# Understanding Collections of Images

COS 521 Final Project Report

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# Contents

# Chapter 1

# Introduction

## 1.1 Abstract

This report explores a variety of image properties which make it possible to understand and explore collections of images. In particular, we look at how properties like color distribution, saturation, sharpness, and detail can be extracted and compared between images. To perform these task we use methods such as Fast Fourier Transform (FFT) and Singular Value Decomposition (SVD). We seek to understand how the theoretical underpinnings of these two algorithms affect the way the images are formed in the first place. Ultimately, we provide a way of decomposing the image into mathematical notation (a descriptor) that differentiates well between a collection of images.

## 1.2 Background Work

I'll look at this section closer tmrw, definietly needs some expanding. Will add some color histogram stuff – Maciej

There are many possible situations in which we would need to understand and compare image structure. For example, one might like to search for a location in which a photograph was taken, by looking at all the other available images, and finding the image closest to the search image. Alternatively, one may want to cluster images based on their content, and see what categories the image collection can be decomposed to. Both of these would be easy problems to solve, if the images were annotated with words: textual search is a well-solved problem. However, when the images are not labelled (this is known as unsupervised learning), and the image collection is extremely large, it is impractical to label the images by hand. For such problems, it is important to analyze image content automatically.

Existing methods of image search by analyzing content of the image include Google Goggles and Google Image Search, both are based on similar technology [1], which checks for distinctive points, analyzes lines and textures and finally creates a mathematical model of the image. While the exact implementation is not available, Google Image Search does not provide clustering capabilities of analyzing existing input datasets. A more closer work is that of Oliva and Torralba [2] which create a GIST descriptor (cite Freedmans work!!!), and use perceptual dimensions (naturalness, openness, roughness, expansion, ruggedness) to classify natural images, for example, pictures of coasts, mountains or cities. The authors work with a low dimensional representation of the scene which is collectively known as the *Spatial Envelope*. The properties of the spatial envelope are estimated by means of Discrete Fourier Transform (DFT), Windowed Fourier transform (WFT), as well as spectral properties of the image are estimates (!!!! Expand).

Having completed a graduate course in algorithms, our goal was to understand the results that were obtained by such a study, and specifically answer the question: why can we estimate proporties of the spatial envelope the way [2] does?

# Chapter 2

# Methods

In this work, we focused on three directions to analyze the image structure, being color distributions, frequency distribution and geometrical structure. Color distribution is widely analyzed using color histograms, frequencies can be easily investigated using Fast Fourier Transform. For geometrical structure we have decided to explore the Singular Value Decomposition of the image in hope of discovering some useful properties ( see chapter 2.5 ).

## 2.1 Data

The dataset that we have used for exploring the image structure has been provided with work by Oliva and Torralba [2]. The dataset contains images from 8 categories, Expand!

## 2.2 Implementation

When decomposing images using SVD or working with the FFT of an image, we chose to work in grayscale. The choice to do so was motivated by the desire to explore the physical meaning of the decomposition or transformation in the context of images. Though it is entirely possible to separate the red, green, and blue channels and work with each separately, it is difficult to determine whether relative differences in colors or deeper properties of the decomposition/transformation are leading to the observed descriptor performance. To do the conversion we used matlab's built-in *rgb2gray* function, which removes hue and saturation information but preserves luminance. Above sentence is not exactly true - recovering luminance vs preserving color is a huge area of work, basically look up for integral images if interested. Will have to tweak the section a bit – [Steve] - that sentence was lifted from the matlab documentation, see: `http://www.mathworks.com/help/images/ref/rgb2gray.html`, that is the extent of my knowledge, so feel free to update.

## 2.3 Color Analysis

### 2.3.1 Color Spaces

In computer graphics the subset of colors which can be displayed by computer monitor is called *gamut*[6]. There are many models which aim to represent the gamut — one of the most common and well known such models is known as the RGB model, in which color at every pixel is a combination of three intensity values, of the Red, Green, and Blue base colors. RGB models how devices produce colors, so our initial intuition was to look at models that model human vision more closely.

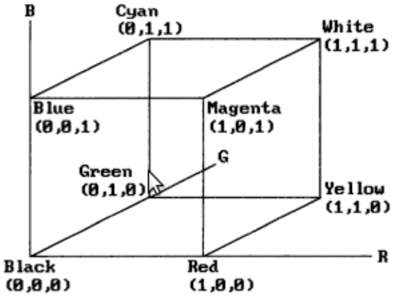
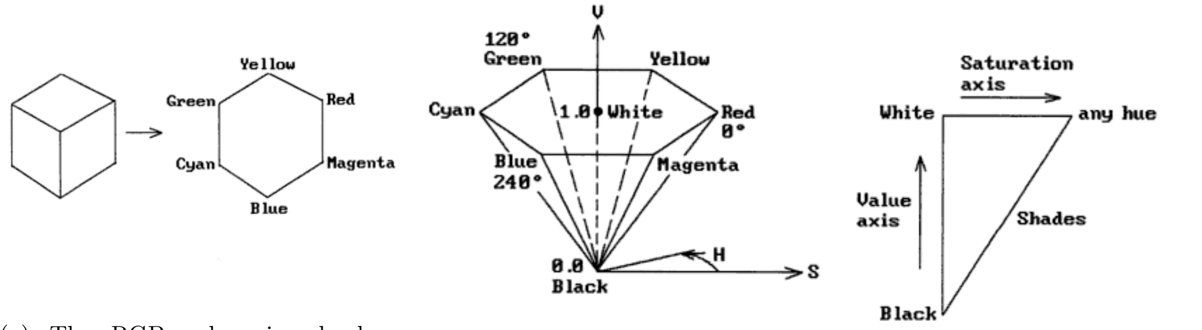


Figure 2.1: RGB Cube Model

One of such models is Hue-Saturation-Value model, which were designed to give more intuitive in terms of human color perception, and thus is widely used as color picker in many photo-editing and digital painting packages [6]. HSV model is a fairly straightforward transformation from the RGB model, as shown in 2.2.



(a) The RGB cube viewed along Black-White diagonal

(b) Recoordination

(c) Hue-Saturation Triangle

Figure 2.2: Transformation of the RGB cube to HSV color space

In particular, the hue varies from 0° to 360°, and represents the color. The saturation and value are numbers in the range $[0, 1]$ which represent how far is the hue from white and black, respectively.

@Elena - if you have some nice figure for L*a*b we can put it here

Another color space which we have considered in this work was the $L^*a^*b^*$ color space which also has been designed to model the human perception closely. $L^*$ parameter control the overall lightness of the image, ranging from $L = 0$ being black, and $L = 100$ yielding white. Both $a^*$ and $b^*$ range from negative to positive values, where negative $a*$ yields, and positive values give colors closer magenta. For $b*$ negative values give blue color, while positive values relate to yellow. Since $L^*a^*b^*$ aims to be most complete color model that represents all colors that are perceived by human vision it was a clear choice to investigate what improvements we might get from the incorporation L*a*b* in our color descriptors, which will be introduced in next section.

### 2.3.2 Color-based Descriptors

The color information is widely used for image retrial, since as a descriptor it has a nice properties of being rotationally invariant, as well as not being dependent on image resolution. As mentioned in previous section, color closely relate to how we percieve images as similar. Also similar objects will yield similar colors, especially in terms of natural objects ( i.e. forest images will always be mostly green, landscapes will always show a blue sky etc. ).

A very crude way of comparing images in terms of colors is to average each of the channels in specific color space, and use it as a 3-value descriptor, which will be a point in a color space. Figure 2.4 shows points plotted in $\mathbb{R}_3$. What is interesting about all the color spaces is the fact that we can easily navigate through them and have a good understanding how single parameter affects overall color appearance.

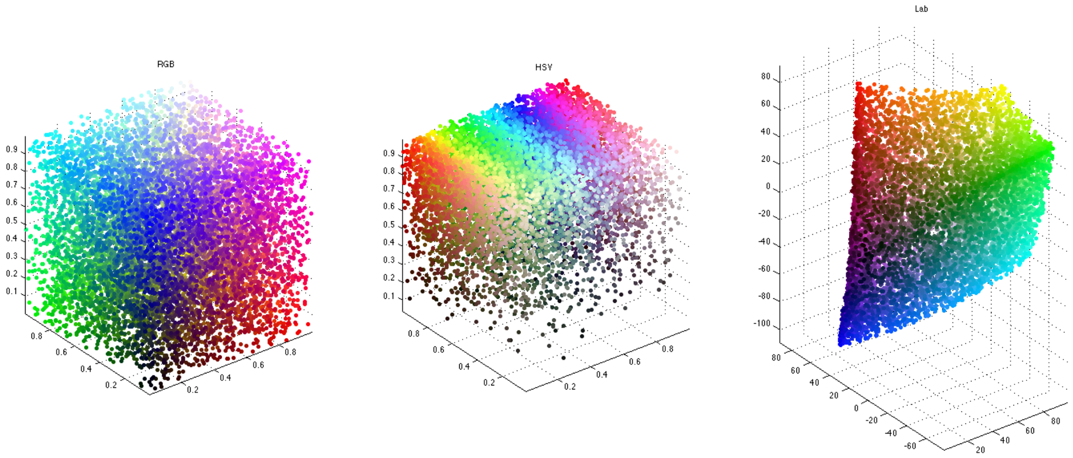


Figure 2.3: Points in $\mathbb{R}_3$, plotted according to their values in RGB, HSV and $L^*a^*b^*$ color spaces

This gave us the inspiration to look for similar, simple parameters that relate to overall image structure, using FFT and SVD ( see sections 2.4 and 2.5 respectively). However by a simple averaging we are not obtaining information regarding the overall color distribution. A more common way of investigating the color distribution is *normalized color histogram*. To create it we first create bins - given the number of bins $n$ we create a set of uniformly spaced ranges $R = \{r_1, r_2, ..., r_n\}$. Now for each pixel $p_i$ we determine its value $v_i$ and determine into which $r_k$ it falls to and increment value at the relating bin. After visiting all pixels $p_i$ we normalize each bin by total number of pixels. This produces a length $n$ vector that better describe the the overall value distribution. Above structure of course describes the procedure for gray scale images, however it is trivial to extend it to color images where we create a histogram for each color channel, and then concatenate resulting vectors.

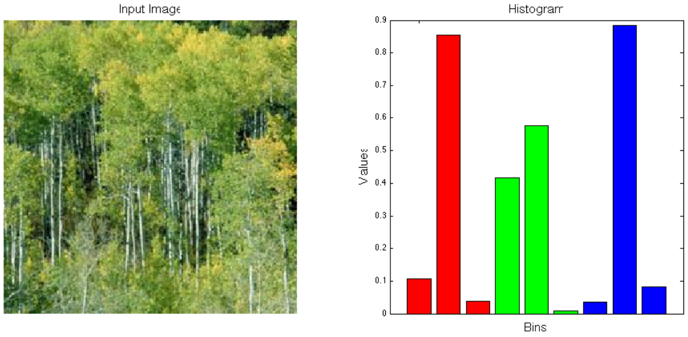


Figure 2.4: Image and its color histogram in RGB, with $n = 3$ bins per channel.

We then designed our descriptor for each color space. As noted in [7], hue is more important for color perception than saturation and lightness, and thus the HSV histogram have been always computed giving more bins to hue channel (i.e. provided that the required length of descriptor is $n$, the number of bins for hue would be given by $\frac{n}{2}$, while other two parameters $\frac{n}{4}$. Similarly we conjecture that in terms of $L^*a^*b^*$ color space the lightness is less important than parameters describing colors, so we have also modified the number of bins for each channel accordingly ( $L^* = \frac{2*n}{8}$,$a^* = \frac{3*n}{8}$, $b^* = \frac{3*n}{8}$ )

To sum up for color descriptors we have looked both at simple averaging of values as well as normalized color histograms. All these have been implemented to work with each of the color spaces discussed in section 2.3.1. In chapter 3 we discuss the overall performance of all descriptors discussed here. Also note that in literature it is common to note that uniform quantization does not take the spatial relationship between pixels into account, and investigate the techniques to overcome this drawback [8]. However because of time limitation we have decided to settle for simpler approach.

## 2.4 Fourier Transform

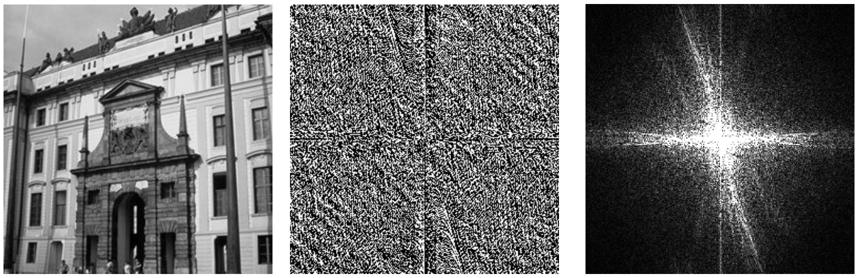
A 2-dimensional, Discrete Fourier Transform(DFT) of an grayscale image is a transformation from the spatial domain to the frequency domain. In the spatial domain, image is represented by function $f(x, y)$ on all relevant points $(x, y)$ in $\mathbb{R}^2$. In the frequncy domain, the image is represented by a function $F(u, v)$ where $u$ and $v$ are frequency values. It follows that in the frequency decomposition, an image is represented by a matrix of complex values $F$, where $F(u, v)$ encodes the amplitude and the phase of the frequencies $u$ and $v$. Mathematically, the 2-D Fourier Transform is defined as:

$$F(u, v) = \int\int_{-\infty}^{\infty} f(x, y)e^{-j2\pi(ux+vy)}dxdy \tag{2.1}$$

For $n$ points, we can compute the 1-D DFT efficiently in $O(n \log n)$ operations. The Matlab implements 2-D FFT in funcion *fft2*, which is extremely fast, and we had no computational time issues when basing our descriptors on these computations.

### 2.4.1 Amplitude and Phase Analysis

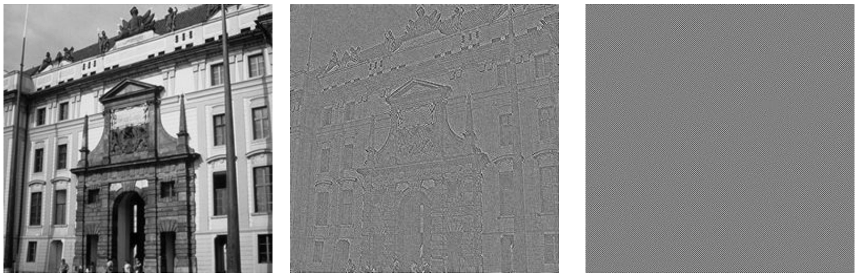
Consider the decomposition of an image of a building in a city from the GIST dataset into its power(amplitude) and phase spectra.



(a) Original image (b) Phase $\Phi(x, y)$ (c) Amplitude $A(x, y)$

Figure 2.5: Phase and power spectra of building image

As Oliva writes in [2], the phase image represents local properties of the image. It contains information relative to the form and the position of image components. This can be further understood by taking the true phase values of the image, and setting all the amplitude values to 1 (effectively taking out all amplitude variation and flattening the image), or taking true amplitude values and randomizing the phase:



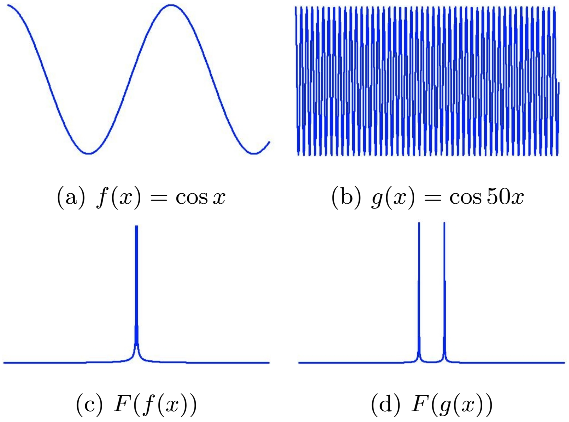
(a) Original image (b) Flat amplitude (c) Randomized phase

Figure 2.6: Analysis of Contributions of both Phase and Amplitude to an Image

Note that a reconstructed image in which the amplitude information was not preserved retains the information about the edges and outlines in the original image. However, after Oliva [2], the amplitude image (power spectra) informs us on global properties of the image structure, telling us about the orientation, smoothness, lenght and strenght of the edges that appear in the image. Two images that are similar might have different local information in terms of contours appearing in them, however they should have similar power spectra. This insight motivates our decision to investigate the amplitude image for our descriptor, since our goal is do capture the global properties of the image.

### 2.4.2 Localization fact

To understand the properties of the 2-D Fourier Transform more deeply, let us first analyze the 1-D variant first. Consider the following functions, and their representations in the Fourier domain (2.7). A plot of $f(x) = \cos x$ is not very well localized in the spatial domain, that is, we can inutitively state that function is streteched along the x-axis. In more mathematical terms we can say that a small change in the function domain $\delta_x$ relates to small change $\delta_y$ in the function image. However, when we analyze same function in the Fourier domain, we notice that its representation is extremly packed — for $F(cos(x))$, the representation is just a single peak. Conversely, $g(x) = \cos 50x$ is well localized in the spatial domain, i.e. function can be thought of being squeezed along x-axis. Again, more strict definition would be that small change in the function domain $\delta_x$, relates to dramatic change$\delta_y$, i.e. $\delta_y \gg \delta_x$. However, in the Fourier domain, we observe inverted behaviour - the peaks of the $F(g(x))$ spread apart away from the zero frequency. This simple fact regarding the behavior of Fourier Transfrom suggested us a way of how to describes images in the Fourier domain, analyzing its amplitude image $A(x, y)$. Images containing a lot of high frequency periodicy (buildings, trees) will have more contribution from these (high) frequencies, while images showing open landscapes will mostly consist of low frequencies. Similarly to our approach with color descriptors, we can try to generate bins of different frequencies, and then normalize them to get overall contribution from each frequency range.



(a) $f(x) = \cos x$

(b) $g(x) = \cos 50x$

(c) $F(f(x))$

(d) $F(g(x))$

Figure 2.7: Example of localization properties of different functions in the spatial and Fourier domains. Images localized in the spatial domain are not localized in the Fourier domain, and vice versa.

To test the above analysis we have created a simple, single-value descriptor, that was just looking at the contribution from the low frequencies - the descriptor $d(I)$ is simply given by :

$$d(I) = \sum_{x^2+y^2 \in r_0} A(x, y)$$

where $r_0 = \{0, \omega_1\}$ is a range of frequencies from zero to some small frequency $\omega_1$. Idea is again that images with lots of high frequency information, will have less contribution from low frequencies, and thus returning smaller value $d(I)$. We sorted the images according the increasing contribution of low frequencies 2.8. The images should be read as one line that starts at top-left corner and ends in bottom-right.

One can see that the images on the left side of the spectrum, with a lot of contribution from high frequencies and not so much from low frequencies have many small details: they show leaves, rock incisions on the mountain, and fine building facade. In comparison, the images at the right side of the spectrum are images of open country, roads, and beaches. These images are simple, in the sense that they have a dominant horizon line and not so much small detail. It follows that these images are described mostly by low frequencies, and not by high frequencies. Notice that this analysis discards a lot of information about the distribution of frequency contribution.

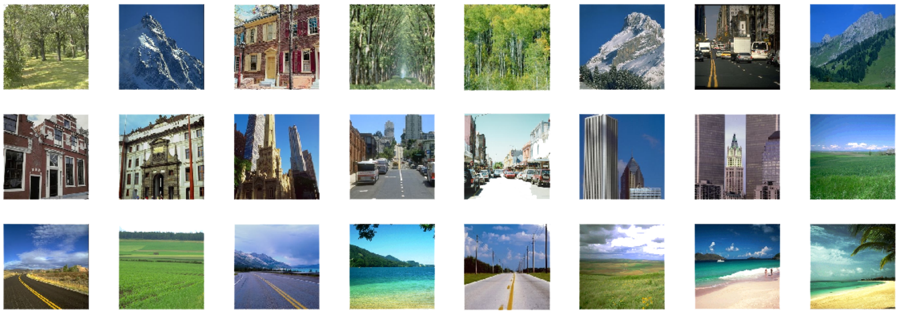


Figure 2.8: Ordering of a subset of the dataset by increasing contribution of low frequencies

### 2.4.3 Frequency-Based Descriptor

Given that simple descriptor described in the previous section allowed us to create quite meaningful ordering of images we decided to extend it to give us better information regarding the overall frequency distribution, similarly to our color descriptors. The said extension, simply consider a set of disjoint, uniformly spaced, frequency ranges $\{r_0, r_1, ..., r_n\}$, see fig. 2.9.

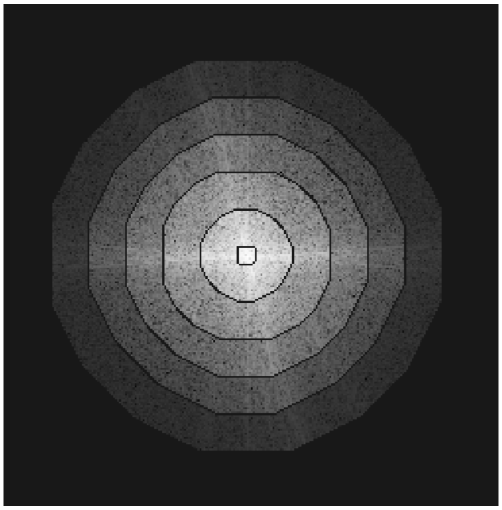


Figure 2.9: Frequency Descripor Visualization

Our descirptor is then a vector $\vec{d}(I)$, of n-values, where each value is simple averaging of frequency values in specific bin, that is then normalized by $N = \sum_{x}, yA(x, y)$, the averaging of all values in an image. This way, each element $d_i$ give us percentage contribution from each frequency range. We will discuss the performance in chapter 3.

Or add the figures with performance here

## 2.5 Singular Value Decomposition

A deeper understanding of Singular Value Decomposition (SVD) in the context of images allows the creation of a descriptor that captures overall image complexity in a relatively small descriptor length. SVD is a factorization of any real or complex 2-dimensional matrix. Since images will always be represented by real matrices, we ignore the complex case in our analysis.

Consider an $m \times n$ matrix $A$. The singular values of $A$ correspond to the non-zero square roots of the eigenvalues from $AA^T$ and $A^TA$. The matrix $AA^T$ is spanned by the row space of A, and the matrix $A^TA$ is spanned by the column space of $A$ [3]. The row space and column space being the set of all linear combinations of row vectors and column vectors of $A$, respectively.

SVD conveniently decomposes $A$, separating out singular values, row space eigenvectors, and column space eigenvectors into three different matrices. The SVD of $A$ is defined as:

$$A_{m \times n} = U_{m \times m} S_{m \times n} V^T_{n \times n}$$

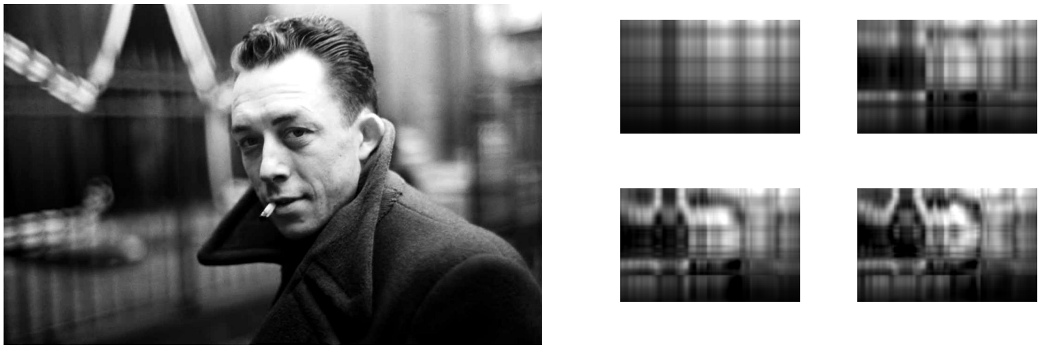
where $S$ is a diagonal $m \times n$ matrix with the singular values of $A$ on the main diagonal (in decreasing order). The columns of $U$ are the eigenvectors of $AA^T$ and the columns of $V$ are the eigenvectors of $A^TA$ [3]. $U$ and $V$ are known as the left and right singular vectors of $A$, respectively.

The rank of a matrix is informally defined as a measure of the ”nondegenerateness” of system of linear equations encoded by that matrix, or more formally is equal to the size of the row space or column space of the matrix. Linearly independent singular vectors correspond to non-degenerate singular values. The rank of a matrix is thus equal to the number of non-degenerate singular values of the matrix, and is also equal to the number of linearly independent vectors in $U$ or in $V$. If all singular values are non-degenerate, $A$ is a full rank matrix and the SVD of $A$ is unique.

### 2.5.1 SVD and Compression

It is useful to think of each singular value in $S$ as scaling the row and column singular vectors from $U$ and $V$ to generate an 'eigenimage' [4], or the contribution of that specific singular value to the overall image. This construction is what enables image compression through a low-rank matrix approximation. Let $\tilde{A}$ represent the $k$-approximation of $A$, where $rank(\tilde{A}) = k$. Thus:

$$\tilde{A} = U(:, 1:k)\tilde{S}V(:, 1:k)^T$$



(a) Original image

(b) Rank 1 to 4 from top left to bottom right

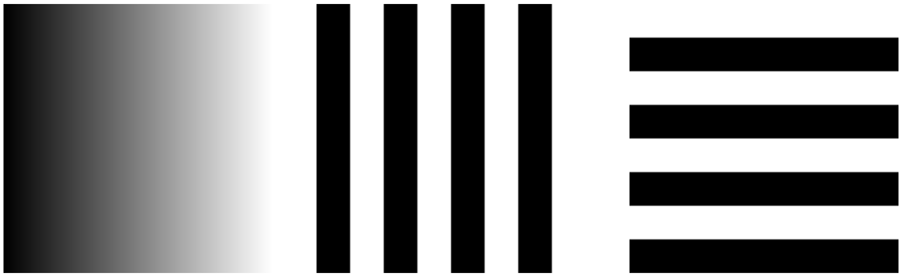
Figure 2.10: Rank k-approximations of an image using SVD

The performance of SVD in image compression shows that there is a significant amount of visual information in low rank approximations of images. Our desire is to capture this low rank information in a concise descriptor for use in image retrieval. An examination of both the singular values and the singular vectors follows.

### 2.5.2 Singular Values

Image retrieval using all singular values of a matrix has been shown to perform better than a simple distance metric, such as the Mahalanobis distance, Manhattan distance, or Euclidean distance [5]. Several factors contribute to this performance, but relate to the fact that the distribution of singular values in $S$ is connected to the rank of $A$, the image matrix.

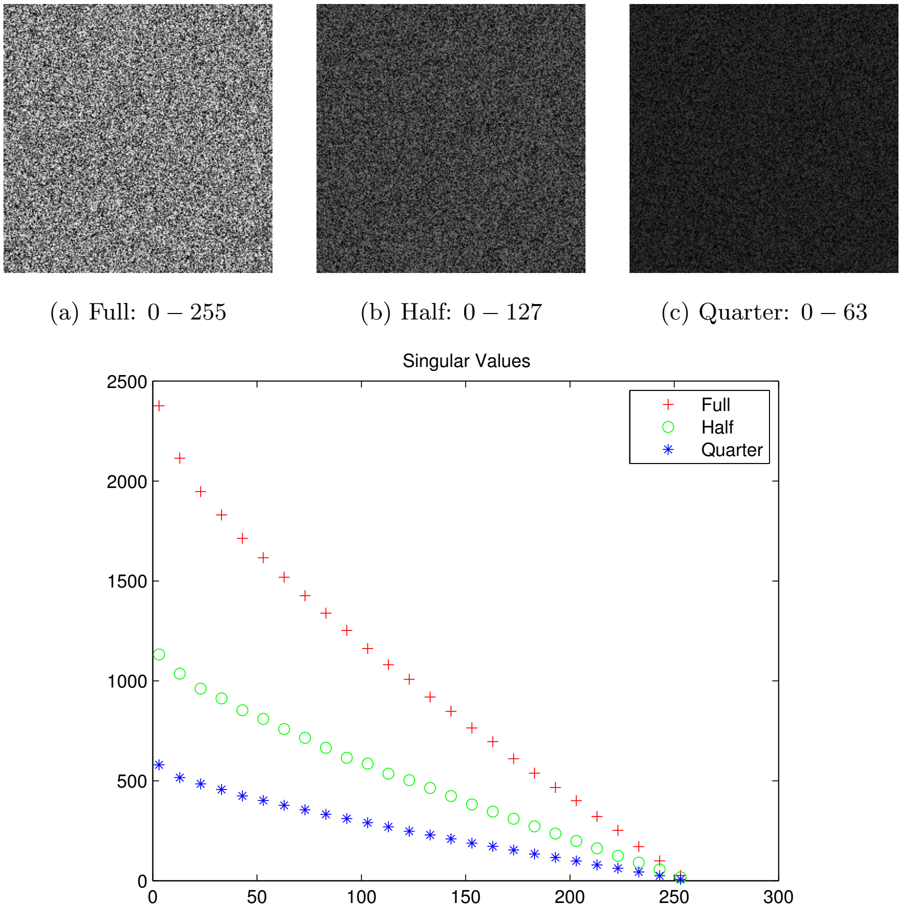
The alignment of an images' strongest color contours to either the horizontal or vertical axis allow it to be fully captured in the left or right singular vectors of an image and thus requires few singular values. The means the image's matrix representation is of lower rank and thus loses less information by ignoring the smallest singular values. In an extreme example, the three images in Figure 2.11 can be fully represented using only the top singular value since they are all rank 1 matrices.



(a) $S(1,1) = 37726$ (b) $S(1,1) = 46041$ (c) $S(1,1) = 46041$

Figure 2.11: Rank 1 images and their top singular value

On the opposite end of the spectrum, consider the randomly generated images in Figure 2.12. These three images are full rank and require all 256 singular values for a full reconstruction. The plot of the singular values from these images reveals that the range of intensity in the image directly correlates to the magnitude of the singular values needed, but has no correlation to the number of values needed.



(a) Full: $0 - 255$ (b) Half: $0 - 127$ (c) Quarter: $0 - 63$

(d) Plot of singular values (1st value omitted for scaling)

Figure 2.12: (a-c): Random images with given intensity range (d): Singular values of (a-c)

Figure 2.11 shows very simple rank 1 images that only require the 1st singular value, whereas Figure 2.12 shows that full rank images requires all singular values. To get a better understanding of the required number of singular values we look at what happens when there is symmetry in the image. Figure 2.13 shows the plot of singular values from several random images which span the full intensity range from $0-255$, but have symmetry in various directions. In all images, the symmetry is about a central axis, so it covers half of the image.

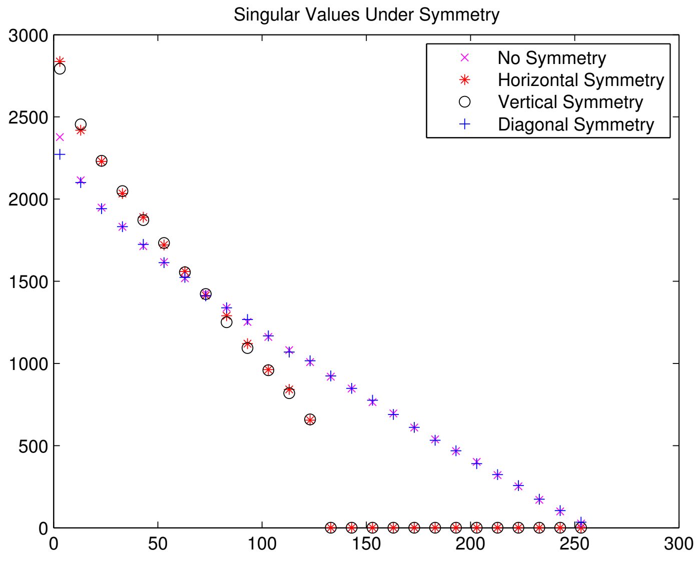


Figure 2.13: Plot of singular values under symmetry (1st value omitted for scaling)

Figure 2.13 shows that the amount of symmetry within an image can greatly reduce the number of singular values required for reconstruction (in this case by half, since half of the image was repeated). However, this also shows a major issue with directly using the output of SVD for an image descriptor. An image which has symmetry off-axis at a 45° angle will still have linearly independent vectors in $U$ and $V$ as the image is still full rank. Despite the fact that half of the image is repeated, the distribution of singular values is nearly identical to the image with no symmetry. It should be noted that 45° is chosen because it represents the worst-case off-axis example. Angles above and below will experience some level of symmetry in either the horizontal or vertical direction.

# Chapter 3

# Analysis

[Steve] I suppose that here we should give a brief summary of k-means clustering and multidimensional scaling, and then show the results of the descriptor (with and without windowing) and an analysis of that? I was planning to investigate the properties of SVD and show some basic performance results in the SVD section....but that presents the problem that I evaluate my results with k-means and k-means won't be introduced yet..

E: I think we can introduce K-means, embedding, search by query, and any other similar things we used to test descriptors with in the Methods section, all under one section, say testing framework?

# Chapter 4

# Suggestions for Further Work

Just some quick ideas - our exploration assumed that each property is indepenend which is obviously not true, and it would be interesting to look at similarities/ collerations between SVD and FT. Our exploration was purely based on properties of algorithms and visual exploration using MDS and k-nearest neighbour searches. Our performance for retrival is not really measured other that providing visual examples - some improvements on this front would be nice. Images are extremly high dimensional space, and our analysis might not be entirely revealing - using some machine learning techniques (vide Zeyu and Shuran ) could help us to learn some more about mentioned dimensionality independence etc. This project have not really focused on the color investigation very deeply where we have settled on euclidean distance between the RGB values. investigation of the non-linear distance metrics in HSV and Lab spaces might yield better results.

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