Blind Signal Separation (BSS) Problem solved with NMF.

In this exercise, you are given a file named **SoundSourceData.mat**, contained into the folder **data**. You are asked to apply NMF to that problem.

We want to use NMF to solve the so-called Blind Signal Separation (BSS) Problem. In particular, we will consider the two dimensional case in which we have m sound sources inside a unit disc. The position of the sources are given by the two dimensional vectors

$$y^{(1)} = (y_1^{(1)}, y_2^{(1)}), y^{(2)} = (y_1^{(2)}, y_2^{(2)}), \dots, y^{(m)} = (y_1^{(m)}, y_2^{(m)})$$

The emitted sound is collected by a series of n microphones, located at certain coordinates

$$x^{(1)} = (x_1^{(1)}, x_2^{(1)}), x^{(2)} = (x_1^{(2)}, x_2^{(2)}), \dots, x^{(m)} = (x_1^{(m)}, x_2^{(m)})$$

positioned at the boundary of the unit disc.

Each microphone measures the sound it receives at p different amount of time, and the sound collected by the l-th microphone at time t_i is named $b_l(t_i)$.

Consider the data matrix X such that $X_{i,j} = b_i(t_j)$, of shape $n \times p$. We will suppose that the sound measured by each microphone is corrupted by noise.

Both the data matrix X of dimension $n \times p$ and the true sound source (in our example, there are $m = 4$ sound sources), are collected into the file **SoundSourceData.mat**.

1. Import the needed libraries (remember the function **scipy.io.loadmat** to load **.mat** files).

2. Use the functions of scipy.io to load the content of **SoundSourceData.mat**, and use it to identify the dataset X and the true sound source matrix F. Use those matrix to find the values for n and p in this case (remember that $n \times p$ is the shape of X).

3. To solve convergence problems, set equals to 0 the values contained in X which are strictly negative (those values exists because of the presence of noise).

4. Import the utility function NMF contained into the **folder ./utils/NMF.py** and use it to compute the Non-Negative Matrix Factorization for the matrix X. Try different values for the number of iterations. Remember that the NMF of a positive matrix $X \in R_+^{n \times p}$ is the matrix decomposition

$$X = WH \qquad where \;\; W \in R_+^{n \times m}, H \in R_+^{m \times p}$$

that minimizes the error measured in Frobenius norm

$$\|X - WH\|_F$$

among the set of positive matrices.

Below you will find the documentation on how to use the utility function NMF of NMF.py.

The NMF function takes as input:

X: the data matrix.

m: the number of source (known in advance).

T (optional): the number of iterations of the algorithm. The Default value is T=1000

tau (optional): the stopping criterion. The Default value is tau=1e-2.

return_error (optional): a boolean variable. if True, also returns the error vector. The Default value is False.

And returns:

(W, H): tuple containing the NMF decomposition of X, where the shape of W is n x m, while the shape of H is m x p.

err (only if return_error = True): an array of length T (the number of iterations) that contains the behavior of the error during the iterations.

5. Using a subplot, show a 2×4 table of plots, where in the first line the rows of F are shown (the true sound sources), while in the second one, you show the rows of H (the

approximated sound sources).

6. Comment the obtained results.

In [1]:

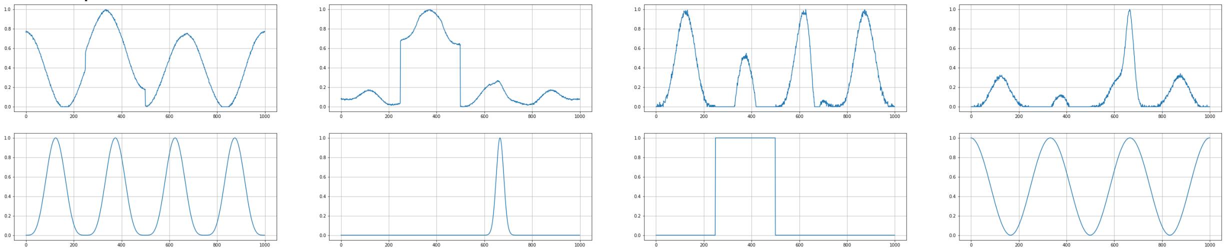
```
import scipy.io
import numpy as np
import matplotlib.pyplot as plt


data = scipy.io.loadmat('./data/SoundSourceData.mat')


# Split the acquired data into F and X.
F = data['F']
X = data['X']

# Variables
def _NMF(m,T):

    # sound sources

    np.random.seed(42)

    print("hypothesized sources: ",m, "real sources: ", 4, "Iterations: ",T)
    # microphones, times
    n, p = X.shape
    print("microphones: ",n, "times: ",p)
    X[X<0] = 0  # Set equal to zero, all negative elements of X

    from utils import NMF

    # NMF Algorithm
    (W, H), err = NMF.NMF(X, m, T,  return_error=True )

    # Visualize the Results (since you don't know how to use matplotlib, the
    # following part of the code is already complete).

    plt.figure(figsize=(50, 10))
    for i in range(m):
        plt.subplot(2, m, i+1)
        plt.plot(H[i, :])
        plt.grid()
    for i in range(4):
        plt.subplot(2, 4, 4+i+1)
        plt.plot(F[i, :])
        plt.grid()
    plt.show()
```

In [2]:

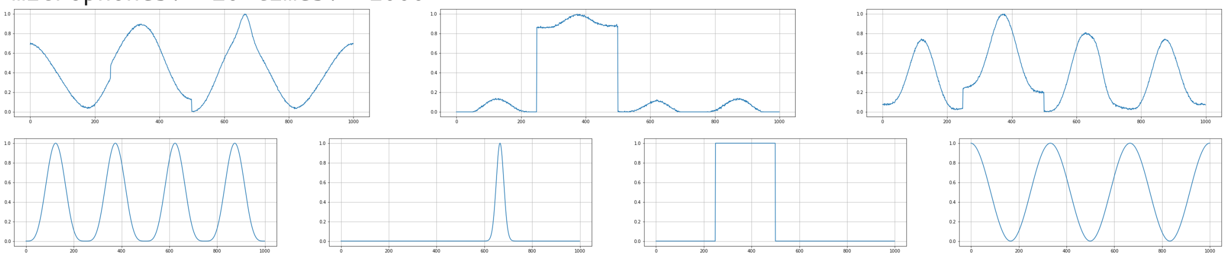
```
T1 = 2
T2 = 1000
T3 = 2000

x = [T1, T2, T3]

err4 = []

print("######     Hypotizing right number of sources (4=4)     ######")
err4.append(_NMF(4,T1))
print("######     Hypotizing less sources (3<4)     ######")
_NMF(3,T1)
```

```
print("######     Hypotizing more sources (5>4)     ######")
_NMF(5,T1)
```

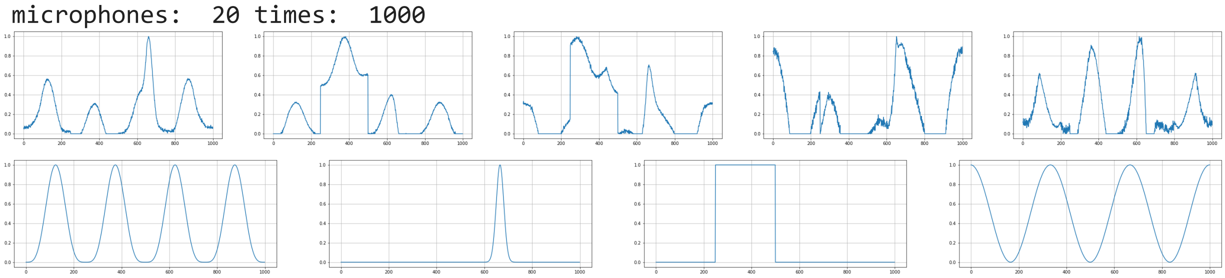
```
######     Hypotizing right number of sources (4=4)     ######
hypothesized sources:  4 real sources:  4 Iterations:  2
microphones:  20 times:  1000
```



```
######     Hypotizing less sources (3<4)     ######
hypothesized sources:  3 real sources:  4 Iterations:  2
microphones:  20 times:  1000
```



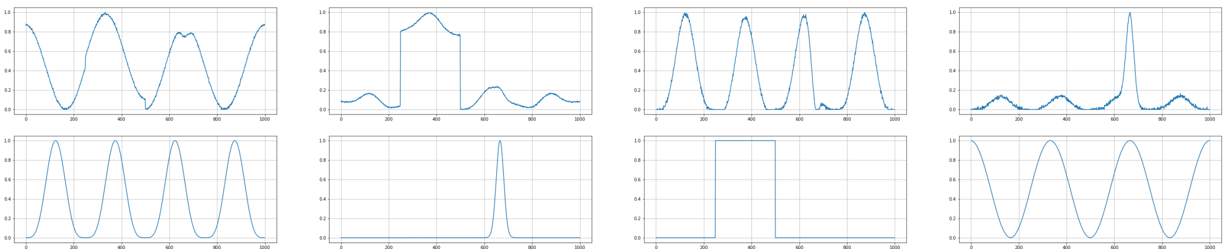
```
######     Hypotizing more sources (5>4)     ######
hypothesized sources:  5 real sources:  4 Iterations:  2
microphones:  20 times:  1000
```



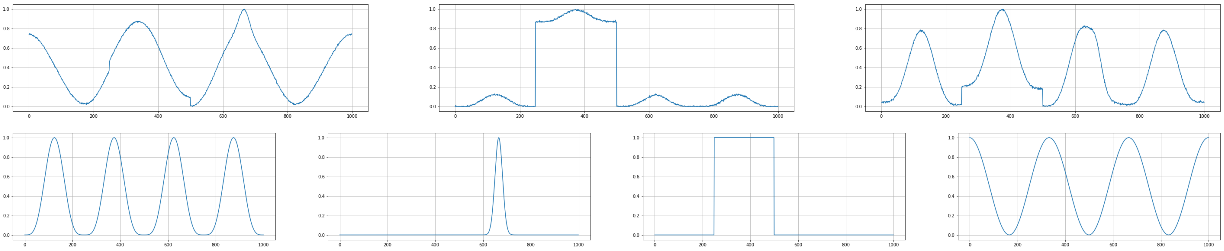
In [3]:

```
print("######     Hypotizing right number of sources (4=4)     ######")
err4.append(_NMF(4,T2))
print("######     Hypotizing less sources (3<4)     ######")
_NMF(3,T2)
print("######     Hypotizing more sources (5>4)     ######")
_NMF(5,T2)
```

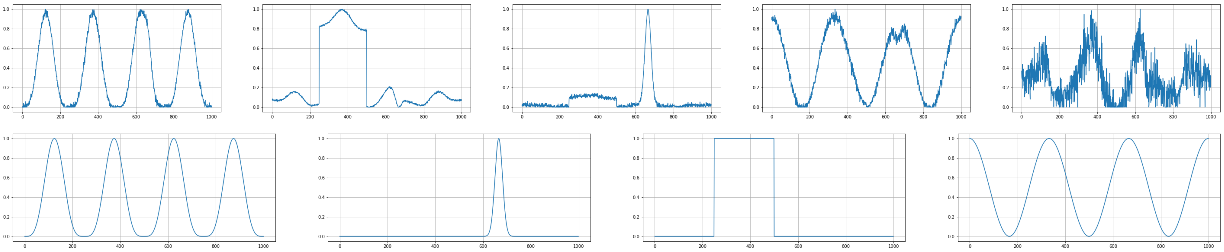
```
######     Hypotizing right number of sources (4=4)     ######
hypothesized sources:  4 real sources:  4 Iterations:  1000
microphones:  20 times:  1000
Correct!
```



```
######     Hypotizing less sources (3<4)     ######
hypothesized sources:  3 real sources:  4 Iterations:  1000
microphones:  20 times:  1000
Correct!
```



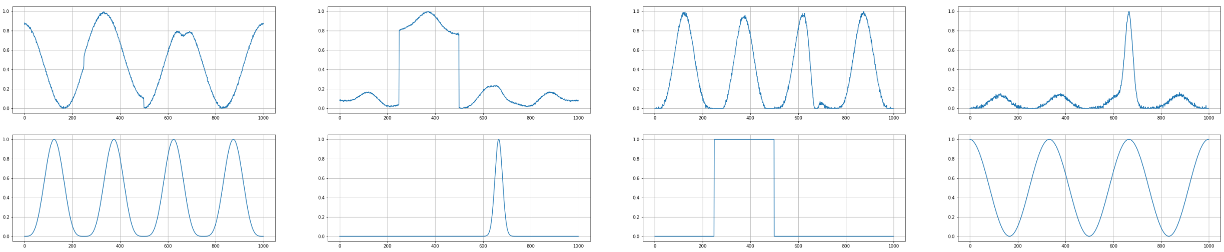
```
######     Hypotizing more sources (5>4)      ######
hypothesized sources:  5 real sources:  4 Iterations:  1000
microphones:  20 times:  1000
Correct!
```



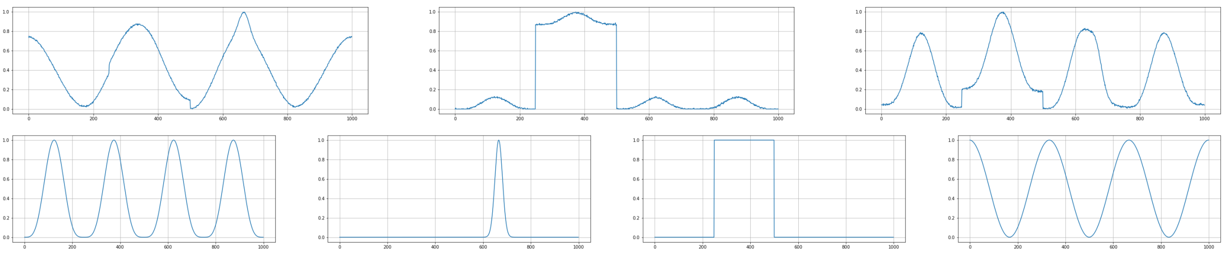
In [4]:

```python
print("######     Hypotizing right number of sources (4=4)      ######")
_NMF(4,T3)
print("######     Hypotizing less sources (3<4)      ######")
_NMF(3,T3)
print("######     Hypotizing more sources (5>4)      ######")
_NMF(5,T3)
```

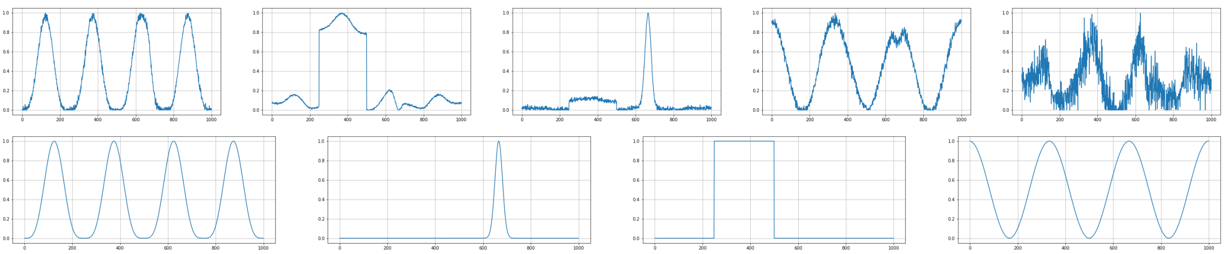
```
######     Hypotizing right number of sources (4=4)      ######
hypothesized sources:  4 real sources:  4 Iterations:  2000
microphones:  20 times:  1000
Correct!
```



```
######     Hypotizing less sources (3<4)      ######
hypothesized sources:  3 real sources:  4 Iterations:  2000
microphones:  20 times:  1000
Correct!
```



```
######     Hypotizing more sources (5>4)      ######
hypothesized sources:  5 real sources:  4 Iterations:  2000
microphones:  20 times:  1000
Correct!
```



We can observe that if we use the right number of sources, there's a 1-to-1 correspondency between the original and the recovered sounds, even with a low noise. If, instead, we try to recover less/more signals we get signals that are a mixture between the original tracks. If we try to recover more tracks than the original ones, then we can easily see an oscillating behavior. The number of iterations doesn't make a big difference in the results, unless it is really small (just 2-3 iteration).

With respect to the *SVD*, the *NMF*

```
 1. works just with matrixes of positive numbers
 2. has just positive numbers in W and H (here we used just H)
 2. is an iterative algorithm
 3. SVD is a more ‘insightful’ factorization technique - NMF gives
only U and V matrices, but SVD gives a Sigma matrix also along with
these two.Sigma gives us insights into the amount of information each
eigen vector holds. Such info is not available in NMF.
```

In [ ]: