Power transmited :

→ the Power transmited along the waveguide Can be obtained as $W_T = \iint P_z \, ds$

→ where P_z is the Power density along the Z-direction "direction of Propagation"

$$\begin{bmatrix} \hat{x} & \hat{y} & \hat{z} \\ E_x & E_y & E_z \\ H_x^* & H_y^* & H_z^* \end{bmatrix}$$

$$P_z = \frac{1}{2} Re\left\{ \vec{E} \times \vec{H^*} \right\} = \frac{1}{2} Re\left\{ E_x H_y^* - E_y H_x^* \right\} \quad watt/m^2$$

$$\therefore P_z = \frac{1}{2} \eta \left(|H_y|^2 + |H_x|^2 \right)$$

where η is η_{TM} or η_{TE} $\rightsquigarrow \therefore W_T = \frac{1}{2}\eta \int_0^b \int_0^a (H_y^2 + H_x^2) \cdot dx\,dy$

Ex: Find the transmitted Power in a rectangular waveguide For a TM_{mn} mode :

$H_z = 0$, $E_z = E_o \sin(\frac{m\pi}{a} x) \sin(\frac{n\pi}{b} y)\, e^{-j\beta z}$

$$\begin{vmatrix} E_x \\ H_y \\ E_y \\ H_x \end{vmatrix} = \frac{1}{K_c^2} \begin{vmatrix} -j\beta & -j\omega\mu & 0 & 0 \\ -j\omega\epsilon & -j\beta & 0 & 0 \\ 0 & 0 & -j\beta & j\omega\mu \\ 0 & 0 & j\omega\epsilon & -j\beta \end{vmatrix} \times \begin{vmatrix} \frac{\partial E_z}{\partial x} \\ \frac{\partial H_z}{\partial y} \\ \frac{\partial E_z}{\partial y} \\ \frac{\partial H_z}{\partial x} \end{vmatrix}$$

$W_T = \frac{1}{2}\eta_{TM} \int_0^b \int_0^a (H_x^2 + H_y^2) \cdot dx\,dy$

$\therefore H_x = j\omega\epsilon/K_c^2 \cdot \frac{\partial E_z}{\partial y} = j\omega\epsilon/K_c^2 \cdot E_o \frac{n\pi}{b} \cdot \sin(\frac{m\pi}{a} x) \cos(\frac{n\pi}{b} y)\, e^{-j\beta z}$

$\therefore H_y = -j\omega\epsilon/K_c^2 \cdot \frac{\partial E_z}{\partial x} = -j\omega\epsilon/K_c^2 \cdot E_o \frac{m\pi}{a} \cdot \cos(\frac{m\pi}{a} x) \sin(\frac{n\pi}{b} y)\, e^{-j\beta z}$

$$\therefore W_T = \frac{1}{2}\eta_{TM} \int_0^b \int_0^a \Big[\left(\frac{\omega\epsilon}{K_c^2}\right)^2 (E_o)^2 \left(\frac{n\pi}{b}\right)^2 \sin^2(\frac{m\pi}{a} x) \cos^2(\frac{n\pi}{b} y)\, dx\,dy \Big]$$
$$+ \Big[\left(\frac{\omega\epsilon}{K_c^2}\right)^2 (E_o)^2 \left(\frac{m\pi}{a}\right)^2 \cos^2(\frac{m\pi}{a} x) \sin^2(\frac{n\pi}{b} y)\, dx\,dy \Big]$$

$\star \int_0^a \sin^2(\ldots x)\, dx = \frac{a}{2}$

$\star \int_0^b \cos^2(\ldots y)\, dy = \frac{b}{2}$

$$\therefore W_T = \frac{1}{2}\eta_{TM} \left(\frac{\omega\epsilon}{K_c^2}\right)^2 (E_o)^2 \left[\left(\frac{n\pi}{b}\right)^2 \left(\frac{a}{2}\right)\left(\frac{b}{2}\right) + \left(\frac{m\pi}{a}\right)^2 \left(\frac{a}{2}\right)\left(\frac{b}{2}\right) \right]$$

$$\therefore W_T = \frac{1}{2}\eta_{TM} \left(\frac{\omega\epsilon}{K_c^2}\right)^2 (E_o)^2 \left(\frac{a}{2}\right)\left(\frac{b}{2}\right) \left[\left(\frac{n\pi}{b}\right)^2 + \left(\frac{m\pi}{a}\right)^2 \right]$$

$$\therefore W_T = \frac{ab}{8}\eta_{TM} \left(\frac{\omega\epsilon}{K_c^2}\right)^2 (E_o)^2 (K_c^2)$$

$$\therefore \boxed{W_T = \frac{ab}{8}\eta_{TM} \left(\frac{\omega\epsilon}{K_c}\right)^2 (E_o)^2} \quad watt$$

Ex: Find W_T For a TE_{mn} mode:

TE_{mn}: $H_z = H_o \cos(\frac{m\pi}{a}x) \cos(\frac{n\pi}{b}y)\, e^{-j\beta z}$, $E_z = 0$

From the matrix: $H_x = \frac{j\beta}{K_c^2} H_o \left(\frac{m\pi}{a}\right) \sin(\frac{m\pi}{a}x) \cos(\frac{n\pi}{b}y)\, e^{-j\beta z}$

$H_y = \frac{j\beta}{K_c^2} H_o \left(\frac{n\pi}{b}\right) \cos(\frac{m\pi}{a}x) \sin(\frac{n\pi}{b}y)\, e^{-j\beta z}$

$$W_T = \frac{1}{2}\eta_{TE} \int_0^a \int_0^b \Big[\left(\frac{\beta}{K_c^2}\right)^2 |H_o|^2 \left(\frac{m\pi}{a}\right)^2 \sin^2(\tfrac{m\pi}{a}x) \cos^2(\tfrac{n\pi}{b}y)\, dx\,dy$$
$$+ \left(\frac{\beta}{K_c^2}\right)^2 |H_o|^2 \left(\frac{n\pi}{b}\right)^2 \cos^2(\tfrac{m\pi}{a}x) \sin^2(\tfrac{n\pi}{b}y)\, dx\,dy \Big]$$

$$W_T = \frac{1}{2}\eta_{TE} \left(\frac{\beta}{K_c^2}\right)^2 |H_o|^2 \cdot \frac{ab}{4} \underbrace{\left(\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 \right)}_{K_c^2}$$

$$W_T = \frac{ab}{8}\eta_{TE} \left(\frac{\beta}{K_c}\right)^2 |H_o|^2 \quad watt$$

$$\therefore \boxed{W_T = \frac{ab}{8}\eta_{TE} \left(\frac{\beta}{K_c}\right)^2 (H_o)^2} \quad watt$$

Ex: Find W_T For dominate Mode TE_{10} : $m = 1$, $n = 0$

$E_z = 0$, $H_z = H_o \cos(\frac{\pi}{a} x)\, e^{-j\beta z}$, $H_y = 0$, $K_c^2 = \frac{\pi^2}{a^2}$

$\therefore W_T = \frac{1}{2}\eta \iint (H_x^2 + H_y^2) \cdot dx\,dy$

$\therefore H_x = -j\beta/K_c^2 \cdot \frac{\partial H_z}{\partial x} = +j\beta/K_c^2 \cdot H_o \cdot \frac{\pi}{a} \cdot \sin(\frac{\pi}{a} x)\, e^{-j\beta z}$

$$\therefore W_T = \frac{1}{2}\eta \cdot \left(\frac{\beta}{K_c^2}\right)^2 (H_o)^2 \left(\frac{\pi}{a}\right)^2 \int_0^b \int_0^a \sin^2(\frac{\pi}{a} x)\, dx\,dy$$

$$\therefore W_T = \frac{1}{2}\eta \left(\frac{\beta}{K_c^2}\right)^2 (H_o)^2 \left(\frac{\pi}{a}\right)^2 \left(\frac{a}{2}\right)(b)$$

$$\therefore W_T = \frac{ab}{4}\eta_{TE} (H_o)^2 \left(\frac{\beta}{K_c^2}\right)^2 K_c^2$$

$$\therefore \boxed{W_T = \frac{ab}{4}\eta_{TE} (H_o)^2 \left(\frac{\beta}{K_c}\right)^2}$$

Waveguide losses :

- they are due to :

→ 1) Dielectric losses ($\alpha_d \neq 0$) "not a Perfect dielectric" → Small and can be neglected

→ 2) Wall Losses ($\alpha_{wall} \neq \infty$) "not a Perfect Conductor"

↳ Considered as an attenuation Factor α_w and Can be obtained .

$$\alpha_w = \frac{P_L}{2W_T} \qquad \text{where} \quad P_L = \frac{1}{2} R_s \iint_{\substack{wall \\ area}} |J_s|^2\, ds$$

→ R_s is the waveguide wall resistance (given) , $|J_s|$ is the sheet current density (A/m) and is equal to the tangent magnetic field to the sheet "wall"

→ W_T is the transmitted Power $W_T = \frac{1}{2}\eta \int_0^b \int_0^a (H_x^2 + H_y^2)\, dx\,dy$

Ex: Find α_w For a TE_{mn} mode:

$$\begin{vmatrix} E_x \\ H_y \\ E_y \\ H_x \end{vmatrix} = \frac{1}{K_c^2} \begin{vmatrix} -j\beta & -j\omega\mu & 0 & 0 \\ -j\omega\epsilon & -j\beta & 0 & 0 \\ 0 & 0 & -j\beta & j\omega\mu \\ 0 & 0 & j\omega\epsilon & -j\beta \end{vmatrix} \times \begin{vmatrix} \cancel{\frac{\partial E_z}{\partial x}} \to 0 \\ \frac{\partial H_z}{\partial y} \\ \cancel{\frac{\partial E_z}{\partial y}} \to 0 \\ \frac{\partial H_z}{\partial x} \end{vmatrix}$$

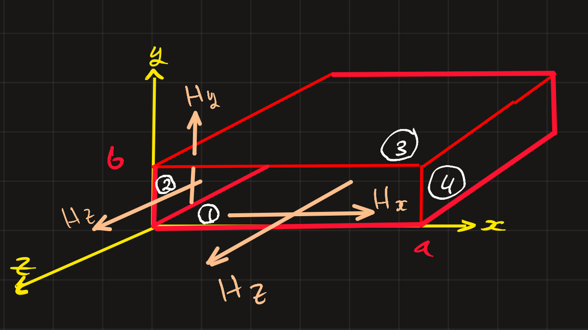
→ $\alpha_w = \frac{P_L}{2W_T}$, $P_L = \frac{1}{2} R_s \iint |J_s|^2\, ds$, $W_T = \frac{1}{2}\eta \int_0^b \int_0^a (H_x^2 + H_y^2)$

→ $E_z = 0$, $H_z = H_o \cos(\frac{\pi m}{a} x) \cos(\frac{\pi n}{b} y)\, e^{-j\beta z}$

→ $H_x = +j\beta/K_c^2 \cdot H_o \cdot \frac{\pi m}{a} \sin(\frac{\pi m}{a} x) \cos(\frac{\pi n}{b} y)\, e^{-j\beta z}$

→ $H_y = -j\beta/K_c^2 \cdot H_o \cdot \frac{\pi n}{b} \sin(\frac{\pi n}{b} y) \cos(\frac{\pi m}{a} x)\, e^{-j\beta z}$

→ $\boxed{W_T = \frac{ab}{8}\eta_{TE} (H_o)^2 \left(\frac{\beta}{K_c}\right)^2}$ ① $\quad \eta_{TE} = \frac{\eta}{\sqrt{(1 - F_c/F)^2}}$, $\eta = \sqrt{\mu/\epsilon}$



- For P_L , the J_s is due the Magnetic Field tangent to the sheet Surface "the wall"

$\star$ $P_L = P_1 + P_2 + P_3 + P_4$, $P_1 = P_3$ $\quad P_2 = P_4$

$\therefore P_L = 2(P_1 + P_2)$

$$\rightarrow P_{L_1} = \frac{1}{2} R_s \int_0^1 \int_0^a \left(|H_x|^2_{y=0} + |H_z|^2_{y=0} \right) dx\,dz$$
$$= \frac{1}{2} R_s \iint \left\{ \left(\frac{\beta}{K_c^2}\right)^2 (H_o)^2 \left(\frac{\pi m}{a}\right)^2 \sin^2(\frac{\pi m}{a} x) + (H_o)^2 \cos^2(\frac{\pi m}{a} x) \right\} dx\,dz$$
$$= \frac{1}{2} R_s (H_o)^2 \left[\left(\frac{\beta}{K_c^2}\right)^2 \left(\frac{\pi m}{a}\right)^2 \left(\frac{a}{2}\right) + \left(\frac{a}{2}\right) \right]$$

$$\therefore \boxed{P_{L_1} = \frac{1}{2} R_s (H_o)^2 \left(\frac{a}{2}\right) \left[1 + \left(\frac{\beta}{K_c^2}\right)^2 \left(\frac{\pi m}{a}\right)^2 \right]} \quad ②$$

$$\star\ P_{L_2} = \frac{1}{2} R_s \int_0^1 \int_0^b \left(|H_z|^2_{x=0} + |H_y|_{x=0} \right) dy\,dz$$
$$= \frac{1}{2} R_s (H_o)^2 \int_0^1 \int_0^b \left[\cos^2(\frac{\pi n}{b} y) + \left(\frac{\beta}{K_c^2}\right)^2 \left(\frac{\pi n}{b}\right)^2 \sin^2(\frac{\pi n}{b} y) \right] dy\,dz$$
$$= \frac{1}{2} R_s (H_o)^2 \left[\frac{b}{2} + \left(\frac{\beta}{K_c^2}\right)^2 \left(\frac{\pi n}{b}\right)^2 \left(\frac{b}{2}\right) \right]$$

$$\therefore \boxed{P_{L_2} = \frac{1}{2} R_s (H_o)^2 \left(\frac{b}{2}\right) \left[1 + \left(\frac{\beta}{K_c^2}\right)^2 \left(\frac{\pi n}{b}\right)^2 \right]} \quad ③$$

$$\rightarrow \therefore \alpha_w = \frac{2(P_{L_1} + P_{L_2})}{2W_T} = \checkmark$$

Ex: Find α_w For a TM_{mn} mode :

$H_z = 0$

$E_z = E_o \sin(\frac{m\pi}{a}x) \sin(\frac{n\pi}{b}y)\, e^{-j\beta z}$

From the matrix:

$H_x = \frac{j\omega\epsilon}{K_c^2} \left(\frac{n\pi}{b}\right) E_o \sin(\frac{m\pi}{a}x) \cos(\frac{n\pi}{b}y)\, e^{-j\beta z}$

$H_y = \frac{-j\omega\epsilon}{K_c^2} \left(\frac{m\pi}{a}\right) E_o \cos(\frac{m\pi}{a}x) \sin(\frac{n\pi}{b}y)\, e^{-j\beta z}$

$P_L = 2(P_{L_1} + P_{L_2})$

$P_{L_1} = \frac{1}{2} R_s \int_0^a \int_0^1 |H_x|^2_{y=0}\, dx\,dz$

$= \frac{1}{2} R_s \int_0^a \int_0^1 \left(\frac{\omega\epsilon}{K_c^2}\right)^2 |E_o|^2 \left(\frac{n\pi}{b}\right)^2 \sin^2(\frac{m\pi}{a}x)\, dx\,dz$

$P_{L_1} = \frac{1}{2} R_s \frac{(\omega\epsilon)^2}{K_c^4} |E_o|^2 \left(\frac{n\pi}{b}\right)^2 \cdot \frac{a}{2}$

$P_{L_2} = \frac{1}{2} R_s \int_0^b \int_0^1 |H_y|^2_{x=0}\, dy\,dz$

$= \frac{1}{2} R_s \int_0^b \int_0^1 \left(\frac{\omega\epsilon}{K_c^2}\right)^2 |E_o|^2 \cdot \left(\frac{m\pi}{a}\right)^2 \sin^2(\frac{n\pi}{b}y)\, dy\,dz$

$= \frac{1}{2} R_s \frac{(\omega\epsilon)^2}{K_c^4} |E_o|^2 \left(\frac{m\pi}{a}\right)^2 \cdot \frac{b}{2}$

$P_L = 2(P_{L_1} + P_{L_2})$

W_T is obtained before. $\left(W_T = \frac{ab}{8}\eta_{TM} \frac{(\omega\epsilon)^2}{K_c^2} |E_o|^2 \right)$

$$\therefore \alpha_w = \frac{2(P_{L_1} + P_{L_2})}{2W_T} = \checkmark$$

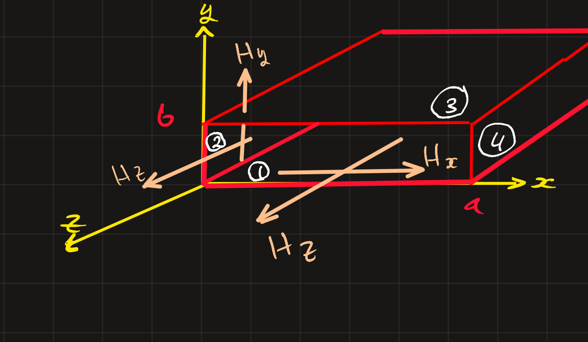
Ex: Find α_w for the dominate mode (TE_{10}) :

- $E_z = 0$, $H_z = H_o \cos(\frac{\pi}{a} x)\, e^{-j\beta z}$, $H_y = 0$

→ $\alpha_w = \frac{P_L}{2W_T}$, $P_L = \frac{1}{2} R_s \iint |J_s|^2\, ds$, $W_T = \frac{1}{2}\eta \int_0^b \int_0^a H_x^2\, dx\,dy$

- $H_x = +j\beta/K_c^2 \cdot H_o \cdot \left(\frac{\pi}{a}\right) \sin(\frac{\pi}{a} x)\, e^{-j\beta z}$

$\therefore \boxed{W_T = \frac{ab}{4}\eta_{TE_{10}} (H_o)^2 \left(\frac{\beta}{K_c^2}\right)^2}$ ① $\quad \eta_{TE_{10}} = \frac{\eta}{\sqrt{(1 - F_c/F)^2}}$, $\eta = \sqrt{\mu/\epsilon}$



- $P_L = P_{L_1} + P_{L_2} + P_{L_3} + P_{L_4}$, $P_{L_1} = P_{L_3}$ $\quad P_{L_2} = P_{L_4}$

$\therefore P_L = 2(P_{L_1} + P_{L_2})$

$$\rightarrow P_{L_1} = \frac{1}{2} R_s \int_0^1 \int_0^a \left(|H_x|^2_{y=0} + |H_z|_{y=0} \right) dx\,dz$$
$$= \frac{1}{2} R_s \cdot H_o^2 \int_0^1 \int_0^a \left[\left(\frac{\beta}{K_c^2}\right)^2 \left(\frac{\pi}{a}\right)^2 \sin^2(\frac{\pi}{a} x) + \cos^2(\frac{\pi}{a} x) \right] dx\,dz$$
$$= \frac{1}{2} R_s \cdot H_o^2 \left[\left(\frac{\beta}{K_c^2}\right)^2 \left(\frac{\pi}{a}\right)^2 \left(\frac{a}{2}\right) + \frac{a}{2} \right]$$

$$\therefore \boxed{P_{L_1} = \frac{1}{2} R_s \cdot H_o^2 \cdot \left(\frac{a}{2}\right) \left[1 + \left(\frac{\beta}{K_c}\right)^2 \right]} \qquad K_c^2 = \left(\frac{\pi}{a}\right)^2$$

$$\rightarrow P_{L_2} = \frac{1}{2} R_s \int_0^1 \int_0^b \left(|H_z|^2_{x=0} \right) dy\,dz$$
$$= \frac{1}{2} R_s \cdot H_o^2 \int_0^1 \int_0^b dy\,dz$$

$$\therefore \boxed{P_{L_2} = \frac{1}{2} R_s (H_o)^2 \cdot b}$$

$$\therefore \alpha_w = \frac{2(P_{L_1} + P_{L_2})}{2W} = \checkmark$$