

3.4.2 Show that the forward absolute error  $x(\varepsilon) - x$  is given by

$$x(\varepsilon) - x = \varepsilon A^{-1}(e - Ex(\varepsilon))$$

From Golub and van Loan,  $(A + \varepsilon E)x(\varepsilon) = b + \varepsilon e$   
 for  $A, E \in \mathbb{R}^{n \times n}$  and  $x, b, e \in \mathbb{R}^n$ .

Starting with  $Ax = b$ , left-multiply  $A^{-1}$  for  $x = \underbrace{A^{-1}b}_{x(0)}$ . Also

$$(A + \varepsilon E)x(\varepsilon) = b + \varepsilon e \Rightarrow (I + A^{-1}\varepsilon E)x(\varepsilon) = \underbrace{A^{-1}(b + \varepsilon e)}$$

$$x(\varepsilon) + A^{-1}\varepsilon Ex(\varepsilon) = \underbrace{A^{-1}b}_{x(0)} + A^{-1}\varepsilon e \Rightarrow$$

$$x(\varepsilon) - \vec{x} = A^{-1}\varepsilon e - \underbrace{A^{-1}\varepsilon Ex(\varepsilon)}$$

$$x(\varepsilon) - \vec{x} = A^{-1}\varepsilon(e - Ex(\varepsilon))$$

$$\Rightarrow x(\varepsilon) - \vec{x} = \varepsilon A^{-1}(e - Ex(\varepsilon)) \text{ since } \varepsilon \in \mathbb{R}.$$