Parabolic Example

$$\frac{\partial^2 u(x,t)}{\partial^2 x} - \frac{\partial u(x,t)}{\partial t} = 0$$

$$u(0,t) = u(1,t) = 0 \qquad u(x,0) = \sin(\pi x)$$

$$Use \quad h = 0.25, \quad k = 0.25 \ \text{ to find } u(x,t) \text{ for } x \in [0,1], t \in [0,1]$$

$$\lambda = \frac{k}{h^2} = 4$$

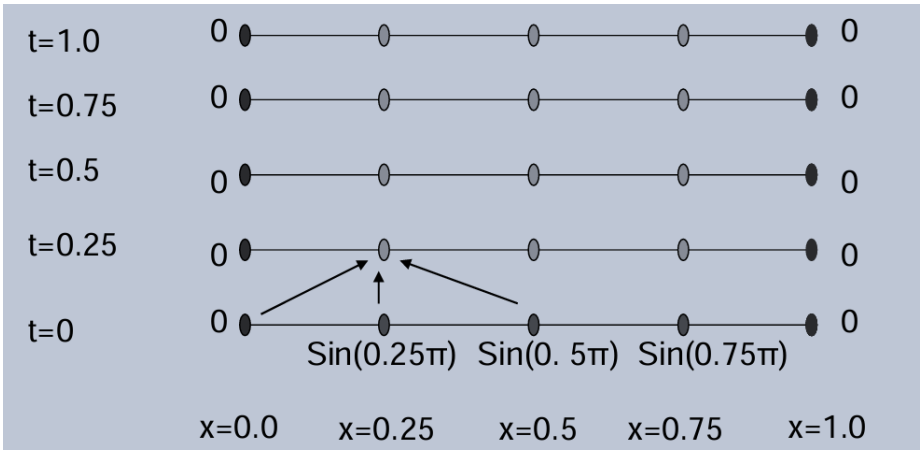
$$\frac{\partial^2 u(x,t)}{\partial x^2} - \frac{\partial u(x,t)}{\partial t} = 0$$

$$\frac{u(x-h,t) - 2u(x,t) + u(x+h,t)}{h^2} - \frac{u(x,t+k) - u(x,t)}{k} = 0$$

$$16\big(u(x-h,t) - 2u(x,t) + u(x+h,t)\big) - 4\big(u(x,t+k) - u(x,t)\big) = 0$$

$$u(x,t+k) = 4\ u(x-h,t) - 7\ u(x,t) + 4\ u(x+h,t)$$

$$u(x,t+k) = 4\ u(x-h,t) - 7\ u(x,t) + 4\ u(x+h,t)$$



$$u(0.5,0.25) = 4\ u(0.25,0) - 7\ u(0.5,0) + 4\ u(0.75,0)$$
$$= 4\sin(\pi/4) - 7\sin(\pi/2) + 4\sin(3\pi/4) = -0.1716$$

$$u(0.25,0.25) = 4\ u(0,0) - 7\ u(0.25,0) + 4\ u(0.5,0)$$
$$= 0 - 7\sin(\pi/4) + 4\sin(\pi/2) = -0.9497$$

The obtained results are probably not accurate

because : $1 - 2\lambda = -7$

select $k \le \frac{h^2}{2} = \frac{(0.25)^2}{2} = 0.03125$, choose $k = 0.025$, then $\lambda = \frac{k}{h^2} = 0.4$

$$u(x, t+k) = 0.4\ u(x-h, t) + 0.2\ u(x, t) + 0.4\ u(x+h, t)$$

$$u(0.25,0.025) = 0.4\ u(0,0) + 0.2\ u(0.25,0) + 0.4\ u(0.5,0)$$
$$= 0 + 0.2\sin(\pi/4) + 0.4\sin(\pi/2) = 0.5414$$

$$u(0.5,0.025) = 0.4\ u(0.25,0) + 0.2\ u(0.5,0) + 0.4\ u(0.75,0)$$
$$= 0.4\sin(\pi/4) + 0.2\sin(\pi/2) + 0.4\sin(3\pi/4) = 0.7657$$