3.3.8 : Prove for positive definite matrix A, A^{-1} is positive definite.

since A is symmetric,

$$I = I^T \Rightarrow (AA^{-1}) = (A^{-1}A)^T \Rightarrow A(A)^{-1} = A^T(A^{-1})^T$$

$$\Rightarrow A(A^{-1}) = A(A^{-1})^T \Rightarrow A^{-1}A(A^{-1}) = A^{-1}A(A^{-1})^T$$

$$\Rightarrow A^{-1} = (A^{-1})^T$$ so A^{-1} is symmetric

If A is positive definite, $\lambda > 0$ forall eigenvalues λ of A. Since all eigenvalues of A^{-1} are of the form $\frac{1}{\lambda}$, all eigenvalues of A^{-1} are also positive.

Thus A^{-1} is positive definite.