Understanding Collections of Images

COS 521 Final Project Report

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Chapter 1

Introduction

1.1 Abstract

This report explores a variety of image properties that make it possible to understand vast collections of images. In particular, we look at how image color, saturation, sharpness, and detail can be extracted and compared between images using methods such as Fast Fourier Transform (FFT) and Singular Value Decomposition (SVD). We seek to understand how the theoretical underpinnings of these two algorithms affect the way the images are created in the first place. Ultimately, we provide a way of decomposing the image into mathematical notation (a descriptor) that differentiates well between a collection of images.

1.2 Background Work

There are many possible situations in which we would need to understand and compare image structure. For example, one might like to search for a location in which a photograph was taken, by looking at all the other available images, and finding the image closest to the search image. Alternatively, one may want to cluster images based on their content, and see what categories the image collection can be decomposed to. Both of these would be easy problems to solve, if the images were annotated with words: textual search is a well-solved problem. However, when the images are not labelled (this is known as unsupervised learning), and the image collection is extremely large, it is impractical to label the images by hand. For such problems, it is important to analyze image content automatically.

Existing methods of image search by analyzing content of the image include Google Goggles and Google Image Search, both are based on similar technology [1], which checks for distinctive points, analyzes lines and textures and finally creates a mathematical model of the image. While the exact implementation is not available, Google Image Search does not provide clustering capabilities of analyzing existing input datasets. A more closer work is that of Oliva and Torralba [2] which create a GIST descriptor (cite Freedmans work!!!), and use perceptual dimensions (naturalness, openness, roughness, expansion, ruggedness) to classify natural images, for example, pictures of coasts, mountains or cities. The authors work with a low dimensional representation of the scene which is collectively known as the *Spatial Envelope*. The properties of the spatial envelope are estimated by means of Discrete Fourier Transform (DFT), Windowed Fourier transform (WFT), as well as spectral properties of the image are estimates (!!!! Expand).

Having completed a graduate course in algorithms, our goal was to understand the results that were obtained by such a study, and specifically answer the question: why can we estimate proporties of the spatial envelope the way [2] does?

Chapter 2

Methods

Three directions of image analysis were analyzed and tested. We first started with a collection of color images that were analyzed by the method of Color Histograms. We also analyzed the properties of the FFT method and the SVD method on the corresponding greyscale images. The resulting descriptors were then compared against each other and subsequently combined into a joint descriptor, that used the information from all three methods to describe images.

2.1 Data

We tested the descriptors on two data-sets ?????

2.2 Implementation

When factoring images using SVD or working with the FFT of an image, we chose to work in grayscale. The choice to do so was motivated by the desire to explore the physical meaning of the decomposition or transformation in the context of images. Though it is entirely possible to separate the red, green, and blue channels and work with each separately, it is difficult to determine whether relative differences in colors or deeper properties of the decomposition/transformation are leading to the observed descriptor performance.

To do the conversion we used matlab's built-in *rgb2gray* function, which removes hue and saturation information but preserves luminance. [[I originally was going to discuss this in the SVD section, but I think it fits well here.]]

2.3 Color Analysis

2.4 Fast Fourier Transform (FFT)

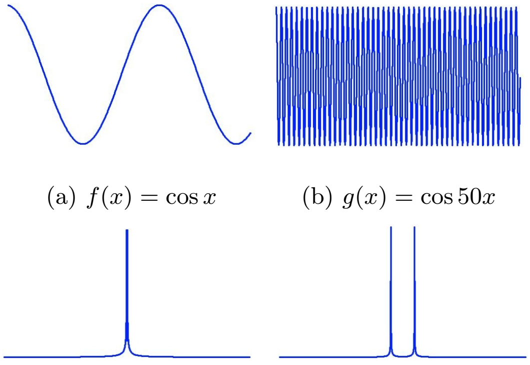
(????? what is the 2D FFT in terms of FFT) A 2-dimensional FFT of a (grayscale) image is a transformation from the spatial domain to the frequency domain. In the spatial domain, an image is represented by function $f(x, y)$ on all relevant points (x, y) in $\mathbb{R}^2$. In the frequncy domain, the image is represented by a function $F(u, v)$ where u and v are frequency values. It follows that in the frequency decomposition, an image is represented by a matrix of complex values F, where $F(u, v)$ encodes the amplitude and the phase of the frequencies u and v. Mathematically, the 2-D FFT is defined as:

$$F(u,v) = \int\int_{-\infty}^{\infty} f(x,y) e^{-j2\pi(ux+vy)} dxdy \tag{2.1}$$

For n points, we can compute the 1-D DFT efficiently in $O(n \log n)$ operations. The Matlab implemetation of the 2-D FFT is extremely fast, and we had no computational time issues with basing descriptors on these computations.

2.4.1 Localization

To understand the properties of the 2-D FFT, we started by analyzying the 1-D FFT first. Consider the following functions, and their representations in the Fourier domain (2.3). A plot of $f(x) = \cos x$ is not localized in the spatial domain, but is localized in the Fourier domain: it is represented by a single peak. Conversely, $g(x) = \cos 50x$ is localized in the spatial domain, as it consists of oscillations around $x = 0$. In the Fourier domain, the same function is no longer localized. This example is representative of the behavior of FFT, and suggested us a way of how to tackle images in the Fourier domain.



(a) $f(x) = \cos x$ (b) $g(x) = \cos 50x$

(c) $F(f(x))$ (d) $F(g(x))$

Figure 2.1: Example of localization properties of different functions in the spatial and Fourier domains. Images localized in the spatial domain are not localized in the Fourier domain, and vice versa.

The given analysis yielded an easy descriptor, in which we sorted the images according the increasing contribution of low frequencies (alternatively, the decreasing contribution of high frequencies) 2.2 (!!!!!! Maciej, how exactly did you generate this?? Is this based on amplitude?). The images should be read as one line that wraps from right to left. One can see that the images on the left side of the spectrum, with a lot of contribution from high frequencies and not so much from low frequencies have many small details: they show leaves, rock incisions on the mountain, and fine building facade. In comparison, the images at the right side of the spectrum are images of open country, roads, and beaches. These images are simple, in the sense that they have a dominant horizon line and not so much small detail. It follows that these images are

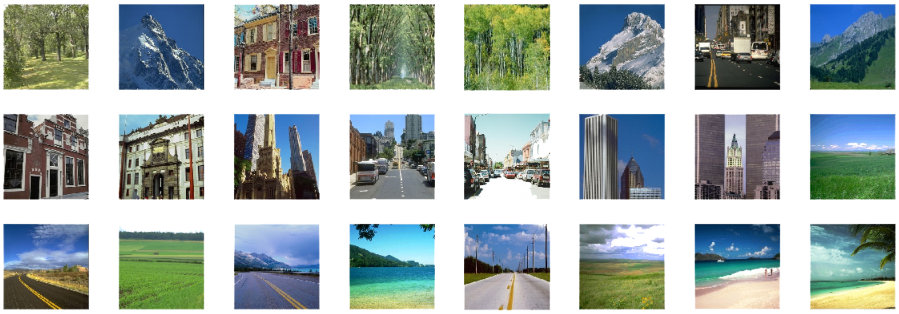
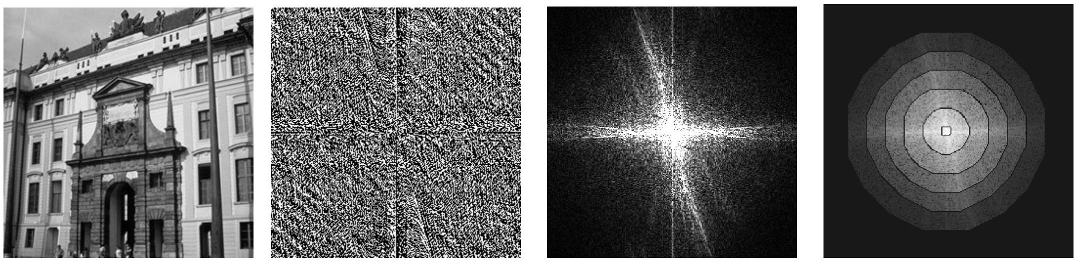


Figure 2.2: Ordering of a Subset of the ???? Dataset by Increasing Contribution of Low Frequencies

described mostly by low frequencies, and not by high frequencies.

2.4.2 Amplitude and Phase Analysis

Consider the decomposition of an image of a building in a city from the (???????????) GIST dataset into itd frequency and amplitude images.



(a) Original image (b) Phase (c) Amplitude (d) Frequency Bins

Figure 2.3: Example of localization properties of different functions in the spatial and Fourier domains. Images localized in the spatial domain are not localized in the Fourier domain, and vice versa.

2.5 Singular Value Decomposition

A deeper understanding of Singular value decomposition (SVD) in the context of images allows the creation of a descriptor that captures overall image complexity in a relatively small descriptor length. SVD is a factorization of any real or complex 2-dimensional matrix. Since images will always be represented by real matrices, we ignore the complex case in our analysis. An $m \times n$ matrix A has an SVD defined by:

$$A_{m\times n} = U_{m\times m} S_{m\times n} V^T_{n\times n}$$

where S is a diagonal $m \times n$ matrix with the singular values of A on the main diagonal (in decreasing order). The columns of U and V are known as the left and right singular vectors of A, respectively. U and V are orthonormal matrices, and the columns are basis vectors for the reconstruction. It is useful to think of each singular value in S as scaling the basis vectors from U and V to generate an 'eigenimage' [3], or the contribution of that specific singular value to the overall image.

[[Perhaps it makes sense to go a bit more in depth here on the mathematical definition of singular values given on https://en.wikipedia.org/wiki/Singular_value and https://en.wikipedia.org/wiki/Singular_value_decomposition#Singular_values.2C_singular_vectors.2C_and_their_relation_to_the_SVD]]

With this perspective, we

Chapter 3

Analysis

Chapter 4

Suggestions for Further Work

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