Motivation

This homework includes a very limited knowledge used/given in your coursework. However, it is planned to give you an insight to solve real engineering problems and show you the versatility of numerical approaches to them. Using numerical approach instead of analytical approach provides more flexibility and complexity to the problem while making its solution possible. The homework will also force you to use programming/scripting languages such as Python or MATLAB which will be very helpful for your next course's assignments too.

Since all of those above, most of the knowledge, equations, definitions etc. are already given in the "**Problem Definition**" part. Your work is to implement this knowledge to a simple computer program which will have around 100-120 code lines.

Good luck!

Problem Definition

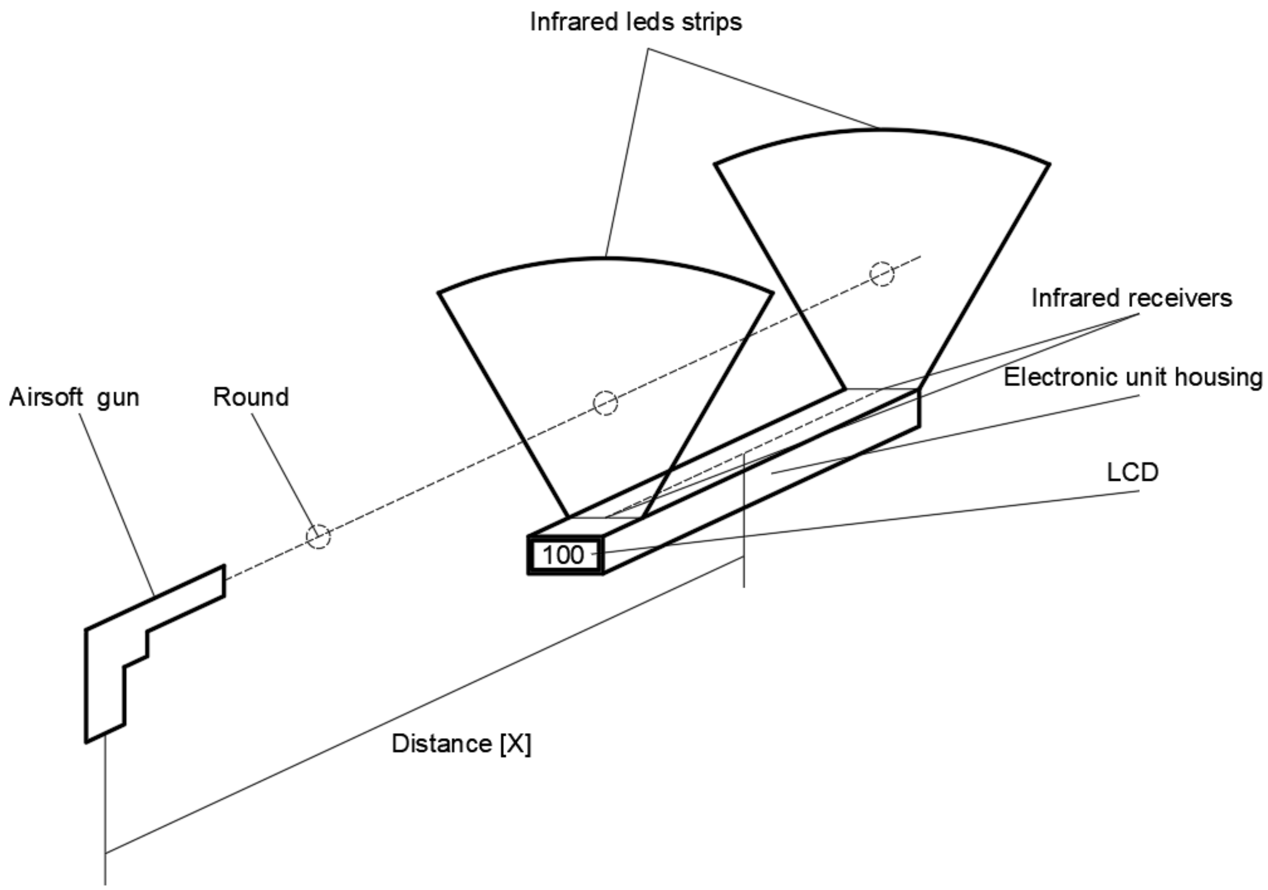

Figure 1: Infrared radiation based projectile speed measurement system scheme.

Small arm round speeds can be measured using infrared radiation-based measurement units. Figure 1 shows this kind of an experiment setup. In this setup; after the gun is fired, the round first passes through the gap between the first led strip and the infrared receivers. During that passage, the amount of light received at the receivers is decreased. Round continues to move and the same thing occurs at the gap between the second led strip and the infrared receivers. Since the round has a finite speed, there is a time gap between the light decrement of the first and the second strips which can be used to calculate the average speed of the round between the strips. The used sensors are generally sensitive to light conditions and prone to have measurement errors.

With a similar setup with Figure 1, an airsoft pistol's round speeds at different distances (@1,6,11 and 16 meters) to the pistol after they are fired are measured as in Table 1.

Table 1: Measured round longitudinal speeds at different distances.

Shot	**v_x [m/s]**			
	@ 1 meter	**@ 6 meters**	**@ 11 meters**	**@ 16 meters**
1	109.5	90.0	76.8	65.2
2	110.0	88.5	73.6	64.0
3	109.3	88.0	74.6	63.6
4	112.4	88.5	76.2	-
5	108.0	87.5	76.1	-
6	112.7	88.5	81.0	-
7	107.4	87.9	75.9	-
8	106.9	89.6	74.9	-
9	105.2	91.0	76.4	-
10	106.9	88.9	78.9	-

Airsoft guns, use spherical rounds made of plastic material. The rounds are mainly subjected to aerodynamic and gravitational forces during their flight. The measurement above is made in order to predict the drag coefficient (C_D) of the airsoft rounds. After this, the necessary parameters for a gas or an electric mechanism to provide a good range while providing safe round velocity at target person to prevent severe injury can be found. The kinematic and dynamic FBD (Free body diagram) of the airsoft round is given in Figure 2.

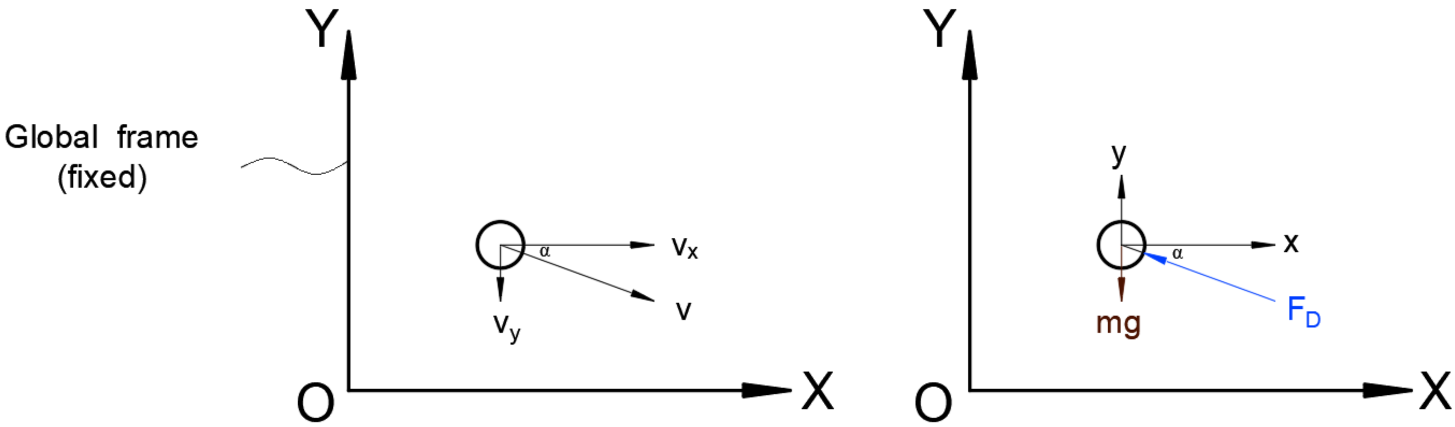

Figure 2: Kinematic (left) and dynamic (right) FBD of the problem.

Aerodynamic drag force F_D is modelled as a second order equation given in (1):

$$F_D = \frac{1}{2}\rho C_D A_r v^2 \tag{1}$$

Where $\boldsymbol{\rho}$ is the air density that will be calculated using the test room absolute pressure, temperature and relative humidity measurements, $\boldsymbol{A_r}$ is the projection area of the spherical rounds and $\boldsymbol{v}$ is the round speed. Necessary initial conditions and parameters are given in Table 2.

Although some physical phenomenon is included in the model described till here, a lot of them are also neglected for the sake of simplicity and possibility of the solution:

- The air is windless for the duration of motion.
- "Magnus effect" is neglected, rounds do not have spinning motion which causes lift or side forces to their body.

Question

An airsoft gun whose barrel is being held parallel to ground is fired at predetermined distances through a measurement system, whose results are presented in Table 1.

a) **(10%) Plot** the round's longitudinal velocity (v_x) versus its distance from the pistol using the experiment data. Fit a 2nd order polynomial curve to your graph and estimate the initial velocity of the round with respect to your fitted curve. You can get **10 extra points** if you use your own code for curve fitting.

b) **(20%) Write down** and **classify** the equations of planar motion of an airsoft round using FBDs in Figure 2.

c) **(30%)** Use **Euler forward formula** to solve the system whose longitudinal velocity values (v_x) at given experimental distances must be consistent with the measurements. Determine your own time step for your solver program**.** Make an initial guess for the drag coefficient using literature. Find the correct value of the aerodynamic drag coefficient either by:

- **I.** Trial and error
- **II.** or determining an error metric between the measurement and your model solution, then minimizing that error by changing your drag coefficient estimation programmatically. You can get **10 extra points** if you use this method.

Give error metrics of your solution with respect to the measurements.

d) **(20%)** Plot

- **I.** Both experimental and your model's longitudinal velocity results versus distance on the same graph.
- **II.** Trajectory of the airsoft round.

e) **(10%)** State the range of the pistol and the final velocity of the round. Calculate the energy of the round when it hits to a target at a distance of 20 meters.

f) **(10%)** Elaborate on your results. (C_D and calculated energy on target.)

Table 2: Parameters and initial conditions for the solution.

Parameter	Definition	Value	Unit
$\mathbf{m_{round}}$	Mass of the round.	(20 + id_1*) E-2	[g]
d	Diameter of the round.	6	[mm]
T	Temperature of the test room.	16	[ºC]
ϕ	Relative humidity of the test room air.	58	[%]
p	Absolute pressure of the test room.	1024	[mbar]
$\mathbf{X_0}$	Initial position of the round. (wrt. Global frame)	0	[m]
$\mathbf{Y_0}$		1.5	[m]

* id_1 : last digit of your student ID number.