Topics

- Time value of money
- Present value and Future values of cash flows
- Interest rate, time, conversion period
- Annuity, types of annuity
- Uneven cash flow
- Loan Amortization schedule

Time value of money

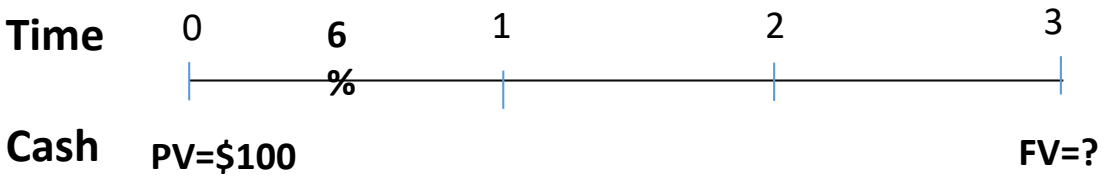
The principles and computations used to revalue cash payoffs at different times so they are stated in dollars of the same time period.

The time value of money (TVM) is the idea that money available at the present time is worth more than the same amount in the future due to its potential earning capacity.

Ref: http://www.investopedia.com/terms/t/timevalueofmoney.asp?lgl=no-infinite

Cash Flow Time lines

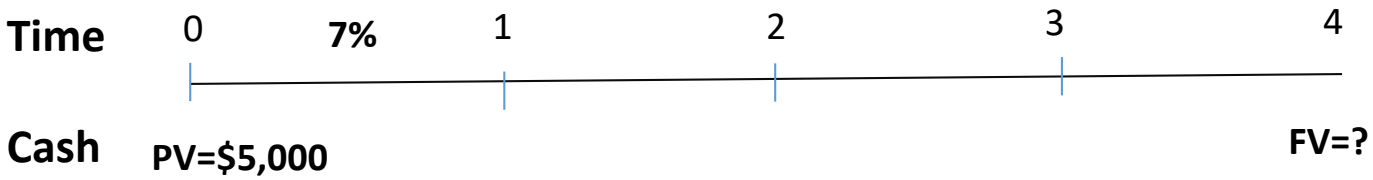
An important tool used in time value of money analysis, it is a graphical representation to show the timing of cash flows.



Practice

Draw a cash flow timeline to illustrate the following situation:

You invest $5,000 today in a four-year savings instrument that pays 7% interest each year.


Time
0
7%
1
2
3
4
Cash
PV=$5,000
FV=?

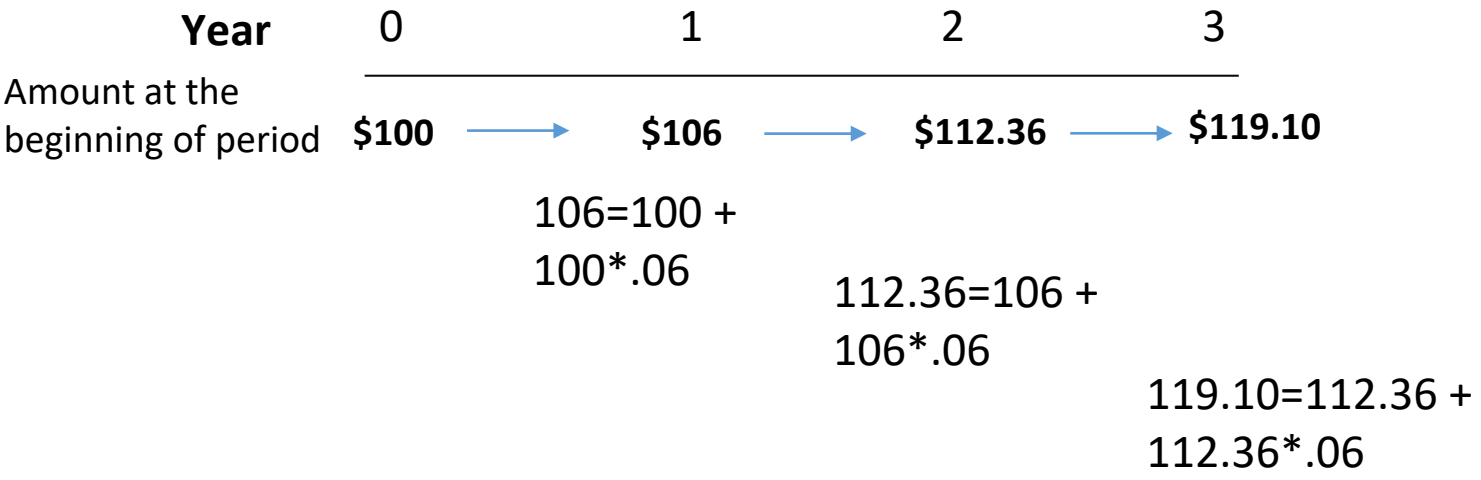
Future value

A dollar in hand today is worth more than a dollar to be received in the future because if you had money now , you could invest, earn interest and end up with more than one dollar in the future.

The amount to which a cash flow or series of cash flows will grow over a given period of time when compounded at a given interest rate.

Future value

- Suppose, you plan to deposit $100 in a bank that pays a guaranteed 6% interest each year. How much you would have at the end of year 3?



Future value

We can use formula to calculate future values.

$FV = PV\ (1+r)^n$

- FV = Future value
- PV = Present value
- r = Interest rate
- n = Period

So $FV=\$100\ (1+.06)^3 = \$119.10$

Practice

Assume that you invest $2,500 today. How much will this amount be worth in five years if the interest rate is 4%? How would your answer change if the interest rate is 6%?

Calculate using the formula.

$FV = PV (1+r)^n$

= 2,500 (1+ 0.04)^5 = $ 3,041.63 (Answer)

$FV = PV (1+r)^n$

= 2,500 (1+0.06)^5 = $ 3,345.56 (Answer)

Present value

The process of finding present value is called discounting.

5% Interest rate

Year	0	1	2	3
Amount at the beginning of period	**$100** ←	**$105** ←	**$110.25** ←	**$115.7625**

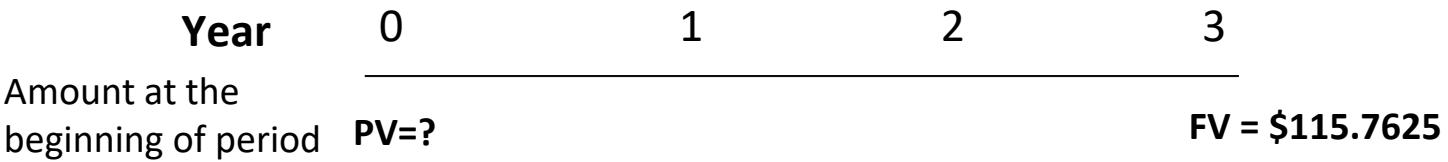
110.25=115.7625 / (1+.05)

105 = 110.25/ (1+.05)

100 =105/ (1+.05)

Present value

$$PV = FV / (1+r)^n$$



$$PV = 115.7625/ (1+.05)^3$$

$$= 100$$

Practice

Assume that you have the opportunity to purchase an investment that promises to pay you $3,041.63 in five years and your opportunity cost is 4%, How much should you be willing to pay for this investment today? How your answer change if your opportunity cost is 6%?

Answer

PV = 3,041.63/ (1+.04)^5

= $2,499.99

PV = 3,041.63/ (1+.06)^5

= $2,272.88

Interest rate

Suppose a bond has a cost of $100 and it will return $150 after 10 years, calculate required rate of return.

$FV = PV (1+r)^n$

$\$150 = \$ 100 (1+r)^{10}$

$150/100= (1+r)^{10}$

$1.5 = (1+r)^{10}$

$(1.5)^{1/10} = [(1+r)^{10}]^{1/10}$

$1.0413 = 1+r$

$1.0413-1=r$

So, r = .0413 or 4.13% (Answer)

Practice

- Suppose you just called the East key State bank and found that the balance in your savings account is $1,269.50. If you deposited $800 six years ago, what rate of return have you earned on the savings account?

Answer

$FV = PV (1+r)^n$

$\$1,269.50 = \$ 800 (1+r)^6$

$1269.50/800 = (1+r)^6$

$1.586 = (1+r)^6$

$(1.586)^{1/6} = [(1+r)^6]^{1/6}$

$1.0799 = 1+r$

$1.0799-1=r$

So, r = 0.0799 or 7.99% (Answer)

Time

- Suppose, you believe that you could retire comfortably if you had $1 mln, you want to find out how long it will take you to acquire $1 mln, assuming you now have $500,000 invested at 4.5%? (1 mln = 1,000,000)

$FV = PV (1+r)^n$

$1{,}000{,}000 = 500{,}000 (1+.045)^n$

$2 = (1.045)^n$

$\ln 2 = \ln (1.045)^n$

$\ln 2 = n \ln(1.045)$

$\ln 2 / \ln (1.045) = n$

$15.74 = n$

so, N = 15.74 years. (Answer)

Practice

- Assume you can invest $1,000,000 today at 4.5% interest. If you plan to sell the investment when its value reaches $1,500,000 for how long will your money have to be invested?

Answer

$FV = PV (1+r)^n$

$1{,}500{,}000 = 1{,}000{,}000 (1+.045)^n$

$1.5 = (1.045)^n$

$\ln 1.5 = \ln (1.045)^n$

$\ln 1.5 = n \ln(1.045)$

$\ln 1.5 / \ln (1.045) = n$

$9.21 = n$

so, N = 9.21 years. (Answer)

Conversion period

- When interest is compounded more often than once a year, interest rate and time need to be modified as per compounding frequency.

- Compounding can be monthly, quarterly, semi -annually etc.

Practice

- Find the future value of $500 at 8% compounded quarterly for 10 years.

$FV = PV\ (1+r)^n$

$= 500\ (1+ 0.08/4)^{10x4}$

$= \$1,104.02$

If compounded monthly, $FV = 500\ (1+ .08/12)^{10x12}$

$= \$1,109.82$

Thank you