CONTROL SYSTEMS ASSIGNMENT 1

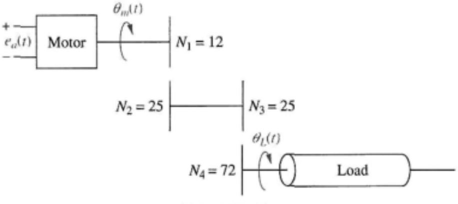
J.Eshwari
EE19BTECH11042
Electrical Engineering
IIT Hyderabad.

September 8, 2020

QUESTION 44

A dc motor develops 55 N-m of torque at a speed of 600 rad/s when 12 volts are applied. It stalls out at this voltage with 100 N-m of torque. If the inertia and damping of the armature are 7 kg-m$^2 and 3N-m-s/rad, respectively, find the transfer function, G(s) = \theta_L(s)/E_a(s),$

of this motor if it drives an inertia load of 105 kg-m^2 through a gear train, as shown in the below figure.

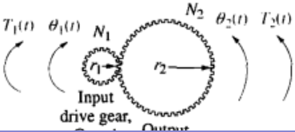


PREREQUISITES

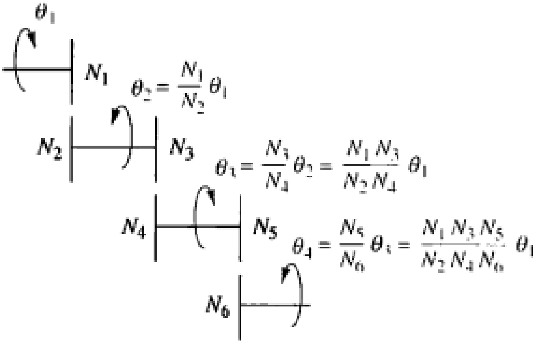
CONCEPT-1

1.As the gears turn, the distance traveled along each gear's circumference is the same.
2.Since the ratio of the number of teeth along the circumference is in the same proportion as the ratio of the radii.
3.The ratio of the angular displacement of the gears is inversely proportional to the ratio of the number of teeth.
4.If we assume there is no gearloss, the rotational energy is conserved i.e torque times angular displacement.

$$\frac{\theta_2}{\theta_1} = \frac{r_1}{r_2} = \frac{N_1}{N_2} = \frac{T_1}{T_2} \tag{2.1}$$

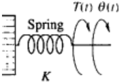
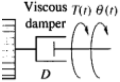
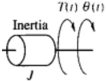


CONCEPT -2 In order to eliminate gears with large radii, a **gear train** is used to implement large gear ratios by cascading smaller gear ratios.For gear trains,the equivalent gear ratio is the product of the individual gear ratios(from concept 1). We assume there is no gear loss.



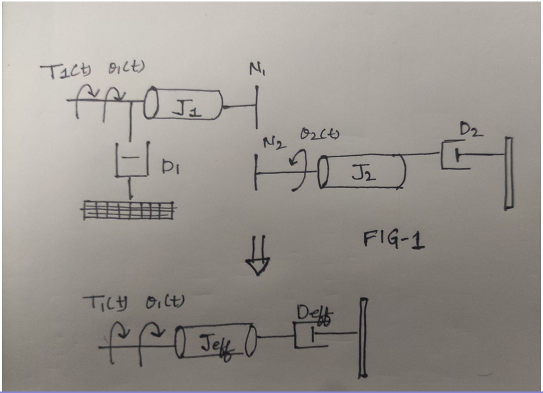
CONCEPT -3 Rotational mechanical system equations in time domain and impedences in frequency domain:-

TABLE 2.5 Torque-angular velocity, torque-angular displacement, and impedance rotational relationships for springs, viscous dampers, and inertia

Component	Torque-angular velocity	Torque-angular displacement	Impedence $Z_M(s) = T(s)/\theta(s)$
	$T(t) = K\int_0^t \omega(\tau)d\tau$	$T(t) = K\theta(t)$	K
	$T(t) = D\omega(t)$	$T(t) = D\frac{d\theta(t)}{dt}$	Ds
	$T(t) = J\frac{d\omega(t)}{dt}$	$T(t) = J\frac{d^2\theta(t)}{dt^2}$	Js^2

Note: The following set of symbols and units is used throughout this book: $T(t)$ – N-m (newton-meters), $\theta(t)$ – rad(radians), $\omega(t)$ – rad/s(radians/second), K – N-m/rad(newton- meters/radian), D – N-m-s/rad (newton- meters-seconds/radian). J – kg-m^2(kilograms-meters^2 – newton-meters-seconds^2/radian).

CONCEPT -4 The below shown figure-1 has inertia J_1 *and damping*D_1 *at armature* , J_2 *and damping* D_2 *at load*.
Figure-2 represents the equivalent system without gears.
To obtain the system 2 form system 1 :-
The systems are attached by gears,it means only one degree of freedom and one equation to solve.



Using concept 3 to find the toques in frequecny domain

$$\frac{T_1(s) - (J_1 \times s^2 + D_1 \times s) \times \theta_1(s)}{(J_2 \times s^2 + D_2 \times s) \times \theta_2(s)} = \frac{N_1}{N_2} \tag{2.2}$$

Considering Concept-1, Eqaution 0.2 can be further simplified as follows

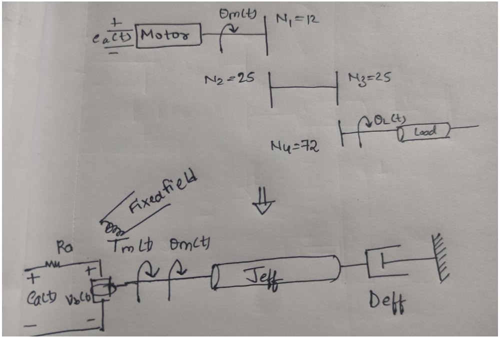
$$T_1(s) = [(J_1 + J_2(\frac{N_1}{N_2})^2)s^2] + [(D_1 + D_2(\frac{N_1}{N_2})^2)s]\theta_1(s) \tag{2.3}$$

Therefore

$$J_{eff} = J_1 + J_2(\frac{N_1}{N_2})^2 \tag{2.4}$$

$$D_{eff} = D_1 + D_2(\frac{N_1}{N_2})^2 \tag{2.5}$$

SOLUTION



In the above figure, from concept 4

Given that inertia at armature is 7 kg-m^2; $load\ is\ 105kg - m^2$.

$$J_{eff} = J_{armature} + J_{load}\left(\frac{N_1 N_3}{N_2 N_4}\right)^2 = 9.92kg - m^2$$

$$D_{eff} = 3N - m - s/rad(armaturedampingonly)$$

Considering the circuit with voltage e(t), armature current as i(t) and the system The torque developed by the motor is proportional to the armature current in freqency domain is as follows.

$$T_m(s) = K_t I_a(s) \tag{3.1}$$

The current-carrying armature is rotating in a magnetic field, its voltage(back-emf) is proportional to speed.

$$v_b(t) = K_b \frac{d\theta_m}{dt} \tag{3.2}$$

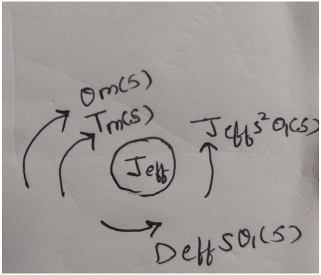
In frequency domain it can be written as

$$V_b(s) = K_b s\theta(s) \tag{3.3}$$

The loop equation around the Laplace transformed armature circuit

$$R_a I_a(s) + L_a s I_a(s) + V_b(s) = E_a(s) \tag{3.4}$$

Considering the free body diagram as shown below,



$$T_m(s) = (J_{eff}s^2 + D_{eff}s) \times \theta_m(s) \tag{3.5}$$

Substituting 3.1,3.2,3.5 in 3.4 equation it boils down to:-
Inductance of inductor is low is taken into account.

$$\frac{\theta_m(s)}{E_a(s)} = \frac{\frac{K_t}{R_aJ_{eff}}}{s[s + \frac{1}{J_{eff}}(D_{eff} + \frac{K_tK_b}{R_a})} \tag{3.6}$$

The electrical constants of the motor's transfer function can be found as follows Considering the armature circuit equation in time domain;

$$\frac{R_a}{K_t} T_m(t) + K_b w_m(t) = e_c(t) \tag{3.7}$$

When voltage is constant, dropping the time function,

$$T_m = -\frac{K_b K_t}{R_a} w_m + \frac{K_t}{R_a} e_a \tag{3.8}$$

At zero angular velocity, the torque intercept is known as T$_{stall}$

$$\frac{K_t}{R_a} = \frac{T_{stall}}{e_a} \tag{3.9}$$

The angular velocity occurring when the torque is zero is called the no- load speed,w$_{no-load}$.

$$K_b = \frac{e_a}{w_{no-load}} \tag{3.10}$$

From the data at 12 V armature circuit voltage,
$T_{stall} = 100$, *Therefore*,
$\dfrac{K_t}{R_a} = \dfrac{T_{stall}}{e_a} = \dfrac{100}{12} N - m/v$
Taking the equation 3.8 at values $T_m = 55$ *and* $w_m = 600$
$55 = -K_b \times \dfrac{100}{12} \times 600 + \dfrac{100}{12} \times 12$
$K_b = 0.009 v - s/rad$
Substituting the results in 3.6 with
J_{eff} *as* $9.92 kgm^2$ *and* D_{eff} *as* $3N - m - s/rad$

$$\frac{\theta_m(s)}{E_a(s)} = \frac{\frac{100}{12}\frac{1}{9.92}}{s(s + \frac{1}{9.92}(\frac{100}{12})0.009)} = \frac{0.84}{s(s + 0.31)} \tag{3.11}$$

But we require the transfer function,

$$G(s) = \frac{\theta_L(s)}{T_m(s)}$$

From concept 2,

$$\theta_L = \frac{N_1 N_3}{N_2 N_4} \times \theta_m$$

$N_1 = 12, N_2 = N_3 = 25, N_4 = 72$

$$\theta_L = \frac{1}{6}\theta_m$$

Therefore,

$$G(s) = \frac{\theta_L(s)}{T_m(s)} = \frac{0.14}{s(s + 0.31)}$$