HW-4

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DERS ADI : COMPUTATIONAL FLUID DYNAMICS

DERS YÜRÜTÜCÜSÜ : SERTAÇ ÇADIRCI

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The inviscid Burgers Equation solution with Mac Cormack method

What is the Mac Cormack Method?

According to the Bernard, MacCormack's method, renowned for over two decades, is a finite-difference technique widely employed in solving compressible flow and various other scenarios. It offers both explicit and implicit variants, with the former predating the latter by over a decade, marking a significant milestone in computational fluid dynamics. Both versions operate by advancing in time, enabling solutions to hyperbolic and parabolic equations. The explicit approach, particularly favored for its simplicity and straightforward implementation, employs forward differencing for time derivatives in both prediction and correction phases. This characteristic proves advantageous for equations featuring nonlinear advective terms, eliminating the necessity for Jacobian matrices typical in one-step explicit schemes like that of Lax & Wendroff. However, conventional time-marching necessitates the presence of a time derivative in each equation to be tackled (1992).

What is the Inviscid Burgers Equation?

The inviscid Burgers' equation is a fundamental model in fluid dynamics, capturing the essential dynamics of one-dimensional, inviscid flow. Formulated as a nonlinear partial differential equation, it describes how fluid velocity evolves over time and space. The equation's simplicity belies its significance, as it exhibits complex behavior such as shock wave formation and wave steepening. Introduced as a simplification of the Navier-Stokes equations, it serves as a cornerstone in the study of hyperbolic conservation laws and nonlinear wave propagation.

Discretization

$$u(x,0)=\begin{cases} 1 & , \quad x<0.25 \\ 1.25-x & , \quad 0.25\le x\le 1.25 \\ 0 & , \quad x>1.25 \end{cases}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}=0 \qquad \begin{matrix} u(0.0,t)=1.0 \\ u(4.0,t)=0.0 \end{matrix}$$

$$\text{hint: } E=\frac{u^2}{2}$$

for $0 \leq x \leq 4.0$

Predictor step:

$$u_i^p = u_i^j - \frac{dt}{dx} * 0.5 * ((u_{i+1}^j)^2 - (u_i^j)^2)$$

Corrector step:

$$u_i^{j+1} = 0.5 * \left(u_i^j + u_i^p - \frac{dt}{dx} * 0.5 * \left((u_i^p)^2 - (u_{i-1}^p)^2\right)\right)$$

Graphics

a.) Δt = 0.01

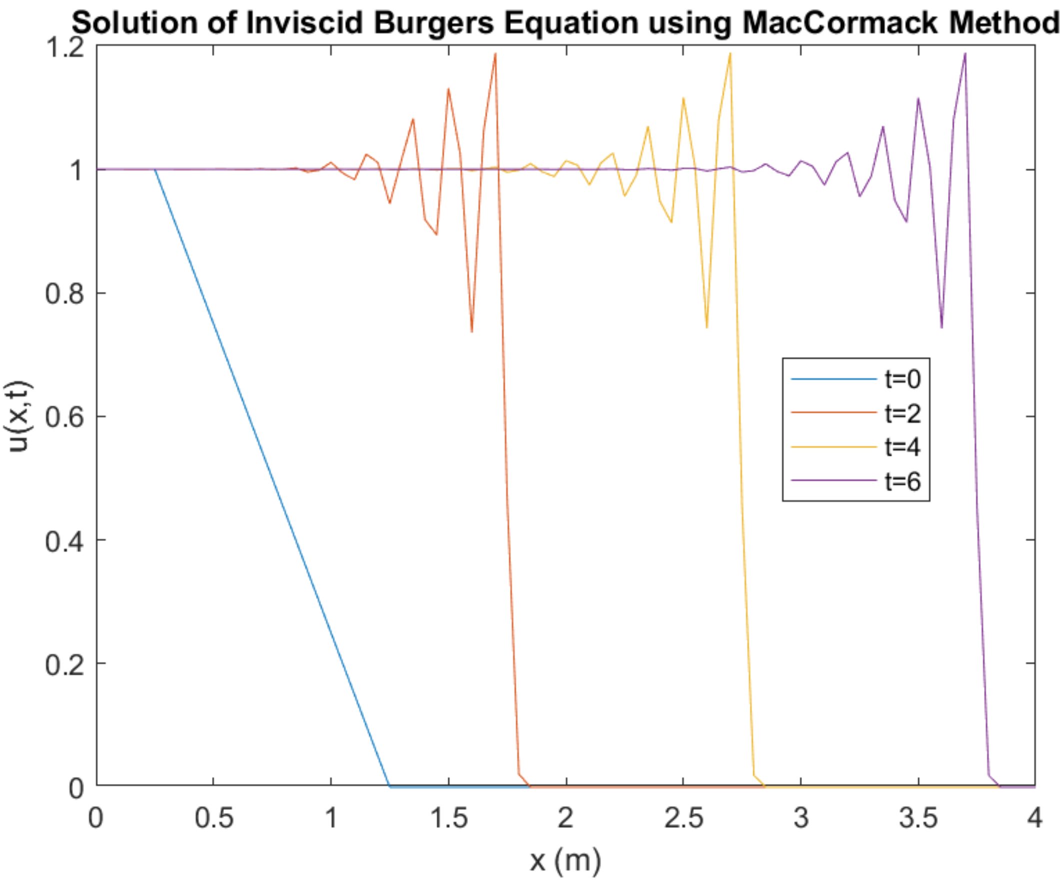

Figure 1. for Δt = 0.01

b.) Δt = 0.025

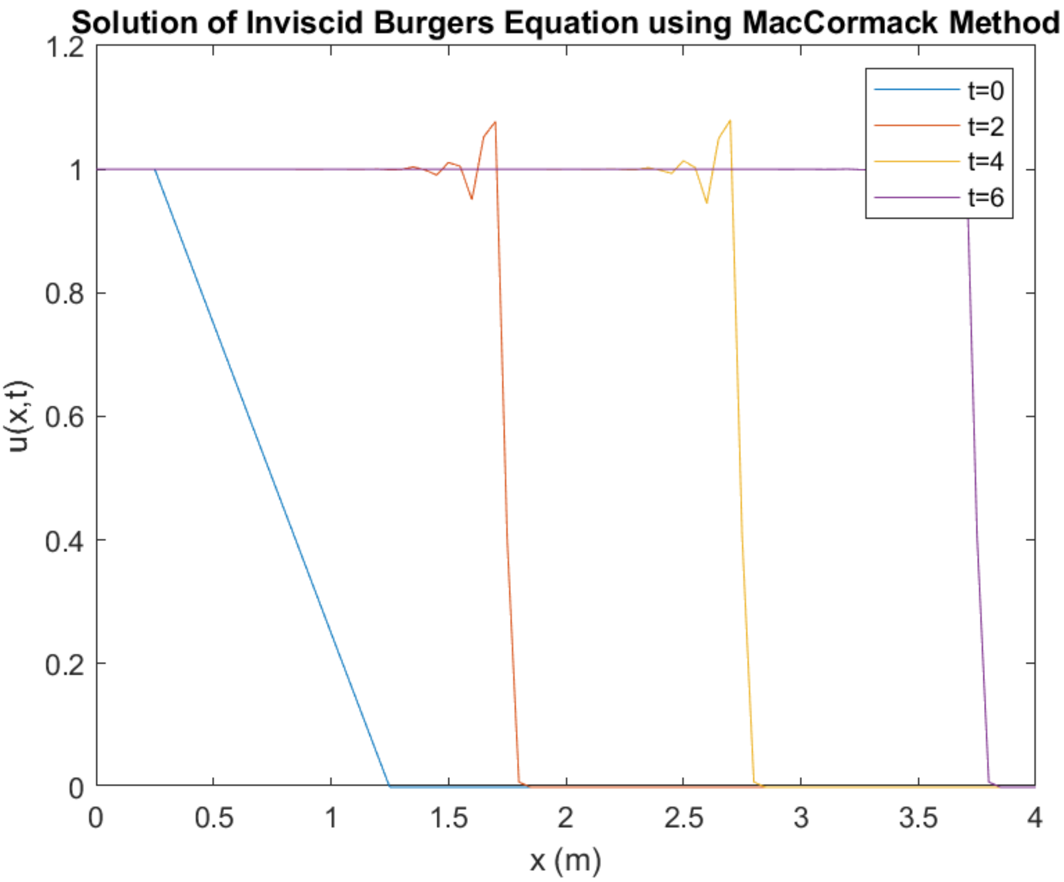

Figure 2. for Δt = 0.025

c.) Δt= 0.1

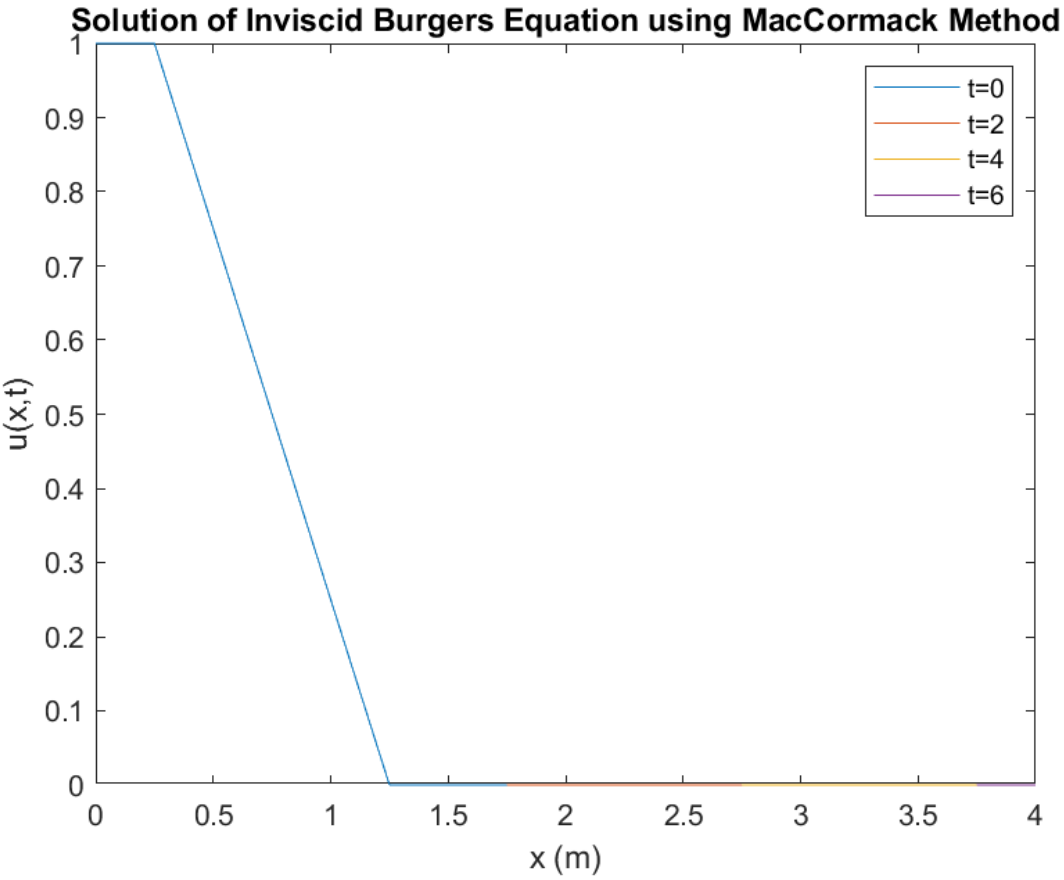

Figure 3. for Δt = 0.1

Conclusion

As time progresses, the solution develops a wave, which is a discontinuity in the solution. The wave propagates to the right, and the solution behind the wave becomes steeper. The Mac Cormack method is able to capture the wave formation reasonably well, but it does introduce some numerical dissipation, which is the artificial smearing of the wave.

CFL condition

$$C = u\frac{\Delta t}{\Delta x} \leq 1 \qquad \#Courant\ Number$$

This condition should be satisfied.

In this problem, because of $\Delta x = 0.05$, Δt can be maximum 0.05. If Δt is above 0.05, the graph is not stable, see Figure 3.

Second-order wave equation using the Lax-Wendroff method.

What is the Lax-Wendroff Method?

The Lax-Wendroff method is a numerical approach for solving hyperbolic partial differential equations, which are common in wave propagation problems. It utilizes finite differences to approximate derivatives, offering second-order accuracy in both space and time. As an explicit method, it calculates solutions at the next time step based on current data, making implementation straightforward. With its compact stencil requiring information from only three grid points, it's computationally efficient. However, it can suffer from stability issues and introduce dispersion, affecting accuracy in wave propagation problems. Despite these drawbacks, it remains a widely used technique due to its balance between simplicity and accuracy, serving as a benchmark for higher-order finite difference schemes.

Discretization

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}; \quad 0 < x < 1 \quad 0 < t$$

Below is the version of the equation to be used in the program.

$$u_i^{j+1} = u_i^j + \frac{dt}{2 * dx}\left(u_{i+1}^j - u_{i-1}^j\right) + \frac{dt^2}{2 * dx^2} * (u_{i+1}^j - 2 * u_i^j + u_{i-1}^j)$$

Graphics

Analytical solution

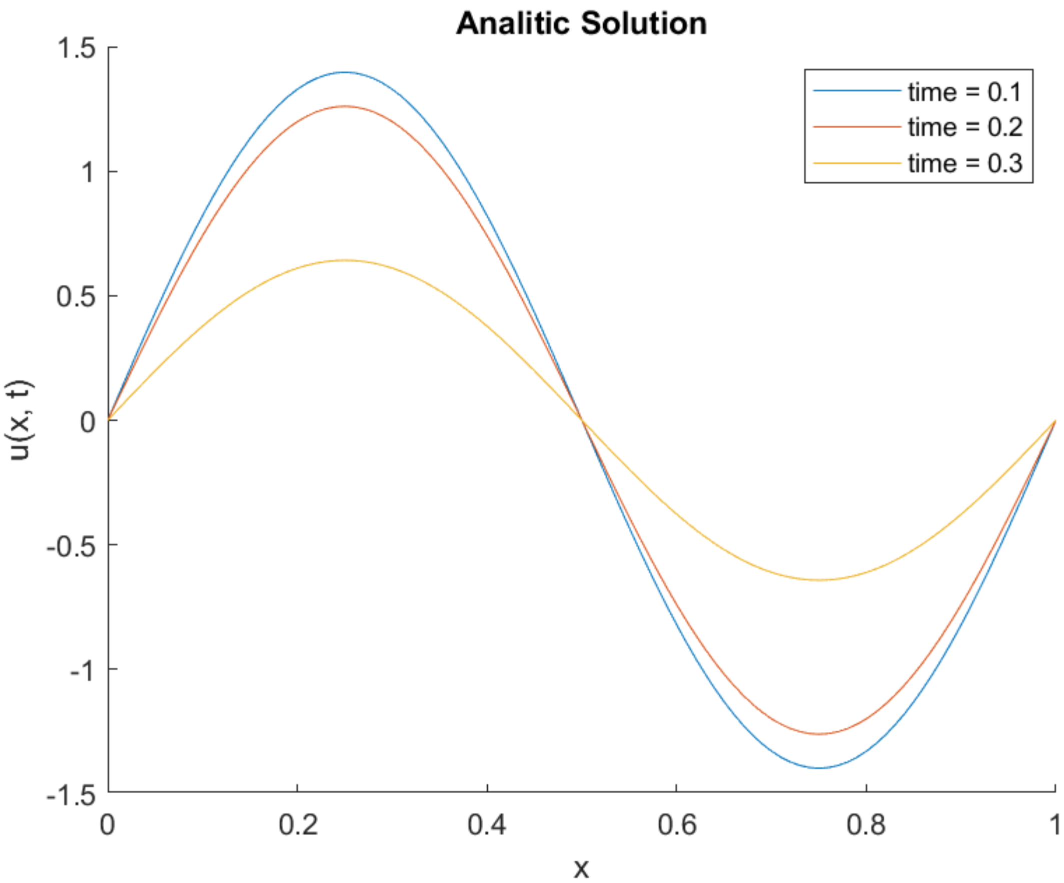

Figure 4. The graphs of the analytical solution.

Numerical Solution

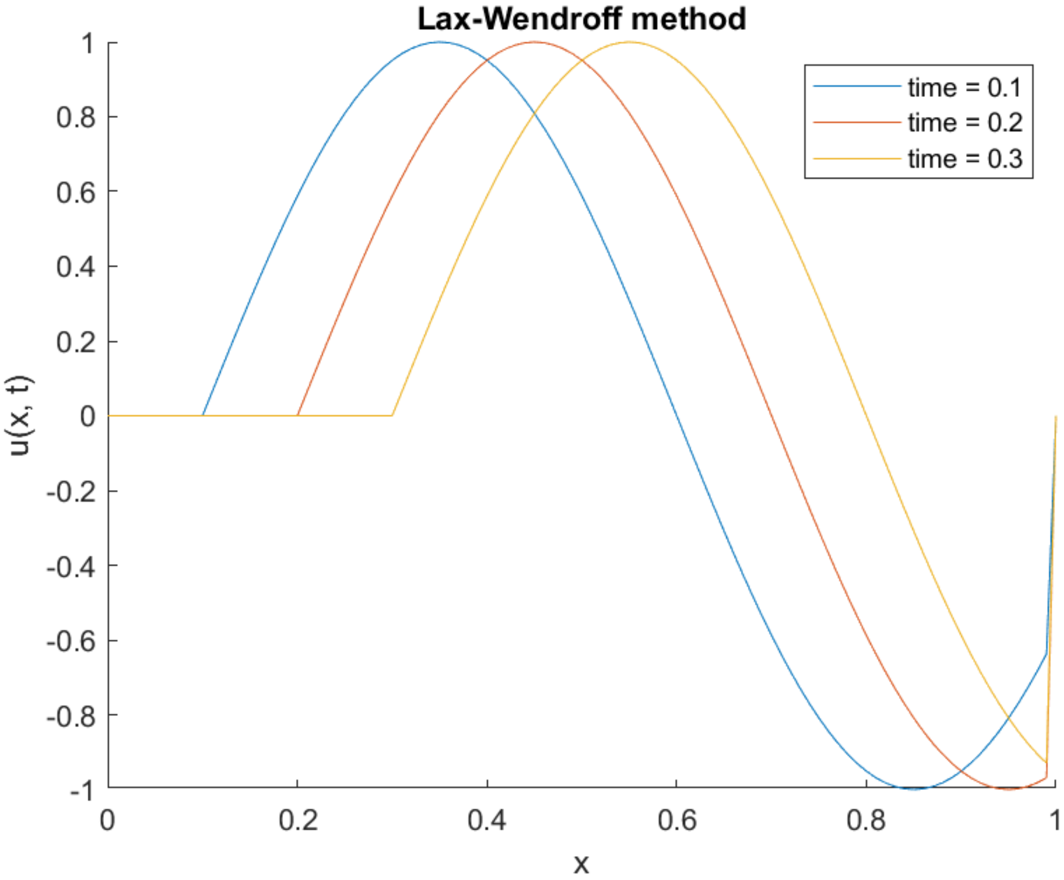

Figure 5. The graphs of the numerical solution.

Conclusion

As can be seen from the graphs, when the solution was made with the LaxWendroff method, it was seen that the value of the graphs was 0 from the beginning to the time step. It can be said that there is a problem with the initial or boundary conditions here. Apart from these, the general situation of the graphics is similar.

Reference List

Bernard, R. S. (1992). A MacCormack scheme for incompressible flow. *Computers & Mathematics With Applications*, *24*(5–6), 151–168. https://doi.org/10.1016/0898-1221(92)90046-k

CFD Lecture Notes, Çadırcı, S.

R.W. MacCormack, A numerical method for solving the equations of compressible viscous flow, AIAA J. 20 (9), 1275-1281 (1982).