

Mathematics for business decisions

Lecture 5

Chapter 3 Percents

Textbook: Business Mathematics by Gary Clendenen and Stanley Salzman

Percents

Similar to fractions and decimals, percents represent parts of a whole.

Percents (**hundredths**) mean parts out of 100.

Written using a percent sign (%).

For example 1% means 1 of 100 equal parts.

The number 12% is read “twelve percent.”

Percents

Decimal
Fraction

12% = 12 out of 100 equal parts .12

$$\frac{12}{100}$$

25% = 25 out of 100 equal parts .25

$$\frac{25}{100}$$

50% = 50 out of 100 equal parts .50

$$\frac{50}{100}$$

Percents

	<i>Decimal</i>	<i>Fraction</i>
100% = 100 out of 100 equal parts	1.00	$\frac{100}{100}$
150% = 150 out of 100 equal parts	1.50	$\frac{150}{100}$

Converting a decimal number to a percent

Step 1 Move the decimal point two digits to the right attaching trailing zeros as needed.

You are effectively multiplying by 100.

Step 2 Attach a percent sign.

$$.75 = 75\%$$

Example

Change the following decimals to percents.

$$(a) \quad .35 = 35\%$$

$$(b) \quad .42 = 42\%$$

$$(c) \quad .58 = 58\%$$

Example

Change the following decimals to percents.

(a) $.8 = 80\%$

(b) $2.6 = 260\%$

(c) $.1 = 10\%$

(d) $4 = 400\%$

Example

Change the following decimals to percents.

(a) $.625 = 62.5\%$

(b) $.0057 = .57\%$

(c) $1.25 = 125\%$

Write a Fraction as a Percent

Write the fraction as a decimal.

Move the decimal point two places to the right.

Add a percent sign (%).

Example

A marketing manager is given the following data in fraction form and must change the data to percents.

(a) $\frac{1}{4}$

(b) $\frac{3}{8}$

(c) $\frac{4}{5}$

Example

First write each fraction as a decimal, and then write the decimal as a percent.

$$(a) \quad \frac{1}{4} = .25 = 25\%$$

$$(b) \quad \frac{3}{8} = .375 = 37.5\%$$

$$(c) \quad \frac{4}{5} = .8 = 80\%$$

Write a Fraction as a Percent

(Second Method)

Multiply the fraction by 100%.

$$\frac{4}{5} \times 100\% = \frac{400\%}{5} = 80\%$$

Write a Percent as a Decimal

Step 1 Drop the percent sign.

Step 2 Move the decimal point two places to the left. You are effectively dividing by 100.

$$25\% = .25$$

Example

To calculate some insurance claims, an insurance agent must change the following percents to decimals.

$$(a) \ 35\% = .35$$

$$(d) \ 37\frac{1}{2}\% = .375$$

$$(b) \ 50\% = .5$$

$$(c) \ 325\% = 3.25$$

Write a Percent as a Fraction

Change the percent to a decimal.

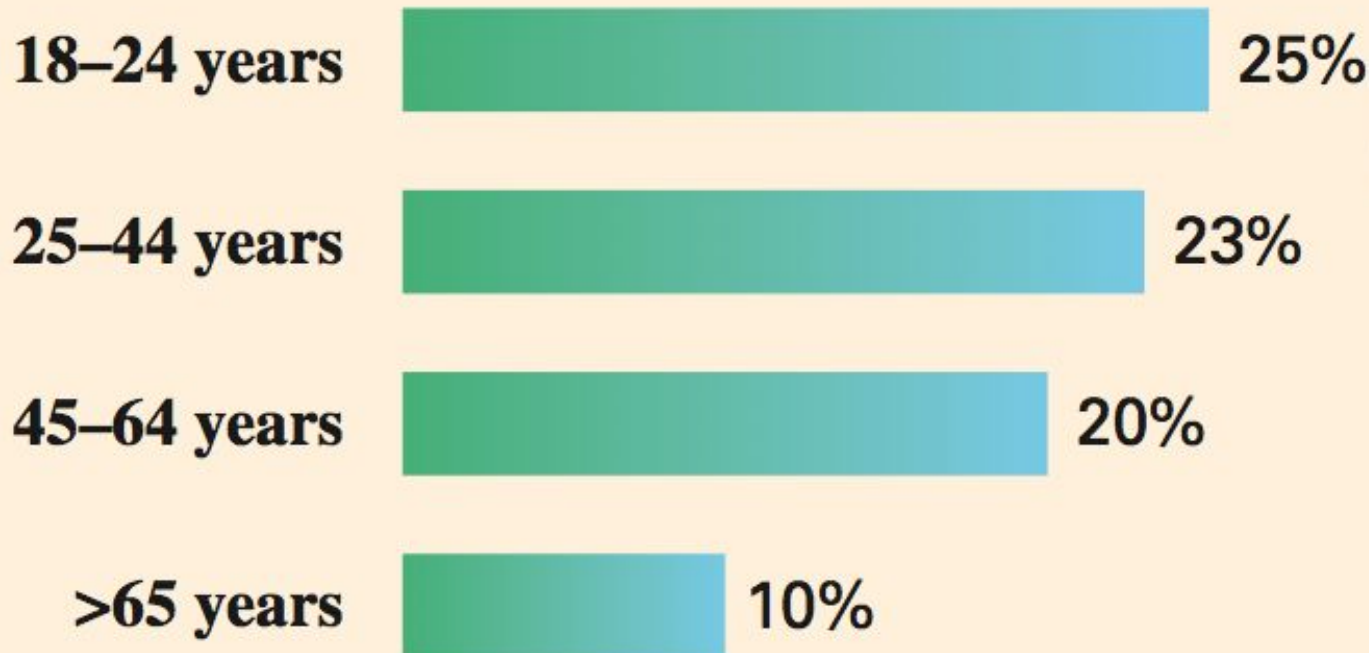
Write the decimal as a fraction in lowest terms.

Example

Even though smoking is believed to account for 1 of every 5 deaths in the United States, more than 45 million Americans smoke. The bar chart shows the percent of people in each age group that smoke. Convert each percent to a fraction and reduce to lowest terms.

Numbers in the News

Percent that Smoke by Age



DATA: *America Lung Association*

Example

Write each percent as a decimal and then as a fraction in lowest terms.

$$(a) \quad 25\% = .25 = \frac{25}{100} = \frac{1}{4}$$

$$(b) \quad 23\% = .23 = \frac{23}{100}$$

Example

Write each percent as a decimal and then as a fraction in lowest terms.

$$(c) \quad 20\% = .20 = \frac{20}{100} = \frac{1}{5}$$

$$(d) \quad 10\% = .10 = \frac{10}{100} = \frac{1}{10}$$

Write a Fractional Percent as a Decimal

Change the fraction to a decimal followed by a percent sign.

Move the decimal point two places to the left.

Drop the percent sign.

Example

The following percents appear in a newspaper article. Write each fractional percent as a decimal.

$$(a) \quad \frac{1}{5}\% = .2\% = .002$$

$$(b) \quad \frac{3}{4}\% = .75\% = .0075$$

$$(c) \quad \frac{5}{8}\% = .625 = .00625$$

Fraction, Decimal, Percent Equivalents

$$\frac{1}{100} = .01 = 1\%$$

$$\frac{1}{50} = .02 = 2\%$$

$$\frac{1}{25} = .04 = 4\%$$

$$\frac{1}{20} = .05 = 5\%$$

Fraction, Decimal, Percent Equivalents

$$\frac{1}{16} = .0625 = 6.25\% = 6\frac{1}{4}\%$$

$$\frac{1}{12} = .08\overline{3} = 8\frac{1}{3}\%$$

$$\frac{1}{10} = .1 = 10\%$$

$$\frac{1}{9} = .1\overline{1} = 11\frac{1}{9}\%$$

Fraction, Decimal, Percent Equivalents

$$\frac{1}{8} = .125 = 12.5\% = 12\frac{1}{2}\%$$

$$\frac{1}{7} = .1428 = 14\frac{2}{7}\%$$

$$\frac{1}{6} = .166\bar{6} = 16\frac{2}{3}\%$$

$$\frac{3}{16} = .1875 = 18.75\% = 18\frac{3}{4}\%$$

Fraction, Decimal, Percent Equivalents

$$\frac{1}{5} = .2 = 20\%$$

$$\frac{1}{4} = .25 = 25\%$$

$$\frac{3}{10} = .3 = 30\%$$

$$\frac{5}{16} = .3125 = 31.25\%$$

Fraction, Decimal, Percent Equivalents

$$\frac{1}{3} = .333\overline{3} = 3\frac{1}{3}\%$$

$$\frac{3}{8} = .375 = 37.5 = 37\frac{1}{2}\%$$

$$\frac{2}{5} = .4 = 40\%$$

$$\frac{7}{16} = .4375 = 43.75\% = 43\frac{3}{4}\%$$

Fraction, Decimal, Percent Equivalents

$$\frac{1}{2} = .5 = 50\%$$

$$\frac{9}{16} = .5625 = 56.25\% = 56\frac{1}{4}\%$$

$$\frac{3}{5} = .6 = 60\%$$

$$\frac{5}{8} = .625 = 62.5\% = 62\frac{1}{2}\%$$

Fraction, Decimal, Percent Equivalents

$$\frac{2}{3} = .666\overline{6} = 66\frac{2}{3}\%$$

$$\frac{11}{16} = .6875 = 68.75\% = 68\frac{3}{4}\%$$

$$\frac{7}{10} = .7 = 70\%$$

$$\frac{3}{4} = .75 = 75\%$$

Fraction, Decimal, Percent Equivalents

$$\frac{4}{5} = .8 = 80\%$$

$$\frac{13}{16} = .8125 = 81.25\% = 81\frac{1}{4}\%$$

$$\frac{5}{6} = .833\overline{3} = 83\frac{1}{3}\%$$

$$\frac{7}{8} = .875 = 87.5\% = 87\frac{1}{2}\%$$

Fraction, Decimal, Percent Equivalents

$$\frac{9}{10} = .9 = 90\%$$

$$\frac{15}{16} = .9375 = 93.75\% = 93\frac{3}{4}\%$$

$$1 = 1.00 = 100\%$$

$$1\frac{1}{10} = 1.1 = 110\%$$

Fraction, Decimal, Percent Equivalents

$$1\frac{1}{4} = 1.25 = 125\%$$

$$1\frac{1}{3} = 1.333\overline{3} = 133\frac{1}{3}\%$$

$$1\frac{1}{2} = 1.50 = 150\%$$

$$1\frac{2}{3} = 1.666\overline{6} = 166\frac{2}{3}\%$$

Fraction, Decimal, Percent Equivalents

$$1\frac{3}{4} = 1.75 = 175\%$$

$$2 = 2.00 = 200\%$$

Know the Three Components of a Percent Problem

1. **Base:** The whole or total, starting point, or that to which something is being compared.
2. **Rate:** A number followed by % or percent.
3. **Part:** The result of multiplying the base and the rate. The part is a *part* of the base. For example, sales tax is a part of total sales.

Learn the Basic Percent Formula

$$P = B \times R$$

$$\text{Part} = \text{Base} \times \text{Rate}$$

or

$$P = R \times B$$

$$\text{Part} = \text{Rate} \times \text{Base}$$

Solve for Part

Use one of the formulas:

$$P = B \times R \quad \text{or}$$

$$P = R \times B$$

Substitute the known values.

Carry out the calculation.

Example

Solve for part, using $P = B \times R$.

(a) 4% of 50

(b) 1.2% of 180

(c) 140% of 225

(d) $\frac{1}{4}\%$ of 560

Example

Solve for part, using $P = B \times R$.

(a) 4% of 50

$$\begin{array}{r} 50 \\ \times .04 \\ \hline 2.00 \end{array}$$

(b) 1.2% of 180

$$\begin{array}{r} 180 \\ \times .012 \\ \hline 2.160 \end{array}$$

Example

Solve for part, using $P = B \times R$.

(c) 140% of 225

$$\begin{array}{r} 225 \\ \times 1.4 \\ \hline 315.0 \end{array}$$

(d) $\frac{1}{4}\%$ of 560

$$\begin{array}{r} 560 \\ \times .0025 \\ \hline 1.4000 \end{array}$$

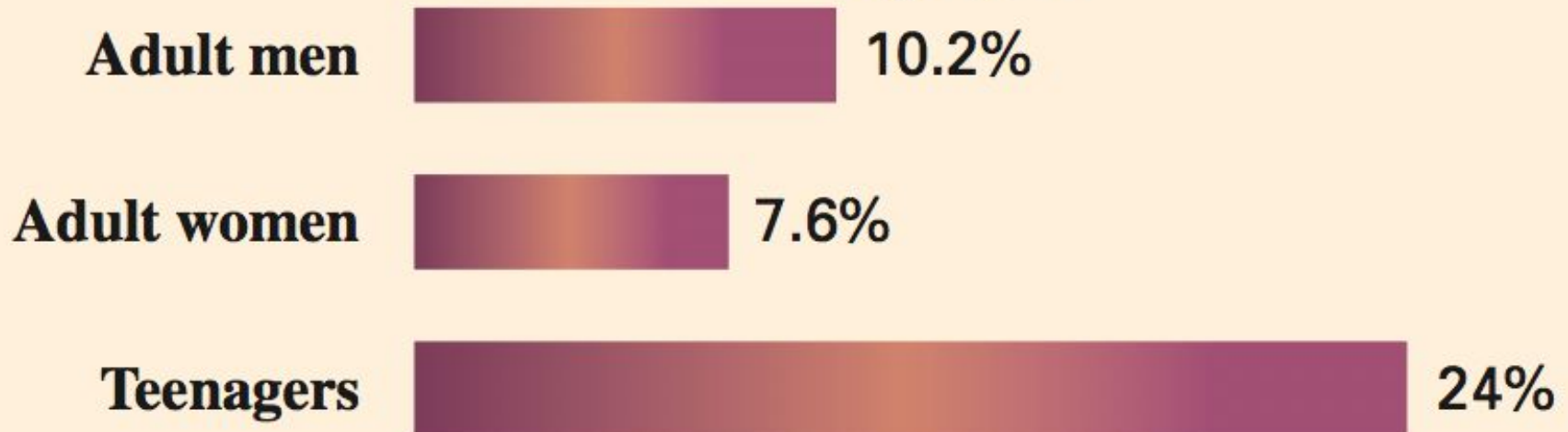
Example

The bar graph on the next slide shows the unemployment rate by category in the midst of a serious recession. Use the data provided to estimate the number of unemployed teenagers out of a total of roughly 32,000 working-age teenagers in one city.

Numbers in the News

Who Has Been Hurt by the Recession?

Percent Unemployed



DATA: *Bureau of Labor Statistics*

Example

The base is 32,000. The rate for unemployed teenagers is 24%. The number of unemployed teenagers is part of the whole, so part (P) is the unknown.

$$P = B \times R$$

$$P = 32,000 \times 24\%$$

$$P = 32,000 \times .24 = 7680$$

About 7680 of the 32,000 working-age teenagers in the city are unemployed.

Recognize the Terms Associated with Base, Rate, and Part

Percent problems have similarities.

Some phrases are associated with the base.

Some phrases lead to the part.

% or percent identifies the rate.

Recognize the Terms Associated with Base, Rate, and Part

Usually indicates
the base (*B*)

Usually indicates
the part (*P*)

Sales

Sales tax

Investment

Return on Investment

Savings

Interest

Retail price

Discount

Last year's figure

Increase or decrease

Old salary

Raise

Earnings

Expenditures

Calculate Sales Tax

Good example of finding part.

States, counties, and cities tax retail sales.

Sales tax is a percent of the sale.

The formula is:

$$P = B \times R$$

$$\text{Sales tax} = \text{Sales} \times \text{Sales tax rate}$$

Example

Becky Smith finally saved enough to buy the guitar she had dreamed about. Her goal was to start a band with her two sisters as backup and a friend as a drummer. The list price on the guitar was \$1199.99 and the sales tax was 8.5%. Find the sales tax and total cost.

Example

The whole (B) is \$1199.99, and the rate (R) is 8.5%.

$$P = B \times R$$

$$P = 1199.99 \times 8.5\%$$

$$P = 1199.99 \times .085 = 101.99915$$

Total is cost of guitar plus sales tax.

$$\text{Total} = \$1199.99 + \$102.00 = \$1301.99$$

Identify Rate, Base and Part

Base tends to be preceded by the word *of* or *on*;
tends to be the whole.

Rate is followed by a percent sign (%) or the word
percent.

Part is in the same units as the base and is usually
a portion of the base.

Learn the Standard Format of Percent Problems

Written in the form “% of whole is/are part.”

Rate	Whole	Part
7.5% of the	total	is the sales tax
8.5% of the	workers	are unemployed
74% of the	students	are full-time students
18% of the	children	are obese

Example

Identify the whole and rate in the following; then find the part.

- (a) A refrigerator with an original price of \$949 was marked down 10%.
- (b) Expenses for the weekend were 92% of total sales of \$1850.
- (c) Corporate income taxes were 30% of total profit of \$18,240,000.

Example

- (a) A refrigerator with an original price of \$949 was marked down 10%.

$$\text{Base} \times \text{Rate} = \text{Part}$$

$$\$949 \times 10\% = \$94.90 \text{ discount}$$

- (b) Expenses for the weekend were 92% of total sales of \$1850.

$$\text{Base} \times \text{Rate} = \text{Part}$$

$$\$1850 \times 92\% = \$1702 \text{ expenses}$$

Example

- (c) Corporate income taxes were 30% of total profit of \$18,240,000.

Base × Rate = Part

$$\begin{array}{l} \$18,240,000 \times 30\% = \$5,472,000 \\ \text{corporate income taxes} \end{array}$$

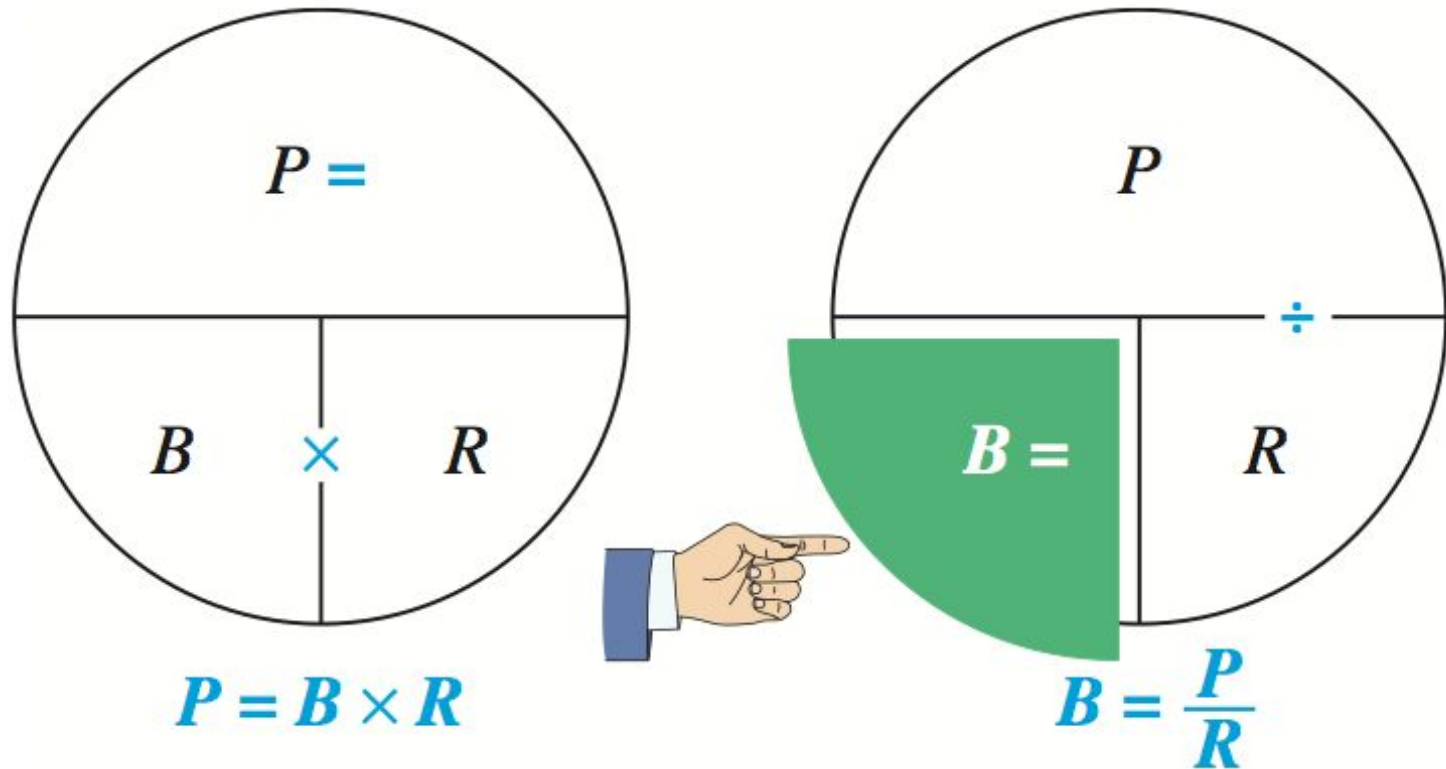
Use the Basic Percent Formula to Solve for Base

Rate and part are given, but the base, or starting point, must be found.

The formula $P = B \times R$ can be used to get the **formula for base**.

Use circle diagram on the next slide to learn how to use this formula.

To find the formula for base, cover B with your finger.



This leaves P divided by R , so $B = \frac{P}{R}$.

Use the Basic Percent Formula to Solve for Base

$$\text{Base} = \frac{\text{Part}}{\text{Rate}} \quad \text{or} \quad B = \frac{P}{R}$$

Example

Solve for base, using the formula

$$B = \frac{P}{R}.$$

(a) 8 is 4% of _____

(b) 135 is 15% of _____

(c) 1.25 is 25% of _____

Example

(a) 8 is 4% of _____ $B = \frac{8}{.04} = 200$

(b) 135 is 15% of _____ $B = \frac{135}{.15} = 900$

(c) 1.25 is 25% of _____ $B = \frac{1.25}{.25} = 5$

Find Sales when Tax Amount and Tax are Known

In business problems involving sales tax, the amount of sales is always the base.

We use the formula

$$B = \frac{P}{R}.$$

Example

One week, a Famous Footware store collected sales taxes of \$780. If the sales tax rate is 5%, find total sales for the week.

Example

The rate of tax collection is 5%, and taxes collected are a part of total sales. The rate in this problem is 5%, the part is \$780, and the base, or total sales, must be found.

$$R \times B = P$$

% of something is something

5% of total sales is \$780 (tax)

Use the formula:

$$B = \frac{P}{R} = \frac{780}{.05} = \$15,600 \text{ total sales}$$

Find the Investment when Interest Payment and Rate of Interest are Known

In problems involving investments, the amount of money invested is always the base.

We use the formula

$$B = \frac{P}{R}.$$

Example

Roberta Gonzales received \$162.50 in interest from an account that paid 3.25% interest for the year. Find the amount of money invested in the account.

The part (P) is the \$162.50 in interest and the rate (R) is 3.25%. The whole, or base (B), is unknown.

$$B = \frac{P}{R} = \frac{\$162.50}{3.25\%} = \frac{\$162.50}{.0325} = \$5000$$

The original investment was \$5000.

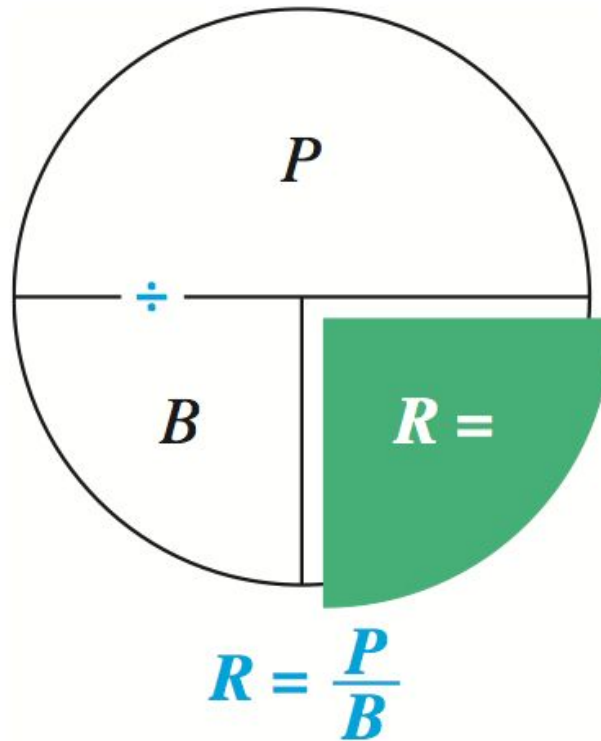
Use the Basic Percent Formula to Solve for Rate

Part and base are given, and the rate must be found.

The **formula for rate** is found from the formula
 $P = B \times R$.

Use circle diagram on the next slide to learn how to use this formula.

Cover R .



We get part \div base, or $\frac{P}{B}$.

Use the Basic Percent Formula to Solve for Rate

$$\text{Rate} = \frac{\text{Part}}{\text{Base}} \quad \text{or} \quad R = \frac{P}{B}$$

Use the Basic Percent Formula to Solve for P , B , or R

Need to know the values of two of the three components.

To find P :

$$\text{Use } P = B \times R.$$

Use the Basic Percent Formula to Solve for P , B , or R

To find R :

$$P = B \times R.$$

$$\frac{P}{B} = \frac{B \times R}{B}$$

$$\frac{P}{B} = R, \quad \text{or} \quad R = \frac{P}{B}$$

Use the Basic Percent Formula to Solve for P , B , or R

To find B :

$$P = B \times R.$$

$$\frac{P}{R} = \frac{B \times R}{R}$$

$$\frac{P}{R} = B, \text{ or } B = \frac{P}{R}$$

Example

Solve for rate.

(a) 26 is _____% of 104

(b) _____% of 300 is 60

(c) 54 is _____% of 12

Example

(a) 26 is _____% of 104 $\frac{26}{104} = .25 = 25\%$

(b) _____% of 300 is 60 $\frac{60}{300} = .2 = 20\%$

(c) 54 is _____% of 12 $\frac{54}{12} = 4.5 = 450\%$

Solve for Rate in Application Problems

In application problems, to find the rate,

we use the formula $R = \frac{P}{B}.$

Example

An investment officer at Graham Bank placed \$3,000,000 in a government security. It remained there for one year and earned \$96,000 in interest. Find the rate of return.

Example

The whole is \$3,000,000 and the part is \$96,000.
Find the rate as follows.

$$R = \frac{P}{B} = \frac{\$96,000}{\$3,000,000} = .032 = 3.2\%$$

The investment earned 3.2% for the year.

Example

One of the toughest races in the world is the 26.2-mile-long Boston marathon. After running 22 miles, Sheila James is exhausted. Find the percent of the race that she must still run.

Distance yet to be run

$$= 26.2 - 22 = 4.2 \text{ miles}$$

$$R = \frac{P}{B} = \frac{4.2}{26.2} = .160 = 16\%$$

James still has to run 16% of the total distance.

Learn to Identify an Increase or Decrease Problem

Businesses look at how amounts change, either up or down.

Business: percent by which sales have **increased** or **decreased**.

Consumer: percent by which an item has changed.

Identify an Increase Problem

The base (100%) *plus* some portion of the base gives a new value, which is the part.

an increase of

more than

greater than

$$\begin{array}{ccc} \text{Original} & + & \text{Increase} & = & \text{New value} \\ \text{(base)} & & & & \text{(part)} \end{array}$$

Identify a Decrease Problem

The part equals the base (100%) *minus* some portion of the base (gives new value).

after a decrease of

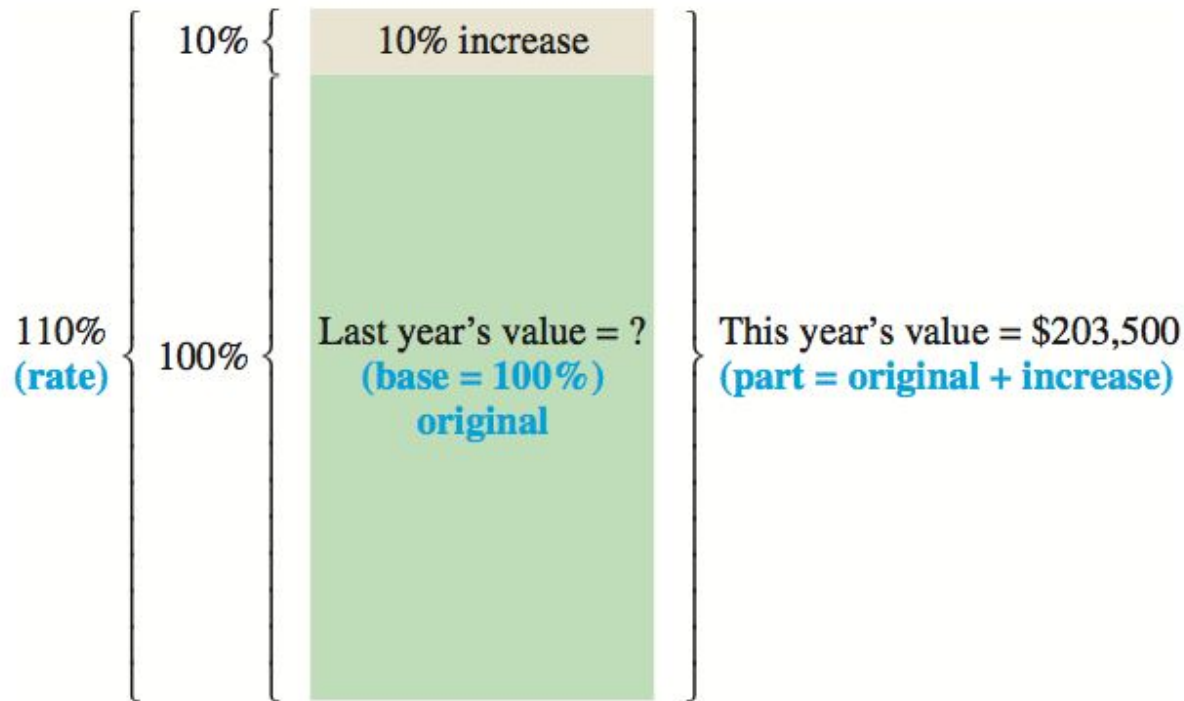
less than

after a reduction of

$$\begin{array}{ccc} \text{Original} & - & \text{Decrease} & = & \text{New value} \\ \text{(base)} & & & & \text{(part)} \end{array}$$

Apply the Basic Diagram for Increase Word Problems

A diagram, such as this one, can be used to help solve an increase word problem.



Use the Basic Percent Formula to Solve for Base in Increase Problems

To find the Rate, use

$$\text{Original} + \text{Increase} = \text{New value}$$

To find the Base, use

$$B = \frac{P}{R}$$

Example

The value of a home sold by Tom Dugally this year is \$203,500, which is 10% more than last year's value. Find the value of the home last year.

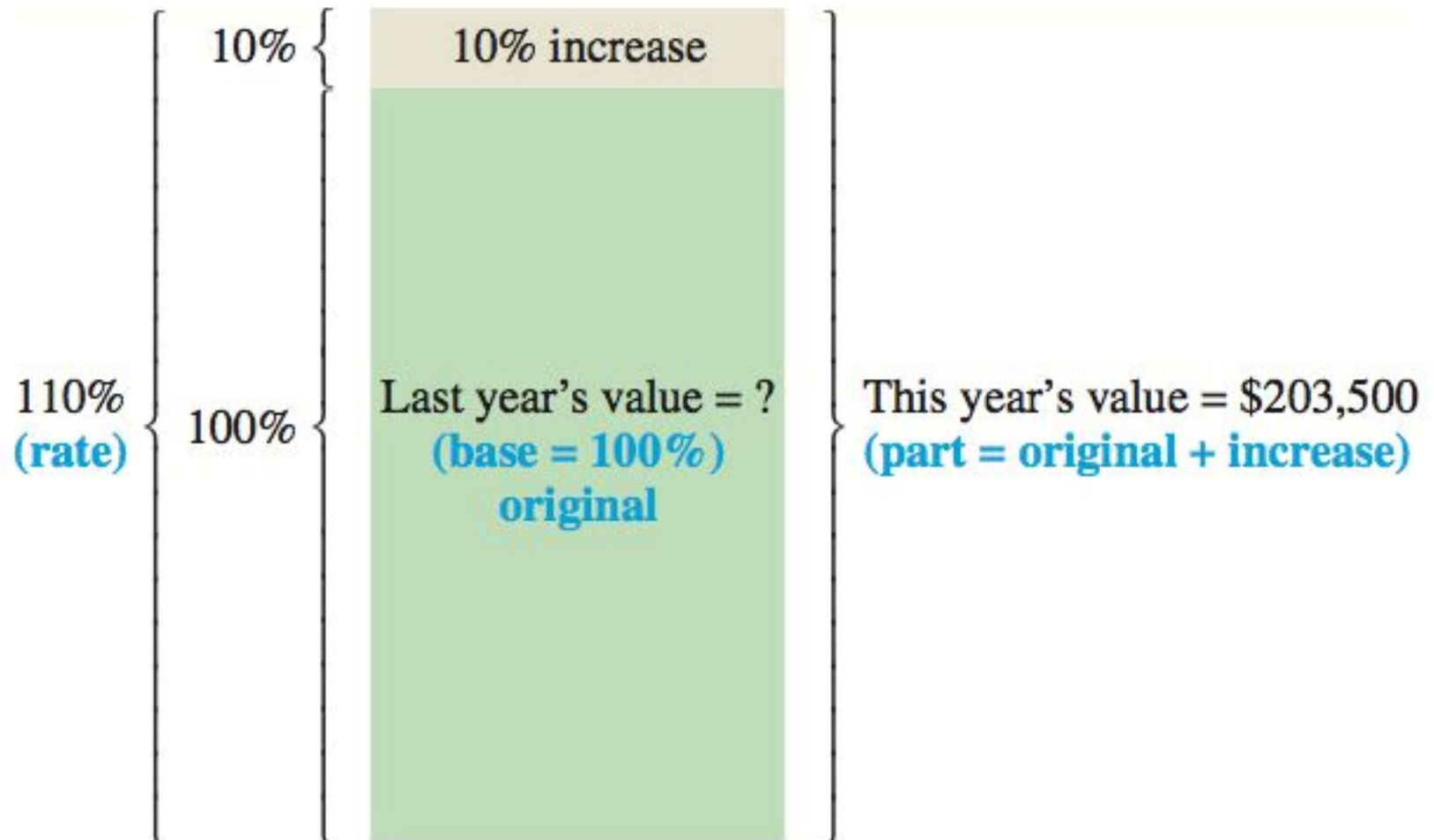
Example

The base is the starting point, or that to which something is compared. In this case the base is last year's value. Call base 100%, and remember that

Original (base) + Increase = New value

Apply the basic diagram for an increase.

Example



Example

Original + Increase = New value

$$100\% + 10\% = 110\%$$

Substitute part = \$203,500 and rate = 110%

$$B = \frac{\text{Part}}{\text{Rate}} = \frac{\$203,500}{110\%}$$

$$B = \frac{\$203,500}{1.1} = \$185,000$$

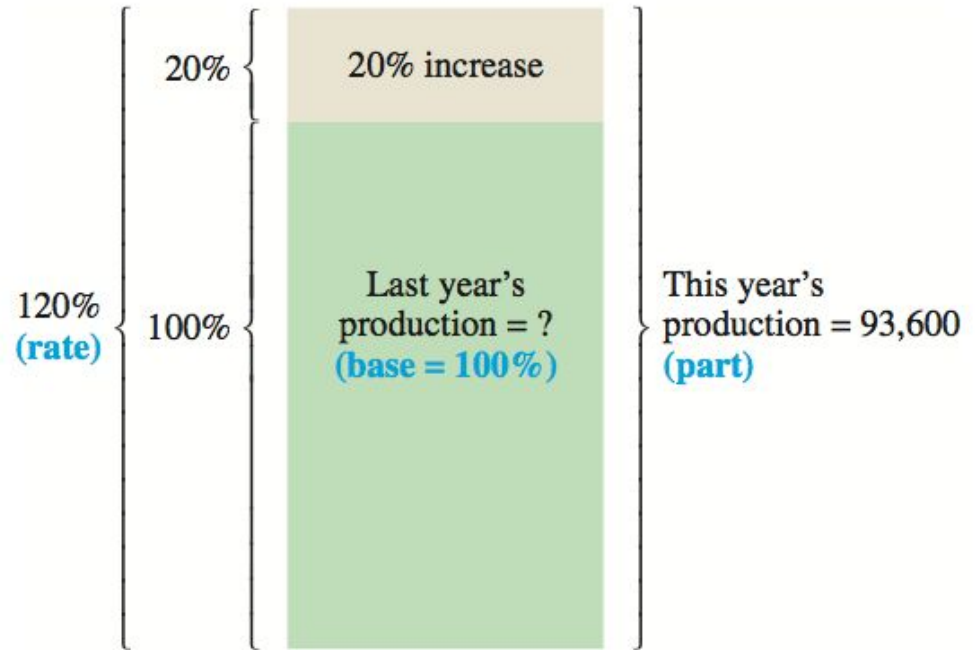
The value of the house last year was \$185,000.

Example

Due to increased demand for a patented process that will help analyze the DNA of mice, Biotics Genome has increased production of testing kits by 20% per year for each of the two past years. This year's production is 93,600 kits. Find the number of kits produced two years ago.

Example

The two increases cannot be added together because the increases are from two different years, or two separate bases. The problem must be solved in two steps. First, use a diagram to find last year's production.



Example

Part = 93,600 and

Rate = 100% + 20% = 120%

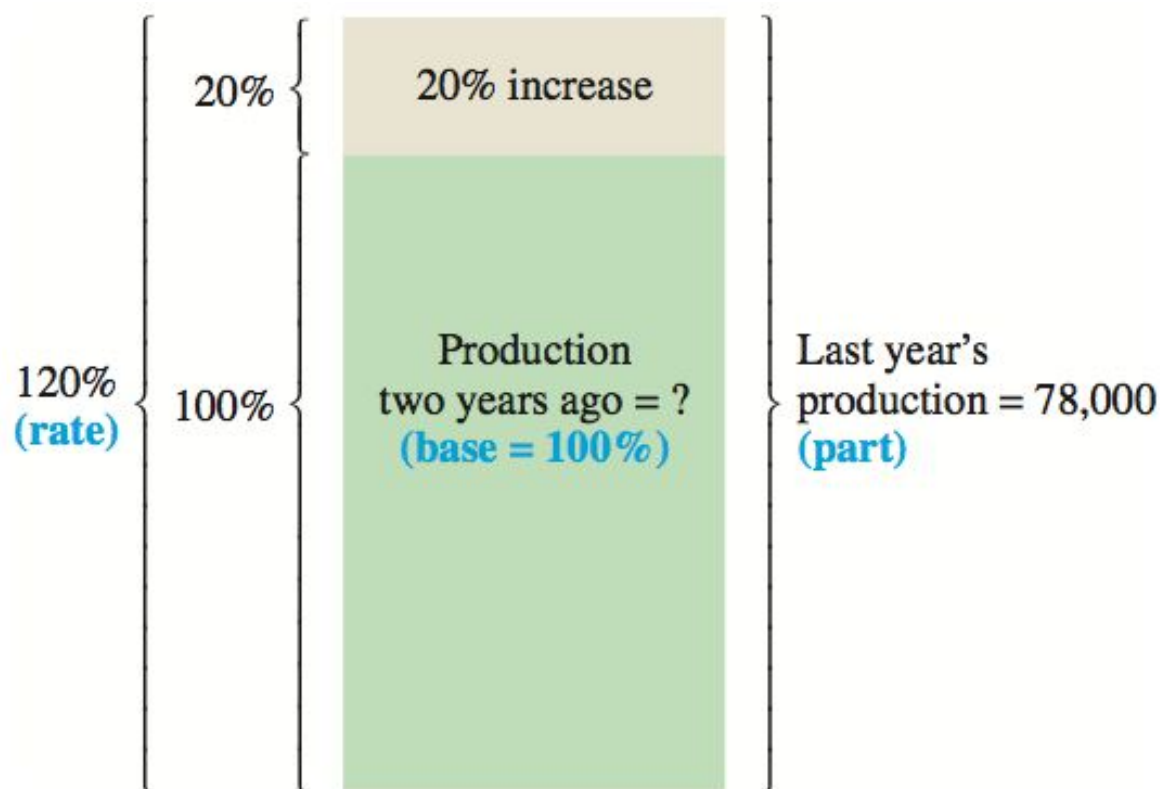
$$B = \frac{\text{Part}}{\text{Rate}} = \frac{93,600}{120\%}$$

$$B = \frac{93,600}{1.2} = 78,000$$

Production last year was 78,000 kits.

Example

Production for the preceding year (two years ago) must now be found.



Example

Part = 78,000 and

Rate = 100% + 20% = 120%

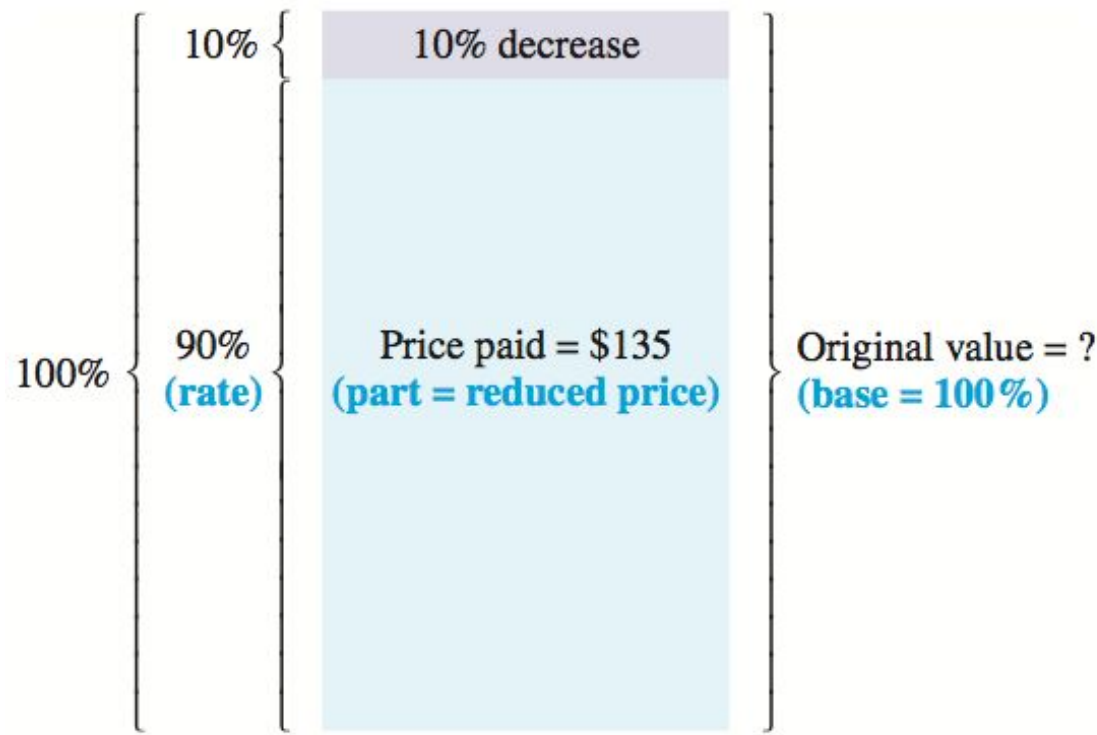
$$B = \frac{\text{Part}}{\text{Rate}} = \frac{78,000}{120\%}$$

$$B = \frac{78,000}{1.2} = 65,000$$

Production two years ago was 65,000 kits.

Apply the Basic Diagram for Decrease Word Problems

A diagram, such as this one, can be used to help solve a decrease word problem.



Use the Basic Percent Formula to Solve for Base in Decrease Problems

To find the Rate, use

$$\text{Original} - \text{Increase} = \text{New value}$$

To find the Base, use

$$B = \frac{P}{R}$$

Example

After Nike deducted 10% from the price of a pair of competition running shoes, Katie Small paid \$135. What was the original price of the shoes?

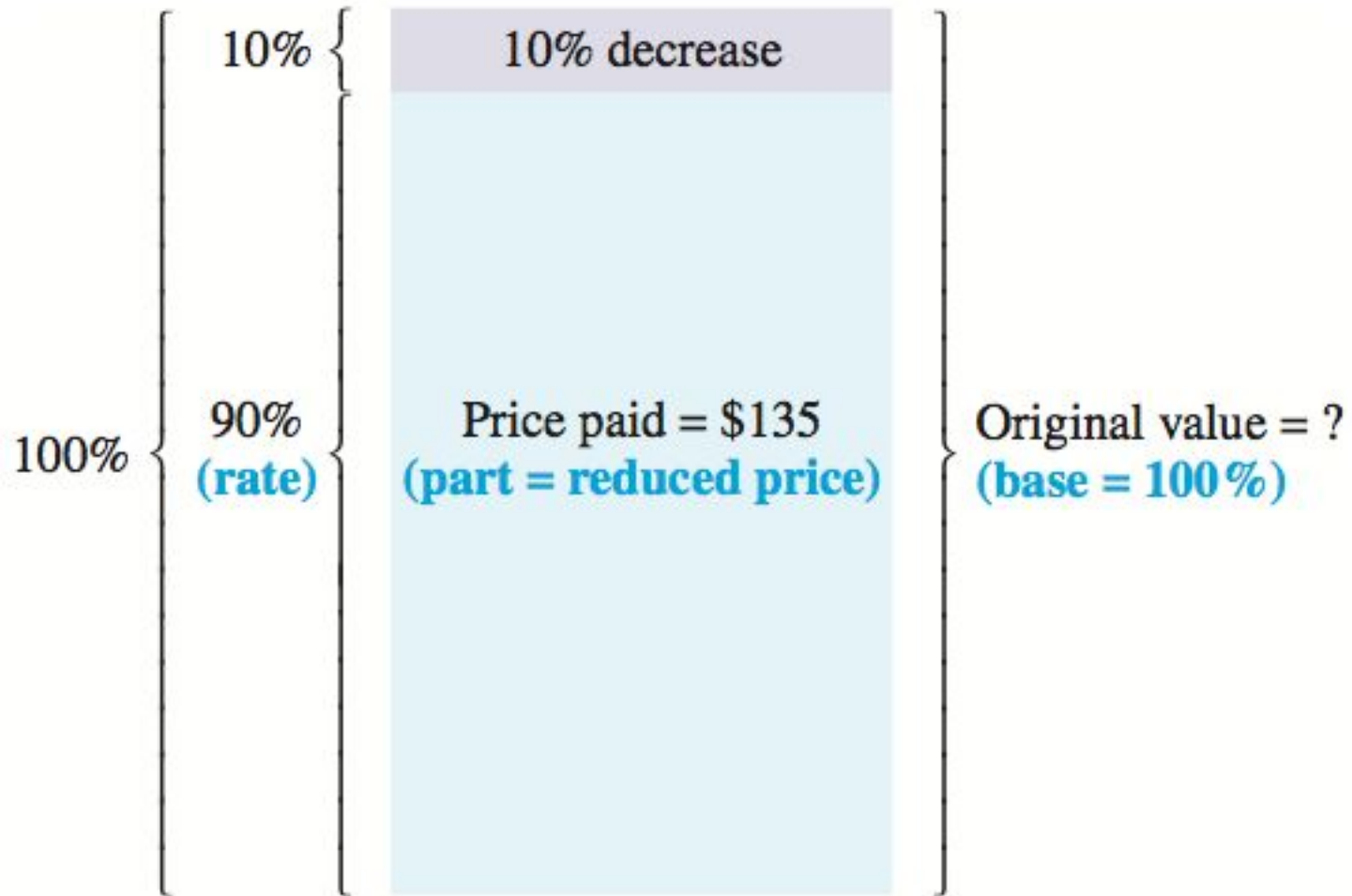
Example

Use a diagram again, and remember that base is the starting point, which is the original price. As always, the base is 100%. Use the decrease formula.

Original (base) – Increase = New value

Apply the basic diagram for a decrease.

Example



Example

Original – Increase = New value

$$100\% - 10\% = 90\%$$

Substitute part = \$135 and rate = 90%

$$B = \frac{\text{Part}}{\text{Rate}} = \frac{135}{90\%}$$

$$B = \frac{135}{.9} = \$150$$

The original price was \$150.