Title: Design of Chebyshev IIR filter

AIM: To design chebyshev IIR filters.

Objective: To design chebyshev IIR filters using MATlab.

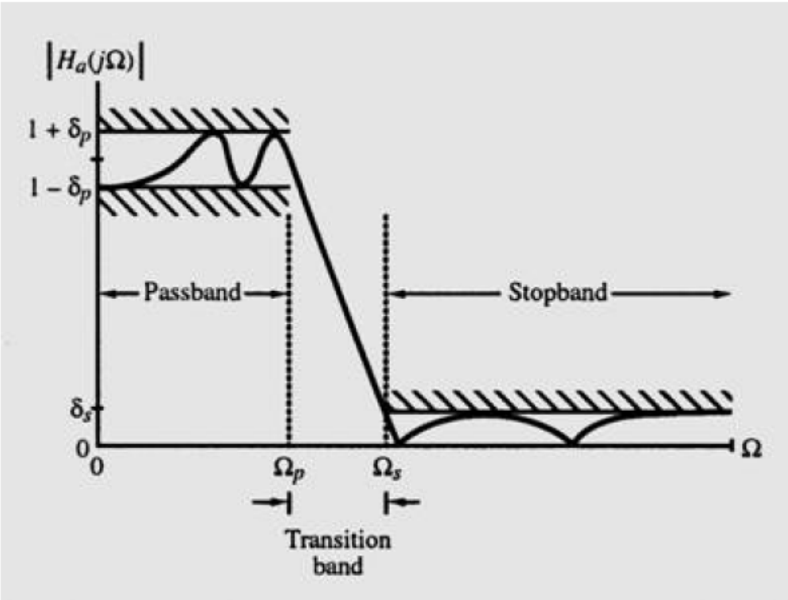
1.Write Matlab code to design a Chebyshev IIR highpass filter with the following specification:

A 0.5 dB ripple in passband at the frequency of 3,000Hz.
B 25 dB attenuation at the frequency of 1,000 Hz and
C Sampling frequency of 8,000 Hz.

Description
Low pass filter

The magnitude response can be expressed as

$$Magnitude = \begin{cases} 1-\delta_p \le |H(j\Omega)| \le 1 & for\ 0 \le \Omega \le \Omega_p \\ 0 \le |H(j\Omega)| \le \delta_s & for\ |\Omega| \ge \Omega_s \end{cases}$$



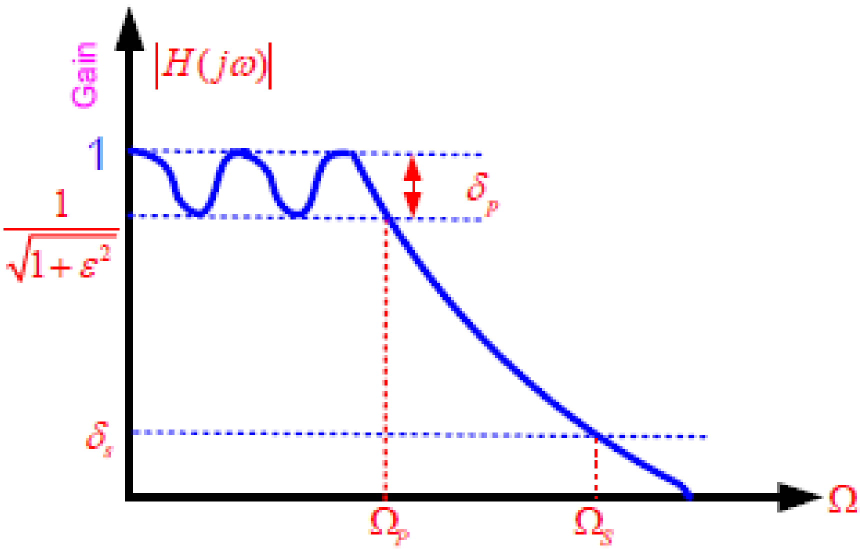
Chebyshev low pass filter

The magnitude frequency response of Chebyshev filter is

$$|H(j\Omega)|^2 = \frac{1}{\left[1+\epsilon^2 T_n^2\left(\frac{\Omega}{\Omega_p}\right)\right]}$$

Properties of Chebyshev filter

- If $\Omega)_p = 1$ rad/sec then it is called as type-I normalized Chebyshev lowpass filter.
- $H_N(j\Omega)|^2|_{\Omega=0} = 1$ for all N
- $|H(j0)| = 1$ for odd N and $|H(j0)| = \frac{1}{\sqrt{1+\epsilon^2}}$ for even N
- The filter has uniform ripples in the passband and is monotonic outside the passband.
- The sum of the number of maxima and minima in the passband equals the order of the filter.



Design steps for Chebyshev filter:

From the given specifications

1. Determine the order of the Filter
2. Determine the Normalized Chebyshev lowpass filter transfer function
3. From analog lowpass to lowpass frequency transformation, find the desired transfer function by substituting the following

$$H_a(s) = H_N(s)|_{s \rightarrow \frac{s}{\Omega_C}}$$

where $\Omega_C = \Omega_P$

Order of the Filter

K_p Gain or Magnitude at passband in normal value(without dB) for frequency Ω_p
K_s Gain or Magnitude at passband in normal value(without dB) for frequency Ω_s

$$N_1 = \frac{cosh^{-1}\left[\left[\frac{(1/K_s^2)-1}{(1/K_p^2)-1}\right]^{\frac{1}{2}}\right]}{cosh^{-1}\left(\frac{\Omega_s}{\Omega_p}\right)}$$

K_p Gain or Magnitude at passband in dB for frequency Ω_p
K_s Gain or Magnitude at passband in dB for frequency Ω_s

$$N_1 = \frac{cosh^{-1}\left[\left[\frac{10^{0.1K_s}-1}{10^{0.1K_p}-1}\right]^{\frac{1}{2}}\right]}{cosh^{-1}\left(\frac{\Omega_s}{\Omega_p}\right)}$$

Chose the order of the filter $N > N_1$

Normalized Chebyshev lowpass filter transfer function

When N is Even

$$H(s_n) = \prod_{k=1}^{\frac{N}{2}} \frac{B_k}{s^2 + b_k s + c_k}$$

When N is odd

$$H(s_n) = \frac{B_0}{s + c_0} \prod_{k=1}^{\frac{N-1}{2}} \frac{B_k}{s^2 + b_k s + c_k}$$

- where $b_k = 2y_N sin\left[\frac{(2k-1)\pi}{2N}\right]$, $c_k = y_N^2 + cos^2\left[\frac{(2k-1)\pi}{2N}\right]$
- $c_0 = y_N$

$$y_N = \frac{1}{2}\left[\left[\left(\frac{1}{\epsilon^2}+1\right)^{\frac{1}{2}} + \frac{1}{\epsilon}\right]^{\frac{1}{N}} - \left[\left(\frac{1}{\epsilon^2}+1\right)^{\frac{1}{2}} + \frac{1}{\epsilon}\right]^{-\frac{1}{N}}\right]$$

where $\epsilon = \left[(1/K_p^2) - 1\right]^{\frac{1}{2}}$

When N is Even the values of parameter B_k are evaluated using

$$H(s_n)|_{s=0} = \frac{1}{(1+\epsilon^2)^{\frac{1}{2}}}$$

When N is odd the values of parameter B_k are evaluated using

$$H(s_n)|_{s=0} = 1$$

Design steps for highpass filter:
From the given specifications

1. Determine stopband frequency of the normalized lowpass filter by $\Omega_s = \frac{\Omega_p}{\Omega_s'}$
2. Determine the order of the Filter using

$$N = \frac{log\left[\frac{10^{\frac{-K_p}{10}} - 1}{10^{\frac{-K_S}{10}} - 1}\right]}{2log\left[\frac{\Omega_p}{\Omega_S}\right]}$$

3. Determine the cutoff frequency Ω_C using

$$\Omega_C = \frac{\Omega_s}{\left(10^{\frac{-K_s}{10}} - 1\right)^{\frac{1}{2N}}} \quad OR \quad \Omega_C = \frac{\Omega_p}{\left(10^{\frac{-K_p}{10}} - 1\right)^{\frac{1}{2N}}}$$

4. Determine the transfer function of normalized Butterworth filter by

$$H_N(s) = \frac{1}{\prod_{LHP}(s - s_k)} = \frac{1}{B_N(s)}$$

5. From analog lowpass to high frequency transformation, find the desired transfer function by substituting the following

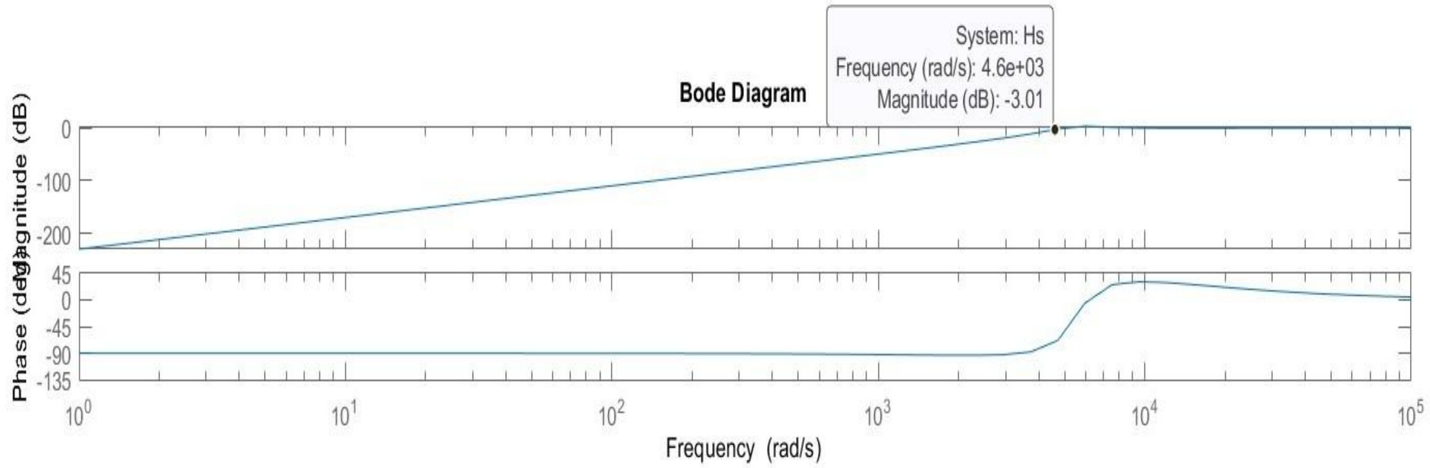
$$H_a(s) = H_N(s)\big|_{s \to \frac{\Omega_p}{\Omega_C S}}$$

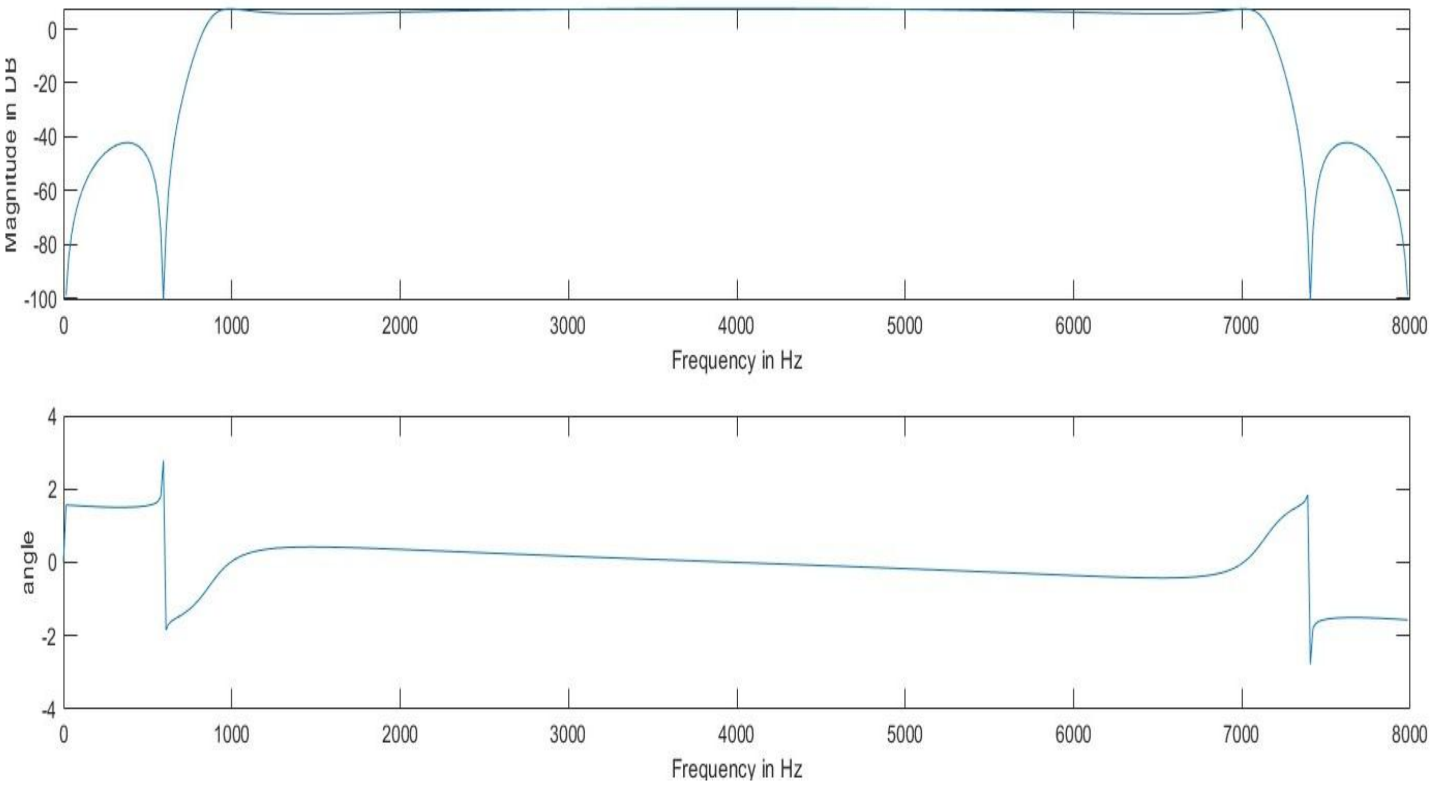
Matlab Program

```
clear;clc ;
KP = 0.5;
KS = 25;
fp = 1000;
fs = 3000;
fsam =8000;
radp = 2*pi*fp;
rads = 2*pi*fs;
% To find the Order of Chebyshev Filter
ks = power(10,-KS/20);
kp = power(10,-KP/20);
Nks = power((1/(ks*ks) - 1),0.5);
Nkp = power((1/(kp*kp) - 1),0.5);
Nrad = rads/radp;
Nnum = acosh(Nks/Nkp);
Ndec = acosh(rads/radp);
N = Nnum/Ndec;
N = ceil(N);
% To find cutoff frequency in rad/sec Chebyshev Filter
radc =  radp;
% To find Transfer function Chebyshev Filter
s = tf("s");
Ynum = power((1/(Nkp*Nkp)+1),0.5)+(1/Nkp);
yn = 0.5*( power(Ynum,1/N)-power(Ynum,-1/N));
Hs = 1;
```

```
if ((N/2) == floor(N/2))
  G0 = 1/power(1+Nkp*Nkp,0.5);
   for k=1:N/2
       bk = 2*yn*sin((2*k-1)*180/N);
       ck = yn*yn+cos((2*k-1)*180/N)*cos((2*k-1)*180/N);
       Hs = Hs * 1/(s*s+bk*s+ck);
   end
    Gain =  G0/evalfr(Hs,0);
    Hs = Gain*Hs;
else
   Hs = 1/(s+yn);
   for k=1:(N-1)/2
       bk = 2*yn*sin((2*k-1)*180/N);
       ck = yn*yn+cos((2*k-1)*180/N)*cos((2*k-1)*180/N);
       Hs = Hs * 1/(s*s+bk*s+ck);
   end
   Gain =  1/evalfr(Hs,0);
   Hs = Gain*Hs;
end
% To transform the transfer function from
% Normalised LPF to un-normalize cutoff HPF
[num,den] = tfdata(Hs);
[num,den] = lp2hp(num{1},den{1},radc);
Hs =  tf(num,den);
subplot(3,1,1);
% To plot the frequency and phase of Transfer function using bode plot
w= logspace(0,5);
bode(Hs,w);
% Sampling of the HPF to Z domain
Hz = c2d(Hs,1/fsam);
[H,W]= freqz(Hz.Numerator{1},Hz.Denominator{1},512,"whole",fsam);
subplot(3,1,2);
plot(W,20*log(abs(H)));
xlabel("Frequency in Hz");
ylabel("Magnitude in DB");
subplot(3,1,3);
plot(W,angle(H));
xlabel("Frequency in Hz");
ylabel("angle");
```

Output of the program





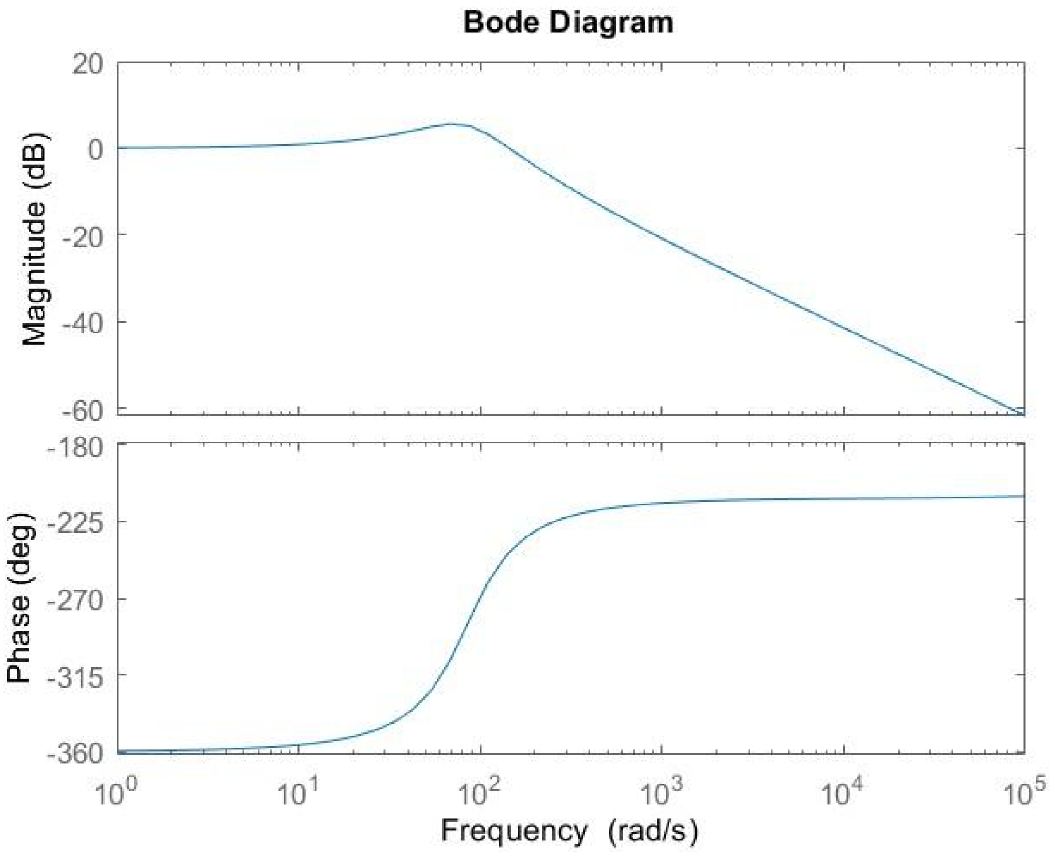
2.Write Matlab code to design a Chebyshev IIR analog low pass filter that has -5.0 dB frequency 150 rad/sec and stopband attenuation 20 dB or greater for all radian frequencies past 300 rad/sec

Matlab Program

```
clear;clc ;
KP = -0.5;
KS = 25;
radp = 150;
rads = 300;
% To find the Order of Chebyshev Filter
ks = power(10,-KS/20);
kp = power(10,-KP/20);
```

```
Nks = power((1/(ks*ks) - 1),0.5);
Nkp = power((1/(kp*kp) - 1),0.5);
Nrad = rads/radp;
Nnum = acosh(Nks/Nkp);
Ndec = acosh(rads/radp);
N = Nnum/Ndec;
N = ceil(N);
% To find cutoff frequency in rad/sec Chebyshev Filter
radc =  radp;
% To find Transfer function Chebyshev Filter
s = tf("s");
Ynum = power((1/(Nkp*Nkp)+1),0.5)+(1/Nkp);
yn = 0.5*( power(Ynum,1/N)-power(Ynum,-1/N));
Hs = 1;
if ((N/2) == floor(N/2))
  G0 = 1/power(1+Nkp*Nkp,0.5);
   for k=1:N/2
       bk = 2*yn*sin((2*k-1)*180/N);
       ck = yn*yn+cos((2*k-1)*180/N)*cos((2*k-1)*180/N);
       Hs = Hs * 1/(s*s+bk*s+ck);
   end
    Gain =  G0/evalfr(Hs,0);
    Hs = Gain*Hs;
else
   Hs = 1/(s+yn);
   for k=1:(N-1)/2
       bk = 2*yn*sin((2*k-1)*180/N);
       ck = yn*yn+cos((2*k-1)*180/N)*cos((2*k-1)*180/N);
       Hs = Hs * 1/(s*s+bk*s+ck);
   end
   Gain =  1/evalfr(Hs,0);
   Hs = Gain*Hs;
end
% To transform the transfer function from
% Normalised LPF to normalise cutoff LPF
[num,den] = tfdata(Hs);
[num,den] = lp2lp(num{1},den{1},radc);
Hs =  tf(num,den);
% To plot the frequency and phase of Transfer function using bode plot
w= logspace(0,5);
bode(Hs,w);
```

Output of the program



Result : The Chebyshev High pass and low pass filters are designed for given filter specification.