# ODE example

First order linear ODEs have the following general form:

$$\frac{dy}{dx} + p(x)y = q(x).$$

**Solution** This is solved by finding an integrating factor (IF). We look for $I(x)$ such that:

$$I(x)\left[\frac{dy}{dx} + p(x)y\right] = \frac{d[I(x)y]}{dx},$$

Then, we have

$$\begin{aligned}
\frac{d[I(x)y]}{dx} &= I(x)q(x), \\
\int d[I(x)y] &= \int q(x)I(x)\,dx + c_1, \\
y(x) &= \frac{1}{I(x)}\left[\int q(x)I(x)\,dx + c_1\right].
\end{aligned}$$

Integrating factors must fulfil:

$$\begin{aligned}
\frac{d(Iy)}{dx} &= I\frac{dy}{dx} + Ipy, \\
I\frac{dy}{dx} + y\frac{dI}{dx} &= I\frac{dy}{dx} + Ipy, \\
\int \frac{dI}{I} &= \int p(x)\,dx + c'.
\end{aligned}$$

So we have:

$$I(x) = Ae^{\int p(x)\,dx},$$

where $A$ is a new arbitrary constant (of integration).

So, we have the following for the general solution:

$$y(x) = e^{-\int p(x)\,dx}\left[\int e^{\int p(x)\,dx}q(x)\,dx + c\right],$$

where $c = c_1/A$ is a new arbitrary constant of integration.