Laptime Simulator Report
University of Manitoba
ePBR20

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Contents

1 Introduction

1.1 Motivation

The team wanted to be able to answer questions regarding the high level architecture of the vehicle. This includes the required pack capacity, ideal motor configuration, how much of a difference regen and an aero package makes, and where to focus efforts in order to maximize points. The goals of these laptime simulations were the following:

- Compare different motors and configurations
- Determine optimal gearing ratios
- Estimate the required accumulator capacity
- Estimate the effect of an aero package
- Estimate the benefit of using regenerative braking
- Perform a points analysis
- Determine the sensitivity of lap times to various vehicle parameters

Laptime simulators such as Optimum Lap already exist. However, there are many limitations of using these. A team developed laptime simulator offers the following advantages:

- All assumptions made are explicit and not hidden
- The simulator can be designed to easily support automation of runs and points analysis
- The simulator can be tailored to the requirements of an electric FSAE team

2 Vehicle Model

2.1 Mechanical Model

The basics of a vehicle model for a lap simulator have already been well explained and documented by other FSAE teams. As a result, they will not be covered in this document. These documents are available below:

- Wisconsin Racing
- Virginia Tech

2.1.1 Weight Distribution

The equation used for calculating the normal force on each tyre can be found in eq. (1).

$$F_z = (mg + F_{downforce}) \cdot Ratio_{FR} + F_{tq,long} + F_{tq,lat} \tag{1}$$

Where:

- F_z is the normal force on each tyre
- m is the mass of the car
- g is the acceleration due to gravity
- $F_{downforce}$ is the aerodynamic downforce
- $Ratio_{FR}$ is the ratio of static weight on the front and rear tyres, due to the location on the center of mass
- $F_{tq,long}$ is the force on the tyres due to the torque applied from longitudinal acceleration and drag (positive on the rear tyres and negative on the front tyres when accelerating)
- $F_{tq,lat}$ is the force on the tyres due to the torque applied from lateral acceleration (positive on the left tyres and negative on the right tyres when turning right)

The simulation will throw an error if any of the normal forces on the tyres are negative.

2.1.2 Available Friction Force

The maximum available longitudinal frictional force on each tyre is calculated based on the friction ellipse. This requires the normal force and the lateral acceleration. The contribution of frictional force of each tyre to the lateral acceleration is assumed to be proportional to the normal force on each tyre, and this is found in eq. (2). The remaining available force can be calculated using eq. (3). If lateral force on a tyre ever exceeds the total available force, the simulation will throw an error.

$$F_y = \frac{F_z \cdot F_{y,total}}{\sum F_z} \tag{2}$$

$$F_{x,max} = F_z \cdot \mu_{long} \cdot \sqrt{1 - \frac{F_y^2}{(F_z \cdot \mu_{lat})^2}} \tag{3}$$

Where:

- $F_{y,total} = m \cdot a_y$, which is the required lateral force during a turn to keep the vehicle from slipping, and a_y is the lateral acceleration
- F_y is the applied lateral frictional force by each tyre
- μ_{long} is the longitudinal coefficient of friction of the tyres
- μ_{lat} is the lateral coefficient of friction of the tyres
- $F_{x,max}$ is the maximum available longitudinal force on each tyre

2.1.3 Applied Torque

From the available frictional force, the required torque for a rear wheel drive vehicle is estimated using eq. (4). This equation is dependent on the acceleration over the time period. While already accelerating, this won't significantly change, however when accelerating from a dead stop, the motor and wheel inertia would be ignored. Therefore, the motor torque is calculated using a lower bound and upper bound of acceleration, and the two results are averaged. The applied motor torque is then the minimum of this averaged value, and the maximum available torque at the operating RPM.

$$\tau_{motor} = \zeta_{gr}\alpha_{wheel}J_{motor} + \frac{\alpha_{wheel} \cdot 4J_{tyre}}{\zeta_{gr}} + \frac{F_{x,total}r_{tyre}}{\zeta_{gr}} \tag{4}$$

Where:

- τ_{motor} is the required motor torque
- ζ_{gr} is the gear ratio between the motor and the wheels
- $\alpha_{wheel} = \frac{a_{x,expected}}{r_{tyre}}$ is the angular acceleration of the wheel, and $a_{x,expected}$ is the expected longitudinal acceleration during the time period
- r_{tyre} is the radius of the tyres
- J_{motor} is the rotational inertia of the motor
- J_{tyre} is the rotational inertia of a wheel
- $F_{x,total}$ is the total available frictional force for the motor

Once the ideal motor torque has been calculated, it can be applied (with the rotational inertia accounted for) using eq. (5).

$$k_{ax} = \frac{\zeta_{gr}J_{motor}}{r_{tyre}} + \frac{4 \cdot J_{tyre}}{\zeta_{gr}r_{tyre}} + \frac{m \cdot r_{tyre}}{\zeta_{gr}}$$

$$\tau_{losses} = (F_{drag} + F_{rr}) \cdot \frac{r_{tyre}}{\zeta_{gr}}$$

$$a_{x,motorlimit} = \frac{\tau_{motor} - \tau_{losses}}{k_{ax}} \tag{5}$$

Where:

- F_{drag} is the force applied to the car due to aerodynamic drag
- F_{rr} is the rolling resistance of the tyres
- $a_{x,motorlimit}$ is the acceleration possible due to the motor torque

2.2 Electrical Model

In order to design an accumulator, two major factors are required which can be obtained from the lap sim: the required pack capacity, and the losses. The lap sim was designed to easily obtain these parameters.

2.2.1 Motor Profile

The motor profile is a series of RPM-Torque pairs. These were taken from the motor datasheet if available, or the torque was assumed to be constant across the operating range.

2.2.2 Motor Losses

Motor losses are assumed to have two components: copper resistance losses, and speed-based losses. The copper losses inside the motor are calculated using eq. (6), where the phase current is determined using fig. 1, found in the motor datasheet. The speed-based losses were determined using fig. 2, also found in the motor datasheet. The total motor losses can than be calculated using eq. (7).

$$P_{loss,res} = 3 \cdot I_{phase,rms}^2 \cdot R_s \tag{6}$$

$$P_{loss,total} = P_{loss,res} + P_{loss,speed} \tag{7}$$

Where:

- $P_{loss,res}$ is the power loss due to the resistance of the copper
- $I_{phase,rms}$ is the RMS current per phase of the motor, found using fig. 1 as a lookup table
- R_s is the stator resistance of the motor
- $P_{loss,speed}$ is the speed-based losses of the motor, found using fig. 2 as a lookup table
- $P_{loss,total}$ is the total losses in the motor

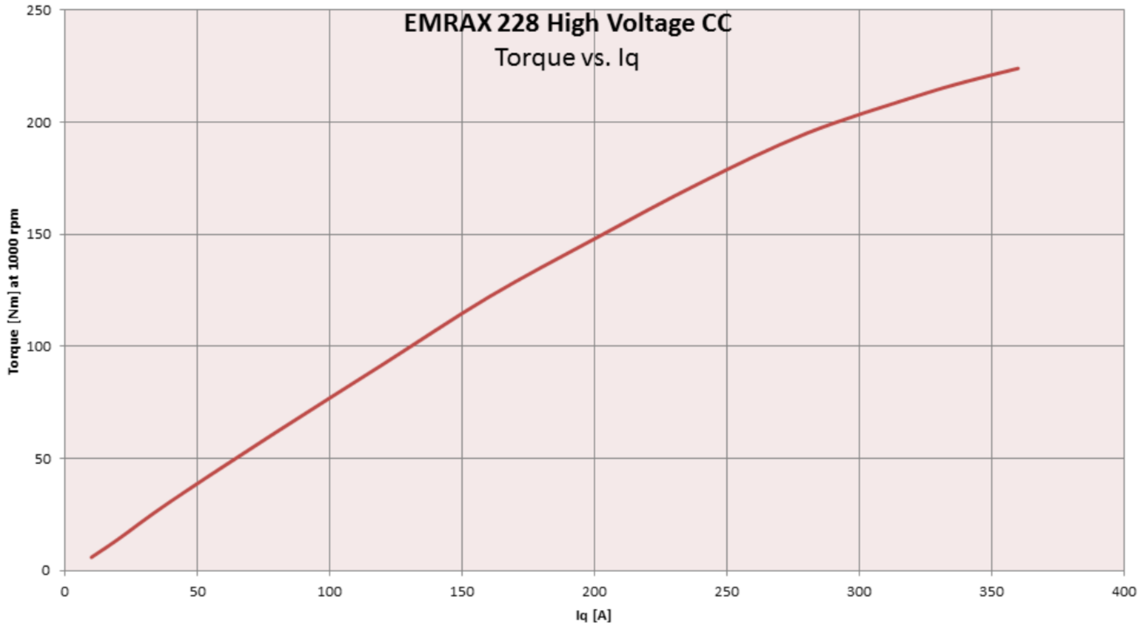

Figure 1: Emrax 228 torque-current curve

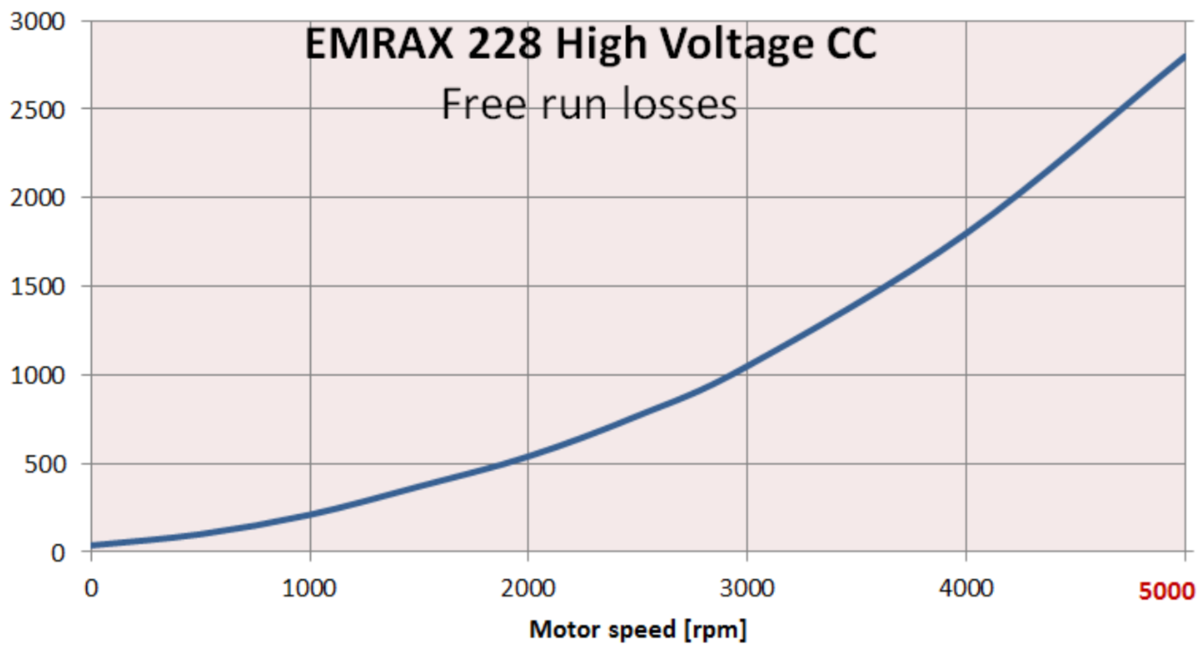

Figure 2: Emrax 228 free run losses

The motor datasheet does provide an efficiency map, found in fig. 3. However, low speed and low torque operating regions are represented on the map. Additionally, there is a difference in efficiency of up to 4% between points in the same region. The efficiency map generated from the model described by eq. (7) is found in fig. 4. It can be seen that the model is not accurate for large torques, however, it is a reasonable approximation for when operating at or below the continuous torque limit. During endurance, this is where the motor would be operating.

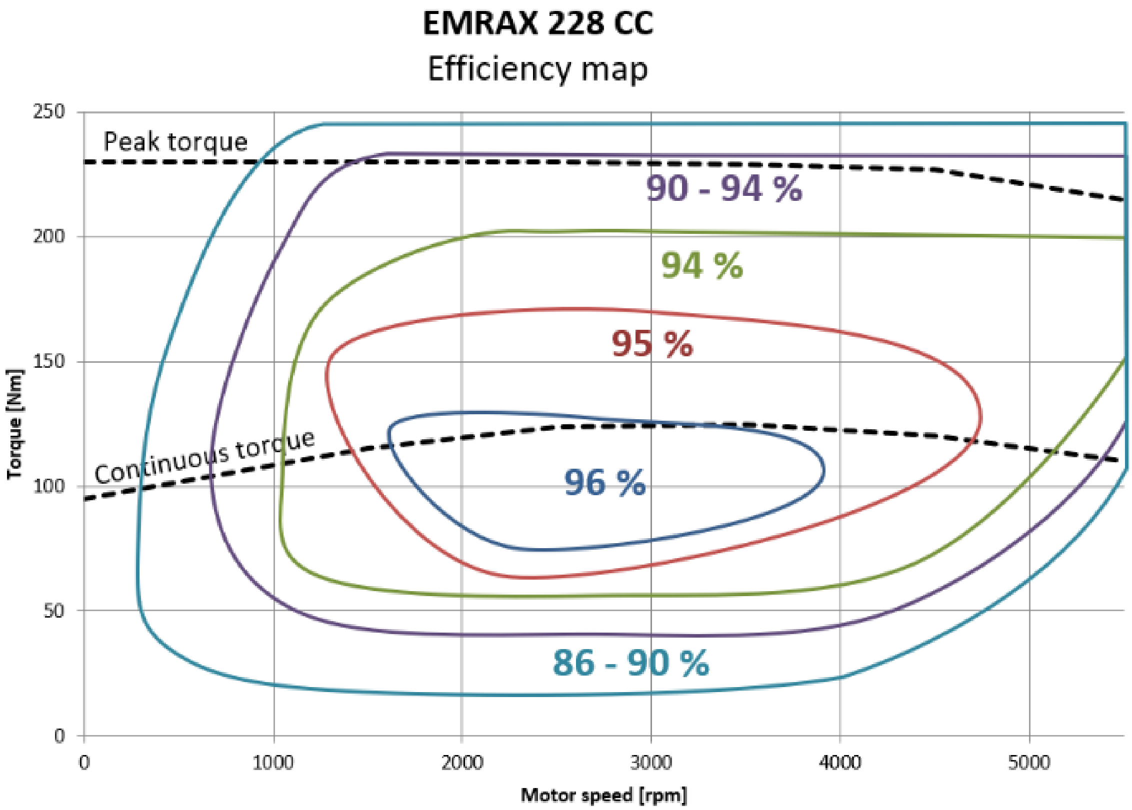

Figure 3: Emrax 228 datasheet efficiency map

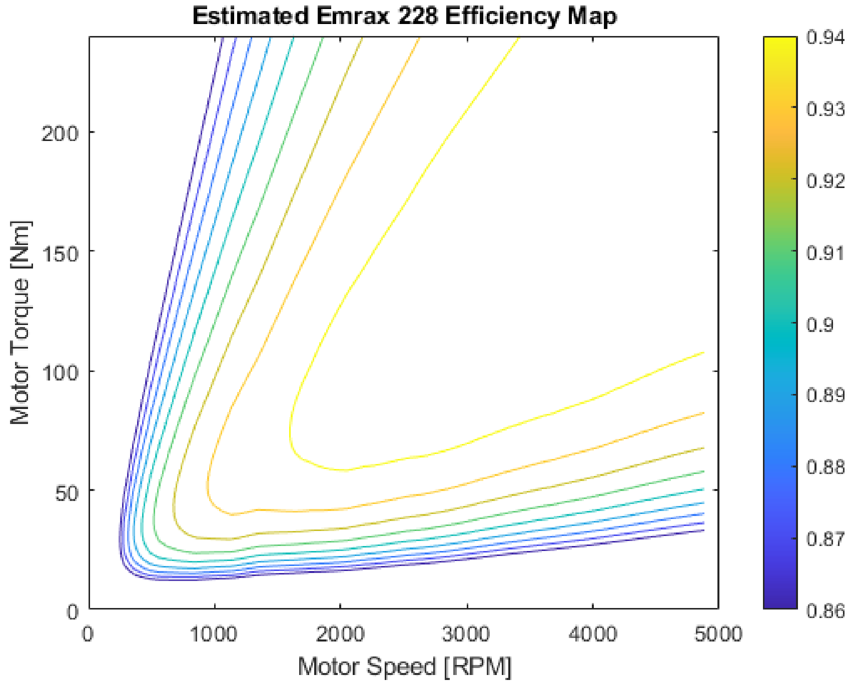

Figure 4: Emrax 228 calculated efficiency map

2.2.3 Accumulator Losses

A constant internal resistance model was used to estimate the accumulator losses. The power leaving the accumulator can be calculated using eq. (8). The accumulator current can then be calculated by solving eq. (8) for I_{acc} giving us eq. (9). The accumulator losses can then be calculated using eq. (10).

$$P_{acc,out} = P_{motor,in} \cdot \eta_{mc} = V_{acc,out} I_{acc} = (V_{acc} - I_{acc} R_{acc}) I_{acc} \tag{8}$$

$$I_{acc} = \frac{1}{2}(\frac{V_{acc}}{R_{acc}} - \sqrt{(\frac{V_{acc}}{R_{acc}})^2 - 4\frac{P_{motor,in} \cdot \eta_{mc}}{R_{acc}}}) \tag{9}$$

$$P_{loss,acc} = I_{acc}^2 R_{acc} \tag{10}$$

Where:

- $P_{acc,out}$ is the power leaving the accumulator
- $P_{motor,in}$ is the input power to the motor
- η_{mc} is the efficiency of the motor controller
- $V_{acc,out}$ is the output voltage of the accumulator
- I_{acc} is the current leaving the accumulator

- V_{acc} is the total open circuit voltage of the accumulator
- R_{acc} is the total internal resistance of the accumulator
- $P_{loss,acc}$ is the power lost in the accumulator

3 Lap Simulator Design

3.1 General Simulation

The simulation was developed in Python. This is due to Python being free and easy for the team to use, and having additional data structures compared to MATLAB, which made logging parameters and writing clean code easier. A forward/backward Euler method is used to solve the set of equations numerically.

3.1.1 Acceleration

At all points in time when accelerating, the vehicle is assumed to be accelerating at the greatest possible rate. This is based on the limits of tire friction, and the torque limit of the motor at the operating velocity. If the vehicle reaches the maximum speed when accelerating (either due to motor speed limit or maximum turning speed) the vehicle will stop accelerating and maintain speed.

3.1.2 Braking

When braking, it is assumed that the vehicle is braking perfectly, with the braking force on each tyre equal to the maximum available frictional force.

3.1.3 Track Input

The track is divided into a series of straight and curved segments. There are four parameters associated with each segment: length, turn radius, turn direction, and weight of segment time to overall time. It has been set up so that accelerometer data can be converted into an Optimum Lap track (using their online tool), and can then be copied over into a CSV file.

3.1.4 Maximum Turning Speed

The simulator determines the maximum turning speed of an upcoming curve using a bisection method. A lateral acceleration is applied to the vehicle, and then is checked to see that the normal force on each tyre is non-negative. The fastest speed at which all tyres are still on the ground and the friction limit is not exceeded, is the maximum speed for the turn.

3.1.5 Cooling & Torque Limits

The limiting effect of high motor, motor controller, and accumulator temperatures on the output power was ignored. During acceleration, skidpad, and autocross, the vehicle is assumed to not be thermally limited, and operating with peak motor torque. During the longer endurance event, the vehicle is assumed to be operating at the continuous torque limit of the motor.

3.2 Segment Protocol

During any segment, the simulator determines if the vehicle will need to brake by the end of the segment. This is only the case for a turn followed by wider turn or straight, and the final segment. Otherwise, the vehicle must brake by the end of the segment. The fastest way through any given segment is to perfectly accelerate, and then start perfectly braking just in time to slow down for the next turn. In order to determine the optimal path, two simulations are run. The first is moving forward in the x-direction and time, starting at the beginning of the segment with the vehicle accelerating. The second is moving backward in the x-direction and time, starting at the end of the segment, with the vehicle braking. For the braking curve, the vehicle starts with a velocity equal to the maximum velocity of the next segment. Outlined in table 1 is the different cases for curve interception and how they are handled. An example of what these curves look like for a straight followed by a curve can be found in fig. 5, and an example turn followed by a turn can be found in fig. 6.

Case	Handling
Braking and acceleration curves intercept	The actual vehicle curve is equal to the acceleration curve before the intercept, and the braking curve afterword.
Curves do not intercept, but the acceleration curve is always below the braking curve	The vehicle has reached the end of the segment, but has not reached the maximum end of segment speed. Only the acceleration curve is used.
Curves do not intercept, but the braking curve is always below the acceleration curve	The vehicle has reached the end of the segment, but has started with too high a speed in order to slow down for the next segment. The previous segment is re-run, with the initial braking velocity set to the velocity of the braking curve when it reached the start of the segment.

Table 1: Acceleration-braking curve intercept handling

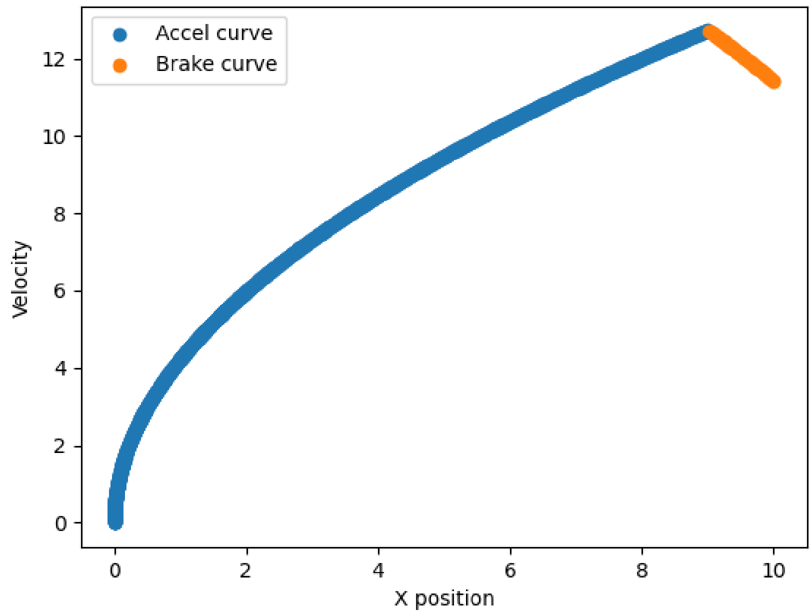

Figure 5: Sample intercept of acceleration and braking curves for a straight followed by a turn

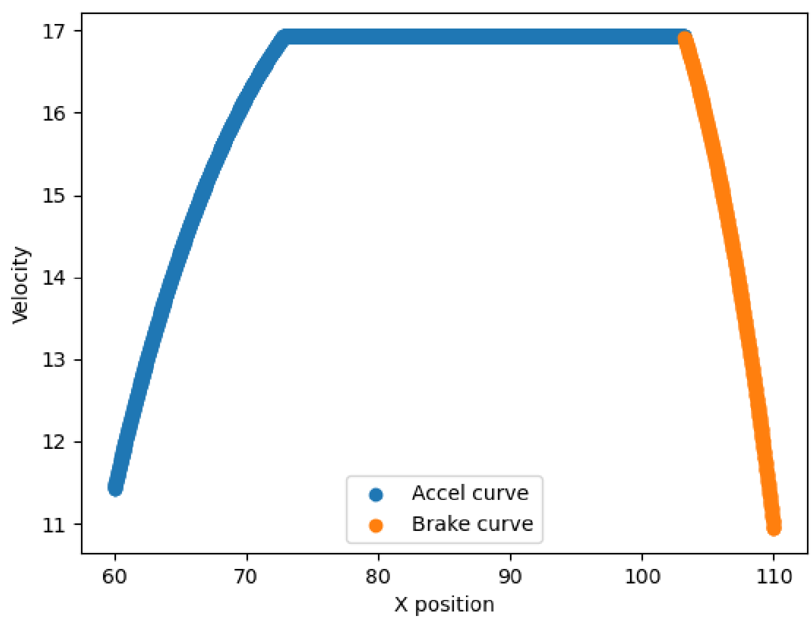

Figure 6: Sample intercept of acceleration and braking curves for a turn followed by a turn

4 Conclusion

4.1 Assumptions

As with any simulation, it is important to understand the assumptions made. These are listed below:

- Acceleration constant across timestep
- Velocity constant across timestep
- Aerodynamic forces always acting on center of mass
- Vehicle inertia acts on center of mass
- Drag assumed always be normal to car, even when turning
- Constant coefficients of tyre friction
- Braking effects of drag ignored when braking
- 100% transmission of torque to ground from motor
- All tyres contribute to lateral friction proportionally to normal force when turning
- Center of mass assumed to be at y = 0
- Ideal differential
- Vehicle pitch does not change
- Vehicle is always centered on the track
- All tyres assumed to take the same path through turns
- Thermal limits on output power ignored
- Operating at peak motor torque during accel, skidpad, and autocross
- Operating at continuous motor torque during endurance
- Accumulator always operates at nominal voltage

4.2 Future Work

The lap simulator would benefit from following items being implemented:

- Load dependent tyre models
- Account for differential
- Thermal models for the motor, controller, and accumulator, and limiting the motor torque based on these values
- Account for decreasing pack voltage limiting maximum motor speed