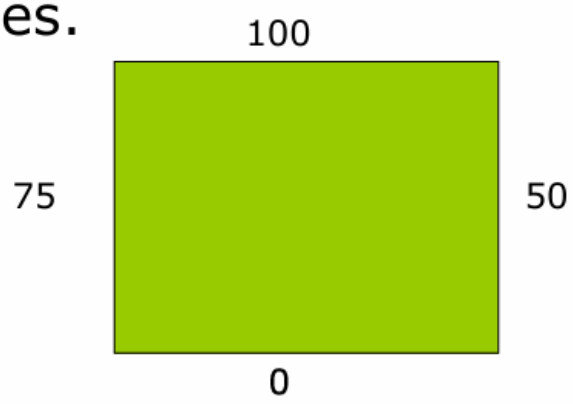
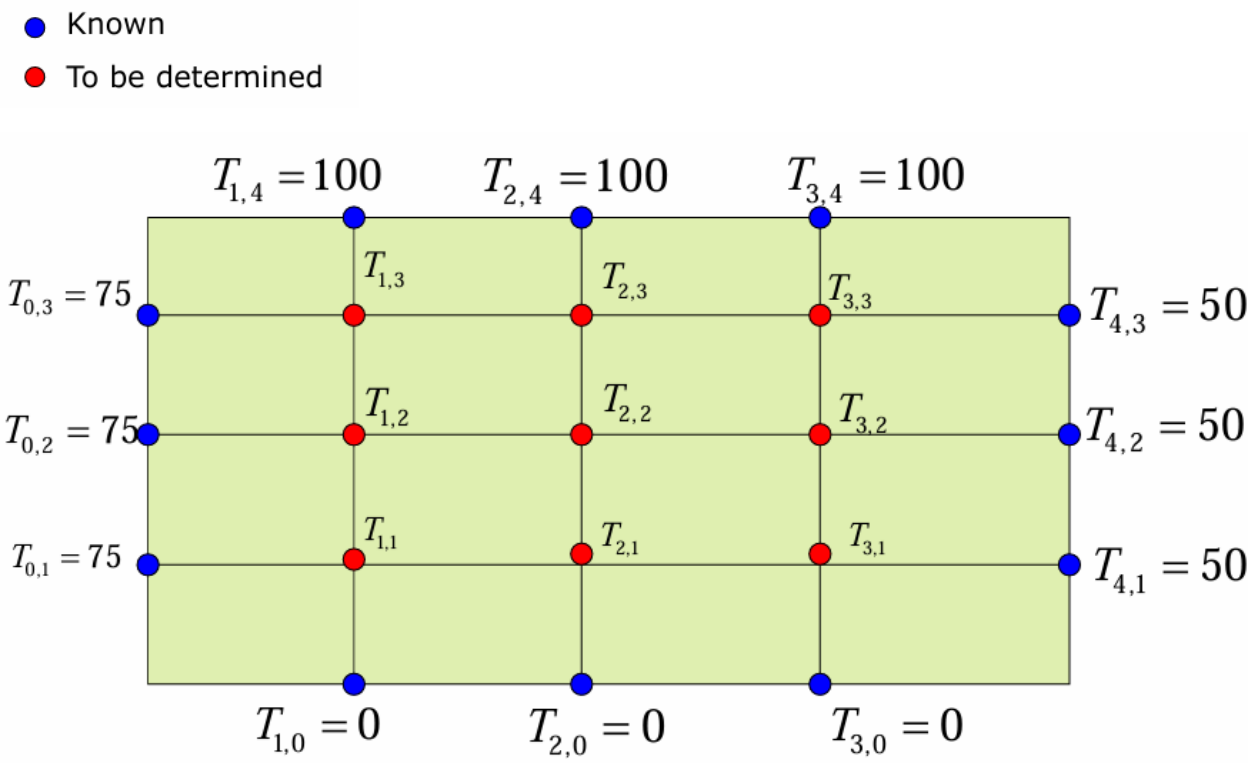
Elliptic Example

It is required to determine the steady state temperature at all points of a heated sheet of metal. The edges of the sheet are kept at a constant temperature: 100, 50, 0, and 75 degrees.



The sheet is divided to 5X5 grids.



$$T_{0,3} + T_{1,4} + T_{1,2} + T_{2,3} - 4T_{1,3} = 0$$

$$75 + 100 + T_{1,2} + T_{2,3} - 4T_{1,3} = 0$$

$$T_{1,3} + T_{2,4} + T_{3,3} + T_{2,2} - 4T_{2,3} = 0$$

$$T_{1,3} + 100 + T_{3,3} + T_{2,2} - 4T_{2,3} = 0$$

The Rest of the Equations

$$\begin{pmatrix} 4 & -1 & 0 & -1 & & & & & \\ -1 & 4 & -1 & 0 & -1 & & & & \\ 0 & -1 & 4 & 0 & 0 & -1 & & & \\ -1 & 0 & 0 & 4 & -1 & 0 & -1 & & \\ & -1 & 0 & -1 & 4 & -1 & 0 & -1 & \\ & & -1 & 0 & -1 & 4 & 0 & 0 & -1 \\ & & & -1 & 0 & 0 & 4 & -1 & 0 \\ & & & & -1 & 0 & -1 & 4 & -1 \\ & & & & & -1 & 0 & -1 & 4 \end{pmatrix} \begin{pmatrix} T_{1,1} \\ T_{2,1} \\ T_{3,1} \\ T_{1,2} \\ T_{2,2} \\ T_{3,2} \\ T_{1,3} \\ T_{2,3} \\ T_{3,3} \end{pmatrix} = \begin{pmatrix} 75 \\ 0 \\ 50 \\ 75 \\ 0 \\ 50 \\ 175 \\ 100 \\ 150 \end{pmatrix}$$