Ray Generation Using a Pinhole Camera Model

Jeppe Revall Frisvad

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Rendering a photorealistic image is to compute the outcome of taking a picture with a digital camera in a modelled scene. To do this, we must be able to model the camera mathematically. The conventional pinhole camera model depends on the parameters given in Table 1. We can use these parameters to generate a ray with origin in the eye point and direction through a pixel in the image, see Figure 1.

Table 1: The parameters in the conventional pinhole camera model.

Extrinsic parameters		Intrinsic parameters	
$\boldsymbol{e}$	Eye point	ϕ	Vertical field of view
$\boldsymbol{p}$	View point	d	Camera constant
$\vec{u}$	Up direction	W, H	Camera resolution

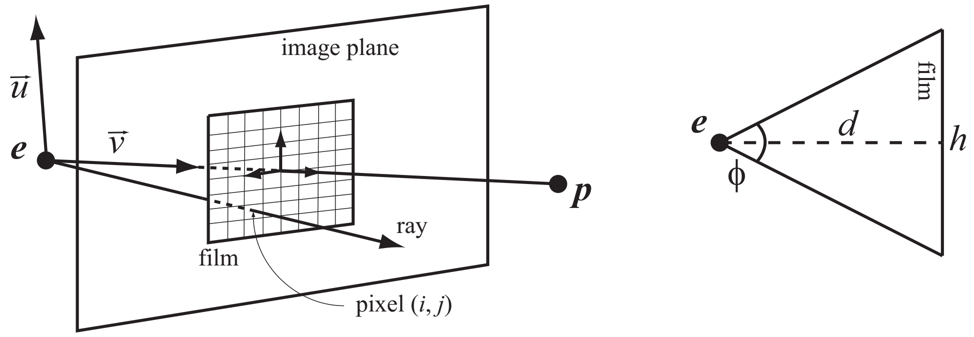

Figure 1: Illustration of how the camera parameters are used to position image plane and film in space, and how we can use this setup to shoot rays through the pixels.

Think about taking a picture. The first thing you do is to position the camera and orient it toward the objects that you wish to photograph. To model this setup, we need a camera position $\boldsymbol{p}$, a viewing direction $\vec{v}$, and an up-vector $\vec{u}$. A digital camera is built around a photoactive charge-coupled device chip (CCD chip) which has a limited number of bins across its area. This means that output images from a digital camera have a specific *resolution* (image width W and height H) measured in number of pixels. Each *pixel* corresponds to a small area of the CCD chip and it has an associated vector which describes the amount of light (of different colours) that reached this area during exposure. To *capture* an image is to expose the chip for a short amount of time and record the vectors for all the pixels. We model the light sensitive area of the CCD chip by a rectangle in the *image plane*, and we will call it the *film*. The viewing direction $\vec{v}$ is normal to the image plane and the up direction of the image plane is given by the up-vector $\vec{u}$. To position the film in space, we center it around the camera position $\boldsymbol{p}$ and move it a distance d in the viewing direction. The distance d depends on the lens system of the camera and is called the *camera constant*. The size of the film (width and height, w and h) is usually specified by an angle in the vertical direction called the *field of view*, ϕ, and an aspect ratio $a = W/H$ such that

$$\begin{aligned} w &= ah \\ h &= 2d\tan(\phi/2) \ . \end{aligned}$$

Using the camera resolution (W, H), we can divide the film into pixels. To compute an image, we trace a number of rays from $\boldsymbol{e}$ through each pixel of the film.

The direction $\vec{\omega}$ of a ray through a pixel of index (i, j) is found by establishing the camera coordinate system [Hughes et al. 2013, Sec. 13.4], and using the convention that the pixel of index $(0, 0)$ is in the bottom left corner of the film. Given pixel index, camera resolution, and the size of the film, we can find the image plane coordinates $(x_{\mathrm{ip}}, y_{\mathrm{ip}})$ of a point in the pixel of index (i, j). The directions of the axes in the camera coordinate system are defined by an orthonormal basis which includes the viewing direction $\vec{v}$. With this basis, the point with coordinates $(x_{\mathrm{ip}}, y_{\mathrm{ip}})$ in the image plane has coordinates $\boldsymbol{q}_{\mathrm{ip}} = (x_{\mathrm{ip}}, y_{\mathrm{ip}}, d)$ in the camera coordinate system, and the direction of a ray from the eye through this point is given by a change of basis from the camera coordinate system to the usual coordinate system.

Given the parameters in Table 1, the orthonormal camera basis $\{\vec{b}_1, \vec{b}_2, \vec{v}\}$ is

$$\vec{v} = \frac{\boldsymbol{p} - \boldsymbol{e}}{|\boldsymbol{p} - \boldsymbol{e}|} \quad , \quad \vec{b}_1 = \frac{\vec{v} \times \vec{u}}{|\vec{v} \times \vec{u}|} \quad , \quad \vec{b}_2 = \vec{b}_1 \times \vec{v} \ ,$$

and the change of basis becomes

$$\boldsymbol{q} = \begin{bmatrix} \vec{b}_1 & \vec{b}_2 & \vec{v} \end{bmatrix} \boldsymbol{q}_{\mathrm{ip}} = \vec{b}_1\, x_{\mathrm{ip}} + \vec{b}_2\, y_{\mathrm{ip}} + \vec{v}\, d \ ,$$

which, after normalization, is the ray direction: $\vec{\omega} = \boldsymbol{q}/|\boldsymbol{q}|$.

References

HUGHES, J. F., VAN DAM, A., MCGUIRE, M., SKLAR, D. F., FOLEY, J. D., FEINER, S. K., AND AKELEY, K. 2013. *Computer Graphics: Principles and Practice*, third ed. Addison-Wesley.