3.4.7 Let $A \in \mathbb{R}^{2\times 2}$

Show correlation between $\det(A)$ and $\kappa_F(A)$ where $\kappa_F(A) = \|A\|_F \|A^{-1}\|_F$.

First, let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ so that $A^{-1} = \frac{1}{\det(A)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

Then $\|A\|_F = \sqrt{a^2+b^2+c^2+d^2}$ and

$$\|A^{-1}\|_F = \frac{1}{\det(A)}\left(\sqrt{(d^2)+(-b)^2+(-c)^2+(a)^2}\right).$$

So $\kappa_F(A) = \dfrac{a^2+b^2+c^2+d^2}{\det(A)}$.

Thus $\det(A)$ and $\kappa(A)$ are inversely proportional for $A \in \mathbb{R}^{2\times 2}$.