Scheduling: a good candidate for quantum annealing?

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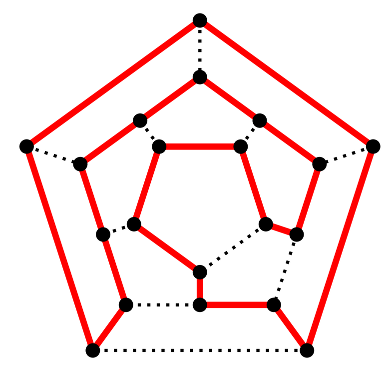
Introduction

- The quantum group at NASA Ames is interested in exploring the potential of **quantum annealing** to solve difficult yet important **scheduling problems**.
- To compare quantum annealing to state-of-the-art classical algorithms, we need to evaluate the performance of each on **parametrized sets of problems.**
- Hen and his team have designed families of intrinsically-hard scheduling problems, where the hardness is controllable by a parameter. These problems derive from the NP-complete **Hamiltonian Path** problem.

Hamiltonian Path problem

Given a graph with M vertices and a fraction p of all the pairs of vertices connected with an edge, determine whether there is a path, moving along the edges, which starts at one of the vertices and visits all of the vertices exactly once.



Constraint satisfaction (SAT) form

- Represent Hamiltonian Path problem on M vertices as a **SAT problem**.
- The latter involves N bits or **Ising spins**.
- Ising spin encoding is essential in order to use D-Wave's quantum annealer.
- Additionally, there are well-studied **classical** algorithms for SAT.
- Unfortunately, a relatively **large number of bits** N is required to represent even small problems. (Is there a more efficient encoding?)

M	N
10	430
12	612
14	1072
16	1350

Algorithms

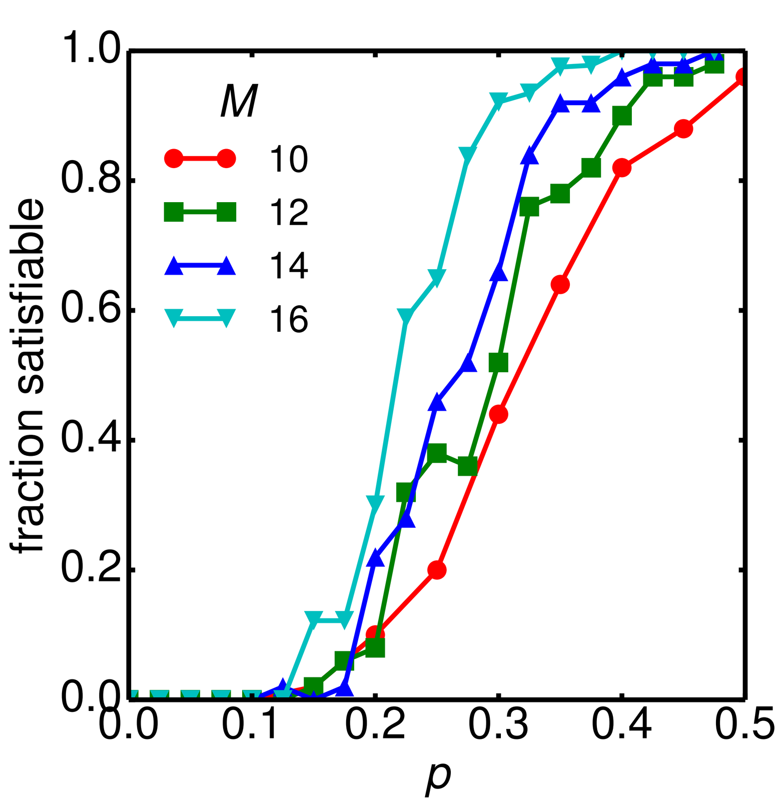
Davis-Putnam-Logemann-Loveland (DPLL) is a **complete** algorithm—it always terminates, even if no solution exists. Because it is basically an exhaustive search of the solution space, it is often not as effective as other methods for large problems.

WALKSAT is a **heuristic** algorithm—there is no guarantee that there is no solution if none is found. Because proposed quantum annealing algorithms share this property, WALKSAT is a good candidate for a fair comparison between classical and quantum algorithms.

Results—Fraction of satisfiable instances

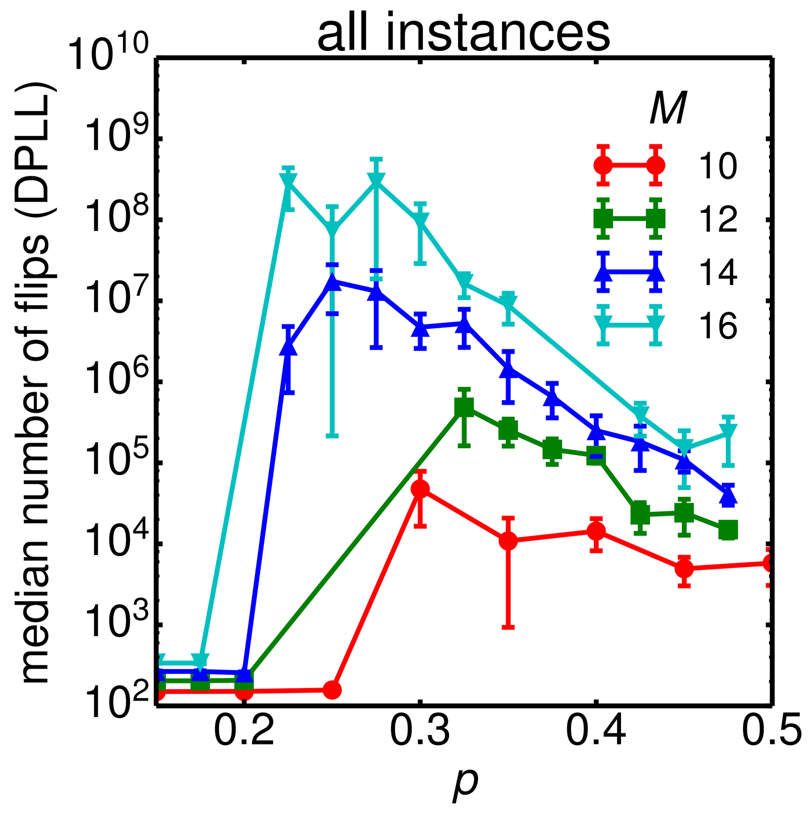
The fraction of problem instances with satisfying assignments varies between 0 at small p and 1 at large p. Two expected features are observed:

1. The "crossover" region gets sharper as M increases.
2. The value of p where the crossover occurs varies as $(\ln M + \ln\ln M)/M$ and so decreases as M increases.

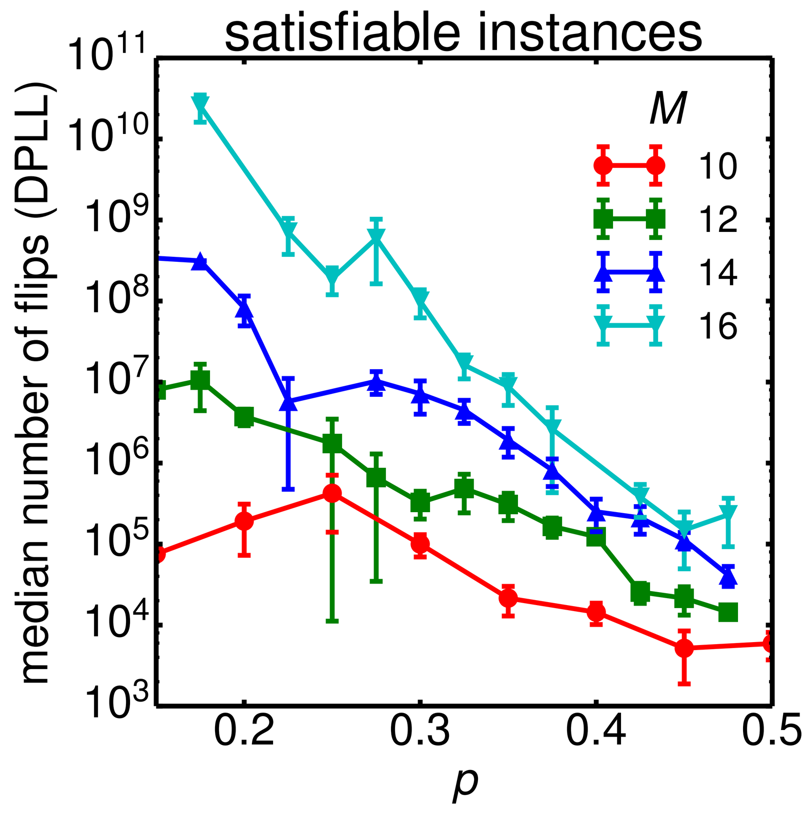


Results—DPLL

The number of basic steps (bit flips) required for DPLL to determine whether or not there is a solution is shown at right (top). There is a peak at intermediate values of p, where there are a small number of solutions that are difficult to find.

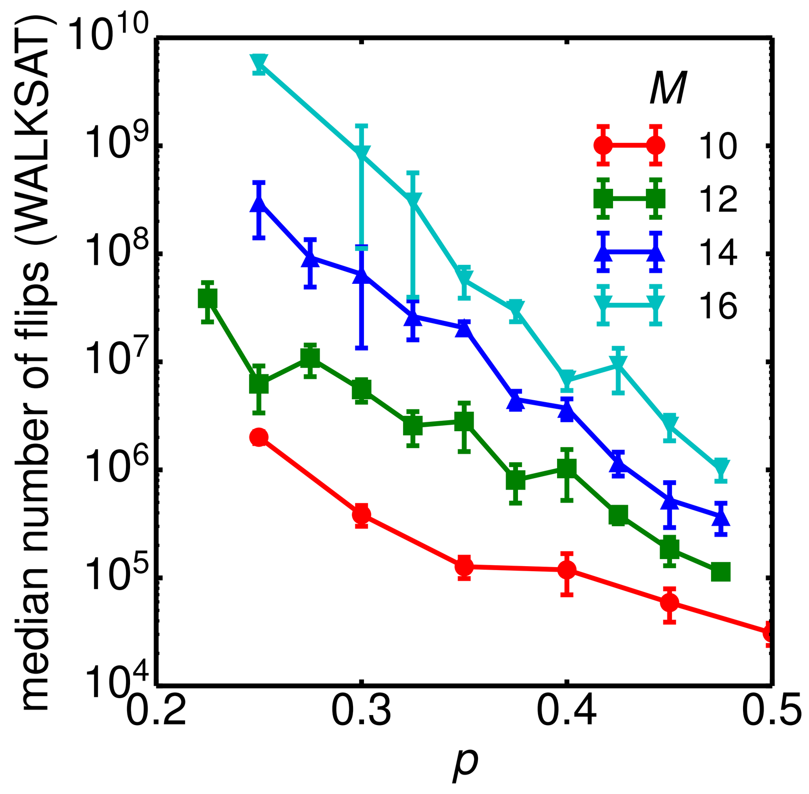


Of the satisfiable instances, the most difficult are those for which p is near the crossover value and hence that have few solutions.



Results—WALKSAT

WALKSAT was tested on instances determined to be satisfiable by DPLL. The results are similar to those for DPLL restricted to satisfiable instances—the most difficult instances are those for which p is near the crossover value.



Conclusions

- The Hamiltonian Path problem is **difficult** for classical algorithms, **even for small problem sizes**. Furthermore, the hardness is **tunable** via a parameter (p) which is independent of problem size.
- The above features, in addition to the resemblance of Hamiltonian Path to scheduling problems, make it a promising candidate for further research using **quantum annealing** techniques.
- **Significant obstacle:** prohibitively large number of bits required to represent Hamiltonian Path as a SAT problem.
- **Future investigation:** study instances small enough to be usable on NASA's (D-Wave) quantum annealer; compare classical results to those of quantum annealing.

References

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