THERMAL ANALSIS OF FRICTION BRAKES

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Lecture Name : **NUMERICAL METHODS**

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Course CRN : **22194**

PROBLEM DEFINITION

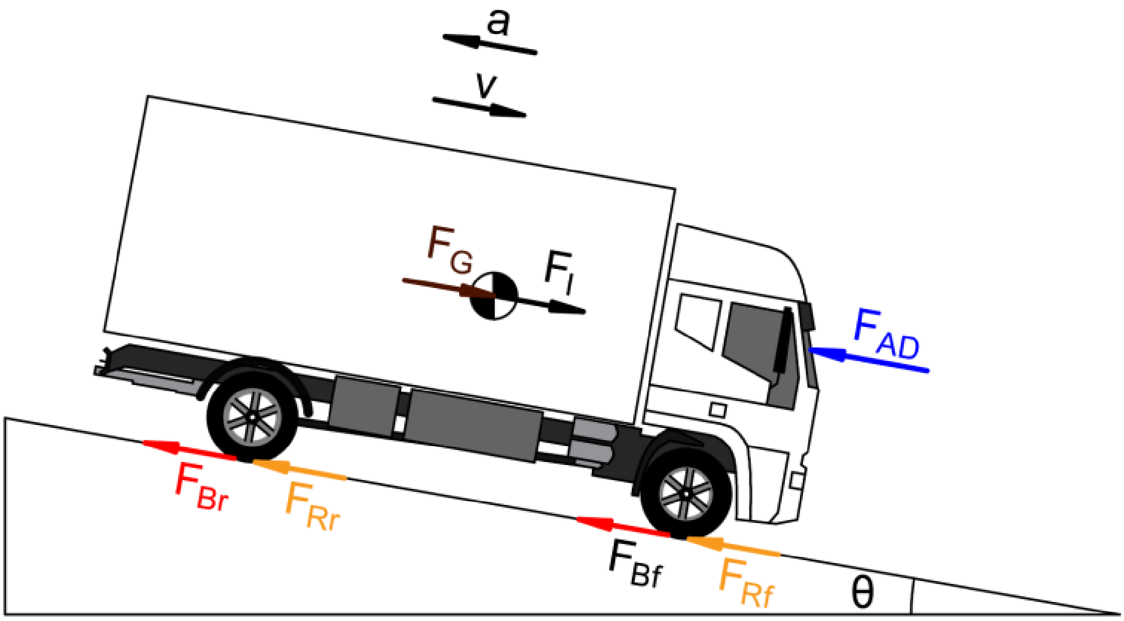

Figure 1: FBD of decelerating truck on a downhill road.

A vehicle's brake system operates by converting the force applied to the braking pedal by the driver, using a mechanism that may include mechanical, electrical, hydraulic, or pneumatic components. This force is then transformed into a frictional force within the braking system. In the case of a disc brake system, this frictional force occurs between the brake pads and discs; moreover, this force creates heat which increases the temperature of the brake system components. Finally, this force is transferred between the tires and the ground to decelerate or bring the vehicle to a stop.

Figure 1 demonstrate a model for the problem. Here, the forces acting on the truck could be seen easily. There are two scenarios for this problem, one is slope angle θ equals 7 degree and the other one is no slope. Also, although in the first scenario truck will go at constant speed, in the second scenario it will stop.

Parameter	Definition	Value	Unit
m	Total mass of the truck	14000	[kg]
λ	Rotational inertia coefficient	1.3	none
g	Gravitational acceleration coefficient	9.80665	[m/s^2]
v_i	Initial speed of Scenario 1	36	[km/h]
v_{flat}	Initial speed of Scenario 2	60	[km/h]
θ	Grade angle for Scenario 1	7	[deg]
A	Cooling area of the brake discs per axle, same for both axles.	0.8	[m^2]
c	Specific heat of brake discs per axle, same for both axles	500	[J/kgK]
h	Coefficient of convection, same for both axles.	60	[W/m^{2}K]
T_a	Ambient temperature	20	[°C]
m_b	Mass of the brake discs per axle, same for both axles	30	[kg]
f_R	Coefficient of rolling resistance	0.01	none
A_p	Projection area of the truck	5	[m^2]
C_D	Aerodynamic drag coefficient	0.5	none
ρ	Air density	1.2	[kg/m^3]
a_{target}	Deceleration value for Scenario 2	6	[m/s^2]

Table 1: Parameters and initial condition for the solution.

SCENARIO-1

There are some assumptions and simplifications for this scenario. These are:

- Heat convection occurs only between the brake discs and the ambient.
- h is assumed to be constant. (Not affected by the disc rotational speed)
- Neglect aerodynamic drag/lift forces and pitch moments.
- There is no engine/retarder brake applied.
- Initial temperature of the brake discs is equal to ambient temperature.
- Abs system is working, there is no slip/lock on any tire of the truck. The normal force between the tires and the ground carries all the force applied on the braking system.
- In this scenario the truck driver is applying brake to keep its speed constant at speed v0 on a downhill road for a distance of 6000 m.

A) Using the FBD in Figure 1, calculate the necessary braking force to keep the truck's speed constant at v_0 during the motion.

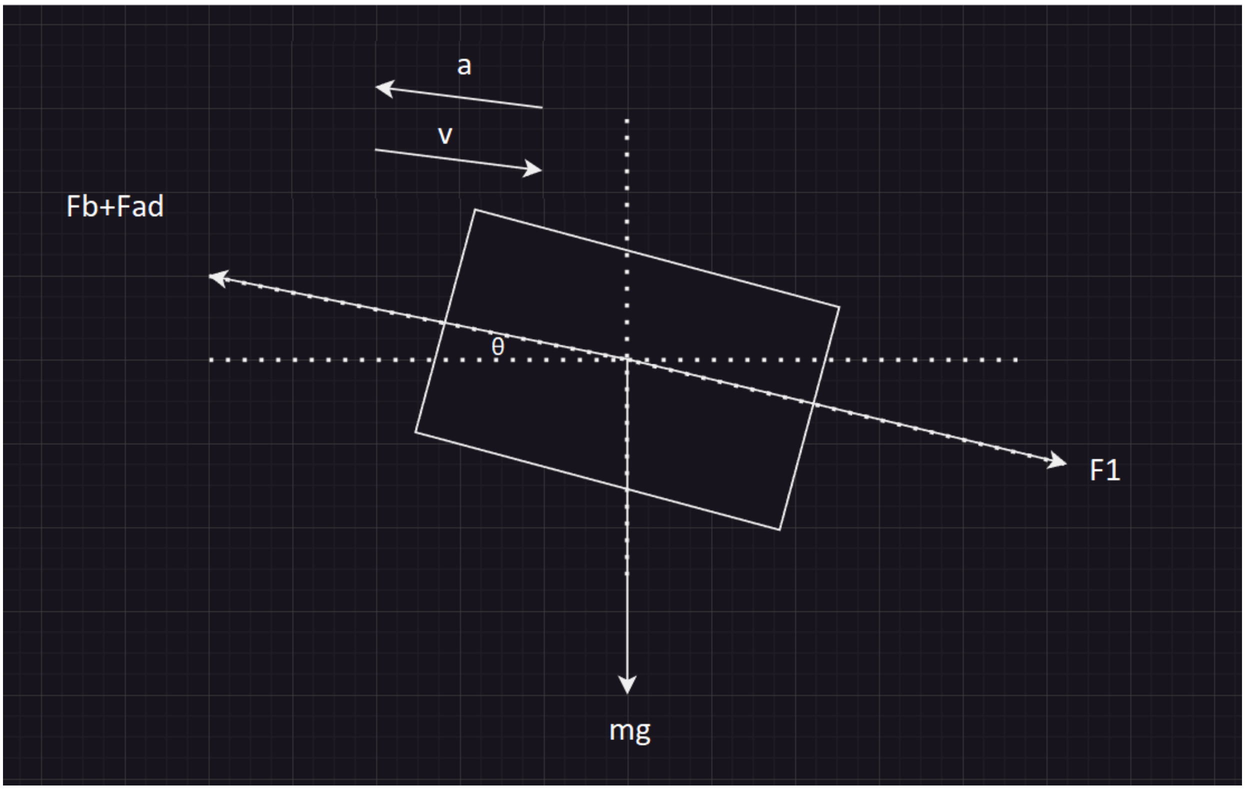

Figure 2: FBD for forces

The following equation will be found from the FBD:

$$-\boldsymbol{F_B} - \boldsymbol{F_R} + \boldsymbol{F_G} + \boldsymbol{F_i} = \boldsymbol{0} \Rightarrow \boldsymbol{F_B} = -\boldsymbol{F_R} + \boldsymbol{F_G} + \boldsymbol{F_i}$$

F_B is found with the following MATLAB codes:

```
1. F_i = lamda*m*a_i;
2. F_G = m*g*sind(theta);
3. F_R = f_r*m*g*cosd(theta);
4. % -F_B-F_R+F_G+F_i = 0 => F_B = -F_R+F_G+F_i
5.
6. F_B = -F_R+F_G+F_i; % that's equal to 15369 N
```

So, F_B is equals to 15369N

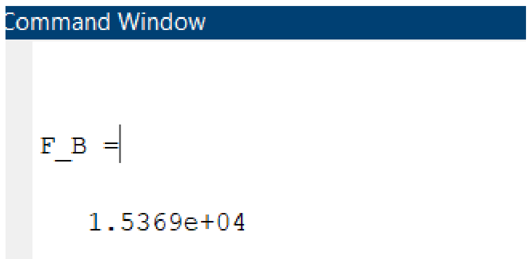

Figure 3: A screenshot of the Command window showing the F_B value.

B) Calculate the total heat energy rate (power) generated on the braking system. Assume that the installed braking system on truck applies 60% of the braking force on the front axle and 40% of the braking force on the rear axle. Calculate the power dissipated on both axles.

Heat generation can be calculated with this formula:

$$P_B = F_B * v_i$$

According to the question, breaking force is distributed as 40% rare axle and 60% front axle. Thus, if the heat generation at each axle is to be calculated, F_B is multiplied by 0.4 for the rare axle and 0.6 for the front axle. Below is a MATLAB code that calculates this:

```
F_B_front = F_B*0.6;
F_B_rear = F_B*0.4;
P_B_front = (F_B_front)*v_i; %that's equal to 92215
P_B_rear = (F_B_rear)*v_i;   %that's equal to 61476
P_B_total = P_B_front + P_B_rear; %that's equal to 153690
```

So, $P_{B,total}$ is equal to 153690 W; $P_{B,rear}$ is equal to 61476 W; $P_{B,front}$ is equal to 92215 W.

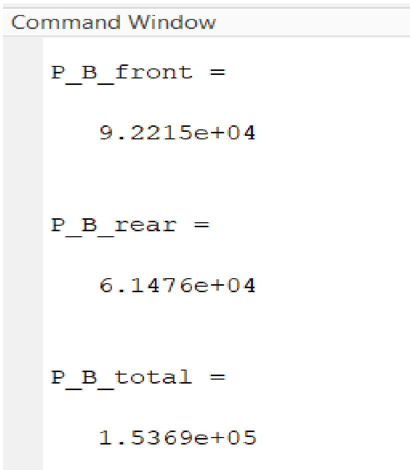

Figure 4: A screenshot of the Command window showing the $P_{B,front}$, $P_{B,rear}$, and $P_{B,total}$

C) Solve the heat equation given in (6) analytically for T and plot brake disc temperatures of the front and rear axles with respect to time on the same graph.

Equation 6 is a first order ordinary differential equation. To solve this equation, following steps are applied:

$$m_b * c * \left(\frac{dT}{dt}\right) + hA * (T - T_a) = P_B$$

$$\left(\frac{dT}{dt}\right) + \frac{h * A}{m_b * c} * T = \frac{P_B + h * A * T_a}{m_b * c}$$

$$\mu = e^{\int [h*A/(m_b*c)]dt} = e^{\frac{h*A*t}{m_b*c}}$$

$$e^{\frac{h*A*t}{m_b*c}} * \left[\left(\frac{dT}{dt}\right) + \frac{h*A}{m_b*c} * T\right] = e^{\frac{h*A*t}{m_b*c}} * \left[\frac{P_B + h*A*T_a}{m_b*c}\right]$$

$$\left(e^{\frac{h*A*t}{m_b*c}} * T\right)' = e^{\frac{h*A*t}{m_b*c}} * \left[\frac{P_B + h*A*T_a}{m_b*c}\right]$$

$$e^{\frac{h*A*t}{m_b*c}} * T = \int \left(e^{\frac{h*A*t}{m_b*c}} * \left[\frac{P_B+h*A*T_a}{m_b*c}\right]\right) * dt = \frac{〚\left(e^{\frac{h*A*t}{m_b*c}} * \frac{P_B+h*A*T_a}{m_b*c}\right)}{\frac{h*A}{m_b*c}}$$

$$T = \frac{P_B}{h*A} + T_a + \frac{C_1}{e^{\frac{h*A*t}{m_b*c}}}$$

To calculate C_1, initial condition has to be used: $T(0) = T_a$

$$T_a = \frac{P_B}{h*A} + T_a + C_1 \rightarrow C_1 = -\frac{P_B}{h*A}$$

Above equation is found by solving ODE. Temperature (T) is a function of second (t).

T is calculated separately for front axle and rear axle.

For front axle discs: $P_{B,front}$ is divided two because one axle has two discs and values are placed.

$$T_{front} = \frac{(P_{B,front}/2)}{h*A_{disc}} + T_a - \frac{\frac{(P_{B,front}/2)}{h*A_{disc}}}{e^{\frac{h*A_{disc}*t}{\left(\frac{m_b}{2}\right)*c}}}$$

$$T_{front} = \frac{\frac{92215}{2}}{60*0.4} + (20+273) + \frac{-\frac{\frac{92215}{2}}{60*0.4}}{e^{\frac{60*0.4}{15*500}*t}}$$

$$T_{front} = 1921.14 + 293 - \frac{1921.14}{e^{3.2*10^{-3}*t}}$$

For rear axle discs: $P_{B,rear}$ is divided two because one axle has two discs and values are placed.

$$T_{rear} = \frac{(P_{B,rear}/2)}{h*A_{disc}} + T_a - \frac{\frac{(P_{B,rear}/2)}{h*A_{disc}}}{e^{\frac{h*A_{disc}*t}{\left(\frac{m_b}{2}\right)*c}}}$$

$$T_{rear} = \frac{\frac{61476}{2}}{60 * 0.4} + (20 + 273) + \frac{-\frac{\frac{61476}{2}}{60 * 0.4}}{e^{\frac{60*0.4}{15*500}*t}}$$

$$T_{rear} = 1280.75 + 293 - \frac{1280.75}{e^{3.2*10^{-3}*t}}$$

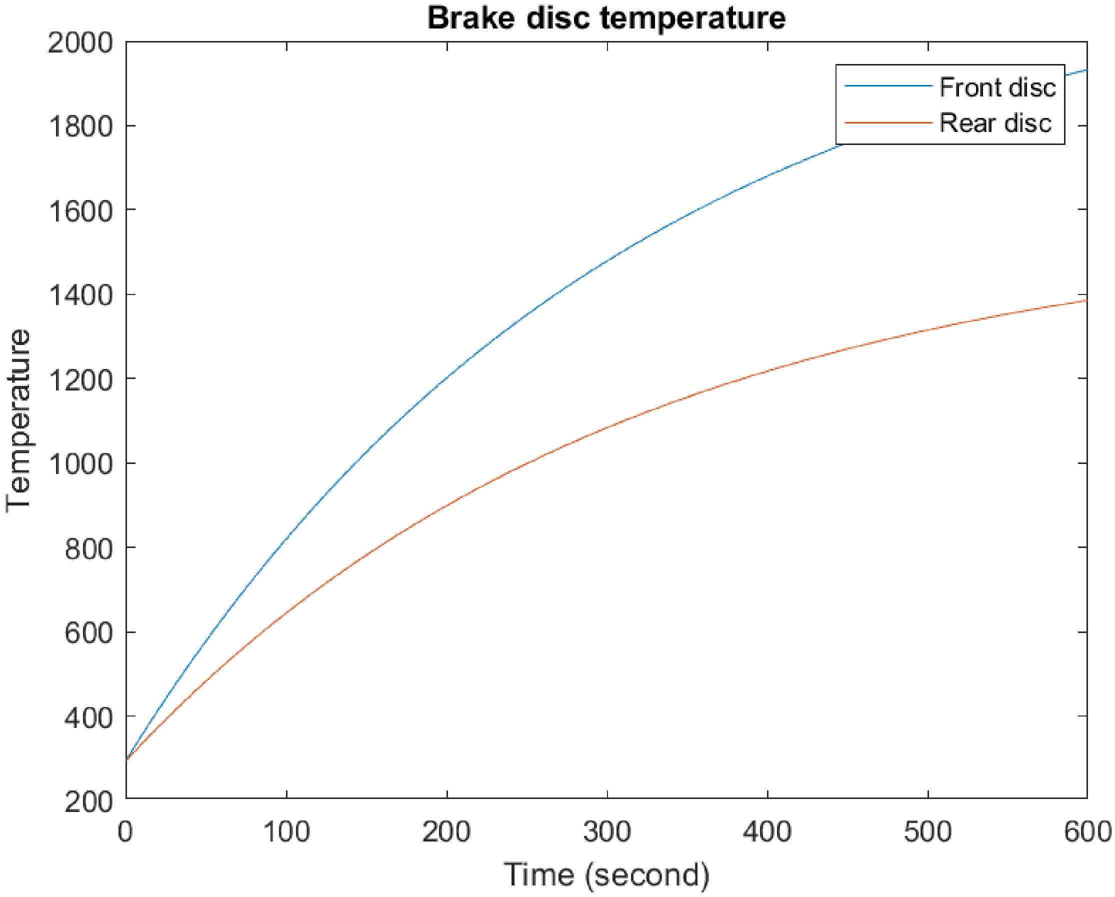

Figure 5: Graph showing the temperature change of the front and rear discs over time. Found with analytical solution.

D) Solve the heat equation given in (6) using the classical (4th order) Runge-Kutta method. Plot brake disc temperatures of the front and rear axles with respect to time on the same graph.

Preparing the equation for the 4th Runge-Kutta Method

$$m_b * c * \left(\frac{dT}{dt}\right) + hA * (T - T_a) = P_B$$

$$m_b * c * \frac{dT}{dt} + h * A * T = P_B + h * A * T_a$$

$$\frac{dT}{dt} = \frac{P_B}{m_b * c} + \frac{h * A}{m_b * c} * (T_a - T)$$

$$f(t) = \frac{P_B}{m_b * c} + \frac{h * A}{m_b * c} * (T_a - T)$$

Runga-Kutta Method formula:

$$y_{j+1} = y_j + \left(\frac{h}{6}\right) * (k_1 + 2 * k_2 + 2 * k_3 + k_4)$$

$$k_1 = f(t_j\,;\ T_j)$$
$$k_2 = f(t_j + \tfrac{h}{2}\,;\ T_j + \tfrac{h}{2} * k_1)$$
$$k_3 = f(t_j + \tfrac{h}{2}\,;\ T_j + \tfrac{h}{2} * k_2)$$
$$k_4 = f(t_j + h\,;\ T_j + h * k_3)$$

If these equations are used separately for the front and rear disc in the program, the output of the program will be as follows:

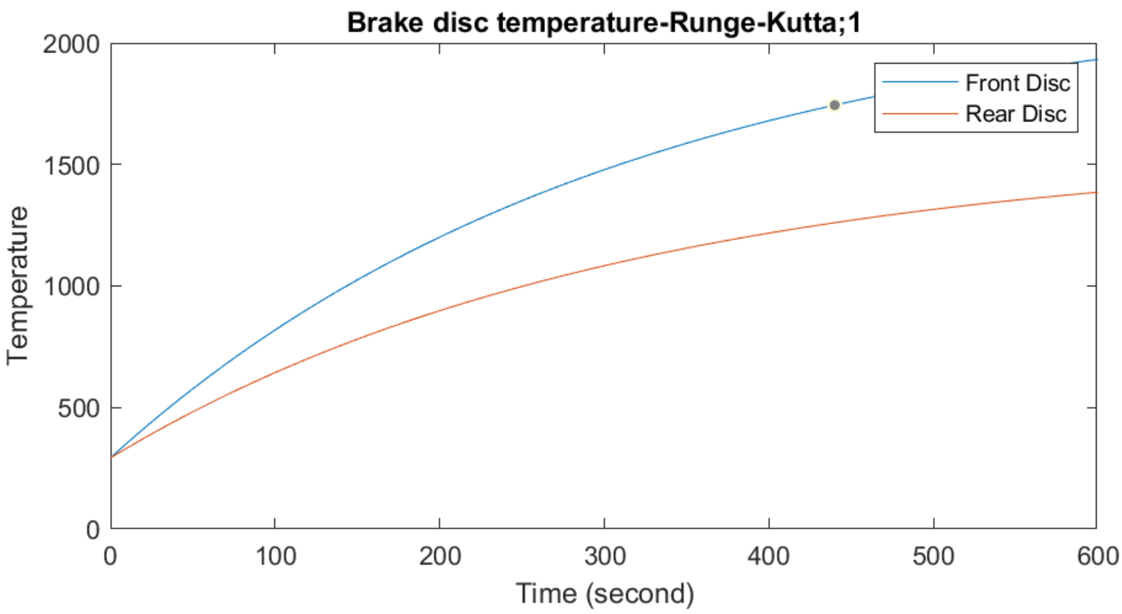

Figure 5: Graph showing the temperature change of the front and rear discs over time. Found with Runge-Kutta method.

Application of Runge-Kutta method in the program:

```
for i = 1: ceil(t_final/h_step_size)

t(i+1) = t(i) + h_step_size;

k_1 = f(t(i), T(i));
k_2 = f(t(i) + 0.5*h_step_size, T(i) + 0.5*k_1*h_step_size);
k_3 = f(t(i) + 0.5*h_step_size, T(i) + 0.5*k_2*h_step_size);
k_4 = f(t(i) + h_step_size, T(i) + k_3*h_step_size);

T(i+1) = T(i) + (h/6)*(k_1 + 2*k_2 + 2*k_3 + k_4);
end
```

E) Compare the analytical and numerical method results an elaborate on them.

Firstly, compere graphics for each disc:

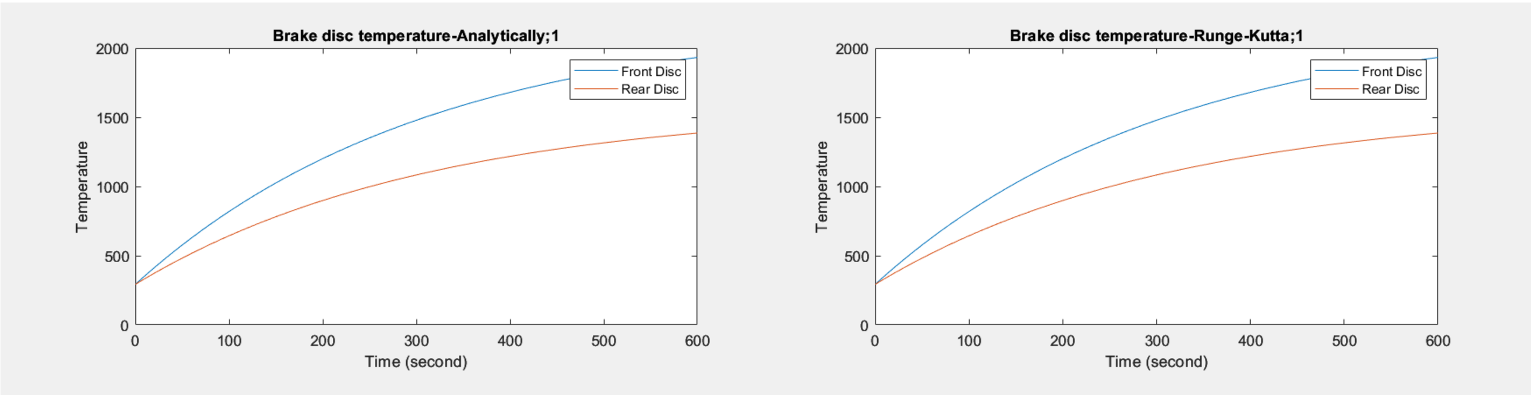

Figure 6: Graphics that help compare analytical solutions and Runge-Kutta method solutions.

For front discs $T_{analyticall}(600)$ = 1932.49 K and $T_{Runge\text{-}Kutta}(600)$= 1932.49 K. So, they are equal and the graphics are almost same. For rear discs $T_{analyticall}(600)$ = 1385.98 K and $T_{Runge\text{-}Kutta}(600)$= 1385.99 K. So they are almost equal and their graphics are almost same.

The analytical solution of a differential equation is to obtain a functional expression of the independent variable in the equation. This gives the exact solution of the equation and is usually expressed in a closed form. The analytical solution clearly shows the exact nature and behavior of the equation.

On the other hand, the Runge-Kutta method are used to approximate differential equations numerically. These methods iteratively calculate the values of the independent variable in the equation with a given step size. Runge-Kutta methods are especially useful for equations that are more complex or for which no analytical solution can be found.

The main difference is accuracy. Because, the analytical solution reflects the exact nature of the equation and gives precise results. Numerical methods such as the Runge-Kutta method give the approximate solution of the equation. The approximate solution may contain a margin of error, so the results are often less precise.

SCENARIO-2

There are some assumptions and simplifications for this scenario. These are:

- Heat convection occurs only between the brake discs and the ambient.
- h is assumed to be constant. (not affected by the disc rotational speed)
- Neglect aerodynamic lift force and pitch moments.
- **Aerodynamic drag force is not neglected.**
- There is no engine/retarder brake applied.
- Initial temperature of the brake discs is equal to ambient temperature.

A) Calculate the brake disc temperatures using the classical (4th order) Runge-Kutta method and plot the values with respect to time on the same graph.

Preparing the equation for the 4^{th} Runge-Kutta Method

$$m_b * c * \left(\frac{dT}{dt}\right) + hA * (T - T_a) = P_B$$

$$m_b * c * \frac{dT}{dt} + h * A * T = P_B + h * A * T_a$$

$$\frac{dT}{dt} = \frac{P_B}{m_b * c} + \frac{h * A}{m_b * c} * (T_a - T)$$

$$f(t) = \frac{P_B}{m_b * c} + \frac{h * A}{m_b * c} * (T_a - T)$$

Runga-Kutta Method formula:

$$y_{j+1} = y_j + \left(\frac{h}{6}\right) * (k_1 + 2 * k_2 + 2 * k_3 + k_4)$$

$k_1 = f(t_j\ ;\ T_j)$

$k_2 = f(t_j + \frac{h}{2}\ ;\ T_j + \frac{h}{2} * k_1)$

$$k_3 = f(t_j + \frac{h}{2} \; ; \; T_j + \frac{h}{2} * k_2)$$
$$k_4 = f(t_j + h \; ; \; T_j + h * k_3)$$

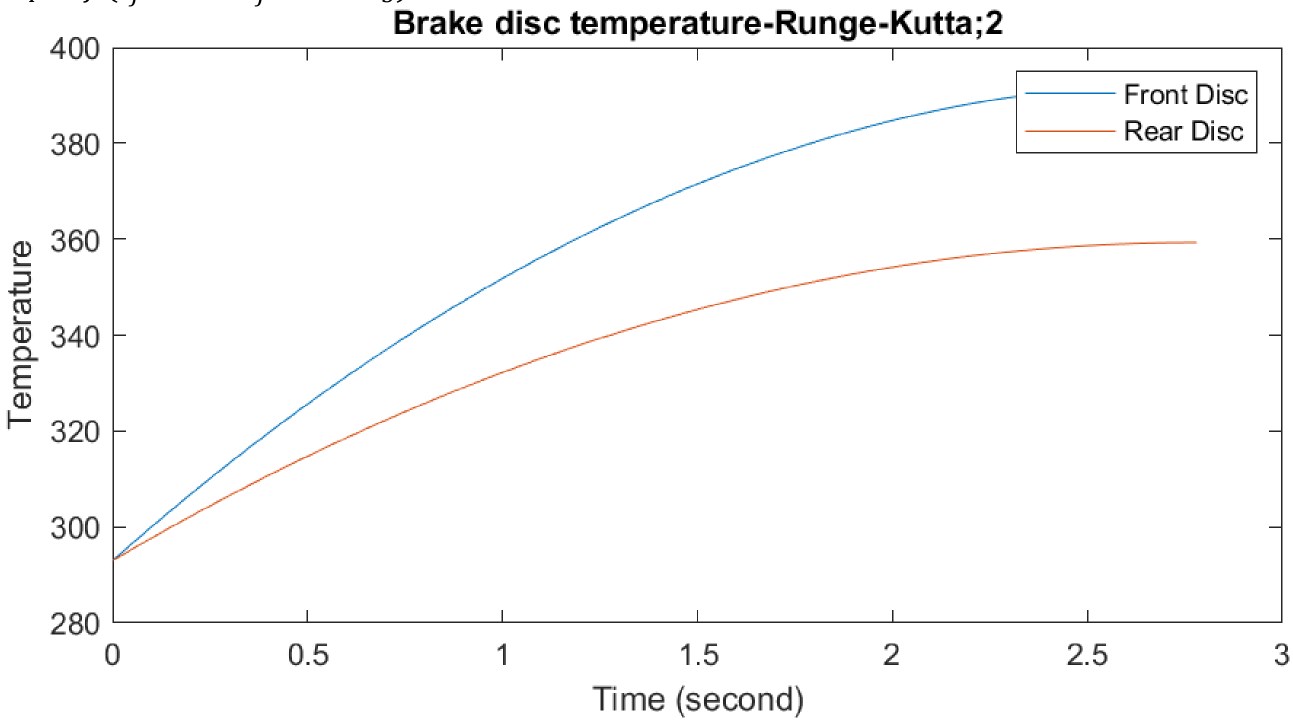

Figure 7: Graph showing the temperature change of the front and rear discs over time. Found with Runge-Kutta Method

B) Elaborate on the use of Runge-Kutta method on this part (Scenario 2) of the problem.

Firstly, aerodynamics drag force is used for this problem and it changes for new velocity value; thus, when aerodynamics force changes, F_B changes. All these forces and P_B must be written in a time loop and Runge-Kutta equations must be changed for new force values. Application of the Runge-Kutta method with changed value is below.

```
for i = 1:ceil(t_final_2 / h_step_size_2)

t(i+1) = t(i) + h_step_size_2;

v_2 = v_flat - a_target * t(i);

F_AD = 0.5 * row * C_D * A_p * v_2^2;
F_B_2 = -F_R_2 + F_G_2 + F_i_2 - F_AD;

F_B_rear_2 = F_B_2*0.4;
P_B_rear_2 = (F_B_rear_2)*v_2;

k_1 = f(t(i), T(i), P_B_rear_2);
k_2 = f(t(i) + 0.5 * h_step_size_2, T(i) + 0.5 * k_1 * h_step_size_2,
P_B_rear_2);
```

```
k_3 = f(t(i) + 0.5 * h_step_size_2, T(i) + 0.5 * k_2 * h_step_size_2,
P_B_rear_2);
k_4 = f(t(i) + h_step_size_2, T(i) + k_3 * h_step_size_2, P_B_rear_2);

T(i+1) = T(i) + (h_step_size_2 / 6) * (k_1 + 2 * k_2 + 2 * k_3 + k_4);
end
```