Ratios

Textbook: Business Mathematics by Gary Clendenen and Stanley Salzman

Define a Ratio

A **ratio** is a quotient of two quantities that is used to *compare* the quantities.

The ratio of the number *a* to the number *b* is written in any of the following ways.

$$a \text{ to } b, \quad a : b, \quad \frac{a}{b}$$

All are pronounced "*a to b*" *or* "*a is to b.*"

This last way of writing a ratio is most common in mathematics, while *a*:*b* is perhaps more common in business.

Example

Write a ratio in the form *a*/*b* for each word phrase. Notice in each example that the number mentioned first is always the numerator.

(a)The ratio of 5 hours to 3 hours is

$$\frac{5}{3}$$

Example

(b) Find the ratio of 5 hours to 3 days.

First convert 3 days to hours. 24 hours in 1 day, 3 days = 3 • 24 = 72 hours

The ratio of 5 hours to 3 days is the quotient of 5 and 72.

$$\frac{5}{72}$$

Example

(c) The ratio of $700,000 in sales to $950,000 in sales is written this way.

$$\frac{\$700,000}{\$950,000}$$

Reduce to write this ratio in lowest terms.

$$\frac{\$700,000}{\$950,000} = \frac{14}{19}$$

Example

Burger King sold the following items in a one-hour period last Friday afternoon.

70 bacon cheeseburgers

15 plain hamburgers

30 salad combos

45 chicken sandwiches

40 fish sandwiches

Example

Write ratios for the following items sold:

(a) bacon cheeseburgers to fish sandwiches

$$\text{(a)} \quad \frac{\text{bacon cheeseburgers}}{\text{fish sandwiches}} = \frac{70}{40} = \frac{7}{4}$$

(b) salad combos to chicken sandwiches

$$\text{(b)} \quad \frac{\text{salad combos}}{\text{chicken sandwiches}} = \frac{30}{45} = \frac{2}{3}$$

Example

Write ratios for the following items sold:

(c) plain hamburgers to salad combos

$$\text{(c)} \quad \frac{\text{plain hamburgers}}{\text{salad combos}} = \frac{15}{30} = \frac{1}{2}$$

(d) fish sandwiches to total items sold

$$\text{(d)} \quad \frac{\text{fish sandwiches}}{\text{total items sold}} = \frac{40}{200} = \frac{1}{5}$$

Set Up a Proportion

A ratio is used to compare two numbers or amounts.

A **proportion** says that two ratios are equal.

$$\frac{3}{4} = \frac{15}{20}$$

Method of Cross Products

The proportion

$$\frac{a}{b} = \frac{c}{d}$$

is true if the cross products $a \cdot d$ and $b \cdot c$ are equal (that is, if $ad = bc$).

Example

Decide whether the following proportions are true.

(a) $\dfrac{3}{5} = \dfrac{12}{20}$

$$\frac{3}{5} = \frac{12}{20}$$

$$3 \times 20 \stackrel{?}{=} 5 \times 12$$

$$60 = 60$$

True

(b) $\dfrac{2}{3} = \dfrac{9}{16}$

$$\frac{2}{3} = \frac{9}{16}$$

$$2 \times 3 \stackrel{?}{=} 9 \times 16$$

$$32 \neq 27$$

False

Solve a Proportion for Unknown Values

Four numbers and/or variables are used in a proportion.

If any three of the values are known, the fourth can be found.

The two methods to solve a proportion are:

1. Multiply both sides by the product of the two denominators or
2. Use the method of cross products.

Example

Find the value of the unknowns. Use the first method for **(a)** and the second method for **(b).**

(a) $\frac{3}{5} = \frac{x}{40}$

(b) $\frac{3}{10} = \frac{5}{k}$

Example

(a) Solve for the unknown by multiplying both sides of the equation by the product of the denominators.

$$\frac{3}{5} = \frac{x}{40}$$

$$\frac{3}{5} \cdot (5 \cdot 40) = \frac{x}{40} \cdot (5 \cdot 40)$$

$$120 = 5x$$

$$\frac{120}{5} = \frac{\cancel{5}x}{\cancel{5}}$$

$$24 = x, \quad \text{or}$$

$$x = 24$$

Example

(b) Solve for the unknown by using the method of cross products.

$$\frac{3}{10} = \frac{5}{k}$$

$$3k = 50$$

$$\frac{\cancel{3}k}{\cancel{3}} = \frac{50}{3}$$

$$k = \frac{50}{3} = 16\frac{2}{3}$$

Example

A food wholesaler charges a restaurant chain \$83 for 3 crates of fresh produce. How much should it charge for 5 crates of produce?

Example

Let x be the cost of 5 crates of produce. Set up a proportion with one ratio the number of crates and the other ratio the costs. Use this pattern.

$$\frac{\text{Crates}}{\text{Crates}} = \frac{\text{Cost}}{\text{Cost}}$$

Substitute the given information.

$$\frac{3}{5} = \frac{83}{x}$$

Example

Use cross products to solve the proportion.

$$\frac{3}{5} = \frac{83}{x}$$

$$3x = 5(83)$$

$$3x = \$415$$

$$x = \$138.33$$

The 5 crates cost \$138.33.

Use Proportions to Solve Problems

Proportions are used in many practical applications.

Example

A firm in Hong Kong and one in Thailand agree to jointly develop an engine-control microchip to be sold to North American auto manufacturers. They agree to split the development costs in a ratio of 8:3 (Hong Kong firm to Thailand firm), resulting in a cost of $9,400,000 to the Hong Kong firm. Find the cost to the Thailand firm.

Example

Let x represent the cost to the Thailand firm, then

$$\frac{8}{3} = \frac{9,400,000}{x}$$

$$8x = 3 \cdot 9,400,000$$

$$8x = 28,200,000$$

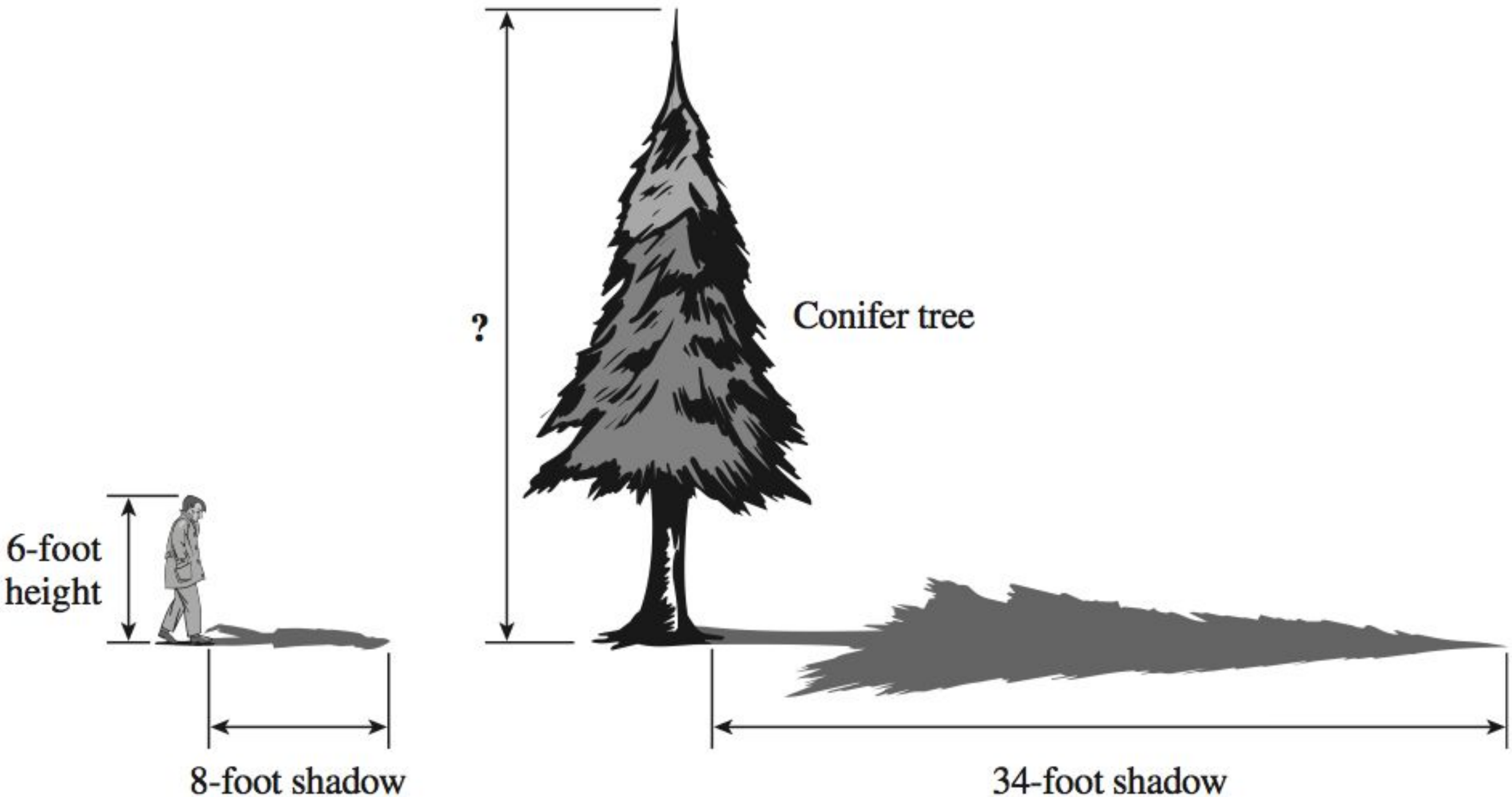
$$x = 3,525,000$$

The Thailand firm's share of the costs is $3,525,000.

Example

Bill Thomas wishes to estimate the amount of timber on some forested land that he owns. One value he needs to estimate is the average height of the trees. One morning, Thomas notices that his own 6-foot body casts an 8-foot shadow at the same time that a typical tree casts a 34-foot shadow. Find the height of the tree.

Example



Example

Set up a proportion in which the height of the tree is given the variable name *x*.

$$\frac{6}{8} = \frac{x}{34}$$

$$6 \cdot 34 = 8 \cdot x$$

$$\frac{204}{8} = \frac{\cancel{8} \cdot x}{\cancel{8}}$$

$$x = 25.5$$

The height of the tree is 25.5 feet.