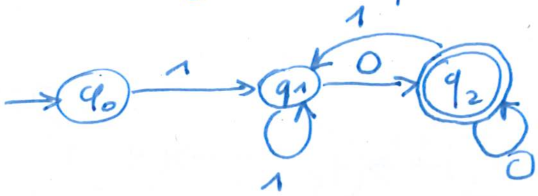
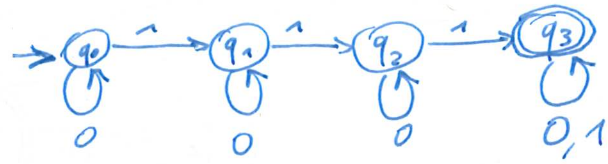
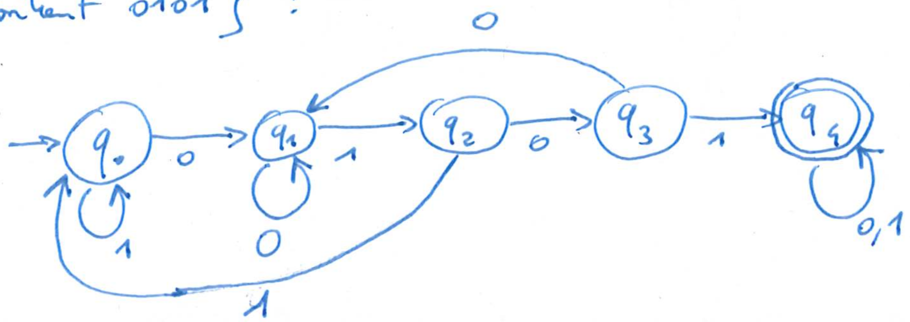
Exercice 6 1. $L_1 = \{u \in \Sigma^* \mid u$ commence par 1 et se termine par 0$\}$



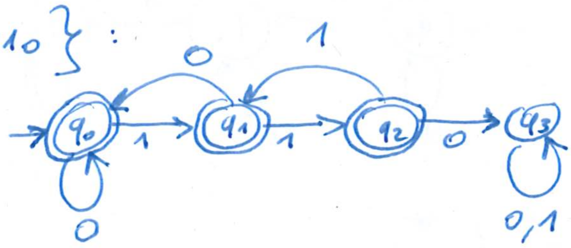
$L_2 = \{u \in \Sigma^* \mid u$ contient au moins 3 "1"$\}$:



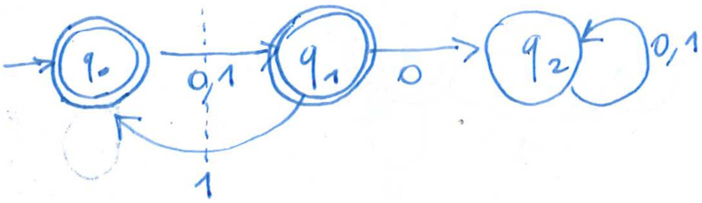
$L_3 = \{u \in \Sigma^* \mid u$ contient 0101$\}$:



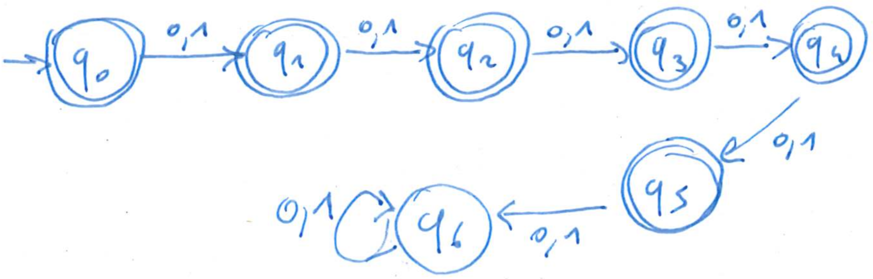
$L_4 = \{u \in \Sigma^* \mid u$ ne contient pas la chaîne 110$\}$:



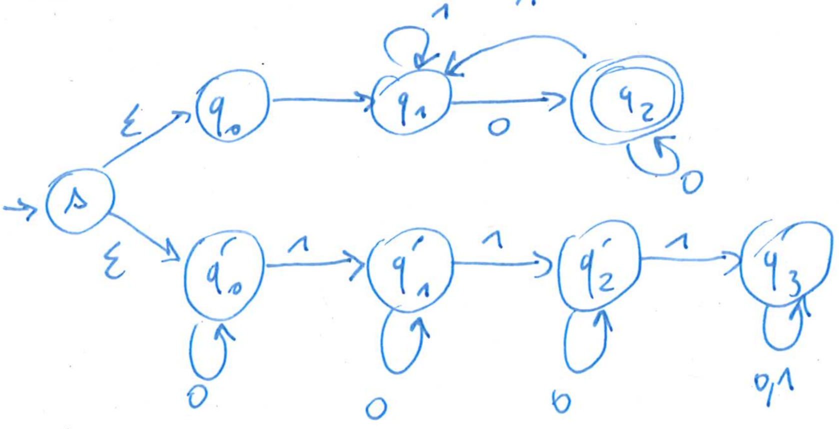
$L_6 = \{u \in \Sigma^* \mid$ toute position paire dans u est occupée par un 1$\}$



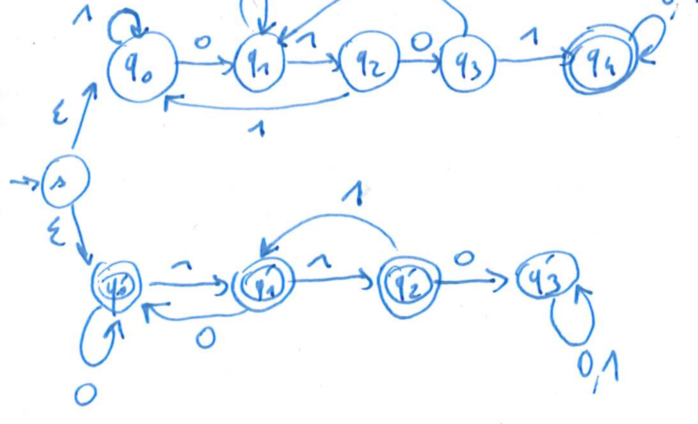
$L_5 = \{u \in \Sigma^*, |u| \leq 5\}$



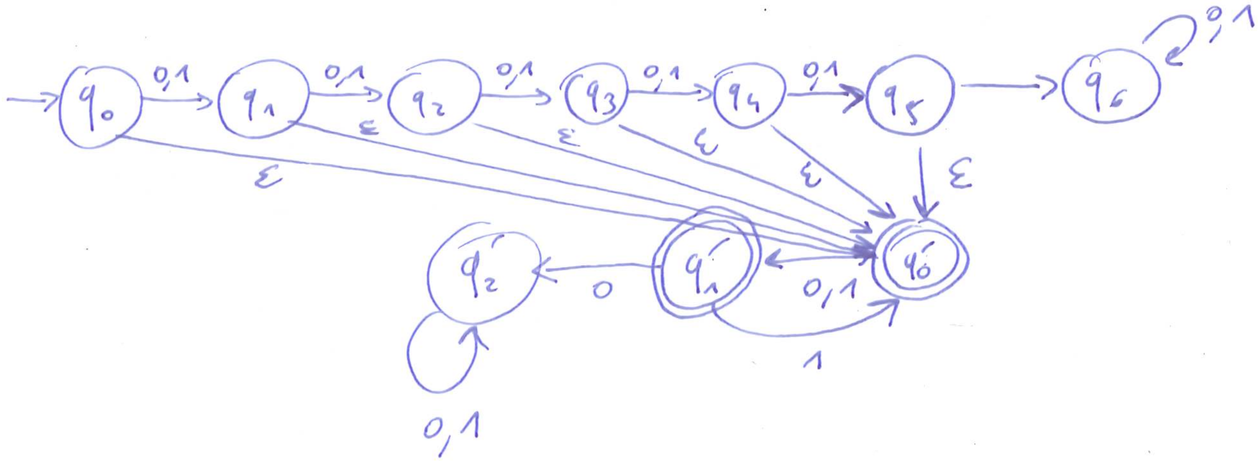
2-a $L_1 \cup L_2$:



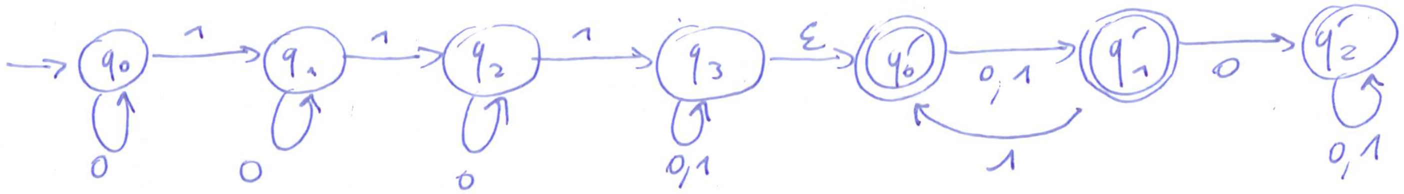
$L_3 \cup L_4$



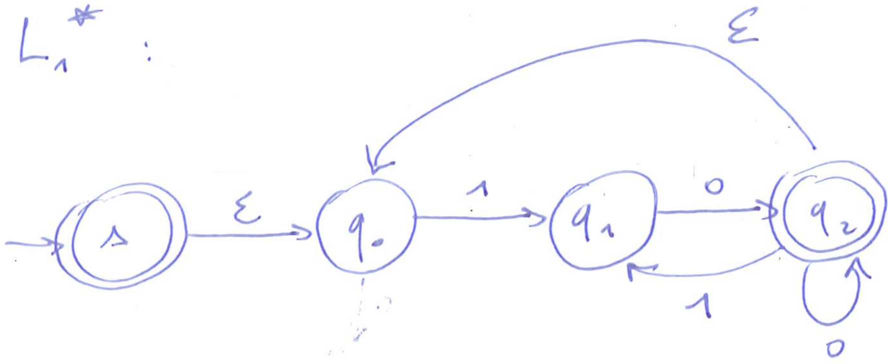
(c) $L_5 L_6$



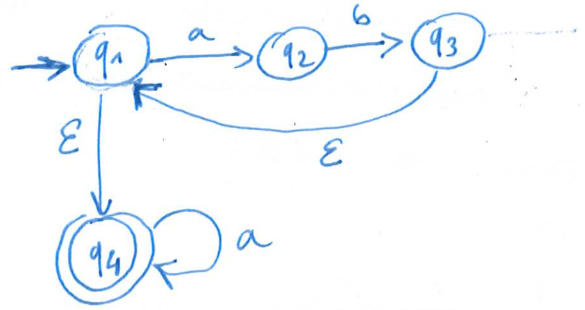
(d) $L_2 L_6$



(e) L_1^* :



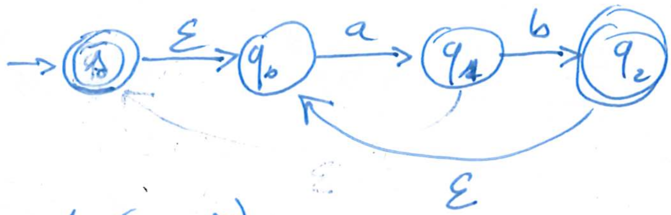
Exercice 7 1.



accepte $\mathcal{L}((ab)^*a^*)$

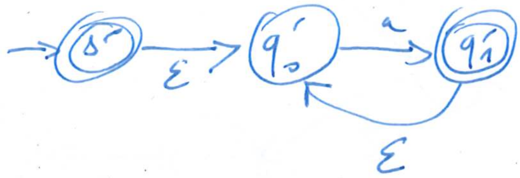
ou : $\mathcal{L}(ab)$:

$\mathcal{L}((ab)^*)$:

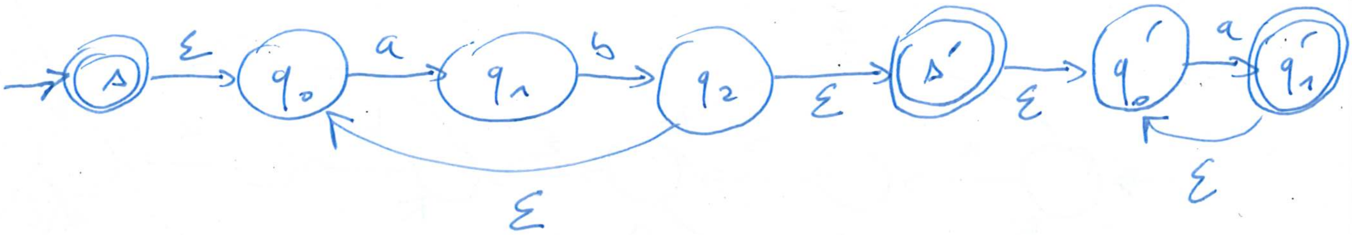


$\mathcal{L}(a)$:

$\mathcal{L}(a^*)$:



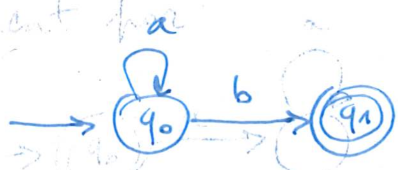
$\mathcal{L}((ab)^*a^*)$:



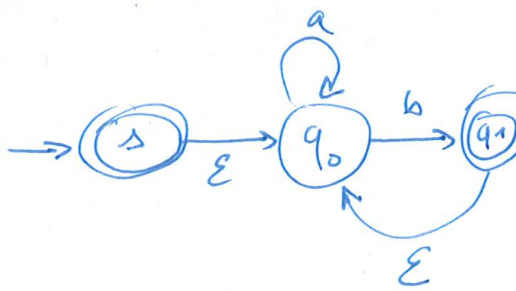
2. $((a^*b)^*a^*)^*$: contient $(b^*a^*)^*$, donc $(a+b)^*$

3. $(a^*b)^*b$:

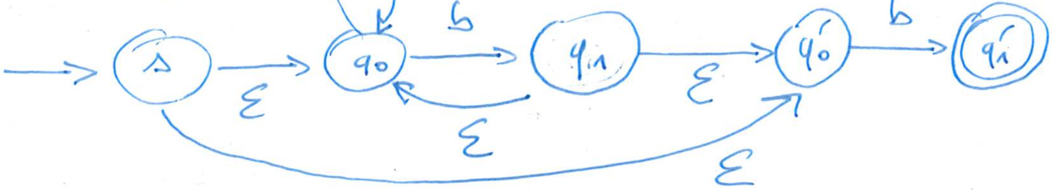
a^*b :



$(a^*b)^*$:

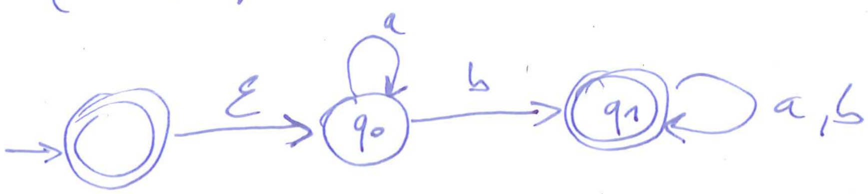


$(a^*b)^*b$:

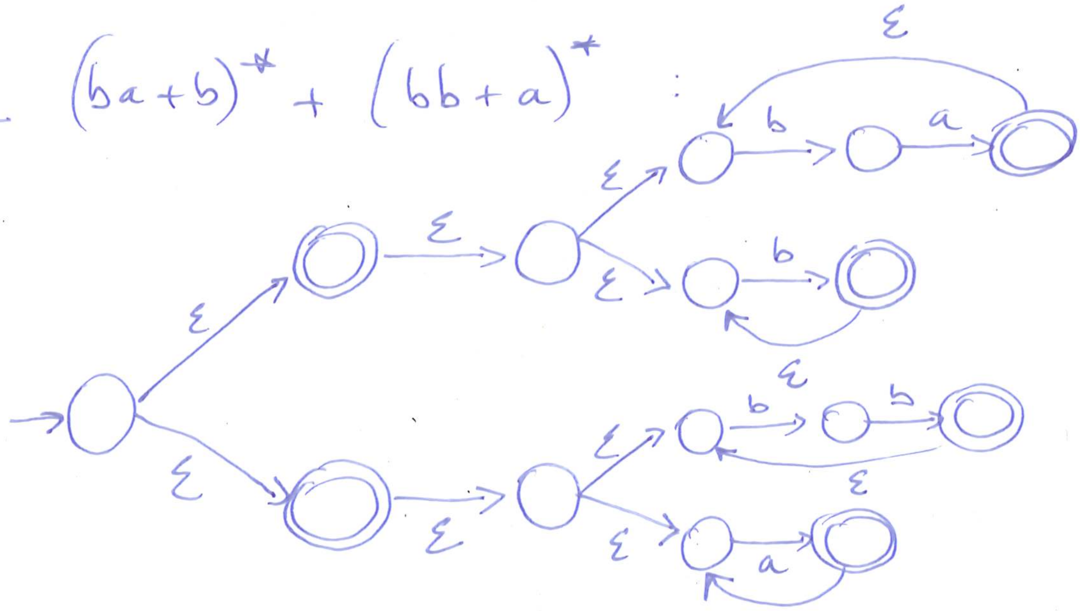


4. $(a^*ba^*)^*$: Le mot vide et tous les mots qui contiennent au moins un b.

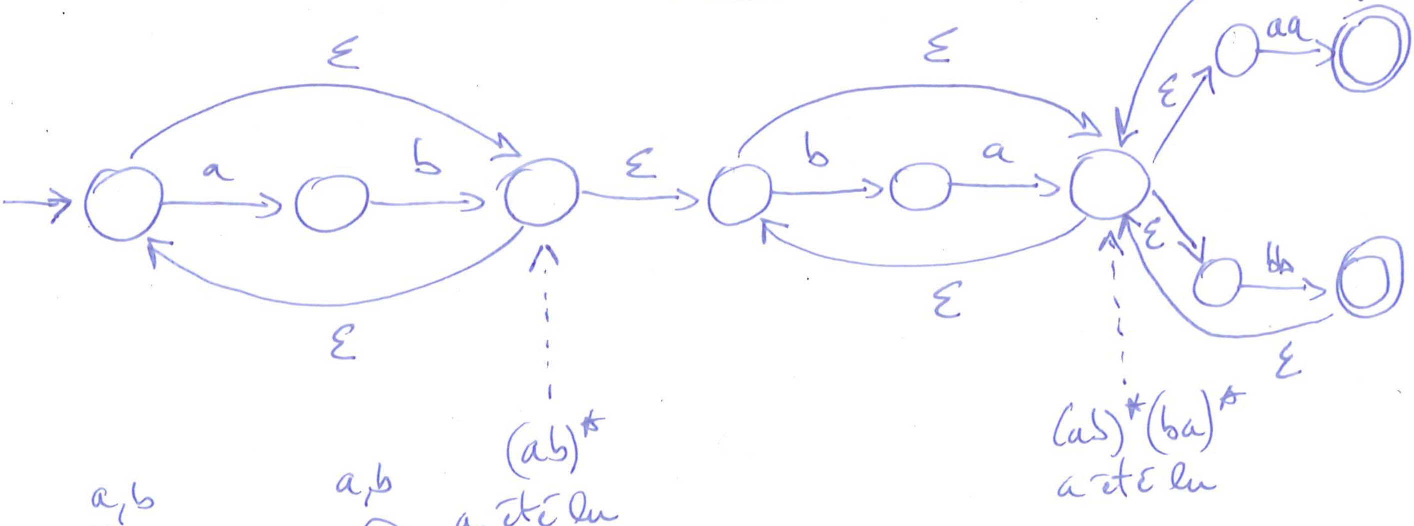
(le complémentaire de $\{a\}^+$



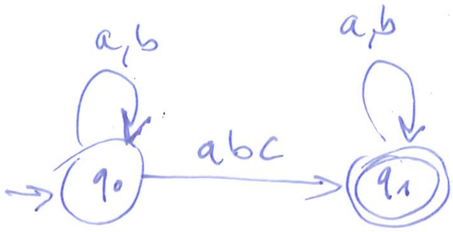
5. $(ba+b)^* + (bb+a)^*$:



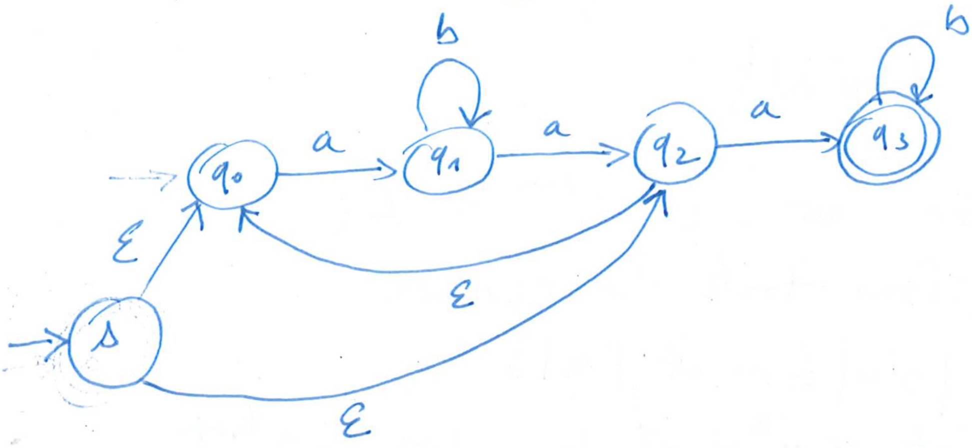
6.



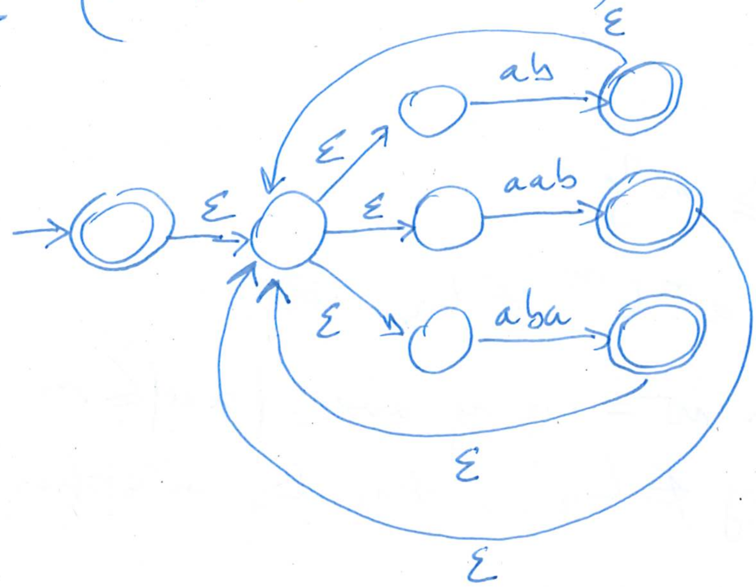
7.



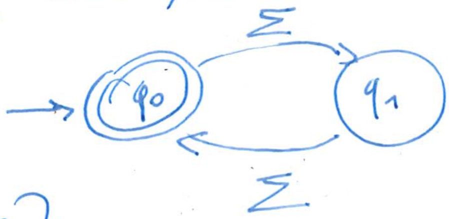
Exercice 7. 8. $(ab^*a)^*ab^*$



9. $(ab + aab + aba)^*$



Exercice 8 1. Oui, il est accepté par l'AF :



$L_1 = \{u \mid |u| \text{ est paire}\}$

2. $L_2 = \{a^p, p \text{ premier}\}$

Non car pour $n \in \mathbb{N}$, il existe $p \in \mathbb{N}$ premier supérieur à n, donc $a^p \in L_2$ et $|a^p| \geq n$,

et pour toute décomposition $a^p = xuy$ avec $|xu| \leq n$ et $u \geq 1$, $|xu^k y| = p + (k-1)|u|$

n'est pas premier pour $k = p+1$:

$|xu^{p+1}y| = p(|u|+1)$,

donc $xu^{p+1}y \notin L_2$ qui ne vérifie pas le lemme de pompage, il n'est pas régulier.

3. $L_3 = \{0^p 1^{2p}, p \in \mathbb{N}\}$

Soit $n \in \mathbb{N}$. Alors $w = 0^n 1^{2n} \in L_3$ et $|w| \geq n$. Pour toute décomposition $w = xuy$ avec $|xu| \leq n$ et $|u| \geq 1$, alors $u = 0^{|u|}$ avec $xu^2y \notin L_3$ car il a trop de 0. L_3 ne vérifie pas le lemme de pompage, il n'est donc pas régulier.

4. $L_4 = \{a^n b^m, n \geq m\}$.

Pour $n \in \mathbb{N}$, $w = a^n b^n \in L_4$ et pour toute décomposition $w = xuy$ avec $|xu| \leq n$ et $|u| \geq 1$, $xu^2y \notin L_4$, donc L_4 n'est pas régulier.