Title: Design of analog Butterworth low pass filter

AIM: To design analog Butterworth low pass filter.

Objective: To design analog Butterworth low pass filter using MATlab.

Write Matlab code to design an analog Butterworth low pass filter to meet the following specifications with T=1 second

$$0.9 \le |H(j\Omega)| \le 1 \; for \; 0 \le \Omega \le 0.2\pi$$

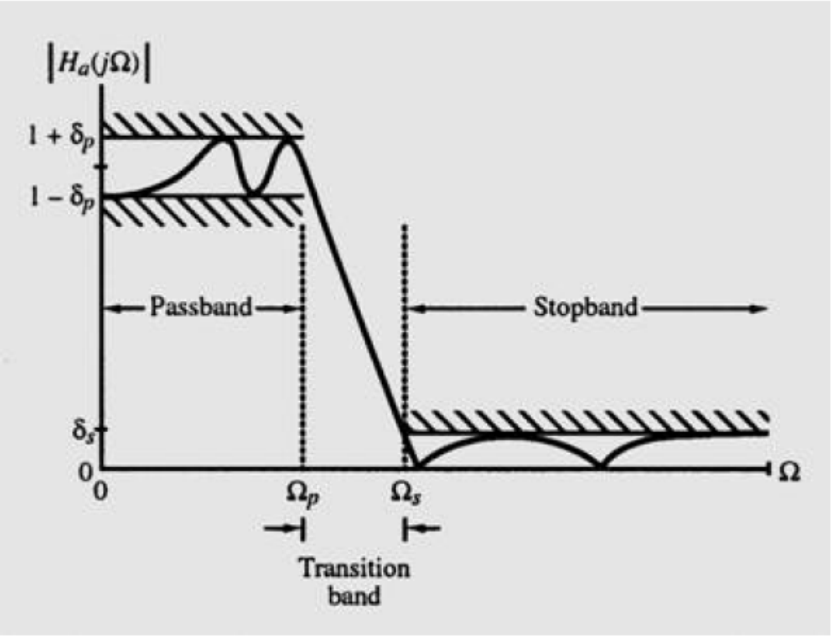
$$|H(j\Omega)| \le 1 \; for \; 0.4\pi \le \Omega \le \pi$$

Description

Low pass filter

The magnitude response can be expressed as

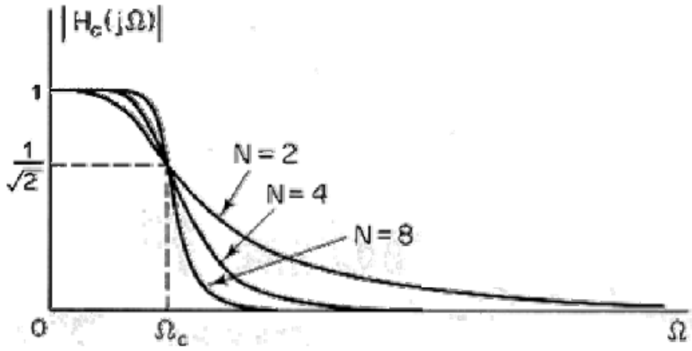
$$Magnitude = \begin{cases} 1-\delta_p \le |H(j\Omega)| \le 1 & for \; 0 \le \Omega \le \Omega_p \\ 0 \le |H(j\Omega) \le \delta_s & for \; |\Omega| \ge \Omega_s \end{cases}$$



Butterworth low pass filter

The magnitude frequency response of Butterworth filter is

$$|H(j\Omega)|^2 = \frac{1}{\left[1+\left(\frac{\Omega}{\Omega_c}\right)^{2N}\right]}$$



$|H_N(j\Omega)|^2 = H_N(j\Omega)H_N(-j\Omega) = \frac{1}{\left[1+\left(\frac{\Omega}{\Omega_c}\right)^{2N}\right]}$

For normalized Butterworth lowpass filter $\Omega_c = 1$

$$H_N(j\Omega)H_N(-j\Omega) = \frac{1}{\left[1+(\Omega)^{2N}\right]}$$

Let $s = j\Omega \therefore \Omega = \frac{s}{j}$

$$H_N(s)H_N(-s) = \frac{1}{1+\left(\frac{s}{j}\right)^{2N}}$$

The poles of are determined by equating the denominator to zero

$$1+\left(\frac{s}{j}\right)^{2N} = 0$$

$$s = (-1)^{\frac{1}{2N}} j$$

-1 can be written as $e^{j\pi(2k+1)}$ where $k = 0, 1 \ldots$ and $j = e^{j\pi/2}$

$$s_k = e^{j\pi\frac{(2k+1)}{2N}} e^{j\pi/2} \qquad k = 0, 1 \ldots 2N-1$$

The poles are placed on a unit circle with radius unity and are placed at angles

$$\begin{aligned} s_k &= 1\angle\frac{k\pi}{N} \qquad k = 0, 1 \ldots 2N-1 \textit{ when } N \textit{ is odd} \\ &= 1\angle\frac{\pi}{2N} + \frac{k\pi}{N} \qquad k = 0, 1 \ldots 2N-1 \textit{ when } N \textit{ is even} \end{aligned}$$

The Order of the Filter is given by the following equation

$$N = \frac{log\left[\frac{10^{\frac{-K_p}{10}}-1}{10^{\frac{-K_s}{10}}-1}\right]}{2log\left[\frac{\Omega_p}{\Omega_s}\right]}$$

The cut off frequency is given by the following equation

$$\Omega_C = \frac{\Omega_p}{\left(10^{\frac{-K_p}{10}} - 1\right)^{\frac{1}{2N}}} \quad OR \ \Omega_C = \frac{\Omega_s}{\left(10^{\frac{-K_s}{10}} - 1\right)^{\frac{1}{2N}}}$$

Therefore the normalised Transfer of Butterworth Low Pass filter is given as

$$H_N(s) = \frac{1}{\prod\limits_{LHP} (s - s_k)} = \frac{1}{B_N(s)}$$

The table below shows the Bn(s) for different filter order N

Order N	Butterworth Polynomial
1	s+1
2	$s^2 + \sqrt{2}s + 1$
3	$(s^2 + s + 1)(s + 1)$
4	$(s^2 + 0.76536s + 1)(s^2 + 1.84776s + 1)$
5	$(s + 1)(s^2 + 0.6180s + 1)(s^2 + 1.6180s + 1)$

The Frequency Transformation for desired cutoff frequency is given

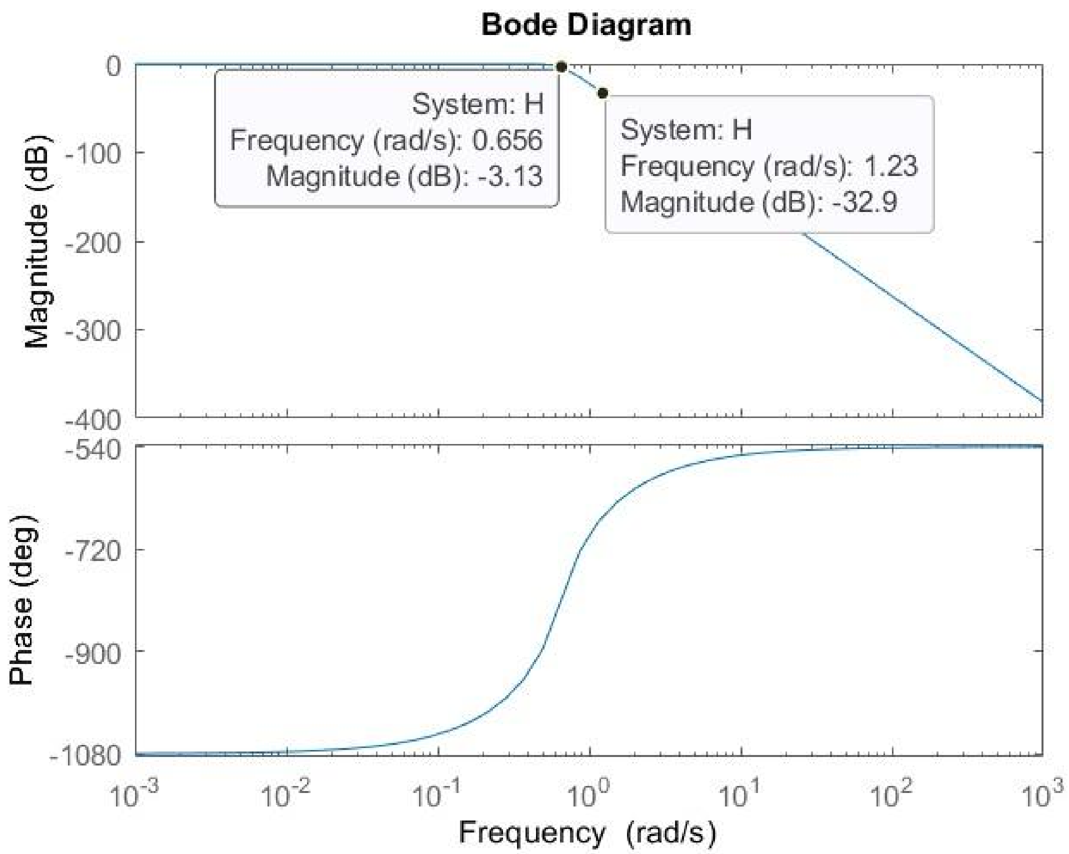
$$H_a(s) = H_N(s)|_{s \to \frac{s}{\Omega_C}}$$

Matlab Program

```
kp = 0.9;  % 0.9<= H(W) <= 1 for 0<= w<= 0.2Pi   , T =1
ks = 1;   %  H(W) <= 1 for 0.4pi<= w<= Pi       , T=1
% For ks = 1 ,N ---> infinity so we assume ks = 0.2
ks = 0.2;
KP = 20*log(kp);
KS = 20*log(ks);
wp = 0.2*pi;
ws = 0.4*pi;
fs =1;  % T=1
% To find the Order of Butterworth Filter
Nkp = power(10,(-KP/10))-1;
Nks = power(10,(-KS/10))-1;
Nnum = log(Nkp/Nks);
Ndec = 2* log(wp/ws);
N = Nnum/Ndec;
N = ceil(N);
% To find the pole of Butterworth Filter
k = 0:2*N-1;
e = (2.*k+1)./(2*N);
p = j*exp(1*j*pi.*e);
```

```
i=1;
for k=1:2*N
   r = real(p);
   if (r(k)<0)
       poles(i)=p(k);
       i=i+1;
   end
end
% To find the cutoff frequency in rad/sec Butterworth Filter
wc = wp/power(Nkp,1/(2*N));
% To find Transfer function Butterworth Filter
s = tf("s");
H = 1;
for k=1:N
   H = H*wc / (s+wc*poles(k));
end
% To plot the frequency and phase of Transfer function using bode plot
w= logspace(-3,3);
bode(H,w);
```

Output of the program



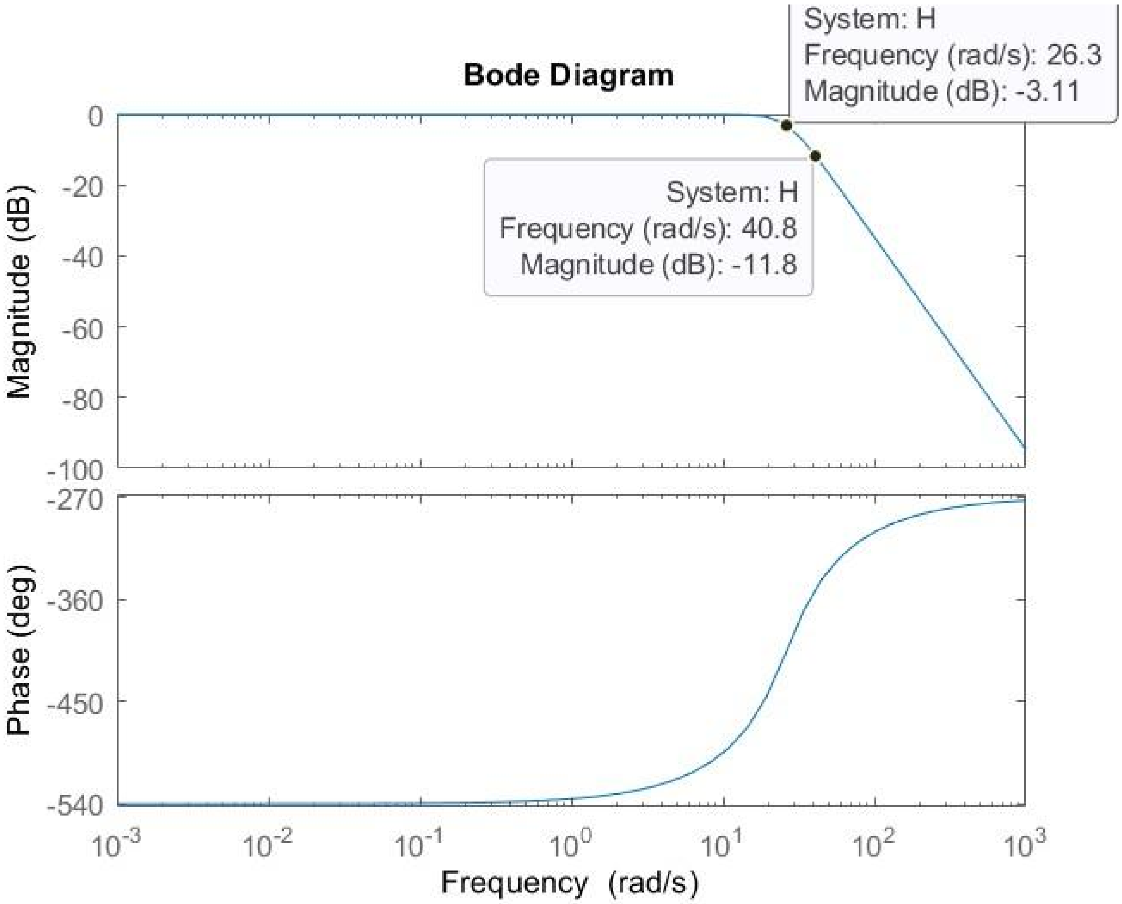
Write Matlab code to design an analog Butterworth using a low pass filter which has -5 dB attenuation at frequency 30rad/sec and at least -10 dB attenuation at 40 rad/sec.

Matlab Program

```
KP = -5;
KS = -10;
wp = 30;
ws = 40;
fs =1;  % T=1
% To find the Order of Butterworth Filter
Nkp = power(10,(-KP/10))-1;
Nks = power(10,(-KS/10))-1;
Nnum = log(Nkp/Nks);
Ndec = 2* log(wp/ws);
N = Nnum/Ndec;
N = ceil(N);
% To find the pole of Butterworth Filter
k = 0:2*N-1;
e = (2.*k+1)./(2*N);
p = j*exp(1*j*pi.*e);

i=1;
for k=1:2*N
   r = real(p);
   if (r(k)<0)
       poles(i)=p(k);
       i=i+1;
   end
end
% To find the cutoff frequency in rad/sec Butterworth Filter
wc = wp/power(Nkp,1/(2*N));
% To find Transfer function Butterworth Filter
s = tf("s");
H = 1;
for k=1:N
   H = H*wc / (s+wc*poles(k));
end
% To plot the frequency and phase of Transfer function using bode plot
w= logspace(-3,3);
bode(H,w);
```

Output of the program



Result : The Butterworth low pass filters are designed for given filter specification.