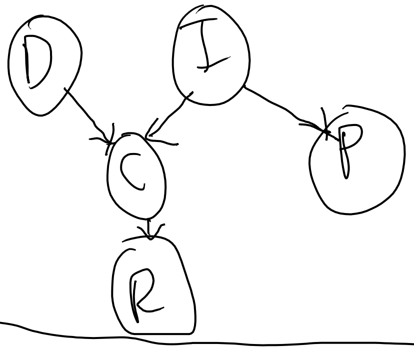
Theorem. Let G, P be a graph and a dist. over the RVs $\{X_1, \dots, X_n\}$. If G is an I-map for P, then P factorizes over G.

Proof. Without loss of generality let's assume that $\{X_1, \dots, X_n\}$ is topologically sorted w.r.t. G.

$*$ $X_i \to X_j$ then $i < j$



D, I, C, P, R

$D \to C$

$I \to C$

So, by the chain rule

$$P(X_1, \dots, X_n) = \prod_{i=1}^{n} P(X_i \mid X_1, \dots, X_{i-1}) \quad *$$

X_1, X_2, X_3

$$P(X_1, X_2, X_3) = P(X_1)P(X_2 \mid X_1)P(X_3 \mid X_1, X_2)$$

Since G is an I-map for P

$$\to I_\ell(G) \subseteq I(P)$$

$$\underbrace{\{(X_i \perp NoDesc_G X_i \mid Pa_G X_i)\}}$$

$$P(X_k \mid \underbrace{X_1, \ldots, X_{k-1}}_{Pa_G X_k \subseteq \{X_1, \ldots, X_{k-1}\}}) = P(X_k \mid Pa_G X_k)$$

Finally,

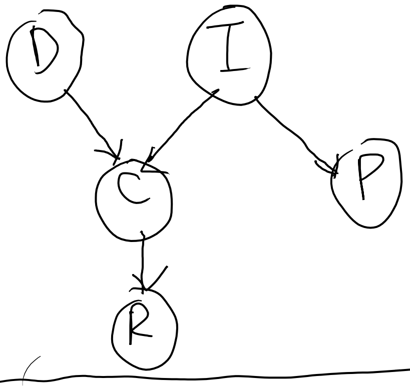
$$P(X_1, \ldots, X_n) = \prod_{i=1}^{n} P(X_i \mid Pa_G X_i)$$

then P factorizes over G $\blacksquare$

Teorema. Sean G, P un grafo y una dist. sobre $\{X_1, \ldots, X_n\}$. Si G es un I-map para P, entonces P factoriza sobre G.

Prueba. Sin pérdida de generalidad supongamos que $X_1, \ldots, X_n$ están ordenados topológica/. respecto a G.

$$X_i \to X_j \Rightarrow i < j$$



D, I, C, P, R

- $D \to C$
- $I \to C$

Usando la regla de la cadena, la dist. conjunta la puedo factorizar como:

$$P(X_1, \ldots, X_n) = \prod_{i=1}^{n} P(X_i \mid X_1, \ldots, X_{i-1}) \quad (*)$$

X_1, X_2, X_3

$$P(X_1, X_2, X_3) = P(X_1)P(X_2 \mid X_1)P(X_3 \mid X_1, X_2)$$

Consideramos el factor k

$$(*)\ P(X_k \mid \underbrace{X_1,\dots,X_{k-1}}) = P(X_k \mid Pa_G X_k)$$

$$\underline{Pa_G X_k} \subseteq \{X_1,\dots,X_{k-1}\}$$

hipo: G I-map para P

$$I_\ell(G) \subseteq I(G) \subseteq I(P)$$

$$\| $$

$$\{(X_i \perp \underline{NoDesc_G X_i} \mid Pa_G X_i)\}.$$

Final/.:

$$\underbrace{P(X_1,\dots,X_n) = \prod_{i=1}^{n} P(X_i \mid Pa_G X_i)}$$

P factoriza sobre G.