# hw4prob3

October 28, 2021

## 1 Problem 3 Homework 4 CSE 250A Probabilistic Reasoning & Learning

### a)

Compute the maximum likelihood estimate of the unigram distribution $P_u(w)$ over words $w$. Print out a table of all the tokens (i.e., words) that start with the letter "M", along with their numerical unigram probabilities (not counts).

[1]:
```
import pandas as pd
import numpy as np
import csv
import matplotlib.pyplot as plt
pd.options.mode.chained_assignment = None

# Import vocabulary and unigrams as pandas dataframes
vocab_data = pd.read_csv('hw4_vocab.txt',sep="\t",quoting=csv.
 ↪QUOTE_NONE,header=None,names=["Words"])
unigram_data = pd.read_csv('hw4_unigram.txt',sep="\t",␣
 ↪header=None,names=["Count"])

# Add a column for the unigram probabilities in the vocab dataframe
vocab_data["Unigram Probs"] = unigram_data["Count"]/unigram_data["Count"].sum()

# Print the words starting with M and their probabilities
vocab_data[vocab_data["Words"].str.startswith("M")]
```

[1]:

| | Words | Unigram Probs |
|---|---|---|
| 53 | MILLION | 0.002073 |
| 68 | MORE | 0.001709 |
| 76 | MR. | 0.001442 |
| 120 | MOST | 0.000788 |
| 121 | MARKET | 0.000780 |
| 125 | MAY | 0.000730 |
| 129 | M. | 0.000703 |
| 130 | MANY | 0.000697 |
| 158 | MADE | 0.000560 |

```
177       MUCH       0.000515
179       MAKE       0.000514
202      MONTH       0.000445
208      MONEY       0.000437
226     MONTHS       0.000406
229         MY       0.000400
246     MONDAY       0.000382
255      MAJOR       0.000371
274   MILITARY       0.000352
286    MEMBERS       0.000336
355      MIGHT       0.000274
365    MEETING       0.000266
369       MUST       0.000267
373         ME       0.000264
374      MARCH       0.000260
384        MAN       0.000253
402        MS.       0.000239
403   MINISTER       0.000240
459     MAKING       0.000212
472       MOVE       0.000210
478      MILES       0.000206
```

**b)**

Compute the maximum likelihood estimate of the bigram distribution $P_b(w'|w)$. Print out a table of the ten most likely words to follow the word "THE", along with their numerical bigram probabilities.

[2]:
```
# Read the bigram file as a dataframe
bigram_data = pd.read_csv('hw4_bigram.
 ↪txt',sep="\t",header=None,names=["w1","w2","count(w1,w2)"])
# Retrieve the index of the word "THE" in the previous dataframe
index = vocab_data.index
condition = vocab_data["Words"] == "THE"
the_index_list = index[condition]
the_index = the_index_list[0]+1

# Make a new dataframe with only the "THE" word as "w1"
the_bigram = bigram_data[bigram_data["w1"]==the_index]

# Save the bigram probabilities as a column in the bigram dataframe
word_sum = the_bigram["count(w1,w2)"].sum()
the_bigram["Bigram Probs"] = the_bigram["count(w1,w2)"]/word_sum
the_bigram = the_bigram.sort_values("Bigram Probs", ascending=False)
ten_words_after_the = the_bigram.head(10)
print(ten_words_after_the)

print("\n We access the words from the vocab dataframe to print them out nicely")
```

```
# Save words and probabilities as lists to print out nicely
bigram_prob_list = ten_words_after_the["Bigram Probs"].tolist()
second_word = []
# Add all second words to list to easily print out
second_word.extend((vocab_data.loc[0,"Words"],vocab_data.
 ↪loc[69,"Words"],vocab_data.loc[78,"Words"],vocab_data.
 ↪loc[72,"Words"],vocab_data.loc[60,"Words"],vocab_data.
 ↪loc[183,"Words"],vocab_data.loc[102,"Words"],vocab_data.
 ↪loc[38,"Words"],vocab_data.loc[307,"Words"],vocab_data.loc[22,"Words"]))
for i in range(len(second_word)):
    if (second_word[i]=="GOVERNMENT"):
        print("Word: {} \t Probability: {}".format(second_word[i],␣
 ↪round(bigram_prob_list[i],6)))
    else:
        print("Word: {} \t \t Probability: {}".format(second_word[i],␣
 ↪round(bigram_prob_list[i],6)))
```

```
      w1   w2  count(w1,w2)  Bigram Probs
993    4    1       2371132      0.615020
1058   4   70         51556      0.013372
1064   4   79         45186      0.011720
1060   4   73         44949      0.011659
1050   4   61         36439      0.009451
1165   4  184         33435      0.008672
1086   4  103         26230      0.006803
1029   4   39         25641      0.006651
1282   4  308         24239      0.006287
1014   4   23         23752      0.006161

 We access the words from the vocab dataframe to print them out nicely
Word: <UNK>             Probability: 0.61502
Word: U.                Probability: 0.013372
Word: FIRST             Probability: 0.01172
Word: COMPANY           Probability: 0.011659
Word: NEW               Probability: 0.009451
Word: UNITED            Probability: 0.008672
Word: GOVERNMENT        Probability: 0.006803
Word: NINETEEN          Probability: 0.006651
Word: SAME              Probability: 0.006287
Word: TWO               Probability: 0.006161
```

**c)**

Consider the sentence **"The stock market fell by one hundred points last week."** Ignoring punctuation, compute and compare the log-likelihoods of this sentence under the unigram and bigram models:

$$\mathcal{L}_u = log[P_u((\textbf{The}))\mathbf{P_u}((\textbf{stock}))\mathbf{P_u}((\textbf{market}))...\mathbf{P_u}((\textbf{last}))\mathbf{P_u}((\textbf{week}))]$$

$$\mathcal{L}_b = log[P_b((\textbf{The}| < \textbf{s} >))\mathbf{P_b}((\textbf{stock}|\textbf{the}))\mathbf{P_b}((\textbf{market}|\textbf{stock}))...\mathbf{P_b}((\textbf{last}|\textbf{points}))\mathbf{P_b}((\textbf{week}|\textbf{last}))]$$

In the equation for the bigram log-likelihood, the token $< s >$ is used to mark the beginning of a sentence. Which model yields the highest log-likelihood?

[3]:
```
# I see now that I should have saved bigram probabilities for all words, so I␣
↪will do that
bigram_probs = []
for index,row in bigram_data.iterrows():
    bigram_probs.append(row["count(w1,w2)"]/unigram_data.
↪iloc[row["w1"]-1]["Count"])
bigram_data["Bigram Probs"] = bigram_probs
```

[4]:
```
sentence1 = "THE STOCK MARKET FELL BY ONE HUNDRED POINTS LAST WEEK"
sentence1 = sentence1.split()
indeces1 = []
for word in sentence1:
    index = vocab_data.index
    condition = vocab_data["Words"] == word
    word_index = index[condition]
    indeces1.append(word_index[0])
```

[5]:
```
# Unigram log-likelihood
Lu = 0
for index in indeces1:
    Lu += np.log(vocab_data.iloc[index]["Unigram Probs"])
print(f"Unigram log-likelihood: {np.round(Lu,5)}")
```

```
Unigram log-likelihood: -64.50944
```

[6]:
```
sentence2 = "<s> THE STOCK MARKET FELL BY ONE HUNDRED POINTS LAST WEEK"
sentence2 = sentence2.split()
indeces2 = []
for word in sentence2:
    index = vocab_data.index
    condition = vocab_data["Words"] == word
    word_index = index[condition]+1
    indeces2.append(word_index[0])
```

[13]:
```
# Bigram log-likelihood
Lb = 0
for i in range(len(indeces2)-1):
    # Last word on the line is .values[0]
    Lb+=np.
↪log(bigram_data[(bigram_data["w1"]==indeces2[i])&(bigram_data["w2"]==indeces2[i+1])]["Bigram␣
↪Probs"].values[0])
```

```
print(f"Bigram log-likelihood: {np.round(Lb,5)}")
```

```
Bigram log-likelihood: -40.91813
```

We observe that the bigram model yields a better log-likelihood.

**d)**

Consider the sentence **"The sixteen officials sold fire insurance."** Ignoring punctuation, compute and compare the log-likelihoods of this sentence under the unigram and bigram models:

$$\mathcal{L}_u = log[P_u(\textbf{The})\mathbf{P_u}(\textbf{sixteen})\mathbf{P_u}(\textbf{officials})...\mathbf{P_u}(\textbf{fire})\mathbf{P_u}(\textbf{insurance})]$$

$$\mathcal{L}_b = log[P_b(\textbf{The}| < \mathbf{s} >)\mathbf{P_b}(\textbf{sixteen}|\textbf{the})\mathbf{P_b}(\textbf{officials}|\textbf{sixteen})...\mathbf{P_b}(\textbf{fire}|\textbf{sold})\mathbf{P_b}(\textbf{insurance}|\textbf{fire})]$$

Which pairs of adjacent words in this sentence are not observed in the training corpus? What effect does this have on the log-likelihood from the bigram model?

[8]:
```
sentence3 = "THE SIXTEEN OFFICIALS SOLD FIRE INSURANCE"
sentence3 = sentence3.split()
indeces3 = []
for word in sentence3:
    index = vocab_data.index
    condition = vocab_data["Words"] == word
    word_index = index[condition]
    indeces3.append(word_index[0])
```

[9]:
```
# Unigram log-likelihood
Lu = 0
for index in indeces3:
    Lu += np.log(vocab_data.iloc[index]["Unigram Probs"])
print(f"Unigram log-likelihood: {np.round(Lu,5)}")
```

```
Unigram log-likelihood: -44.29193
```

[10]:
```
sentence4 = "<s> THE SIXTEEN OFFICIALS SOLD FIRE INSURANCE"
sentence4 = sentence4.split()
indeces4 = []
for word in sentence4:
    index = vocab_data.index
    condition = vocab_data["Words"] == word
    word_index = index[condition]+1
    indeces4.append(word_index[0])
```

[15]:
```
# Bigram log-likelihood
Lb = 0
for i in range(len(indeces4)-1):
    try:
        # Last word on the line is .values[0]
```

```
        prob =␣
↪bigram_data[(bigram_data["w1"]==indeces4[i])&(bigram_data["w2"]==indeces4[i+1])]["Bigram␣
↪Probs"].values[0]
    except IndexError:
        print(f"The combination of {sentence4[i]} and {sentence4[i+1]} is not␣
↪present in the bigram data, resulting in log(0) which is -inf")
        Lb = float('-inf')
    else:
        Lb+=np.log(prob)
print(f"Bigram log-likelihood: {np.round(Lb,5)}")
#print(f"The combination of {sentence4[i]} and {sentence4[i+1]} is not present␣
↪in the bigram data")
```

```
The combination of SIXTEEN and OFFICIALS is not present in the bigram data,
resulting in log(0) which is -inf
The combination of SOLD and FIRE is not present in the bigram data, resulting in
log(0) which is -inf
Bigram log-likelihood: -inf
```

**e)**

Consider the so-called mixture model that predicts words from a weighted interpolation of the unigram and bigram models:

$$P_m(w'|w) = \lambda P_u(w') + (1-\lambda)P_b(w'|w)$$

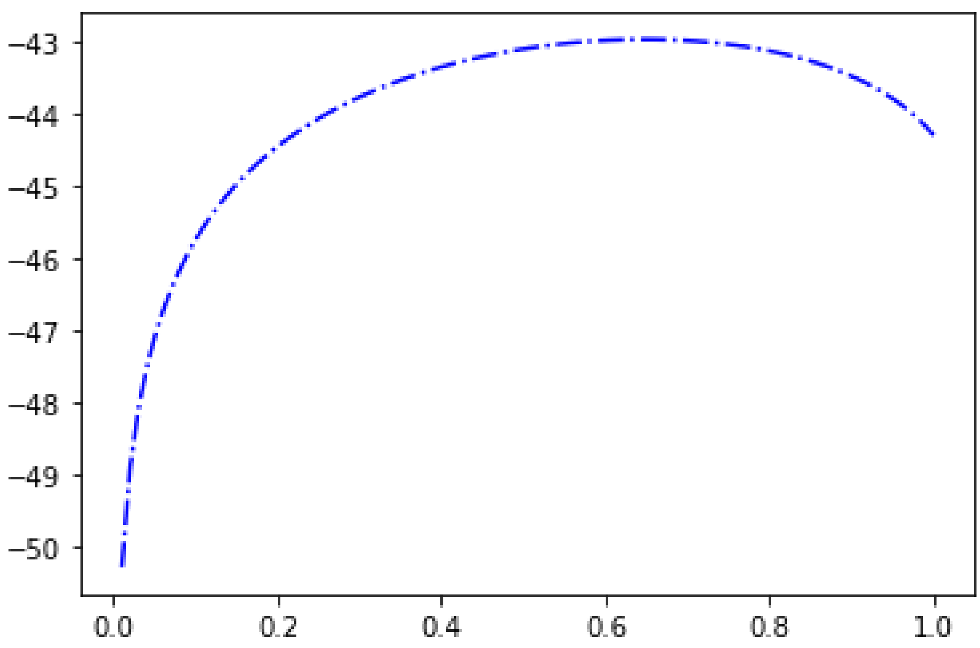
where $\lambda \in [0,1]$ determines how much weight is attached to each prediction. Under this mixture model, the log-likelihood of the sentence from part (d) is given by:

$\mathcal{L}_m = log[P_m(\textbf{The}| < \textbf{s} >)\mathbf{P_m}(\textbf{sixteen}|\textbf{the})\mathbf{P_m}(\textbf{officials}|\textbf{sixteen})...\mathbf{P_m}(\textbf{fire}|\textbf{sold})\mathbf{P_m}(\textbf{insurance}|\textbf{fire})]$.

Compute and plot the value of this log-likelihood $\mathcal{L}_m$ as a function of the parameter $\lambda \in [0,1]$. From your results, deduce the optimal value of $\lambda$ to two significant digits.

[12]:
```
sentence5 = "<s> THE SIXTEEN OFFICIALS SOLD FIRE INSURANCE"
sentence5 = sentence5.split()
indeces5 = []
for word in sentence5:
    index = vocab_data.index
    condition = vocab_data["Words"] == word
    word_index = index[condition]+1
    indeces5.append(word_index[0])

def mixture_prob(lam, p_uni, p_bi):
    return (np.log(lam*p_uni+(1-lam)*p_bi))

def mixture_mle(sentence5, lam):
    p = 0
    for i in range(len(sentence5) - 1):
```

```
        p_uni = vocab_data[vocab_data["Words"]==sentence5[i+1]]["Unigram Probs"].
 ↪values[0]
        try:
            # Last word on the line is .values[0]
           ␣
 ↪p_bi=bigram_data[(bigram_data["w1"]==indeces5[i])&(bigram_data["w2"]==indeces5[i+1])]["Bigram
 ↪Probs"].values[0]
        except IndexError:
            p_bi = 0
        p_ = mixture_prob(lam,p_uni,p_bi)
        p += p_
    return p

x = []
y = []
for i in np.linspace(0.01, 1, 100):
    x.append(i)
    y.append(mixture_mle(sentence5, i))
plt.plot(x, y, 'b-.')
plt.show()

np.array(x)
np.array(y)
print(f"Highest lambda value is {round(x[np.argmax(y)],2)}")
```




```
Highest lambda value is 0.65
```