→ when the wave is transmitted between two mediums, Part of it is transmitted and another is reflected.

→ R: Reflection Cofficient
T: Transmition . . . . .

- $|\vec{E}_t| = T \cdot |\vec{E}_i|$, $|\vec{E}_i|$: mag. of incident electric field , $|\vec{E}_t|$: mag. of transmited elec. field.
- $|\vec{E}_r| = R \cdot |\vec{E}_i|$, ... , $|\vec{E}_r|$: . . . . reflected . . . . .

→ in this chapter we'll replace γ with K "Antena Concept"

$\gamma = \sqrt{-\omega^2 \mu \epsilon}$, $K = \sqrt{\omega^2 \mu \epsilon}$

$\gamma = J\sqrt{\omega^2 \mu \epsilon} = JK$ $\therefore \boxed{\gamma = JK}$

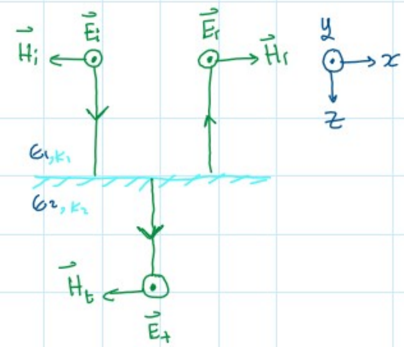
$\therefore \vec{E}_i(z) = E_o e^{-JKz} \hat{x}$ " the Propagation direction is in z "

$\therefore \vec{E}_r(z) = R E_o e^{-JKz} \hat{x}$

*Normal Incidence:

→ Assuming that there are two dielectric media and the Coordinates as shown:

→ the magnetic and electric Fields equations :



- $\vec{E}_i(z) = E_o e^{-JK_1z} (\hat{y})$
- $\vec{E}_r(z) = R E_o e^{+JK_1z} (\hat{y})$
- $\vec{E}_t(z) = T E_o e^{-JK_2z} (\hat{y})$

- $\vec{H}_i(z) = \frac{E_o}{\eta_1} e^{-JK_1z} (-\hat{x})$
- $\vec{H}_r(z) = R \frac{E_o}{\eta_1} e^{+JK_1z} (\hat{x})$
- $\vec{H}_t(z) = T \frac{E_o}{\eta_2} e^{-JK_2z} (-\hat{x})$

- $K_1 = \omega\sqrt{\mu\epsilon_1}$, $K_2 = \omega\sqrt{\mu\epsilon_2}$
- $\eta_1 = \sqrt{\frac{\mu}{\epsilon_1}}$
- $\eta_2 = \sqrt{\frac{\mu}{\epsilon_2}}$

→ we can get a relation between the constants " T, R " only by applying the Boundry Conditions:

→ 1) $E_{t_1} = E_{t_2}\big|_{z=0}$

$\therefore \vec{E}_i(z=0) + \vec{E}_r(z=0) = \vec{E}_t(z=0)$

$\therefore \cancel{E_o} + R\cancel{E_o} = T\cancel{E_o}$

$\therefore \boxed{1 + R = T} \longrightarrow (1)$

→ 2) $H_{t_1} = H_{t_2}\big|_{z=0}$

$\therefore \vec{H}_i(z=0) + \vec{H}_r(z=0) = H_t(z=0)$

$\therefore \frac{E_o}{\eta_1}(-1) + R\frac{E_o}{\eta_1} = T\frac{E_o}{\eta_2}(-1)$

$\therefore -\eta_2 + R\eta_2 = -\eta_1 T \rightarrow \therefore \boxed{\eta_2(1-R) = \eta_1 T} \longrightarrow (2)$

→ $\therefore$ From (1),(2) :

$$R = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \quad , \quad T = \frac{2\eta_2}{\eta_2 + \eta_1}$$

* Wave Propagation in an arbitary direction :

Wave Propagation in an arbatory direction:

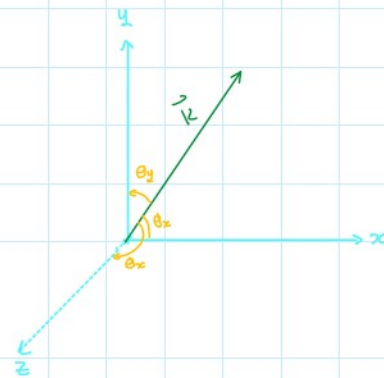
- So Far we were dealing with a wave that Propogates in one direction either x or y or z but waves ussuly Prolagtes in a general direction.

- Helmton equation:

$\nabla^2 \vec{E} + K^2 \vec{E} = 0$, $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$, $K = \omega \sqrt{\frac{\mu}{\epsilon}}$

$\rightarrow \therefore$ general Solution of the wave equation:

$\vec{E} = E_0 e^{\pm jK_x x} e^{\pm jK_y y} e^{\pm jK_z z}$, $K^2 = K_x^2 + K_y^2 + K_z^2$ "Like any Vector"

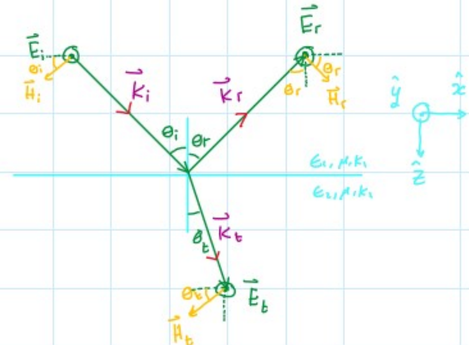


$\rightarrow$ recall that $\vec{r} = \langle x, y, z \rangle$, $\vec{K} = \langle K_x, K_y, K_z \rangle$

$\therefore$ the $\vec{E}$ equation can be written as:

$\therefore \vec{E} = E_0 e^{\pm j\vec{K} \cdot \vec{r}}$, $K_x = K \cos\theta_x$, $K_y = K\cos\theta_y$, $K_z = K\cos\theta_z$

Oblique Incidance:



i) Prepradicalor (Hersontal) Polorization:

- Snell's Law:

$\rightarrow n_i \sin\theta_i = n_t \sin\theta_t$, $n = \sqrt{\epsilon_r}$

$\therefore \sqrt{\epsilon_{r_i}} \sin\theta_i = \sqrt{\epsilon_{r_t}} \sin\theta_t$

- $\vec{E}_i = E_0 e^{-j\vec{k}_i \cdot \vec{r}_i} (\hat{y})$, $\vec{k}_i = k_{i_x}\hat{x} + k_{i_z}\hat{z}$, $\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$

$\therefore \vec{E}_i = E_0 e^{-j[k_{ix} x + k_{iz} z]} (\hat{y})$, $k_{ix} = k_i \sin\theta_i$, $k_{iz} = k_i \cos\theta_i$, $k_i = \omega\sqrt{\mu_0 \epsilon_1}$

- $\vec{E}_r = R_\perp E_0 e^{-j\vec{k}_r \cdot \vec{r}_r} (\hat{y})$, $\vec{k}_r = k_{rx}\hat{x} - k_{rz}\hat{z}$

$\therefore \vec{E}_r = R_\perp E_0 e^{-j[k_{rx} x - k_{rz} z]} (\hat{y})$, $k_{rx} = k_r \sin\theta_r$, $k_{rz} = k_r \cos\theta_r$, $k_r = \omega\sqrt{\mu_0 \epsilon_1} = k_i = k_1$

- $\vec{E}_t = T_\perp E_0 e^{-j\vec{k}_t \cdot \vec{r}_t} (\hat{y})$, $\vec{k}_t = k_{tx}\hat{x} + k_{tz}\hat{z}$

$\therefore \vec{E}_t = T_\perp E_0 e^{-j[k_{tx} x + k_{tz} z]} (\hat{y})$, $k_{tx} = k_t \sin\theta_t$, $k_{tz} = k_t \cos\theta_t$, $k_t = \omega\sqrt{\mu_0 \epsilon_2} = k_2$

BC:

$E_{t_1} = E_{t_2}\big|_{z=0}$

$\therefore \cancel{E_0} e^{-jk_{ix}x} \cancel{e^{-jk_{iz}z}} + R_\perp \cancel{E_0} e^{-jk_{rx}x} \cancel{e^{k_{rz}z}} = T_\perp \cancel{E_0} e^{-jk_{tx}x} \cancel{e^{-jk_{tz}z}}$, $k_i = k_r = k_1$, $k_t = k_2$

$\therefore \boxed{e^{-jk_{1x}x}[1 + R_\perp] = T_\perp e^{-jk_{2x}x}}$

→ need to get relation between k_{1x} & k_{2x}

→ $k_{1x} = k_1 \sin\theta_i = \omega\sqrt{\mu_0\epsilon_1}\,\sin\theta_i$

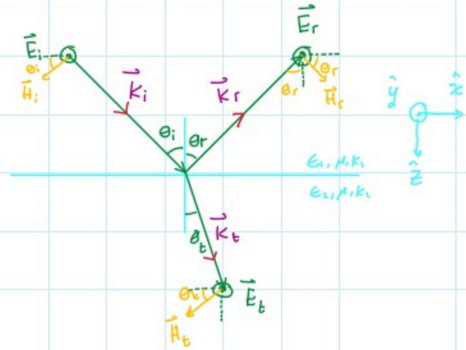
→ $k_{2x} = k_2 \sin\theta_t = \omega\sqrt{\mu_0\epsilon_2}\,\sin\theta_t$

$\therefore k_1 \sin\theta_i = k_2 \sin\theta_t$ "Snell Low"

$\therefore \omega\sqrt{\mu_0\epsilon_1}\,\sin\theta_i = \omega\sqrt{\mu_0\epsilon_2}\,\sin\theta_t$

$\therefore k_{1x} = k_{2x}$

$\therefore \boxed{1 + R_\perp = T_\perp}$ → ①



→ $\vec{H}_i = \frac{E_0}{\eta_1} e^{-j\vec{k}_i\cdot\vec{r}} (-\cos\theta_i\,\hat{x} + \sin\theta_i\,\hat{z})$

$\vec{k}_i\cdot\vec{r} = k_{1x}x + k_{1z}z$

→ $\vec{H}_r = R_\perp \frac{E_0}{\eta_1} e^{-j\vec{k}_r\cdot\vec{r}} (\cos\theta_r\,\hat{x} + \sin\theta_r\,\hat{z})$

$\vec{k}_r\cdot\vec{r} = k_{1x}x - k_{1z}z$

→ $\vec{H}_t = T_\perp \frac{E_0}{\eta_2} e^{-j\vec{k}_t\cdot\vec{r}} (-\cos\theta_t\,\hat{x} + \sin\theta_t\,\hat{z})$

$\vec{k}_t\cdot\vec{r} = k_{2x}x + k_{2z}z$

BC:

$H_{t_1} = H_{t_2}\big|_{z=0}$

$\frac{E_0}{\eta_1} \cancel{e^{-jk_{1x}x}} (-\cos\theta_i) + R_\perp \frac{\cancel{E_0}}{\eta_1} \cancel{e^{-jk_{1x}x}} (\cos\theta_r) = T_\perp \frac{\cancel{E_0}}{\eta_2} \cancel{e^{-jk_{2x}x}} (-\cos\theta_t)$ "$k_{1x} = k_{2x}$"

$\therefore \frac{-1}{\eta_1}\cos\theta_i + \frac{R_\perp}{\eta_1}\cos\theta_r = \frac{-T_\perp}{\eta_2}\cos\theta_t$ "$\theta_i = \theta_r$"

$\therefore -\eta_2\cos\theta_i + \eta_2 R_\perp \cos\theta_r = -\eta_1 T_\perp \cos\theta_t$

$\therefore \boxed{\eta_2 \cos\theta_i [1 - R_\perp] = \eta_1 T_\perp \cos\theta_t}$ → ②

→ sub ① to ②

$\therefore \eta_2\cos\theta_i[1 - R_\perp] = \eta_1[1 + R_\perp]\cos\theta_t$

$\therefore \boxed{R_\perp = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}}$

$\therefore \boxed{T_\perp = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}}$

if $\theta = 0$ → same as normal incident

→ the final form is by making $R_\perp, T_\perp$ as function of θ_i only "because transmit with θ_i"

→ Snell's Low:

$\sqrt{\epsilon_1}\,\sin\theta_i = \sqrt{\epsilon_2}\,\sin\theta_t$

$\therefore \sin\theta_t = \sqrt{\frac{\epsilon_1}{\epsilon_2}}\,\sin\theta_i$

$\therefore \cos\theta_t = \sqrt{1 - \sin^2\theta_t}$

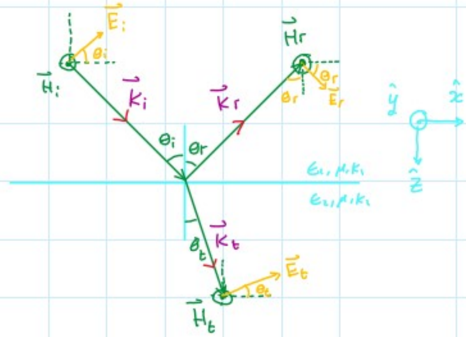
$\therefore \boxed{\cos\theta_t = \sqrt{1 - \frac{\epsilon_1}{\epsilon_2}\sin^2\theta_i}}$ ← replace $\cos\theta_t$ in $R_\perp$ & $T_\perp$

note if lossy medium → replace η with $\eta_c = \sqrt{\frac{\mu}{\epsilon_{eff}}}$, $\epsilon_{eff} = \epsilon - j\frac{\sigma}{\omega}$

II) Paralell (Verticle) Polarization:

~> $\vec{E}$ is Paralell to Plane x (medium and Propogation)

$\cdot\theta_i = \theta_r$, $\sqrt{\epsilon_1}\sin\theta_i = \sqrt{\epsilon_2}\sin\theta_t$



$$\bar{E}_i = E_0 e^{-j\bar{K}_i\cdot\bar{r}} (\cos\theta_i\hat{x} - \sin\theta_i\hat{z})$$
$$K_i = K_{ix}\hat{x} + K_{iz}\hat{z}$$
$$\bar{r} = x\hat{x} + z\hat{z}$$
$$\rightarrow \bar{E}_i = E_0 e^{-j(K_{ix}x + K_{iz}z)}[\cos\theta_i\hat{x} - \sin\theta_i\hat{z}] \rightarrow ①$$

$$\bar{E}_r = (E_0 R_{\parallel}) e^{-j\bar{K}_r\cdot\bar{r}} (\cos\theta_i\hat{x} + \sin\theta_i\hat{z})$$
$$K_r = K_{ix}\hat{x} - K_{iz}\hat{z}$$
$$\bar{r} = x\hat{x} + z\hat{z}$$
$$\rightarrow \bar{E}_r = (E_0 R_{\parallel}) e^{-j(K_{ix}x - K_{iz}z)} (\cos\theta_i\hat{x} + \sin\theta_i\hat{z}) \rightarrow ②$$

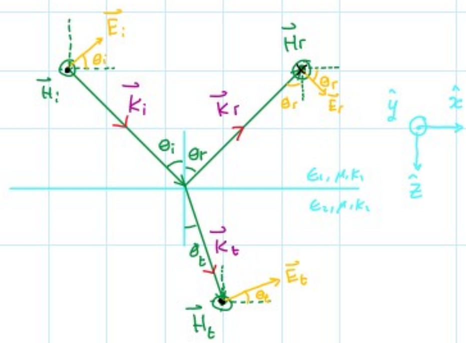
$$\bar{E}_t = (E_0 T_{\parallel}) e^{-j\bar{K}_t\cdot\bar{r}} (\cos\theta_t\hat{x} - \sin\theta_t\hat{z})$$
$$\bar{K}_t = K_{2x}\hat{x} + K_{2z}\hat{z}$$
$$\bar{r} = x\hat{x} + z\hat{z}$$
$$\rightarrow \bar{E}_t = (E_0 T_{\parallel}) e^{-j(K_{2x}x + K_{2z}z)} (\cos\theta_t\hat{x} - \sin\theta_t\hat{z}) \rightarrow ③$$

BC:

$\cdot E_{t_1} = E_{t_2}|_{z=0}$

$\therefore \cancel{E_0} \cancel{e^{-jK_{1x}x}} \cos\theta_i + R_{\parallel}\cancel{E_0}\cancel{e^{-jK_{1x}x}}\cos\theta_i = T_{\parallel}\cancel{E_0}\cancel{e^{-jK_{2x}x}}\cos\theta_t$ " $K_{1x} = K_{2x}$ "

$\therefore$ $\cos\theta_i[1 + R_{\parallel}] = T_{\parallel}\cos\theta_t$ $\rightarrow ①$



$$\bar{H}_i = \frac{E_0}{\eta_1} e^{-j\bar{K}_i\cdot\bar{r}}\hat{y}$$
$$\rightarrow \bar{H}_i = \frac{E_0}{\eta_1} e^{-j(K_{1x}x + K_{1z}z)}\hat{y} \rightarrow ④$$
$$\bar{H}_r = \frac{E_0 R_{\parallel}}{\eta_1} e^{-j\bar{K}_r\cdot\bar{r}}(-\hat{y})$$
$$\rightarrow \bar{H}_r = \frac{E_0 R_{\parallel}}{\eta_1} e^{-j(K_{1x}x - K_{1z}z)}(-\hat{y}) \rightarrow ⑤$$
$$\bar{H}_t = \frac{E_0 T_{\parallel}}{\eta_2} e^{-j\bar{K}_t\cdot\bar{r}}(\hat{y})$$
$$\rightarrow \bar{H}_t = \frac{E_0 T_{\parallel}}{\eta_2} e^{-j(K_{2x}x + K_{2z}z)}\hat{y} \rightarrow ⑥$$

BC:

$H_{t_1} = H_{t_2}|_{z=0}$

BC:

$H_{t_1} = H_{t_2}\big|_{z=0}$

$\therefore \frac{E_0}{\eta_1}\cancel{e^{-jk_{1x}x}} - R_\parallel \frac{E_0}{\eta_1}\cancel{e^{-jk_{1x}x}} = T_\parallel \frac{E_0}{\eta_2}\cancel{e^{-jk_{2x}x}}$

$\therefore \eta_2 - R_\parallel \eta_2 = T_\parallel \eta_1$

$\therefore \boxed{\eta_2[1-R_\parallel] = T_\parallel \eta_1} \rightarrow ②$

→ Sub ② into ①

$\therefore \cos\theta_i [1+R_\parallel] = \frac{\eta_2}{\eta_1}[1-R_\parallel]\cos\theta_t$

$$\boxed{\therefore R_\parallel = \frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}} \quad , \quad \boxed{\therefore T_\parallel = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}}$$

→ $\theta_i \rightleftarrows \theta_t$ → Perpendicular Polarization

→ if $\theta_i = \theta_t = 0$ → normal incidence

→ the final form is by making $R_\perp, T_\perp$ as function of θ_i only "because transmit with θ_i"

→ Snell's law:

$\boxed{\sqrt{\epsilon_1}\sin\theta_i = \sqrt{\epsilon_2}\sin\theta_t}$

$\therefore \sin\theta_t = \sqrt{\frac{\epsilon_1}{\epsilon_2}}\sin\theta_i$

$\therefore \cos\theta_t = \sqrt{1-\sin^2\theta_t}$

$\boxed{\therefore \cos\theta_t = \sqrt{1-\frac{\epsilon_1}{\epsilon_2}\sin^2\theta_i}}$ ← replace $\cos\theta_t$ in $R_\perp$ & $T_\perp$

note if lossy medium → replace η with $\eta_c = \sqrt{\frac{\mu}{\epsilon_{eff}}}$, $\epsilon_{eff} = \epsilon - j\frac{\sigma}{\omega}$

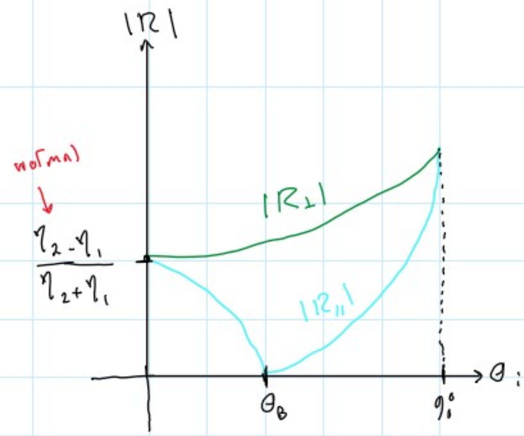
→ notes:

• now we got the equations that discribs the change of R, T as a function of θ_i

→ what we desire is to make R↓↓ "we want the message to be trans. only, no waste in R"

→ some other allications we will see that we need R↑

→ Plot $R_\perp$, $R_\parallel$ as a function of θ_i



→ θ_B is the incident angle at which $R_\parallel = 0$

• what's the value of θ_B??

↳ $\theta_i = \theta_B$, $R_\parallel = 0$, $R_\parallel = \frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t - \eta_1\cos\theta_i}$

$\therefore \eta_2\cos\theta_t = \eta_1\cos\theta_i$

$\therefore \eta_2\sqrt{1-\frac{\epsilon_1}{\epsilon_2}\sin^2\theta_B} = \eta_1\cos\theta_B$

$\therefore [\frac{\cancel{\mu_0}}{\epsilon_2}][1-\frac{\epsilon_1}{\epsilon_2}\sin^2\theta_B] = [\frac{\cancel{\mu_0}}{\epsilon_1}][1-\sin^2\theta_B]$

$\therefore \frac{\epsilon_1}{\epsilon_2} - (\frac{\epsilon_1}{\epsilon_2})^2\sin^2\theta_B = 1-\sin^2\theta_B$

$\therefore \frac{\epsilon_1}{\epsilon_2} - 1 = (\frac{\epsilon_1}{\epsilon_2})^2\sin^2\theta_B - \sin^2\theta_B$

$\therefore \frac{\epsilon_1}{\epsilon_2} - 1 = [(\frac{\epsilon_1}{\epsilon_2})^2 - 1]\sin^2\theta_B$

$\therefore \cancel{\frac{\epsilon_1}{\epsilon_2} - 1} = (\cancel{\frac{\epsilon_1}{\epsilon_2} - 1})(\frac{\epsilon_1}{\epsilon_2}+1)\sin^2\theta_B$

$\therefore (\frac{\epsilon_1+\epsilon_2}{\epsilon_2})\sin^2\theta_B = 1$

$\boxed{\therefore \sin\theta_B = \sqrt{\frac{\epsilon_2}{\epsilon_2+\epsilon_1}}}$

$0 < \theta_B < 90°$

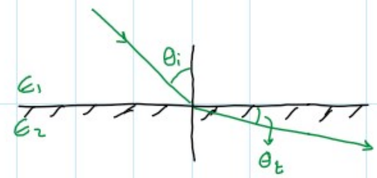
→ therefore, if $\theta_i = \theta_B$ for parallel polarization the wave will be totally transmitted.

* critical angle θ_c:

• consider the case where $\epsilon_1 > \epsilon_2$, according to Snell's law:

$$\sqrt{\epsilon_1}\,\sin\theta_i = \sqrt{\epsilon_2}\,\sin\theta_t$$

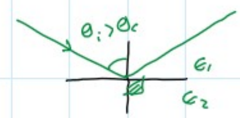
$$\frac{\sin\theta_i}{\sin\theta_t} = \sqrt{\frac{\epsilon_1}{\epsilon_2}} \quad , \therefore \theta_i > \theta_t$$



→ θ_c is the incident angle at which $\theta_t = 0$

→ if $\theta_i > \theta_c$ the wave will be totally reflected

→ we'll get θ_c from Snell's law:



$$\sqrt{\epsilon_1}\,\sin\theta_c = \sqrt{\epsilon_2}\,\sin 90^\circ$$

$$\therefore \sin\theta_c = \sqrt{\frac{\epsilon_2}{\epsilon_1}} \quad , \epsilon_1 > \epsilon_2$$