Lab2

Control

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Section: 7

Step A, Closed-loop Transfer Function

Theoretical:

$$\star\ J\ddot{\theta} + B\dot{\theta} = T_c \quad , \therefore T_c = K(\theta_r - \theta)$$

$$\therefore J\ddot{\theta} + B\dot{\theta} = K(\theta_r - \theta)$$

$$\therefore J\theta(s)s^2 + B\theta(s)s = K\theta_r - K\theta(s)$$

$$\therefore \underset{o/p}{\theta(s)}\left[Js^2 + Bs + K\right] = K\underset{i/p}{\theta_r}$$, we need θ to track θ_r

$$\therefore \boxed{T.F = \frac{\theta(s)}{\theta_r} = \frac{K}{Js^2 + Bs + K} \ \#}$$ Second order system.

Matlab:

```
%------------a)----------------%
j = 600000;
b = 20000;
k = 1;

TF = tf(k, [j b k])


```

```
TF =

              1
  ------------------------
  600000 s^2 + 20000 s + 1

Continuous-time transfer function.
```

Step B, state-space representation

Matlab:

```
%-------------b)------------------%
k=1;
State_Space = ss(TF)
```

```
State_Space =

  A =
                x1         x2
   x1    -0.03333  -0.001707
   x2   0.0009766          0

  B =
            u1
   x1  0.03125
   x2        0

  C =
            x1       x2
   y1        0  0.05461

  D =
       u1
   y1   0

Continuous-time state-space model.
```

Step C, Max value of K to have a stable system

Theoretical:

C)

$J = 600 * 10^3$, $B = 20 * 10^3$

$\therefore 600 * 10^3 S^2 + 20 * 10^3 S + K = 0$

S^2	$600 * 10^3$	K
S^1	$20 * 10^3$	0
S^0	K	

$\therefore$ System is stable for all Values of $K > 0$

scince there's no upper Limit, $\therefore K_{max} = +\infty$

Step D, Max value of K to have Mp < 10%

Theoretical:

D)

K_{max} For $M_p < 0.1$

$\because J s^2 + B s + K = 0$

$\therefore s^2 + \frac{B}{J} s + \frac{K}{J} = 0$

$\therefore 2 \zeta \omega_n = \frac{B}{J}$, $\therefore \omega_n = \sqrt{\frac{K}{J}}$

$\therefore \frac{B}{2 J \zeta} = \sqrt{\frac{K}{J}}$

$\therefore K = J \left(\frac{B}{2 J \zeta} \right)^2$

$\because M_p = e^{\frac{-\pi \zeta}{\sqrt{1-\zeta^2}}} = 0.1$

$\therefore \zeta = 0.59$

$\therefore K = 600 \times 10^3 \left(\frac{20}{2 * 600 * 0.59} \right)^2$

$\boxed{\therefore K_{max} = 479}$

Step E, Max value of K to have rise time < 80 sec

Theoretical:

e)

Values of K to provide rise time < 80 sec

$$\therefore t_r = \frac{\pi - \cos^{-1}(\zeta)}{\omega_n \sqrt{1-\zeta^2}} < 80$$

$$\therefore \frac{\pi - \cos^{-1}(\zeta)}{\frac{B}{2J\zeta}\sqrt{1-\zeta^2}} < 80$$

$$\therefore \zeta = 0.53$$

$$\boxed{\therefore K > 592}$$

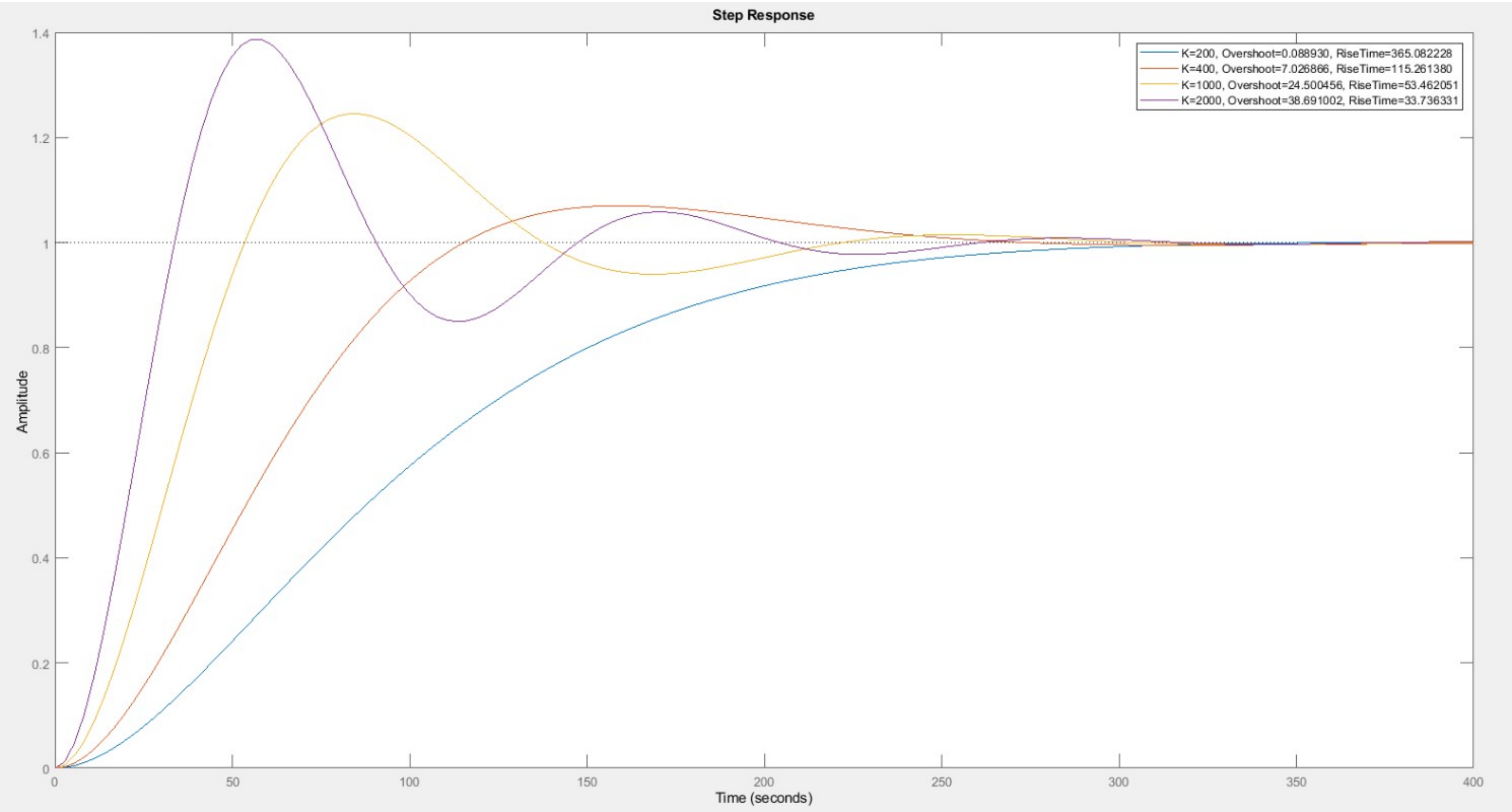
Step F, Plot step response for K=200,400,1000,2000

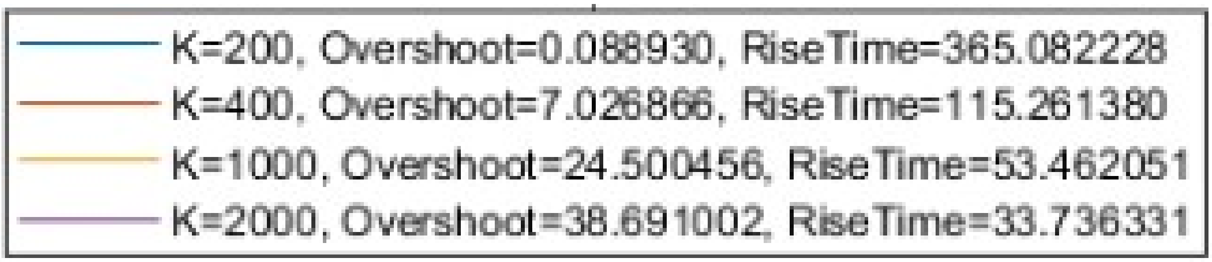
Matlab code

```
%------------f)-----------------%
% as K increases, overshoot increses and rise time decreases
k = 200;
TF1 = tf(k, [j b k]);
info1 = stepinfo(TF1 , 'RiseTimeThreshold', [0 1]);
legend1 = compose('K=200, Overshoot=%f, RiseTime=%f', info1.Overshoot, info1.RiseTime);

k = 400;
TF2 = tf(k, [j b k]);
info2 = stepinfo(TF2, 'RiseTimeThreshold', [0 1]);
legend2 = compose('K=400, Overshoot=%f, RiseTime=%f', info2.Overshoot, info2.RiseTime);

k = 1000;
TF3 = tf(k, [j b k]);
info3 = stepinfo(TF3, 'RiseTimeThreshold', [0 1]);
legend3 = compose('K=1000, Overshoot=%f, RiseTime=%f', info3.Overshoot, info3.RiseTime);

k = 2000;
TF4 = tf(k, [j b k]);
info4 = stepinfo(TF4 , 'RiseTimeThreshold', [0 1]);
legend4 = compose('K=2000, Overshoot=%f, RiseTime=%f', info4.Overshoot, info4.RiseTime);

figure(1);
step(TF1, TF2, TF3 , TF4);
legend(string(legend1) , string(legend2), string(legend3) , string(legend4));
```

Step F, Plot step response for K=200,400,1000,2000

Matlab output image





Do the plots to confirm your calculations in previous parts ?
-Yes, we see that as K increases Rise time decreases confirming our theoretical solution in step e.

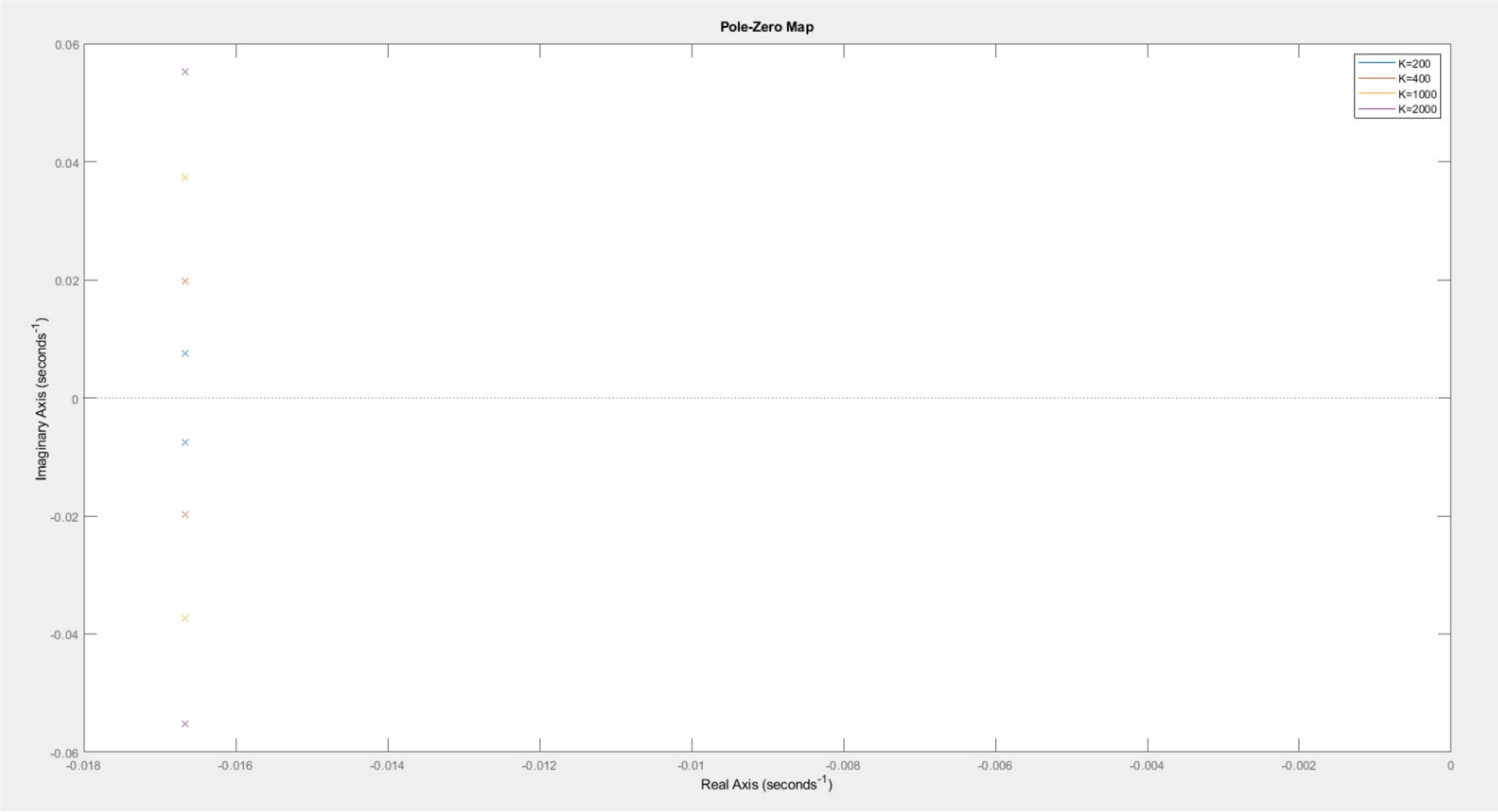
Step G, Plot poles and zeros for K=200,400,1000,2000

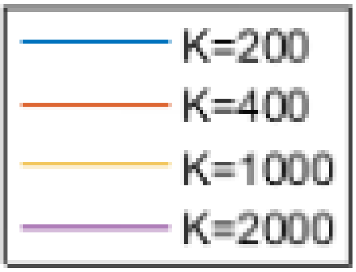
Matlab code

```
%------------g)----------------%
% as we increase K, the poles location increases verticlly
figure(2);
pzplot(TF1, TF2, TF3 , TF4)
legend('K=200', 'K=400', 'K=1000', 'K=2000')
%grid on

```

Step G, Plot step response for K=200,400,1000,2000

Matlab output image





Effect of K on the closed loop zeros and poles?
- As K increases, system poles location increases vertically, Making the system have more oscillating behavior.

Step H, find the steady state error for K=200,400,1000,2000

Theoretical:

h)

Find steady state error for each K

- Step input

$$\rightarrow \theta(s) = T.F. \cdot \frac{1}{s}$$

$$\therefore \theta(s) = \frac{K}{s(Js^2+Bs+K)}$$

$$\text{error} = \theta(\overset{t}{\infty}) - 1$$

$$\theta(\overset{t}{\infty}) = \lim_{s\to 0} s\,\theta(s)$$

$$\therefore \theta(\infty) = \lim_{s\to 0} \frac{K}{Js^2+Bs+K} = \frac{K}{K} = 1$$

$$\boxed{\therefore \text{error} = 1 - 1 = 0}$$

Matlab:

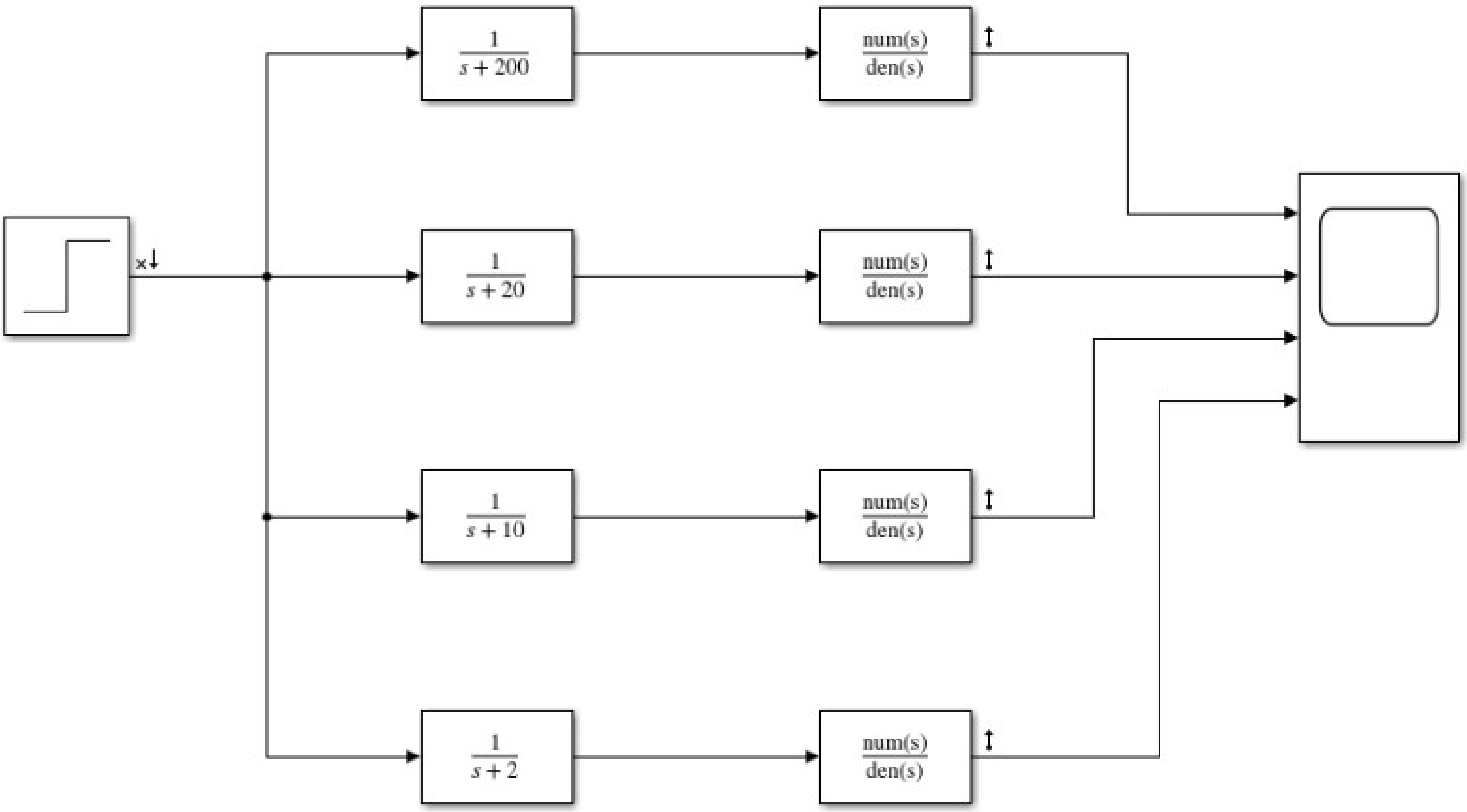
```
%------------h)-------------------%
% error is very close to zero, confirming the theortical solution
[theta1 t] = step(TF1);
ess1 = 1 - theta1(end);

[theta2 t] = step(TF2);
ess2 = 1 - theta2(end);

[theta3 t] = step(TF3);
ess3 = 1 - theta3(end);

[theta4 t] = step(TF4);
ess4 = 1 - theta4(end);
```

```
ess1 =

  -5.1400e-04


ess2 =

    0.0049
```

```
ess3 =

    0.0033


ess4 =

  -8.7153e-04
```

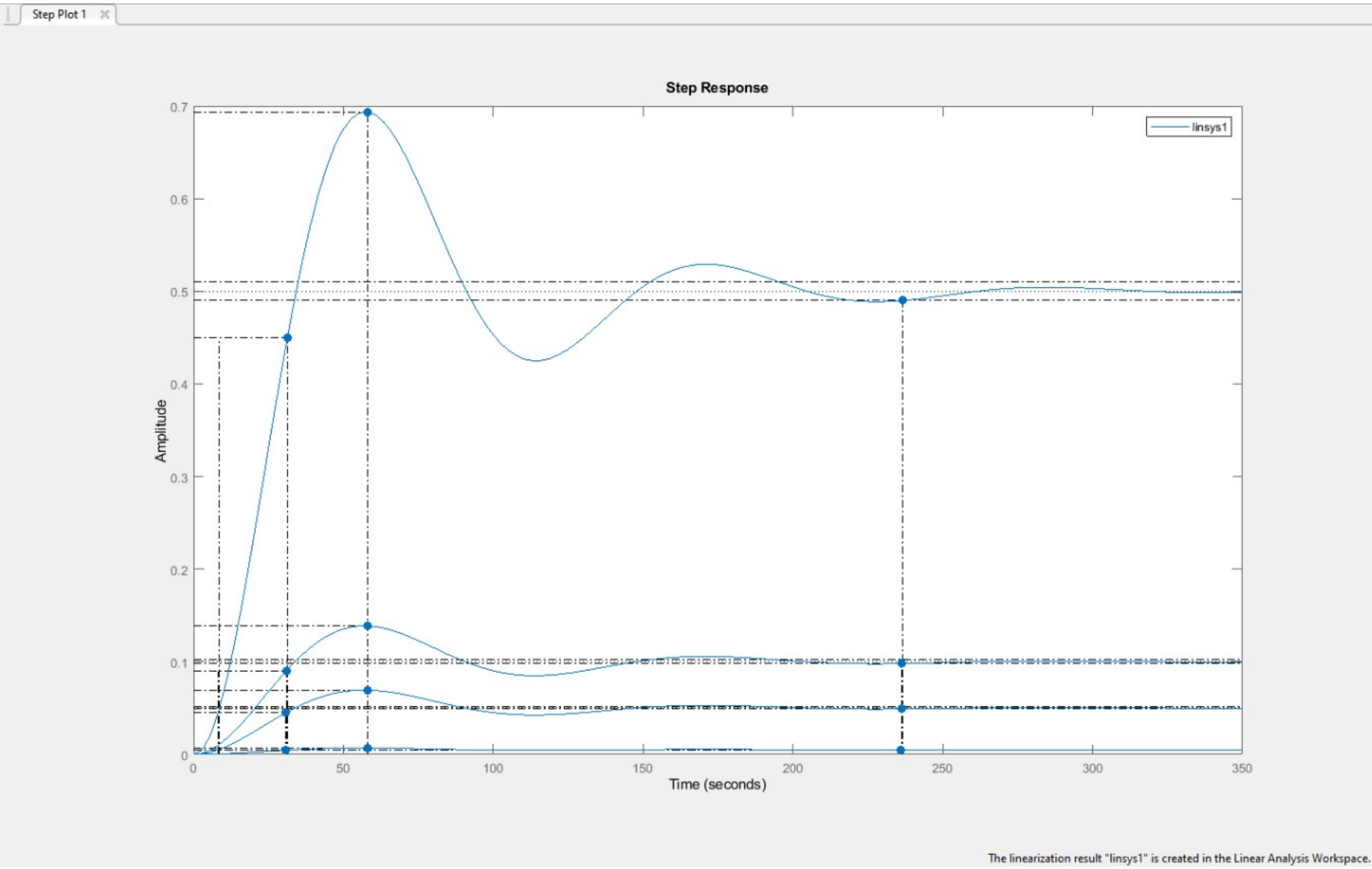
Step I, Using Simulink add poles at -200, -20, -10, -2

Simulink Blocks Image



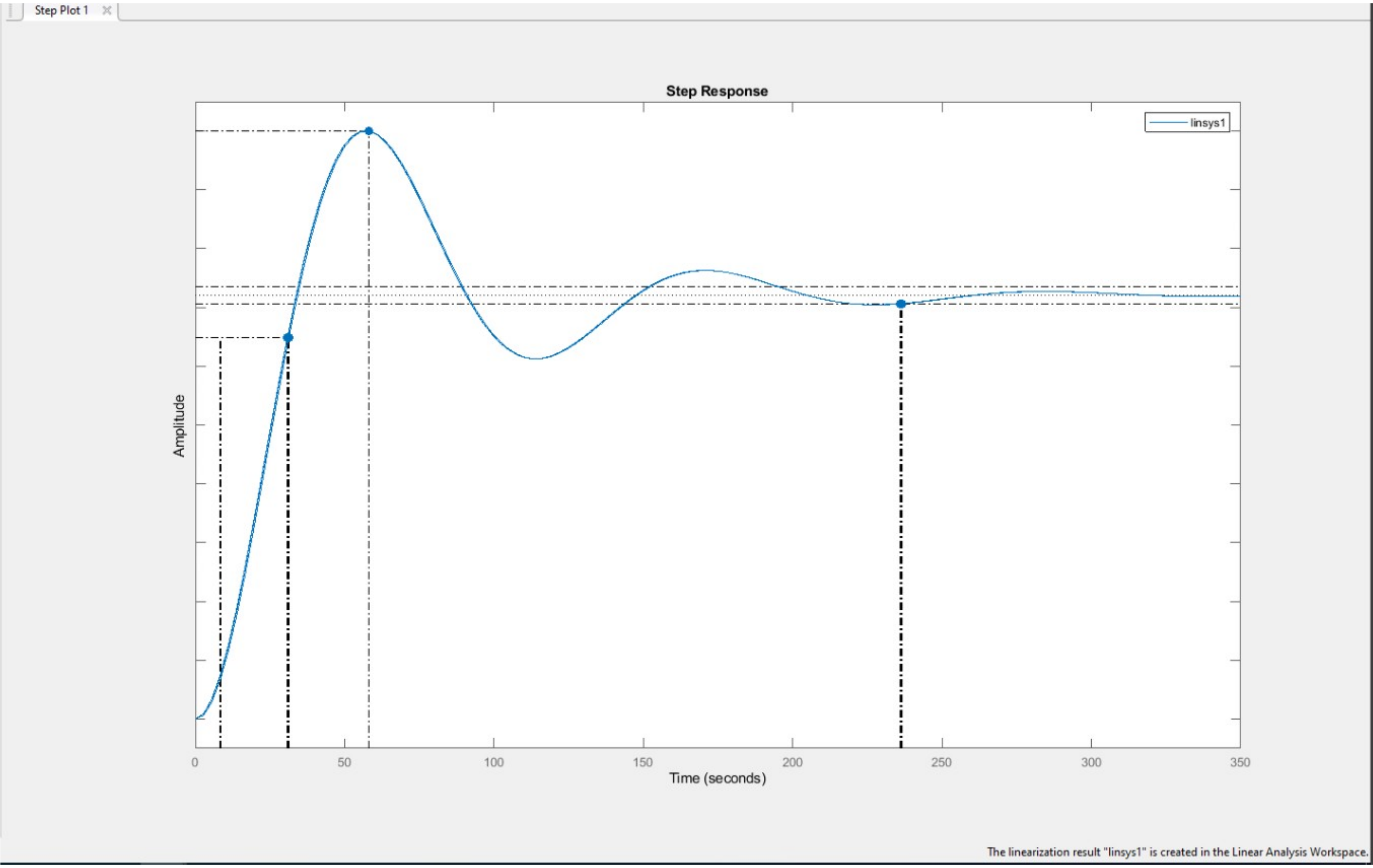
Step I, Using Simulink add poles at -200, -20, -10, -2

Step response of the 4 responses with rise time, overshoot, settling time and peak time.



Step I, Using Simulink add poles at -200, -20, -10, -2

Step response of the 4 responses normalized with rise time, overshoot, settling time and peak time.



Step J, Discuss the effect upon the transient response of the proximity of a higher-order pole to the second-order system

In general the effect of adding poles is makes the step response slower, in our case the original system poles are at -0.017 on the real axis, hence making them the dominant poles, so the response nearly doesn't change because the smallest added pole is 117 time larger than the original system poles.