3.3.7

If matrices A and B are positive definite, the $A+B$ is positive definite.

Proof: For 2 $n \times n$ positive definite matrices, we know the following:

- $A = A^T$ and $B = B^T$
- for diagonal elements a_{ii} in A and b_{ii} in B, $a_{ii} > 0$ and $b_{ii} > 0$

From the first trait, we find $(A+B) = C \Longleftrightarrow (A^T + B^T) = C$ by substitution for some matrix C.
Thus by properties of transposes, where $(A+B)^T = A^T + B^T$, $(A+B)^T = C$. Thus $(A+B)^T = (A+B)$, so $A+B$ is symmetric.

From the second, note that for a sum of matrices,

$$(A+B) = \begin{bmatrix} a_{11}+b_{11} & a_{12}+b_{12} & \cdots & a_{1n}+b_{1n} \\ \vdots & \vdots & & \vdots \\ a_{n1}+b_{n1} & a_{n2}+b_{n2} & \cdots & a_{nn}+b_{nn} \end{bmatrix}.$$

Since diagonal elements $a_{ii} > 0$ and $b_{ii} > 0$,

$$a_{ii} + b_{ii} > 0$$

Thus diagonal elements of $A+B$ are always positive and the symmetric matrix $(A+B)$ is also positive definite.