2. The TM_{mn} mode is propagating through a rectangular waveguide filled by air. Find expressions for the transverse field components, the power transmited and power loss due to wall losses.

[12 points]

$$\begin{bmatrix} E_x \\ H_y \\ E_y \\ H_x \end{bmatrix} = \frac{1}{K_c^2} \begin{bmatrix} -j\beta & -j\omega\mu & 0 & 0 \\ -j\omega\epsilon & -j\beta & 0 & 0 \\ 0 & 0 & -j\beta & j\omega\mu \\ 0 & 0 & j\omega\epsilon & -j\beta \end{bmatrix} \begin{bmatrix} \frac{\partial E_z}{\partial x} \\ \cancel{\frac{\partial H_z}{\partial y}}\ 0 \\ \frac{\partial E_z}{\partial y} \\ \cancel{\frac{\partial H_z}{\partial x}}\ 0 \end{bmatrix}$$

$\rightarrow H_z = 0 \quad , E_z = E_0 \sin(\frac{\pi m}{a} x) \sin(\frac{\pi n}{b} y) e^{-j\beta z}$

$\rightarrow E_x , H_x , E_y , H_y$

$\bullet E_x = -j\beta / K_c^2 \cdot \frac{\partial E_z}{\partial x}$

$= -j\beta / K_c^2 \cdot E_0 \cdot \underbrace{\frac{\pi m}{a} \cos(\frac{\pi m}{a} x) \sin(\frac{\pi n}{b} y) e^{-j\beta z}}$

$\bullet H_y = -j\omega\epsilon / K_c^2 \cdot E_0$ - - - - - - - - -

$\bullet E_y = -j\beta / K_c^2 \cdot E_0 \cdot \underbrace{\frac{\pi n}{b} \cos(\frac{\pi n}{b} y) \sin(\frac{\pi m}{a} x) e^{-j\beta z}}$

$\bullet H_x = j\omega\epsilon / K_c^2 \cdot E_0$ - - - - - - - - -

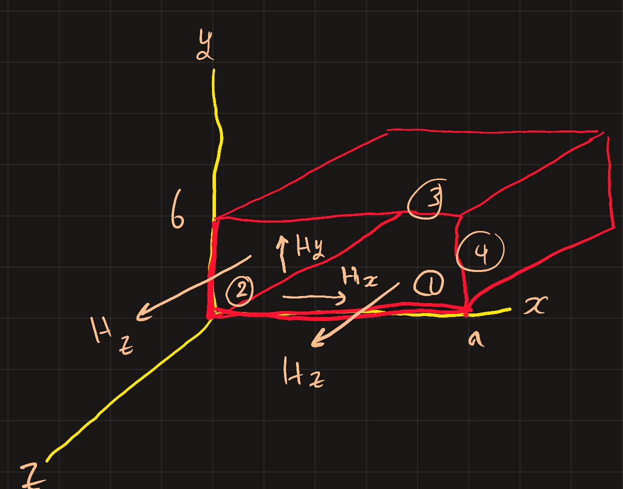
$\star W_T = \frac{1}{2} \eta \int_0^b \int_0^a (H_x^2 + H_y^2)\, dx\, dy$

$= \frac{1}{2} \eta \cdot (\frac{\omega\epsilon}{K_c^2})^2 \cdot (E_0)^2 \iint [(\frac{\pi n}{b})^2 \cos^2(\frac{\pi n}{b} y) \sin^2(\frac{\pi m}{a} x)] + [(\frac{\pi m}{a})^2 \cos^2(\frac{\pi m}{a} x) \sin^2(\frac{\pi n}{b} y)]$

$= \frac{1}{2} \eta \cdot (\frac{\omega\epsilon}{K_c^2})^2 (E_0)^2 [(\frac{\pi n}{b})^2 (\frac{b}{2})(\frac{a}{2}) + (\frac{\pi m}{a})^2 (\frac{a}{2})(\frac{b}{2})]$

$= \frac{ab}{8} \eta \cdot (E_0)^2 (\frac{\omega\epsilon}{K_c^2})^2 [(\frac{\pi n}{b})^2 + (\frac{\pi m}{a})^2]$

$W_T = \frac{ab}{8} \eta \cdot (E_0)^2 (\frac{\omega\epsilon}{K_c})^2$ #



$\star \alpha_w = \frac{P_L}{2 W_T}$

$\rightarrow P_L = P_{L1} + P_{L2} + P_{L3} + P_{L4} = 2(P_{L1} + P_{L2})$

$\bullet P_{L1} = \frac{1}{2} R_s \int_0^1 \int_0^a (H_x^2|_{y=0} + H_z^2|_{y=0})\, dx\, dz$

$\therefore P_{L1} = \frac{1}{2} R_s \cdot (\frac{\omega\epsilon}{K_c^2})^2 (E_0)^2 (\frac{\pi n}{b})^2 \iint \sin^2(\frac{\pi m}{a} x)\, dx\, dz$

$\therefore P_{L1} = \frac{1}{2} R_s (\frac{\omega\epsilon}{K_c^2})^2 (E_0)^2 (\frac{\pi n}{b})^2 (\frac{a}{2})$

$\rightarrow P_{L2} = \frac{1}{2} R_s \int_0^1 \int_0^b (|H_y|^2)\, dy\, dz \quad = \frac{1}{2} R_s \cdot$

$\alpha_w =$