ will no more be constant! You need to recalculate its value on every time step!
- Since the vehicle speed is a function of time **t,** so the aerodynamic drag, be careful to calculate $\mathbf{k_1}$, $\mathbf{k_2}$, $\mathbf{k_3}$ and $\mathbf{k_4}$ terms for Runge-Kutta method!

a) (15 pts.) Calculate the brake disc temperatures using the classical (4th order) **Runge-Kutta** method and plot the values with respect to time on the same graph.

b) (5 pts.) Elaborate on the use of Runge-Kutta method on this part (Scenario 2) of the problem.

Table 2: Necessary parameters and constants.

Parameter	Description	Value	Unit
m	Total mass of the truck	10.000 + id_500*	[kg]
λ	Rotational inertia coefficient	1,3	[-]
g	Gravitational acceleration coefficient	9,80665	[m/s^2]
$\mathbf{v_i}$	Initial speed of Scenario 1	36	[km/h]
$\mathbf{v_{flat}}$	Initial speed of Scenario 2	60	[km/h]
θ	Grade angle for Scenario 1	7	[deg]
A	Cooling area of the brake discs per axle, same for both axles.	0,8	[m^2]
c	Specific heat of brake discs per axle, same for both axles.	500	[J/kgK]
h	Coefficient of convection, same for both axles.	60	[W/m^{2}K]
$\mathbf{T_a}$	Ambient temperature	20	[^{o}C]
$\mathbf{m_b}$	Mass of the brake discs per axle, same for both axles.	30	[kg]
$\mathbf{f_R}$	Coefficient of rolling resistance	0,01	[-]
$\mathbf{A_p}$	Projection area of the truck	5	[m^2]
$\mathbf{C_D}$	Aerodynamic drag coefficient	0,5	[-]
ρ	Air density	1,2	[kg/m^3]
$\mathbf{a_{target}}$	Deceleration value for Scenario 2	6	[m/s^2]

*id_500 : 500 x last digit of your student ID number.

References

[1] Bergman et al. (2011) *Fundamentals of Heat and Mass Transfer*. (7th edition). NJ: Wiley, 2011

[2] Url-1 <https://www.youtube.com/watch?v=MAuVDB-G-HQ&ab_channel=Autotechlabs>, Retrieved in 08.05.2023