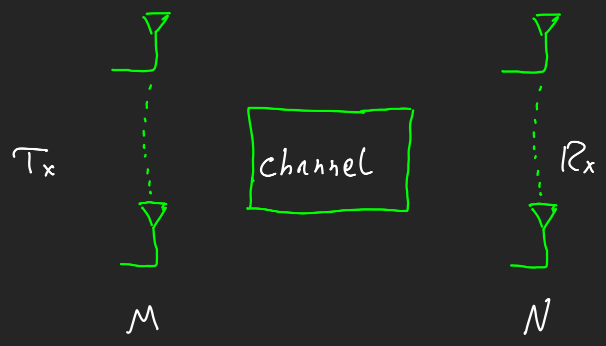
Multiple Input Multiple output

Diversity

→ We use Multiple (M) Antennas at the transmiter and Multiple (N) Antennas at the recieuer to overcome the channel effect in wirless-communication.



★ Narrowband wirlees Fading Channel :

• $y[m] = \underbrace{h[m]} x[m] + n[m]$

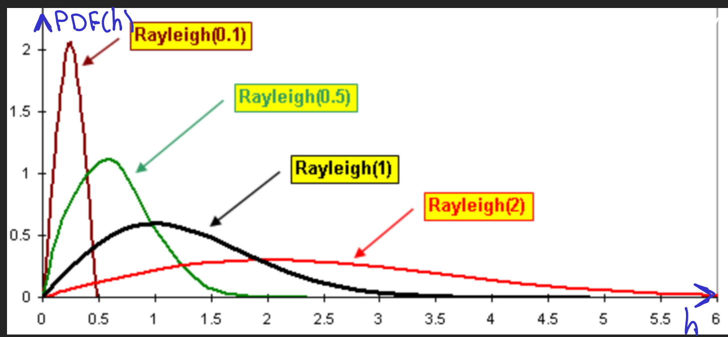
↳ Complex Gaussion, meaning $h[m] \sim CN(0, \sigma^2)$

• So $h[m] = a + jb = |h[m]| e^{j\theta[m]}$

Gaussion Gaussion $\sim$ uniform $[0, 2\pi]$

• $|h[m]| = \sqrt{a^2 + b^2}$ ⟶ the resulting distribution for $|h[m]|$ is Rayleigh distribution.

$f(x) = \frac{x}{\sigma^2} e^{-\frac{x^2}{2\sigma^2}}$ $x \geq 0$



• From the Rayleigh distribution we find that there's a high chance that $h < 1$, this will have an effect of multipling the signal with a Gain < 1 ($h[m] x[m] \downarrow$), which leeds to low signal power.

• $|h|$ can be low enough so that the noise power is bigger (Deep Fading)

★ Deep Fading event:

→ Recieved signal Power $\leq$ Noise Power

$\therefore |h|^2 a^2 \leq \frac{N_0}{2}$, $\because SNR = \frac{a^2}{N_0/2} = \frac{2a^2}{N_0}$

$\therefore |h|^2 \leq \frac{N_0}{2a^2} \leq \frac{1}{SNR}$

→ What is the propability of that happining?

$P\{|h|^2 \leq \frac{1}{SNR}\}$, to find this propability we need to know the distribution (PDF) of $|h|^2$, we know the dist. of $|h|$

$|h| \sim Rayleigh(\sigma^2) \xrightarrow{\therefore \text{from Math}} |h|^2 \sim exponential(\lambda)$ $(\lambda = \frac{\sigma^2}{2})$

$\therefore PDF = f_{|h|^2}(x) = \lambda e^{-\lambda x} = \frac{\sigma^2}{2} e^{-\frac{\sigma^2}{2} x}$ $x \geq 0$

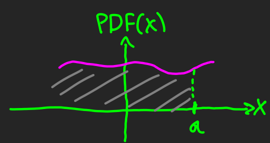
$\therefore P_{df}\{|h|^2 \leq \frac{1}{SNR}\} = \int_0^{\frac{1}{SNR}} \frac{\sigma^2}{2} e^{-\frac{\sigma^2}{2} x} dx = 1 - e^{-\frac{\sigma^2}{2SNR}}$

$\therefore P_{df}\{|h|^2 \leq \frac{1}{SNR}\} = 1 - e^{-\frac{\sigma^2}{2SNR}}$ ※

★ in general for any PDF the

$P\{X < a\} = \int_{-\infty}^{a} PDF(X)$

$\equiv$ Area from $-\infty$ to a



⋆ Coherence Time (N_c)

→ the time interval/Number of symbols at which |h| is constant.

→ In all channels the noise n[m] is random at each time instant, h[m] is random too but it can remain at constant Value for some time interval based on the channel type.

Deterministic

$N_c = \infty$

- h[m] is constant for all symbols
- $N_c >> T_s$

Slow Fading

$N_c \uparrow\uparrow\uparrow$

- h[m] is constant for a number of symbols
- $N_c > T_s$
- ex: $N_c = 10$, h[m] change each 10 symbols "Constant for those 10 symbols"

Fast Fading

$N_c \downarrow\downarrow\downarrow$

- h[m] isn't constant for 2 symbols, sometimes even for 1 symbol.
- $N_c < T_s$

→ Note that even $N_c = 10$ is considered small time, because data is sent at high rate 1M symbols/sec for example so $N_c = 10$ meaning h is constant for 10 symbols, that's $\frac{10}{1000000}$ sec which is 10 msec.

$\therefore y[m] = h\, x[m] + n[m]$, for $m = 0, 1, 2, \ldots, N_c$

→ Now hence h is constant for some time N_c, we can estimate it at the reciever by sending a known symbol between the Tx & Rx, this known symbol is called Pilot P.

$\therefore y[m] = h P + n[m]$

$\therefore \hat{h} = \frac{y[m]}{P}$, $\hat{h}$ won't be exactly h because of the noise added in y[m].

→ We will assume that the reeiver will always have knowledge of h, because the transmitter sends the pilot periodcly.

→ The transmitter may or may not know h, the Tx will know h if the Rx acts as a Tx and sends pilot, but this is costly.

⭑ SISO with Channel Fading

$y = hx + n$

→ For a given h:

$$P_e(h) \le K \cdot Q\left(\frac{d_{min}}{\sqrt{2N_o}}\right)$$

∴ $d_{min} = 2|h|\,a$ in BPSK

$-ah \quad\quad ah$

$Q\left(\sqrt{\frac{4|h|^2 a^2}{2N_o}}\right)$, $SNR = \frac{a^2}{N_o/2}$

∴ $P_e(h) \le K \cdot Q\left(\sqrt{|h|^2 SNR}\right)$

$Q\left(\sqrt{|h|^2 SNR}\right)$

→ Last expression is the P_e for a given h, we are intersted in the average performance $\mathbb{E}\{P_e\}$

$$\therefore P_{e_{avg}} = \mathbb{E}_{|h|^2} P_e(h) = \int P_e(x)\, PDF(|h|^2)\, dx$$

$$\therefore P_{e_{avg}} = \int P_e(x)\, f_{|h|^2}(x)\, dx = \int KQ(2x \cdot SNR)\, f_{|h|^2}(x)\, dx$$

$$\therefore P_{e_{avg}} = K\left[\frac{1}{2} - \frac{1}{2}\sqrt{\frac{SNR}{1+SNR}}\right] \quad \#$$

→ P_e has Linear relation with SNR at high values of SNR, which is not desirble.

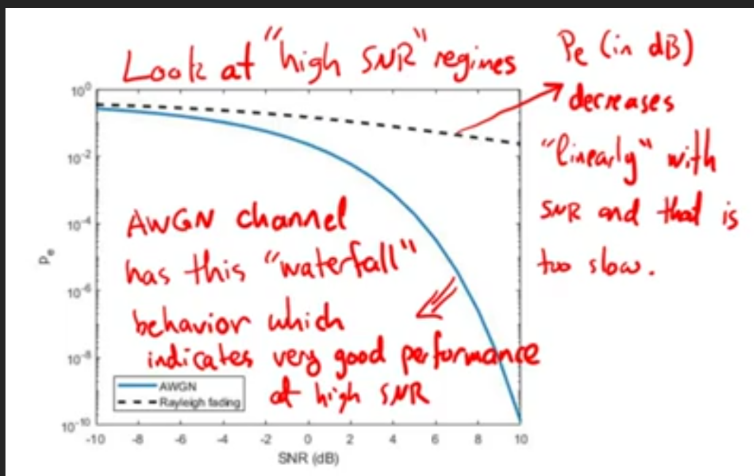
→ this is due to that deep fading probability isn't small.

$\therefore P_{df}\left\{|h|^2 \le \frac{1}{SNR}\right\} = 1 - e^{-\frac{\sigma^2}{2SNR}}$ $\quad$ ∴ $e^{-x} = 1 - x$ if $x >>$ "taylor"

@ high SNR regions:

$$\therefore P_{df}\left\{|h|^2 \le \frac{1}{SNR}\right\} \simeq 1 - \left[1 - \frac{\sigma^2}{2SNR}\right] \simeq \frac{\sigma^2}{2SNR}$$

$\therefore P_{df}\{\dots\} \simeq Const - SNR_{dB}$ "Linear with SNR"



→ Note that we concluded the Linearity with SNR from deep fading prob., we should'de done it from P_e.

but it's easier with deep fading prob and same result will be obtained from both.

→ Next we will enhance the P_e by changing the slope α "Const $- \alpha$ SNR" with MIMO setup.