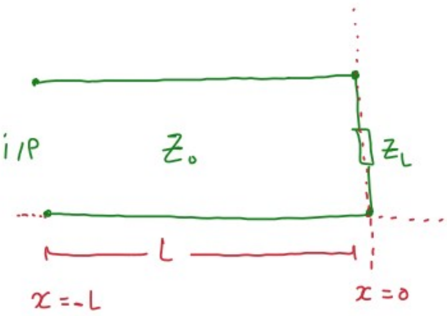
LEC2: Continue CH1

Wednesday, April 6, 2022 4:22 PM



- $\Gamma = \dfrac{V_-}{V_+}$ reflection coff.

- $Z_L = \dfrac{V_x}{I_x}\Big|_{x=0} = \dfrac{V_+ + V_-}{V_+/Z_o - V_-/Z_o} = Z_o \dfrac{V_+ + V_-}{V_+ - V_-}$

$\therefore Z_L = Z_o \dfrac{1 + V_-/V_+}{1 - V_-/V_+} = Z_o \dfrac{1+\Gamma}{1-\Gamma}$

$\therefore \boxed{\Gamma = \dfrac{Z_L - Z_o}{Z_L + Z_o}}$ $\quad\therefore \Gamma = |\Gamma| e^{j\phi}$ $\quad\therefore 0 \leq |\Gamma| \leq 1$

Complex

when $Z_L = Z_o$ " Perfect matching "

Input Impedance $Z_{i/p}$

- $Z_{i/p} = \dfrac{V_x}{I_x}\Big|_{x=-L} = \dfrac{V_+ e^{\gamma L} + V_- e^{-\gamma L}}{\frac{V_+}{Z_o} e^{\gamma L} - \frac{V_-}{Z_o} e^{-\gamma L}}$

$= Z_o \dfrac{e^{\gamma L} + \frac{V_-}{V_+} e^{-\gamma L}}{e^{\gamma L} - \frac{V_-}{V_+} e^{-\gamma L}}$

$= Z_o \dfrac{e^{\gamma L} + \Gamma e^{-\gamma L}}{e^{\gamma L} - \Gamma e^{-\gamma L}}$

$= Z_o \dfrac{e^{\gamma L} + \left(\frac{Z_L - Z_o}{Z_L + Z_o}\right) e^{-\gamma L}}{e^{\gamma L} - \left(\frac{Z_L - Z_o}{Z_L + Z_o}\right) e^{-\gamma L}}$

$\therefore Z_{i/p} = Z_o \dfrac{(Z_L + Z_o) e^{\gamma L} + (Z_L - Z_o) e^{-\gamma L}}{(Z_L + Z_o) e^{\gamma L} - (Z_L - Z_o) e^{-\gamma L}}$

$= Z_o \dfrac{Z_L (e^{\gamma L} + e^{-\gamma L}) + Z_o (e^{\gamma L} - e^{-\gamma L})}{Z_o (e^{\gamma L} + e^{-\gamma L}) + Z_L (e^{\gamma L} - e^{-\gamma L})}$

$= Z_o \dfrac{Z_L + Z_o \left(\frac{e^{\gamma L} - e^{-\gamma L}}{e^{\gamma L} + e^{-\gamma L}}\right)}{Z_o + Z_L \left(\frac{e^{\gamma L} - e^{-\gamma L}}{e^{\gamma L} + e^{-\gamma L}}\right)}$

$\therefore \boxed{Z_{i/p} = Z_o \dfrac{Z_L + Z_o \tanh(\gamma L)}{Z_o + Z_L \tanh(\gamma L)}}$ General : Lossy T.L

Complex

→ For a lossless (ideal) T.L :

- $Z_o = R_o \quad , \gamma = j\beta \quad , \alpha = 0$

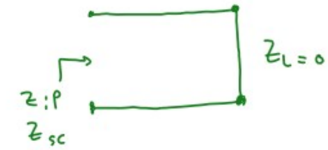
- $\therefore \tanh(\gamma L) = \tanh(j\beta L) = j\tan(\beta L)$

$\therefore \boxed{Z_{i/p} = R_o \dfrac{Z_L + jR_o \tan(\beta L)}{R_o + jZ_L \tan(\beta L)}}$ Still Complex

Short circuit Lossless T.L

- $Z_L = 0 \quad . Z_{i} = Z$

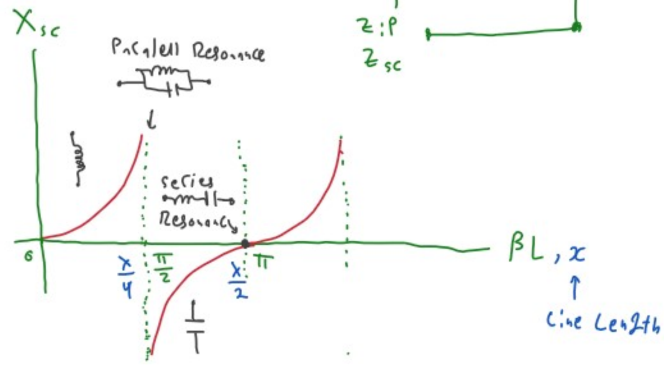
* Short Circuit Lossless T.L



$\cdot Z_L = 0 \quad , Z_{i/p} = Z_{sc}$

$\cdot Z_{sc} = \cancel{R_0} \dfrac{j R_0 \tan(\beta L)}{\cancel{R_0}} = j R_0 \tan(\beta L) = j X_{sc}$ "inductive"

$\therefore X_{sc} = R_0 \tan(\beta L)$



$\cdot \beta x = \frac{\pi}{2}$	$\cdot \beta x = \pi$
$(\frac{2\cancel{\pi}}{\lambda}) x = \frac{\cancel{\pi}}{2}$	$(\frac{2\cancel{\pi}}{\lambda}) x = \cancel{\pi}$
$x = \frac{\lambda}{4}$	$x = \frac{\lambda}{2}$

Notes:

1. if $L < \frac{\lambda}{4}$ ⇝ the Line becomes inductive
2. if $\frac{\lambda}{4} < L < \frac{\lambda}{2}$ ⇝ - - - - - - - - - Capastive
3. if $L = \frac{\lambda}{4}$ ⇝ - - - - - - - - - - Parallel Resonance L, C
4. $L = \frac{\lambda}{2}$ ⇝ - - - - - - - - - Series Resonance L, C

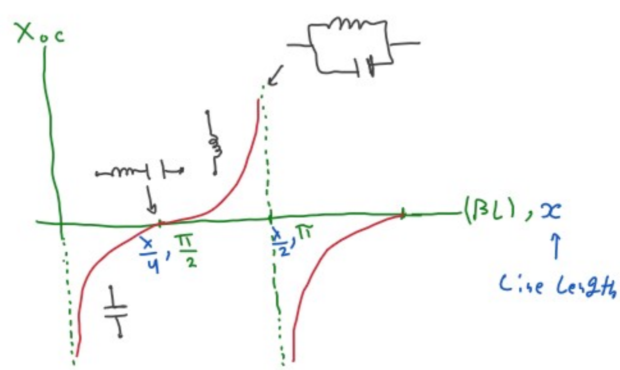
} By changing the line length relative to signal wave length we can control it's behaviour.

* Open Circuit Lossless T.L

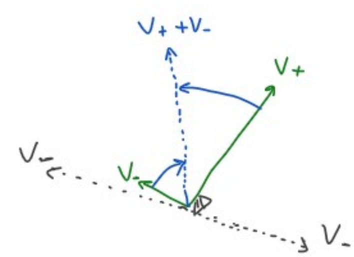
$\because Z_L = \infty$

$\therefore Z_{oc} = R_0 \dfrac{\cancel{Z_L}}{j \cancel{Z_L} \tan(\beta L)} = -j R_0 \cot(\beta L) = j X_{oc}$

$\therefore X_{oc} = -R_0 \cot(\beta L)$



* Voltage Standing Wave Ratio (VSWR)



$\cdot VSWR = \dfrac{V_{max}}{V_{min}} = \dfrac{|V_+| + |V_-|}{|V_+| - |V_-|}$

$= \dfrac{1 + \frac{|V_-|}{|V_+|}}{1 - \frac{|V_-|}{|V_+|}} = \dfrac{1 + |\Gamma|}{1 - |\Gamma|}$

$\therefore VSWR = \dfrac{1 + |\Gamma|}{1 - |\Gamma|}$ "real"

$$\therefore VSWR = \frac{1+|\Gamma|}{1-|\Gamma|} \quad \text{"real"}$$

$$0 \leq \Gamma \leq 1 \quad , \quad 1 < VSWR < \infty$$

$$\cdot \ \beta x_{max-min} = \frac{\pi}{2}$$

$$\therefore \left(\frac{2\cancel{\pi}}{\lambda}\right) x_{max-min} = \frac{\cancel{\pi}}{2}$$

$$\therefore x_{max-min} = \frac{\lambda}{4}$$

$$\therefore x_{Max-Max} = x_{min-min} = x_{p-p} = \lambda/2$$