

ID-3 Algorithm Example:

(1)

Ex: Given data set.

Sl No	Weather	Mood	Study
1	Sunny	Happy	Yes
2	Rainy	Sad	No
3	Sunny	Sad	Yes
4	Rainy	Happy	Yes

(1) Calculate Entropy of dataset (D) for target attribute 'Study'.

$$\begin{aligned}\text{Entropy}(D) &= - \sum_{i=1}^n P_i \log_2(P_i) \\ &= - P(\text{Yes}) \cdot \log_2(P(\text{Yes})) - P(\text{No}) \cdot \log_2(P(\text{No})) \\ &= - \frac{3}{4} \times \log_2\left(\frac{3}{4}\right) - \frac{1}{4} \cdot \log_2\left(\frac{1}{4}\right) \\ &\approx 0.811\end{aligned}$$

(2) Calculate Information Gain for Attribute = Weather.

(a) Entropy of Weather = Sunny.

$$\begin{aligned}E(\text{Sunny}) &= - \frac{2}{2} \log_2\left(\frac{2}{2}\right) - \frac{0}{2} \cdot \log_2\left(\frac{0}{2}\right) \\ &= 0\end{aligned}$$

(b) Entropy of Weather = Rainy.

$$\begin{aligned}E(\text{Rainy}) &= - \frac{1}{2} \cdot \log_2\left(\frac{1}{2}\right) - \frac{1}{2} \log_2\left(\frac{1}{2}\right) \\ &= 1.\end{aligned}$$

~~Information Gain~~

$$\begin{aligned}\text{Avg Entropy (Weather)} &= P(\text{Sunny}) \times E(\text{Sunny}) + P(\text{Rainy}) \times E(\text{Rainy}) \\ &= \frac{2}{4} \times 0 + \frac{2}{4} \times 1 = 0.5\end{aligned}$$

$$\begin{aligned}\text{Information Gain (weather)} &= E[A] - E[\text{weather}] \\ &= 0.811 - 0.5 \\ &= 0.311\end{aligned}$$

(3) Calculate Information Gain of Attribute = Mood.

(a) Entropy of Mood = Happy

$$\begin{aligned}E[\text{Happy}] &= -\frac{2}{2} \log\left(\frac{2}{2}\right) - \frac{0}{2} \log\left(\frac{0}{2}\right) \\ &= 0.\end{aligned}$$

(b) Entropy of Mood = Sad.

$$\begin{aligned}E[\text{Sad}] &= -\frac{1}{2} \log\left(\frac{1}{2}\right) - \frac{1}{2} \log\left(\frac{1}{2}\right) \\ &= 1.\end{aligned}$$

$$\begin{aligned}\text{Avg Entropy [Mood]} &= P[\text{Happy}] \times E[\text{Happy}] \\ &\quad + P[\text{Sad}] \times E[\text{Sad}] \\ &= \frac{2}{4} \times 0 + \frac{2}{4} \times 1 \\ &= 0.5\end{aligned}$$

$$\begin{aligned}\text{Information Gain [Mood]} &= E[A] - E[\text{Mood}] \\ &= 0.811 - 0.5 \\ &= 0.311\end{aligned}$$

(4) Since Gain of Weather & Mood attributes are same, choose any one attribute as ~~split~~ splitting attribute.

Based on this we have following Decision Tree constructed.

