Assignment 4 - s214643

01/12 - 22

1) Multiple choice

A) LU factorization

Answer: 1. Large rounding errors

B) Condition number

Answer: 4. Sensitivity of solution to error

C) Linear system

Answer: 3. 7500

d) Gaussian elimination

Answer: 3. Change: 1 and 3, 2 and 3

2) Newtons method for systems of nonlinear equations

1. Implement FdFhj4:

In [11]:
```
import numpy as np

def FdFhj4(x):
    F = lambda x: np.array([
        2 * x[0] + x[1] + 2 * np.cos(x[0]),
        x[0] + 2* x[1] - np.sin(x[1])
    ], dtype=float)

    hessian = lambda x: np.array([
        [2 - 2 * np.sin(x[0]), 1],
        [1, 2 - np.cos(x[1])]
    ], dtype=float)

    return F(x), hessian(x)

def newt_sys(FdFhj4, x0, itter):
    x = x0.copy()
    estimate = [x.copy()]

    for _ in range(itter):
        f, df = FdFhj4(x)
        h = np.linalg.solve(df, -f).flatten()
        x += h
```

```python
        estimate.append(x.copy())
    return estimate
```

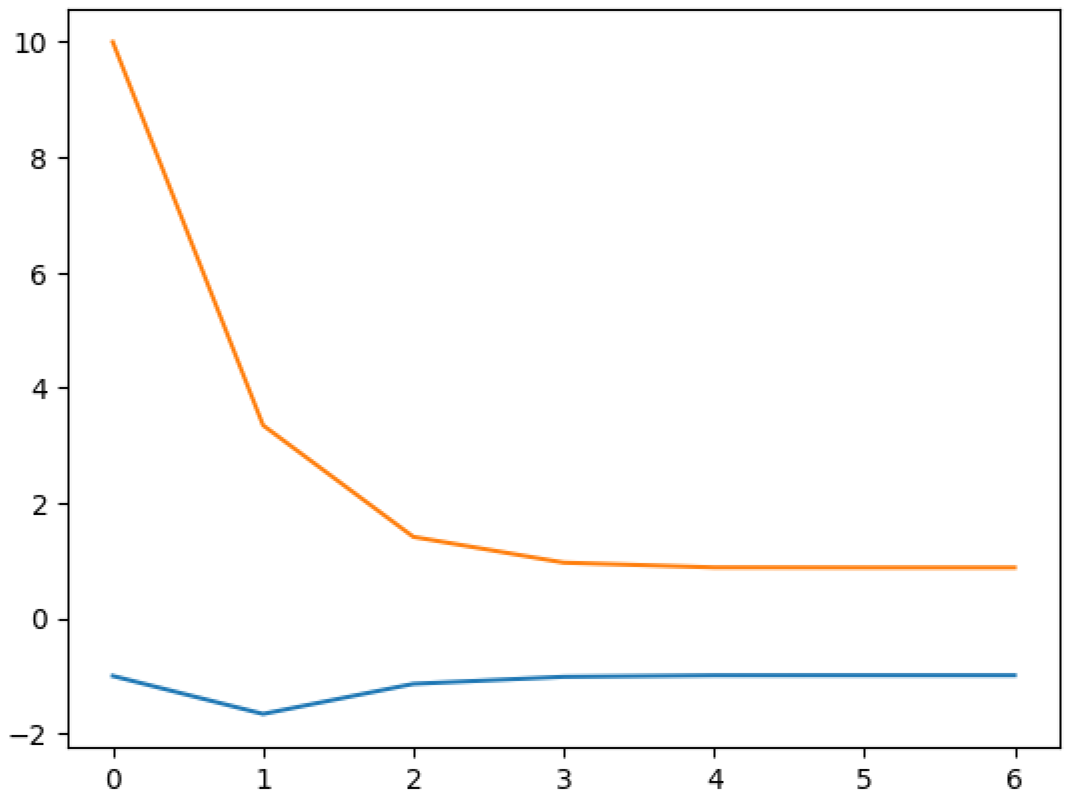
2. Run newton systems and find convergence

In [12]:
```python
x_hat = newt_sys(FdFhj4, np.array([-1, 10], dtype=float), 6)
x_hat
```

Out[12]:
```
[array([-1., 10.]),
 array([-1.65951517,  3.34835148]),
 array([-1.13772814,  1.41319155]),
 array([-1.01449234,  0.96595132]),
 array([-0.99009877,  0.88266467]),
 array([-0.98929344,  0.88002569]),
 array([-0.98929265,  0.88002314])]
```

The function converges on the vector: $\mathbf{x} = [-0.98, 0.88]$

In [9]:
```python
import matplotlib.pyplot as plt
plt.plot(x_hat)
plt.show()
```



3. Convergence

In [21]:
```python
stat_point = np.array([-0.989292652343593, 0.880023136121182])

err = np.max(abs(x_hat - stat_point), axis=1)
rel_err = err[1:] / err[:-1]**2
rel_err
```

Out[21]:
```
array([0.02967669, 0.08751017, 0.3022782 , 0.35775477, 0.36597223,
       0.36658225])
```

Approximation: $C \approx 0.36$

3) Sensitivity analysis

1. Prove that \

We first define $b_m = 0.001$ as the maximum error in the vector.
Then we find the norm of this vector.

$$b_m = max||\delta b||_2$$
$$\left|\left|\begin{bmatrix} b_m \\ b_m \\ \vdots \\ b_m \end{bmatrix}\right|\right| = \sqrt{n \cdot b_m^2} = \sqrt{n} \cdot b_m$$

2. Compute relative error

$$\frac{||\delta x||_2}{||x||_2} = \frac{||\tilde{x} - x||_2}{||x||} \leq \kappa(A)^2 \frac{||\delta b||_2}{||b||_2} = 3.8^2 \frac{\sqrt{100} \cdot 0.001}{1.7} \approx 0.085 \tag{1}$$

So the upper bound for relative error is calculated as the above