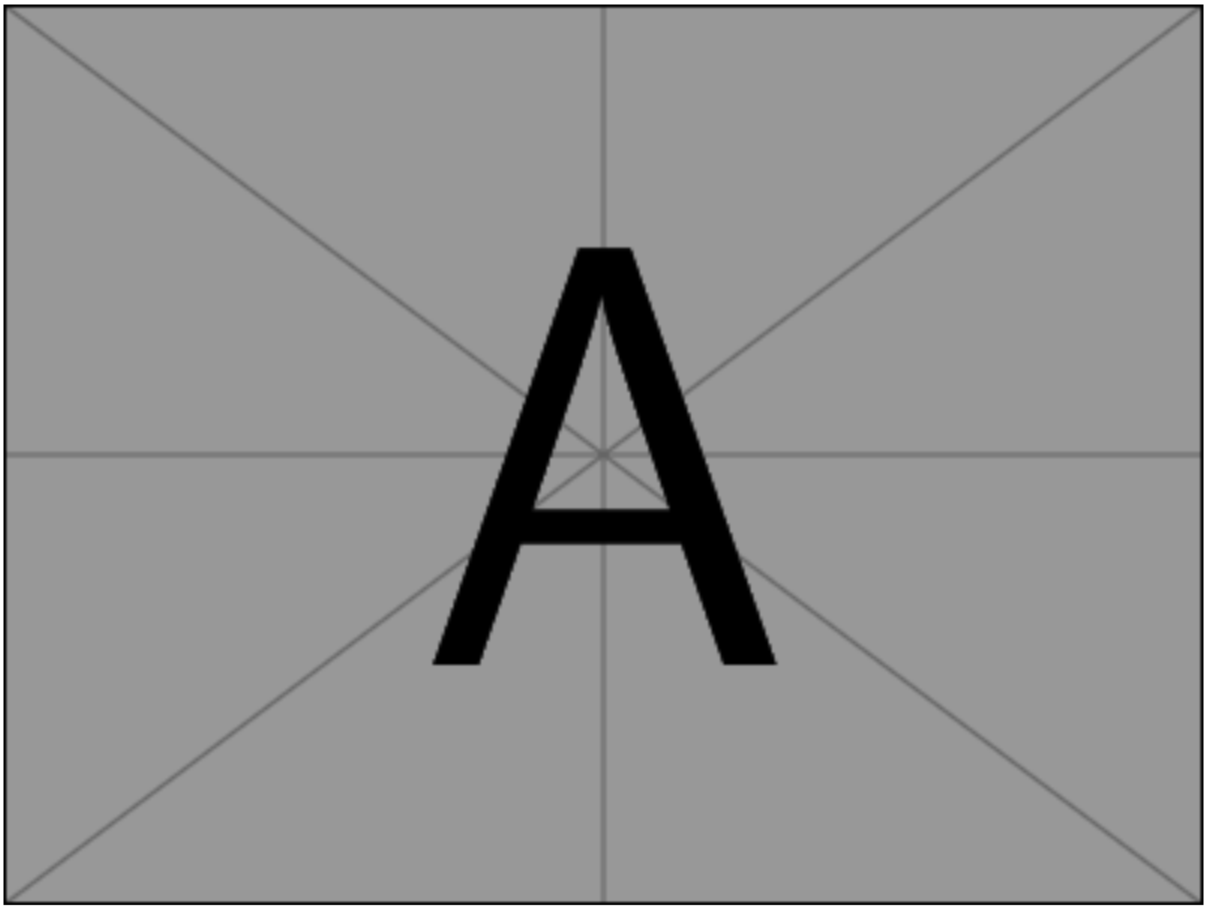
CSE 250A. Assignment 7 —

7.1 Viterbi algorithm

Source code

Plot with correct message:



7.2 Inference in HMMs

Consider a discrete HMM with hidden states S_t, observations O_t, transition matrix $a_{ij} = P(S_{t+1}=j|S_t=i)$ and emission matrix $b_{ik} = P(O_t=k|S_t=i)$. In class, we defined the forward-backward probabilities:

$$\begin{aligned}\alpha_{it} &= P(o_1, o_2, \ldots, o_t, S_t=i), \\ \beta_{it} &= P(o_{t+1}, o_{t+2}, \ldots, o_T|S_t=i),\end{aligned}$$

for a particular observation sequence $\{o_1, o_2, \ldots, o_T\}$ of length T. In terms of these probabilities, which you may assume to be given, as well as the transition and emission matrices of the HMM, show how to (efficiently) compute the following posterior probabilities.

The key tools, as usual, are the product rule, conditional independence (CI), and marginalization.

(a) $P(S_{t+1}=j|S_t=i, o_1, o_2, \ldots, o_T)$

$=$

(b) $P(S_t=i|S_{t+1}=j, o_1, o_2, \ldots, o_T)$

$=$

(c) $P(S_{t-1}=i, S_t=k, S_{t+1}=j | o_1, o_2, \ldots, o_T)$

$=$

(d) $P(S_{t+1}=j | S_{t-1}=i, o_1, o_2, \ldots, o_T)$

$=$

7.3 Conditional independence

Consider the hidden Markov model (HMM) shown below, with hidden states S_t and observations O_t for times $t \in \{1, 2, \ldots, T\}$. State whether the following statements of conditional independence are true or false.

__________ $P(S_t|S_{t-1}) = P(S_t|S_{t-1}, S_{t+1})$

__________ $P(S_t|S_{t-1}) = P(S_t|S_{t-1}, O_{t-1})$

__________ $P(S_t|S_{t-1}) = P(S_t|S_{t-1}, O_t)$

__________ $P(S_t|O_{t-1}) = P(S_t|O_1, O_2, \ldots, O_{t-1})$

__________ $P(O_t|S_{t-1}) = P(O_t|S_{t-1}, O_{t-1})$

__________ $P(O_t|O_{t-1}) = P(O_t|O_1, O_2, \ldots, O_{t-1})$

__________ $P(S_2, S_3, \ldots, S_T|S_1) = \prod_{t=2}^{T} P(S_t|S_{t-1})$

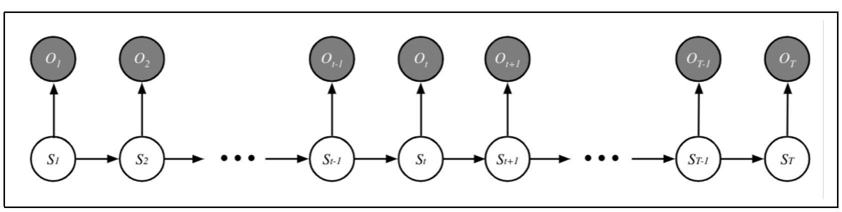
__________ $P(S_1, S_2, \ldots, S_{T-1}|S_T) = \prod_{t=1}^{T-1} P(S_t|S_{t+1})$

__________ $P(S_1, S_2, \ldots, S_T|O_1, O_2, \ldots, O_T) = \prod_{t=1}^{T} P(S_t|O_t)$

__________ $P(S_1, S_2, \ldots, S_T, O_1, O_2, \ldots, O_T) = \prod_{t=1}^{T} P(S_t, O_t)$

__________ $P(O_1, O_2, \ldots, O_T|S_1, S_2, \ldots, S_T) = \prod_{t=1}^{T} P(O_t|S_t)$

__________ $P(O_1, O_2, \ldots, O_T) = \prod_{t=1}^{T} P(O_t|O_1, \ldots, O_{t-1})$



7.4 Belief updating

(a) **Discrete HMMs**

(b) **Continuous dynamical systems**

7.5 V-chain

(a) **Base case**

$$P(Y_1 = j, o_1) \quad =$$

(b) **Forward algorithm**

$$\alpha_{j,t+1} \quad =$$

(c) **Likelihood**

$$P(o_1, o_2, \ldots, o_T) \quad =$$

(d) **Complexity**