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CSE 250A HW#2

2.1

(a) $P(E=1|A=1) = \frac{P(A=1|E=1)\,P(E=1)}{P(A=1)}$

For the numerator: $P(A=1|E=1) = \sum_b P(A=1, B=b \mid E=1) = \sum_b P(B=b|E=1)\cdot P(A=1|B=b, E=1)$

given the assumption that B, E are independent

$P(A=1|E=1) = \sum_b P(B=b)\cdot P(A=1|E=1, B=b)$. $\quad P(E=1) = 0.002$

$= 0.001 \times 0.95 + 0.999 \times 0.29$

$= 0.28971 + 0.00095 \approx 0.291$

For the Denominator: $P(A=1) = \sum_e \sum_b P(A=1, B=b, E=e)$

$= \sum_e \sum_b P(B=b)\,P(E=e|B=b)\cdot P(A=1|B=b, E=e)$

$= \sum_e \sum_b P(B=b)\,P(E=e)\,P(A=1|B=b, E=e)$

$= \sum_e P(E=e)\left(P(B=1)\times P(A=1|B=1, E=e) + P(B=0)\times P(A=1|B=0, E=e)\right)$

$= 0.002\times(0.001\times 0.95 + 0.999\times 0.29) + 0.998\times(0.001\times 0.94 + 0.999\times 0.001)$

$= 0.000581 + 0.001939 \times 0.998$

$= 0.000581 + 0.00193512 \approx 0.0025$

So $P(E=1|A=1) = \frac{0.291 \times 0.002}{0.0025} \approx 0.23$

c

(b) Given from the last problem we have $P(A=1) \approx 0.0025$.

$P(A=1|M=1) = \frac{P(M=1|A=1)\,P(A=1)}{P(M=1)} = \frac{0.7 \times 0.0025}{P(M=1)}$

$P(M=1) = \sum_a P(M=1, A=a) = \sum_a P(A=a)\cdot P(M=1|A=a)$

$= P(A=1)\,P(M=1|A=1) + P(A=0)\,P(M=1|A=0)$

$= 0.0025 \times 0.7 + 0.9975 \times 0.01 = 0.011725$

So $P(A=1|M=1) \approx 0.15$

b

(c) $P(A=1, B=0, E=1) = P(B=0)\,P(A=1|B=0)\cdot P(E=1|A=1, B=0)$

$P(A=1, B=0, E=1) = P(B=0)\cdot P(E=1|B=0)\cdot P(A=1|B=0, E=1)$

$= P(B=0)\,P(E=1)\cdot P(A=1|B=0, E=1)$ given B, E are independent

$= 0.999 \times 0.002 \times 0.29$

≈ 0.00058

$P(A=1|B=0) = \sum_e P(A=1, E=e | B=0) = \sum_e P(E=e|B=0) \cdot P(A=1|E=e, B=0)$

$= \sum_e P(E=e) P(A=1|E=e, B=0) = 0.002 \times 0.29 + 0.998 \times 0.001$ given B,E are independent.

$= 0.00058 + 0.000998 \approx 0.001578$

So $P(E=1|A=1, B=0) = \dfrac{P(A=1, B=0, E=1)}{P(B=0)\,P(A=1|B=0)} = \dfrac{0.00058}{0.999 \times 0.001578} \approx 0.368$

(d) $P(A=1|M=1, J=0) = \dfrac{P(M=1, J=0|A=1) \cdot P(A=1)}{P(M=1, J=0)}$

For the Numerator: $P(M=1, J=0|A=1) = P(M=1|A=1) \cdot P(J=0|A=1)$

$= P(M=1|A=1) \cdot (1 - P(J=1|A=1)) = 0.7 \times 0.1 = 0.07$

For the Denominator:

$P(M=1, J=0) = \sum_a P(M=1, J=0, A=a) = \sum_a P(A=a) \cdot P(M=1, J=0 | A=a)$

$= \sum_a P(A=a) P(M=1|A=a) P(J=0|A=a)$

$= P(A=1) P(M=1|A=1)(1 - P(J=1|A=1)) + P(A=0) P(M=1|A=0)(1 - P(J=1|A=0))$

$= 0.0025 \times 0.7 \times (1 - 0.9) + 0.9975 \times 0.01 \times 0.95$

$= 0.000175 + 0.0095 \approx 0.00965$

So $P(A=1|M=1, J=0) = \dfrac{0.07 \times 0.0025}{0.00965} \approx 0.018$

(e) $P(A=1|M=0) = \dfrac{P(M=0|A=1) \cdot P(A=1)}{P(M=0)}$

As calculated in (c) $P(M=0) = 1 - P(M=1) \approx 0.8985$.

So $P(A=1|M=0) = \dfrac{(1-0.7) \times 0.0025}{0.8985} \approx 0.0008$

(f) $P(A=1|M=0, B=1) = \dfrac{P(M=0|A=1, B=1)\, P(A=1|B=1)}{P(M=0|B=1)}$

For the numerator:

$P(M=0|A=1, B=1) = P(M=0|A=1)$ d-seperation (i) $= 0.3$

$P(A=1|B=1) = 0.94$ (calculated later in the next page)

For the Denominator

$P(M=0|B=1)$

$= \sum_a P(M=0, A=a | B=1) = \sum_a P(A=a|B=1) \cdot P(M=0|A=a, B=1)$

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$P(A=1|B=1) = \sum_e P(A=1, E=e|B=1) = \sum_e P(E=e)\cdot P(A=1|E=e, B=1)$
$= 0.002\times0.95 + 0.998\times0.94$
$= 0.0019 + 0.93812 \cong 0.94$

$= \sum_a P(A=a|B=1)\cdot P(M=0|A=a) = P(A=1|B=1)\cdot P(M=0|A=1) + P(A=0|B=1)\cdot P(M=0|A=0)$
$= 0.94\times0.3 + 0.06\times0.99 = 0.282 + 0.0594 = 0.3414$

So $P(A=1|M=0, B=1) = \frac{0.94\times0.3}{0.3414} \cong 0.826$

2.2

a) From the Belief Network, we have $S_1, S_2, \dots, S_n$ are conditionally independent given D.

$P(D=0|S_1=1, S_2=1, \dots, S_k=1) = \frac{P(S_1=1, S_2=1, \dots, S_k=1|D=0)\cdot P(D=0)}{P(S_1=1, S_2=1, \dots, S_k=1)}$

For the numerator:

$P(S_1=1, S_2=1, \dots, S_k=1|D=0) = \prod_{i=1}^{k} P(S_i=1|D=0) = P(S_1=1|D=0)\cdot\prod_{i=2}^{k} P(S_i=1|D=0) = \prod_{i=2}^{k} P(S_i=1|D=0)$

$= \frac{f(1)}{f(2)}\times\frac{f(2)}{f(3)}\times\cdots\times\frac{f(k-1)}{f(k)} = \frac{f(1)}{f(k)} = \frac{2-1}{2^k+(-1)^k} = \frac{1}{2^k+(-1)^k}$

For the Denominator:

$P(S_1=1, S_2=1, \dots, S_k=1) = \sum_d P(D=d)\cdot P(S_1=1, \dots, S_k=1|D=d)$

$= P(D=0)P(S_1=1, \dots, S_k=1|D=0) + \frac{1}{2}\cdot\frac{1}{2^k+(-1)^k}$

$= \frac{1}{2}\times\prod_{i=1}^{k} P(S_i=1|D=1) + \frac{1}{2}\frac{1}{2^k+(-1)^k} = \frac{1}{2}\times\left(\frac{1}{2}\right)^k + \frac{1}{2}\left(\frac{1}{2^k+(-1)^k}\right)$.

So $P(D=0|S_1=1, \dots, S_k=1) = \left(\frac{\frac{1}{2^k+(-1)^k}\cdot\frac{1}{2}}{\frac{1}{2}\times\left(\frac{1}{2}\right)^k+\frac{1}{2}\times\left(\frac{1}{2^k+(-1)^k}\right)}\right)$

$P(D=1|S_1=1, S_2=1, \dots, S_k=1) = \frac{\frac{1}{2}\times\left(\frac{1}{2}\right)^k}{\frac{1}{2}\times\left(\frac{1}{2}\right)^k+\frac{1}{2}\left(\frac{1}{2^k+(-1)^k}\right)}$

So $r(k) = \frac{\frac{1}{2^k+(-1)^k}}{\left(\frac{1}{2}\right)^k} = \frac{2^k}{2^k+(-1)^k}$.

b) Given $r(k)$. when $k=2t+1$ $t=0,1,2,\cdots$ $r(k)>1$.

when $k=2t$ $t=0,1,2,\cdots$ $r(k)<1$.

So the doctor diagnoses the patient with $D=0$ form of the disease on odd number of days in a month

the doctor diagnoses the patient with $D=1$ form on even number of days in a month.

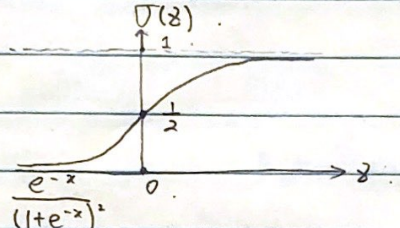
(b) $P(D=0 \mid S_1=1, S_2=1, \cdots, S_k=1)$

$$= \left(\frac{(\frac{1}{2})^k}{(\frac{1}{2})^k + \frac{1}{2^k+(-1)^k}} \right) = \frac{1}{1+\frac{2^k}{2^k+(-1)^k}} = \frac{1}{1+\frac{1}{1+(-\frac{1}{2})^k}}$$

with k getting larger, in which more symptoms are observed, $r(k)$ will gradually converge to 1

this means it will be harder to distinguish $D=0$ and $D=1$ given all the symptoms.

So it will be less certain.



2.3.

$\sigma(z) = (1+\exp(-z))^{-1}$

(a) $\sigma'(z) = \frac{d}{d(z)} \sigma(z) = \frac{e^{-x}}{(1+e^{-x})^2}$

$$= \frac{e^{-z}}{1+e^{-2z}+2e^{-z}} = \frac{1}{e^{z}+e^{-z}+2} = \frac{1}{(1+e^{z})(1+e^{-z})} = \sigma(z) \cdot \sigma(-z)$$

(b) $\sigma(z) + \sigma(-z) = \frac{1}{1+e^{-z}} + \frac{1}{1+e^{z}} = \frac{e^{z}+e^{-z}+2}{e^{z}+e^{-z}+2} = 1$

(c) $L(\sigma(z)) = \log\left(\frac{\sigma(z)}{1-\sigma(z)}\right) = \log\left(\frac{\frac{1}{1+e^{-z}}}{\frac{e^{-z}}{1+e^{-z}}}\right) = \log(e^{z}) = z$

(d) $p_i = P(Y=1 \mid X_i=1, X_j=0 \text{ for all } j\neq i) = \sigma(\sum_{i=1}^{k} w_i x_i) = \sigma(w_i x_i) = \frac{1}{1+e^{-(w_i x_i)}}$

as shown in (c) $L(\sigma(w_i x_i)) = w_i x_i$ given $X_i = x_i = 1$ $L(p_i) = w_i$

2.4

$E = \{puddle\}$. $X, Y = \{month, fall\}, \{rain, fall\}, \{sprinkler, fall\}$.

$E = \{month\}$ $X, Y = \{rain, sprinkler\}$

$E = \{month, puddle\}$. $X, Y = \{rain, fall\}, \{sprinkler, fall\}$

$E = \{rain, sprinkler\}$ $X, Y = \{month, puddle\}, \{month, fall\}$.

$E = \{rain, sprinkler, puddle\}$ $X, Y = \{month, fall\}$

$E = \{sprinkler, puddle\}$ $X, Y = \{month, fall\}, \{rain, fall\}$.

$E = \{rain, puddle\}$ $X, Y = \{month, fall\}$ $\{sprinkler, fall\}$.

$E = \{month, sprinkler, puddle\}$ $X, Y = \{rain, fall\}$.

$E = \{month, rain, puddle\}$ $X, Y = \{sprinkler, fall\}$.

$E = \{rain, sprinkler, fall\}$ $X, Y = \{month, puddle\}$

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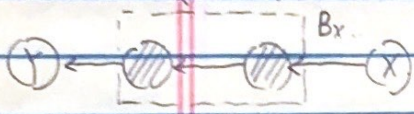
2.5.

$B_X = \{ Parents(X), Childrens(X), \{Parents(childrens(X)) \text{ except } X\} \}$.

1°. $Y \in Childrens(Parents(childrens(X)))$

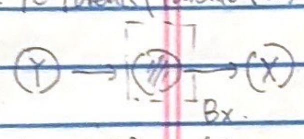


satisfy condition (i) of d-seperation So $P(X,Y|B_X) = P(X|B_X)P(Y|B_X)$

2° $Y \in Parents(Parents(Childrens(X)))$

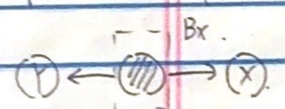
satisfy condition (iii) of d-seperation. So Equality holds.

3°. $Y \in Parents(Parents(X))$



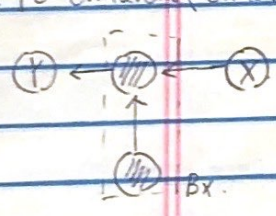
satisfy condition (i) of d-seperation, So Equality holds.

4°. $Y \in Childrens(Parents(X))$



satisfy condition (ii) of d-seperation. So Equality holds.

5°. $Y \in Childrens(Childrens(X))$



satisfy condition (i) of d-seperation. So equality holds.

2.6.

(1) $P(B|G,C) = P(B|G)$ $\boxed{\text{FALSE}}$

(2). $P(F,G|D) = P(F|D)P(G|D)$ $\boxed{\text{FALSE}}$.

(3) $P(A,C) = P(A)P(C)$ $\boxed{\text{TRUE}}$

(4) $P(D|B,F,G) = P(D|B,F,G,A)$ $\boxed{\text{TRUE}}$.

(5) $P(F,H) = P(F)P(H)$ $\boxed{\text{TRUE}}$

(6) $P(D,E|F,H) = P(D|F)\,P(E|H)$ $\boxed{\text{TRUE}}$.

(7). $P(F,C|G) = P(F|G)\cdot P(C|G)$ $\boxed{\text{FALSE}}$

(8) $P(D,E,G) = P(D)\cdot P(E|D)\cdot P(G|D,E)$

$= P(D)\cdot P(E)\cdot P(G|D,E)$ $\boxed{\text{TRUE}}$.

(9) $P(H|C) = P(H|A,B,C,D,F)$ $\boxed{\text{TRUE}}$

(10) $P(H|A,C) = P(H|A,C,G)$ $\boxed{\text{FALSE}}$.

2.7

(1). $P(A|D) = P(A|S)$ $S = \{\phi\}$.

(2) $P(A|B,D) = P(A|S)$. $S = \{B, D, F, E, C\}$

(3) $P(B|D,E) = P(B|S)$ $S = \{D, E, F\}$

(4). $P(E) = P(E|S)$. $S = \{A\}$

(5). $P(E|F) = P(E|S)$ $S = \{\phi\}$.

(6). $P(E|D,F) = P(E|S)$ $S = \{\phi\}$.

(7). $P(E|B,C) = P(E|S)$ $S = \{B, C, A, D\}$

(8). $P(F) = P(F|S)$ $S = \{\phi\}$

(9). $P(F|D) = P(F|S)$ $S = \{\phi\}$

(10) $P(F|D,E) = P(F|S)$ $S = \{D, E, B, A, C\}$

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(a) $P(Z=1|X=0, Y=0) = 1-1 = 0$

$P(Z=1|X=0, Y=1) = 1-(1-p_y) = p_y$. So $\boxed{<}$

(b). $P(Z=1|X=1, Y=0) = 1-(1-p_x) = p_x < p_y$.

So $\boxed{<}$

(c). $P(Z=1|X=1, Y=0) = p_x$ $P(Z=1|X=1, Y=1) = 1-(1-p_x)(1-p_y) = 1-(1-p_x-p_y+p_xp_y)$

$= p_x+p_y-p_xp_y$

$\cancel{p_x}+p_y-p_xp_y-\cancel{p_x} = p_y(1-p_x) > 0$. So $\boxed{<}$

(d) $P(X=1|Z=1) = \dfrac{P(Z=1|X=1)\cdot P(X=1)}{P(Z=1)}$

$P(Z=1|X=1) = \sum_y P(Z=1, Y=y|X=1) = \sum_y P(Y=y|X=1) \cdot P(Z=1|Y=y, X=1)$

$= \sum_y P(Y=y)P(Z=1|Y=y, X=1) = p_y \cdot (p_x+p_y-p_xp_y) + (1-p_y)p_x = \cancel{p_xp_y} + p_y^2 - p_xp_y^2 + p_x - \cancel{p_xp_y}$

$= p_y^2 + p_x - p_xp_y^2$

$P(Z=1) = \sum_x \sum_y P(Z=1, Y=y, X=x)$

$= \sum_x \sum_y P(X=x)P(Y=y) \ P(Z=1|X=x, Y=y)$

$= \sum_x P(X=x) \sum_y P(Y=y) P(Z=1|X=x, Y=y)$

$= (1-p_x)((1-p_y)\cdot 0 + p_y \cdot p_y) + p_x((1-p_y)p_x + p_y(p_x+p_y-p_xp_y))$

$= (1-p_x)p_y^2 + p_x(p_x - \cancel{p_xp_y} + \cancel{p_xp_y} + p_y^2 - p_xp_y^2)$

$= p_y^2 - \cancel{p_xp_y^2} + p_x^2 + \cancel{p_xp_y^2} - p_x^2p_y^2$

$= p_x^2 + p_y^2 - p_x^2p_y^2$

So $P(X=1|Z=1) = \dfrac{p_x(p_y^2+p_x-p_xp_y^2)}{p_x^2+p_y^2-p_x^2p_y^2} = \dfrac{p_xp_y^2+p_x^2-p_x^2p_y^2}{p_x^2+p_y^2-p_x^2p_y^2} = 1 - \dfrac{(1-p_x)p_y^2}{p_x^2+p_y^2-p_x^2p_y^2}$

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$P(X=1|Z=1) - P(X=1)$

$= \frac{P_x P_y^2 + P_x^2 - P_x^2 P_y^2}{P_x^2 + P_y^2 - P_x^2 P_y^2} - \frac{P_x^3 + P_x P_y^2 - P_x^3 P_y^2}{P_x^2 + P_y^2 - P_x^2 P_y^2}$

$= \frac{P_x^2 - P_x^3 + P_x P_y^2 - P_x P_y^2}{P_x^2 + P_y^2 - P_x^2 P_y^2} = \frac{(P_x^2 - P_x^3)(1 - P_y^2)}{P_x^2 + P_y^2 - P_x^2 P_y^2} > 0$.

So. Answer is $\boxed{<}$

(e) Given the Belief Network, X. Y should be independent.

So $P(X=1) = P(X=1|Y=1)$. The answer is $\boxed{=}$

(f) $P(X=1|Y=1, Z=1) = P(X=1, Y=1, Z=1) / P(Y=1, Z=1)$

$P(X=1, Y=1, Z=1) = P(X=1) \cdot P(Y=1|X=1) \times P(Z=1|X=1, Y=1)$

$= P_x P_y (P_x + P_y - P_x P_y)$

$P(Y=1, Z=1) = \sum_x P(X=x, Y=1, Z=1) = P(X=1, Y=1, Z=1) + P(X=0, Y=1, Z=1)$

$= P(X=0) P(Y=1) P(Z=1 | X=0, Y=1) + P_x^2 P_y + P_x P_y^2 - P_x^2 P_y^2$

$= (1 - P_x) \cdot P_y^2 + P_x^2 P_y + P_x P_y^2 - P_x^2 P_y^2 = P_y^2 + P_x^2 P_y - (P_x P_y)^2$

$P(X=1|Y=1, Z=1) = \frac{P_x P_y (P_x + P_y - P_x P_y)}{P_y^2 + P_x^2 P_y - P_x^2 P_y^2}$.

$P(X=1|Z=1) = \frac{P_x P_y^2 + P_x^2 - P_x^2 P_y^2}{P_x^2 + P_y^2 - P_x^2 P_y^2} =$

$P(X=1|Z=1) - P(X=1|Y=1, Z=1) > 0$. So answer is $\boxed{>}$

(g) $P(X=1) P(Y=1) P(Z=1) = P_x P_y (P_x^2 + P_y^2 - P_x^2 P_y^2)$

$P(X=1, Y=1, Z=1) = P_x P_y (P_x + P_y - P_x P_y)$

$P(X=1, Y=1, Z=1) - P(X=1) P(Y=1) P(Z=1) = P_x P_y (P_x + P_y - P_x P_y - P_x^2 - P_y^2 + P_x^2 P_y^2)$.

$P_x + P_y - P_x P_y - P_x^2 - P_y^2 + P_x^2 P_y^2 = P_x(1 - P_x) + P_y(1 - P_y) + P_x P_y (P_x P_y - 1)$

$> P_x(1 - P_x) + P_x P_y (1 - P_y) + P_x^2 P_y^2 - P_x P_y = P_x(1 - P_x) + P_x^2 P_y^2 - P_x P_y^2 = P_x(1 - P_x) - P_y^2 P_x (1 - P_x)$.

$= P_x (1 - P_y^2)(1 - P_x) > 0$.

So the answer is $\boxed{<}$

2.9

(a) By Conditional Version of the Bayes Rule

$$P(C|A.B.D) = \frac{P(D|A.B.C) \cdot P(C|A.B)}{P(D|AB)}.$$

For the numerator:

$P(D|A.B.C) = P(D|B.C)$ given D, A are conditionally independent given B.C (d-seperation (i))

$P(C|A.B) = P(C|A)$ given B.C are condionally independent given A (d-seperation (iii))

For the denominator.

$$P(D|A.B) = \sum_c P(C.D|A.B) = \sum_c P(C|A.B) \cdot P(D|A.B.C)$$
$$= \sum_c P(C|A) \cdot P(D|BC)$$

$$\text{So } P(C|A.B.D) = \frac{P(D|B.C) \cdot P(C|A)}{\sum_c P(C|A) \cdot P(D|B.C)}$$

(b) $P(E|A.B.D) = \sum_c P(E.C|A,B,D)$

$= \sum_c P(C|A,B,D) \cdot P(E|A.B.C,D)$.

E and A are conditionally independent given C (d-seperation (i))

B.D and E are conditionally independent given C (d-seperation (ii))

So $P(E|A.B.C.D) = P(E|C)$ So $P(E|A,B,D) = \sum_c P(C|A,B,D) \cdot P(E|C)$.

(c) $P(G|A,B.D) = \sum_e P(E.G|A.B,D) = \sum_e P(E|A,B.D) \cdot P(G|A.B.E.D)$

given path from A.B.D.C to G comes into E $P(G|A.B.E.D) = P(G|E)$ (d-seperation (i))

So $P(G|A.B.D) = \sum_e P(E|A.B.D) \cdot P(G|E)$

(d) Since all the path from A.B,D to F comes into E So F and AB.D are C.I given E (d-sep (iii))

$P(F|A.B.D.G) = P(G|A.B.D.F) \cdot P(G|A,B,D) / P(F|A.B.D)$

For the numerator $P(G|A.B.D.F) = \sum_e P(G.E|A.B.D.F) = \sum_e P(E|A.B.D.F) \cdot P(G|A.B.E.D.F)$

$= \sum_e P(E|A.B.D.F) \cdot P(G|E.F)$ given G and A,B,D are C.I given E.F (d-seperation (i))

$P(E|A.B.D.F) = P(E|A.B.D)$ given E and F are C.I given A.B.D. (d-seperation (ii))

For the Denominator. $P(F|A.B.D) = \sum_e P(E.F|A.B.D) = \sum_e P(E|A,B,D)\ P(F|A.B,E.D) = \sum_e P(E|A,B.D) \cdot P(F|E)$

$= \sum_e P(E|A,B,D) \cdot P(F)$ given F and E are marginally independent (d-seperation (iii))

$$\text{So } P(F|A,B,D,G) = \frac{\left[\sum_e P(E|A,B,D) \cdot P(G|E.F)\right] \cdot P(G|A,B,D)}{P(F)}$$