Portfolio Optimization of Fantasy Football Teams

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Abstract

In the financial world, portfolio optimization is fundamental to assessing the value of assets commonly measured by their risks and returns. A simple optimization, in particular, includes Modern Portfolio Theory (MPT) [1] that seeks to maximize a portfolio's return at a given level of risk. In this paper, we seek to apply these principles to the sports world through the popular game of fantasy football[2]. As a parallel to MPT, each player is considered an asset where the player points represent the expected returns and the standard deviation becomes the risk. From here, we attempt to balance the risk and return of the entire team of players using the Sharpe ratio [3]. Additional constraints and assumptions are made to account for the domain specific reality of limited position types and team assembly via a drafting process. We develop an iterated process to assess the marginal value a player provides during team assembly while also considering additional factors such as position abundance and player draftibility; ultimately, giving the combined factors the label "urgency score". And finally, we simulate experiments comparing this draft strategy to known baselines and evaluate the performance for a standard fantasy football season.

1 Introduction

What is fantasy football? To put it simply, fantasy football is a series of games. Participants assemble a team (virtually) of current NFL players and score points based on the performance of those sames players in the actual games they play each week of a NFL season. For example, a player may score points for a touchdown scored in a game, a pass caught, etc. Participants will form a league where their teams matchup against each other in head-to-head games each week, the goal being to score more points than their opponent. At the end of the season (coinciding with the end of the NFL regular season), the wins and losses for each team are taillied up and a playoff round is held to determine the league champion. For simplicity, we will claim that the team with the most wins at the end of the season is the league champion instead.

A key aspect of fantasy football is the idea of a draft. No player may be assigned to more than one team in a league. This means that participants must select players to be on their team before the season starts. Similarly, the positions that players can be assigned to are limited. For example, a team will often only be allowed to have 1 quarterback (QB), 2 running backs (RB), 2 wide receivers (WR), 1 tight end (TE), 1 defense (DEF), and 1 kicker (K). And so, it becomes a strategic game of weighing the value of each player not only in terms of the points they are expected to score, but also in terms of the position they play and the scarcity of that position.

How does portfolio optimization come into play? Financial portfolios have a similar structure to fantasy football teams in that they want to balance the risk and return of their assets. Essentially, both want to find the allocation of assets that will produce the most return for a given level of risk. And so, in the following sections, we ask the question; can we create an optimal fantasy football team using the principles of portfolio optimization?

2 Related Work

2.1 Modern Portfolio Theory (MPT) & Sharpe Ratio

The idea behind MPT is to maximize the expected return of a portfolio at a given level of risk. Note this is also referred to as a mean-variance analysis. The expected return $E[R]$ of a portfolio is simply the weighted sum of the expected returns of the assets in the portfolio (commonly measured by annualized returns). The risk σ of the portfolio is measured by the weighted variance of the returns of the individual assets along with the weighted correlations between each asset in the portfolio. The expected return and variance for the portfolio are given below where ρ_{ij} is the correlation between assets i and j.

$$E[R] = \sum_{i=1}^{n} w_i E[R_i] \tag{1}$$

$$\sigma^2 = \sum_{i=1}^{n} w_i^2 \sigma_i^2 + \sum_{i=1}^{n} \sum_{j \neq i}^{n} w_i w_j \sigma_i \sigma_j \rho_{ij} \tag{2}$$

Given a fixed level of risk, solving this problem amounts to finding the optimal weights that produces the maximum return. If we were to look at all possible weight combinations and the corresponding portfolios we could create a plot of the risk versus the expected return, which is commonly known as the efficient frontier. Note how in the figure below, at a given level of risk and an arbitrary portfolio, one would always want to re-allocate to a portfolio with a larger expected return where we observe the efficient frontier.

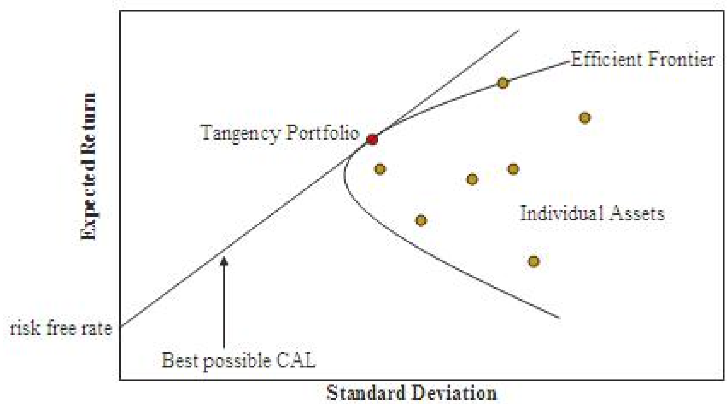

Figure 1: Efficient Frontier for all possible weighted portfolio combinations [4]

But what if we wanted to assess the performance of a portfolio as different levels of risk? The Sharpe ratio is a measure that does this by comparing the expected return of a portfolio to the theoretical risk-free asset (U.S. treasury bonds for example). It is given by the formula:

$$S = \frac{E[R] - R_f}{\sigma} \tag{3}$$

where R_f is the risk-free rate of return, and $E[R]$ and σ are the expected return and risk of the portfolio respectively. The Sharpe ratio allows us to compare the performance of portfolios at different levels of risk and represents the excess return per unit of risk.

Note we can extend this work to a fantasy football setting where we think of the players as assets and the points they score as the expected return. The key difference here will be that our assets no longer can have continuous weights $w_i \in [0, 1]$, but must now be discrete $w_i \in \{0, 1\}$ in measuring the expected return and risk of a given team. Furthermore, we note that, by the nature of the drafting process, the pool of possible assets is limited and dynamic. That is, as players are selected by opposing teams, the pool of available assets shrinks and so the universe of feasible portfolios is reduced. We will explore these limitations in the following sections.

2.2 Geometric Distribution

The geometric distribution is a discrete probability distribution that models the number of trials needed to achieve the first success in a sequence of independent Bernoulli trials. The probability mass function of the geometric distribution is given below where p is a probability of success on a given trial. Note the expected value of the geometric distribution is $E[X] = 1/p$.

$$P(X = k) = (1 - p)^{k-1} p \tag{4}$$

Note when creating teams, if we expect a player to be drafted at a given position k on average then we can easily model this as a geometric distribution. That is, the player was not selected in the $k - 1$ picks prior, but finally was selected at the kth pick. If we let the random variable $X =$"position player is selected in draft" and we know $E[X]$, then $p = \frac{1}{E[X]}$. This will be useful in our iterated optimization process.

3 Methodology

3.1 Limiting Assumptions

To simplify the problem, we make the following assumptions:

1. A league will consist of 12 teams with 1 team being the user team (the team we are drafting).
2. The league will use a snake draft format. That is, if the first picks of each team are numbered $1, 2, \ldots, 12$, the order of picks will be $1, 2, ..., 11, 12, 12, 11, ...2, 1, 1, 2, ...$ and so on. That is, each round the order of the picks will reverse from the round prior to ensure fairness in the draft (this is a common drafting format in fantasy football).
3. Due to data limitations, our teams will be composed of 1 QB, 2 RBs, 2 WRs, 1 TE, and 1 FLEX. A FLEX player can be filled by any RB, WR, or TE. Note that a DEF and K position are often included in fantasy football teams, but excluding them is not unrealistic as the performance of defenses and kickers is often highly variable between and within seasons.
4. Teams will not have any bench players. Modeling the decision making process of who to start and sub out each week is beyond the scope of this work.
5. We look at PPR scoring (1 point per reception) for all players. This scoring format favors WRs generally so it will be interesting to see if the drafting model captures this in some way.
6. NFL seasons have bye weeks where real teams do not play and so neither do their players. Teams will not have bench players to sub in, so any players that have a bye week will simply score 0 points for that week.
7. NFL rookie players (players entering the league for the first time) will not be eligible to be drafted. There is no historical data on these players and so there is nothing for the model to use in evaluating their expected return.
8. Correlations between players can be difficult to estimate. Player performances may not always be perfectly indexed to each other by season and week. For example, one player may have played games from 2016-2020 while another player only played games from 2018-2020. Similarly, players may have been injured or sat out some seasons. And so, we use the following estimator for the correlation between two players $\hat{\rho}_{ij}$:

$$\hat{\rho}_{ij} = \frac{1}{NM} \sum_{k=1}^{N} \sum_{l=1}^{M} \rho_{kl} \tag{5}$$

   where N is the number of seasons player i has played, M is the number of seasons player j has played, and ρ_{kl} is the correlation between the two players in season k and l respectively. That is, we find the product of all seasons played between the two players and average the resulting correlations.

3.2 Motivation

Taking inspiration from MPT, we want to develop a draft strategy that maximizes the Sharpe ratio. But first, a good question to ask is, is this even a good strategy? Note that the following figure shows the relationship between the Sharpe ratio of a team and the expected number of points scored per game. Clearly, there is a strong positive correlation between the two. Note that this can also be seen as the efficient frontier of teams. And so, we have a motivating factor to use the Sharpe ratio as a good measure for building an optimal team. This is intuitive as the more points a team scores on average, the more likely they are to win games.

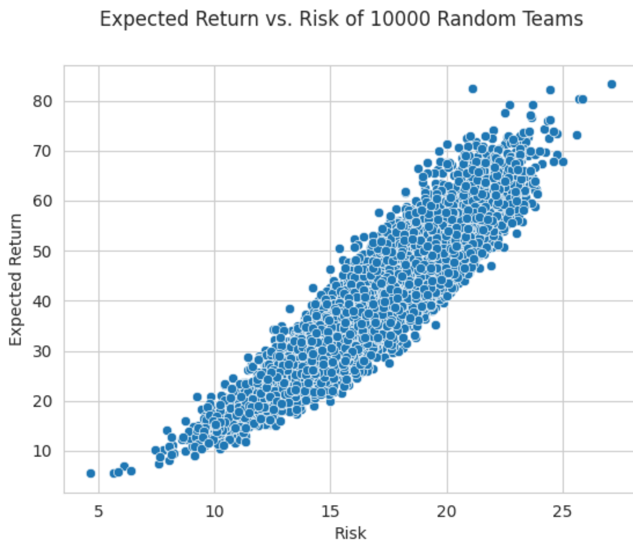

Figure 2: Sharpe Ratio vs. Expected Points Scored per Game

3.3 Draft Strategy

As mentioned above, we want to develop a draft strategy that maximizes the resulting Sharpe ratio of the finalized team. For reasons explained shortly, we will call this the "urgency draft strategy" (UDS) as it will attempt to maximize the "urgency" of drafting a given player with the current pick. We need a baseline to compare this strategy to, and so for every other team in the league, we will employ a "expert draft strategy" (EDS). Every year "expert" fantasy football analysts will create "draft boards" of players ranked for the upcoming season and these are used for drafts in public leagues. We will use the resulting dataset [5] of average draft positions (ADPs) across public leagues for each player as a representation of the EDS. That is, for each pick in the draft, the EDS will select the player with the lowest ADP that has not been selected yet and is eligible to be drafted for that team (e.g. QB slot is not filled). Essentially the EDS will draft an average optimal team for the given pick.

3.3.1 Urgency Draft Strategy (UDS)

A topic we have yet to address is that the maximization of the Sharpe ratio for a team is based on the complete sequence of players drafted. However, we have to make a decision on which player to draft at each pick. And so, there is no guarantee that, given a sequence of players that maximize the Sharpe ratio, the next player in the sequence will be available to be drafted. Instead, we formulate an iterative optimization process for drafting a team. That is, the UDS will attempt to maximize an Urgency Score (US) assigned to every eligible player for the current pick given we know the players that have been drafted and what the next pick will be. Then the US will be comprised of 3 components listed below:

1. **Marginal Sharpe Ratio (MSR)** - Given a team with players $P_1, P_2, \ldots$, this is the new Sharpe ratio of the team if player P_i were to be added to the team.
2. **Positional Scarcity (PS)** - This is the standard deviation of the MSRs of all players that share the same position as the player being selected. This captures the idea that we may want to draft a player sooner it the next best player at that position is significantly worse.
3. **Draft Availability (DA)** - This is the probability that a player will be avaialable to be selected at the next user's pick. This is modeled by a geometric distribution $X \sim G_p$ where p is derived from the ADP of the player as a prior, $p = \frac{1}{ADP}$. If n is the next user pick, then this score computes $P(X \geq n) = 1 - P(X < n) = 1 - (1-p)^{n-1}$. If a player does not have a prior ADP (meaning they were rarely drafted or drafted quite late), then we will assume p is derived by the highest ADP of the current pool of players.

Given a player P_i, the current pick k, and the next pick n, we have the US for a given player as well as the UDS optimization problem:

$$\max_{P_i} US(P_i, n) = \max_{P_i} \text{MSR}(P_i) \cdot \text{PS}(P_i) \cdot \text{DA}(P_i, n) \tag{6}$$

This process is repeated for each pick in the draft until the team is filled.

3.4 Hypothesis

Our hypothesis then becomes, will the UDS outperform the EDS in a fantasy football season measured by the number of wins? Formally, this is:

$$H_0 : \text{UDS wins} = \text{EDS wins} \tag{7}$$

$$H_1 : \text{UDS wins} > \text{EDS wins} \tag{8}$$

In the next section, we will test this hypothesis by running multiple simulations of fantasy football seasons and comparing the results.

4 Experimental Results

Note the code to run the following simulations can be found and reproduced at the following Github repository - GITHUBURLHERE.

4.1 Dataset

We use historical NFL data for all players from the 2013-2022 seasons [6]. The dataset includes the fantasy football PPR points scored by each player in each game as well as the position they are eligible to be drafted at. Any NFL playoff games are removed from the dataset, and missing weeks (such as bye weeks) are filled with 0 points. We will be evaluating the performance of the UDS and EDS on the 2023 season for $N = 100$ trials.

4.2 Results

For each trial, we simulate two 12 team league drafts. One where the user draft strategy uses UDS, and the other where the user draft strategy uses EDS. The non-user (or AI) players will always use EDS for their draft strategy. A random draft order and random league schedule (who plays who each week) is generated for each trial. Note that the EDS will always select the player with the highest ADP and so the same team will be drafted for each trial if given the same pick. Since we don't want this to be deterministic, we introduce a random "noise" index offset for each team's pick that is using the EDS. This will allow for some variability in the drafted teams. We model this by sampling a geometric distribution $I \sim G_{0.9}$ and selecting the player with the $i - 1 \in \mathbb{N}$ lowest ADP.

The sample means and standard deviations of the number of wins for the UDS and EDS are given below. Because the mean number of wins and standard deviations are unknown, we can use a Welch's t-test to compare the two means and assess our hypothesis. Computing the resulting t-test, we get a p-value of 9.40×10^{-19} for $P(\mu_{UDS} \geq \mu_{EDS})$ which is clearly statisitcally significant at extremely low levels. This indicates we should reject the null hypothesis and accept the alternative hypothesis that the UDS outperforms the EDS in terms of the number of wins in a fantasy football season, although the difference is still small.

	UDS	EDS
Mean Wins	9.6	9.14
Standard Deviation	3.440695	2.745133

Table 1: Mean Wins and Standard Deviations for UDS and EDS

Perhaps a more intuitive or visual guide to the slight outperformance in wins by the UDS can be seen in the following figure where we plot the distribution of total points scored by the UDS and EDS teams for each trial respectively. Note that while both distirbutions are approximately normal and center around the same mean, the UDS distribution produces more seasons with higher point totals which should directly translate to more wins; whereas, the EDS distribution is tighter around the mean and produces more "average" scoring seasons.

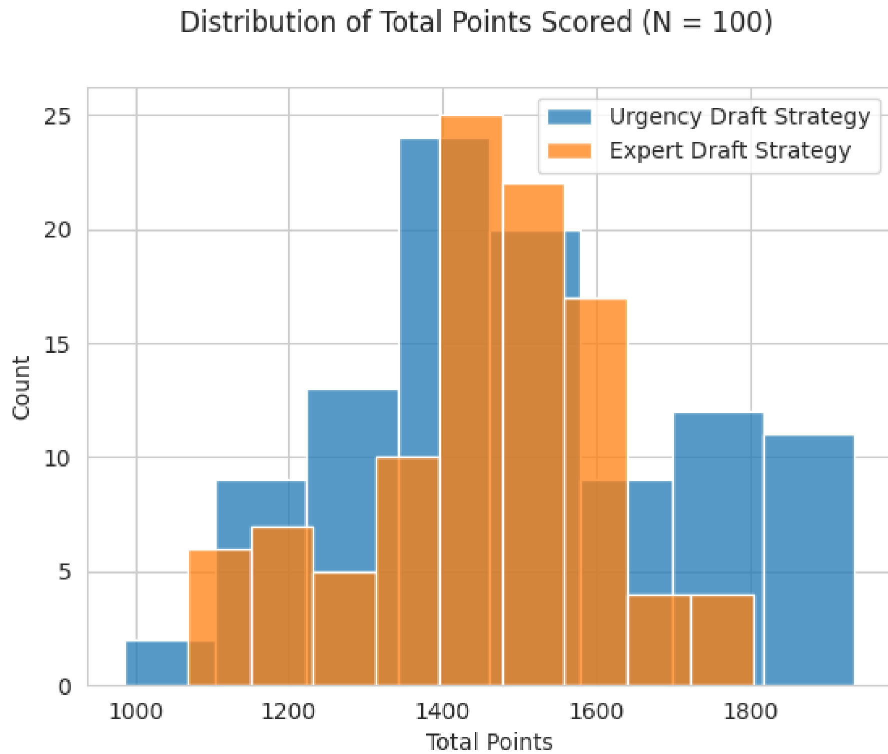

Figure 3: Distribution of Total Points Scored by UDS and EDS Teams

5 Future Work

While the results of the UDS are promising, there are a number of ways to improve the model and some limitations to address.

1. We don't account for bench players in the model. This is very common in fantasy football leagues and can be crucial to managing a team's injury risk. We could extend the model to include bench players and a decision process for who to start each week based on the eligible players' highest Sharpe ratios.

2. Sharpe ratios operate under the assumption of a risk-free rate that by definition has 0 variance. We assumed a risk-free rate of 0 along with a variance of 0. We could extend the model to consider a risk-free rate as the average points scored by all players in a given position that had an ADP higher than the last pick in the draft. This would give a risk-free rate for each position but would also introduce a variance. The Sharpe ratio formula would slightly change but its use in the urgency score would remain the same.

3. All of MPT operates under the assumption that returns are normally distributed. This is troublesome for fantasy football as player performances are often highly variable and can be skewed. Moreover, the risk associated with a player is not entirely symmetric. We would care much more about a player scoring -3σ points below the mean as opposed to $+3\sigma$ points above the mean. We could extend the model to use the idea of downside risk where we replace the Sharpe ratio with the Sortino ratio instead [7]. And then, we would assume that the returns are distributed log-normally as well.

6 Conclusion

In this paper, we have developed a draft strategy for fantasy football teams inspired by MPT and the Sharpe ratio. We have shown that this strategy outperforms the draft strategy of following the experts' average draft position boards at a statistically significant level. We have also discussed a number of ways to improve and refine the model in future work. It would be interesting to see how the model performs in a real fantasy football league and to see if the results hold up over multiple seasons. Hopefully, this work has shown that the principles of portfolio optimization can be applied to somewhat unconventional domains and produce meaningful results.

7 References

[1] - MPT [2] - efficient frontier wiki [3] - https://github.com/nflverse/nfl_data_py [4] - fantasy calculator [5] - https://www.pm-research.com/content/iijinvest/3/3/59 [6] - [7] - Sortino