Maths 761 Lecture 17

Topic for today
Bifurcations in systems with two parameters

Reading for next lecture
Glendinning Chapter 12, Introduction, §12.1 (excluding example using perturbation theory starting halfway down page 341), §12.2, §12.3

Today's handouts
Lecture 17 notes
Worked examples of bifurcations in systems with two parameters

Bifurcations in systems with two parameters

We frequently need to understand the bifurcations in systems such as

$$\dot{x} = f(x; \mu), \quad x \in \mathbf{R}^n, \quad \mu \in \mathbf{R}^m$$

where $m \geq 2$, i.e., when there is more than one parameter in the problem. In this case, the analysis of the system proceeds as before (for instance, we seek equilibrium solutions and determine their types as a function of the relevant parameters, then identify bifurcations), but there are several different ways to present our results. The two main ways of summarising information about solutions are drawing bifurcation diagrams or drawing bifurcation sets and phase portraits.

1. **Bifurcation diagrams** : Define a one-dimensional path through the m-dimensional parameter space and draw a bifurcation diagram showing the bifurcations seen as that path is traversed. For example, in systems with two parameters, it is common to fix one parameter and let the other vary to produce a bifurcation diagram. Other choices of the first (fixed) parameter value may lead to other qualitatively different bifurcation diagrams.

2. **Bifurcation sets** : If $m = 2$ (or if all but two of the parameters are fixed) then a bifurcation set can be constructed. This is a picture in the *parameter space*. For each bifurcation found, a curve is drawn in the parameter space showing the locus of that bifurcation. The curves divide up the parameter space into regions; all choices of parameter within a region result in (qualitatively) the same dynamics. The qualitative dynamics associated with each region is indicated by drawing a representative phase portrait in each region. Thus, a bifurcation set with phase portraits summarises two types of information: the parameter values for which bifurcations occur; and the qualitative dynamics occuring in the various regions between the bifurcation curves.