• Power transmited and losses:

→ the Power transmited along the waveguide Can be obtained as $W_T = \iint P_z \, ds$

→ where P_z is the Power density along the Z-direction "direction of Propagation"

$$\begin{bmatrix} \hat{x} & \hat{y} & \hat{z} \\ E_x & E_y & E_z \\ H_x^* & H_y^* & H_z^* \end{bmatrix}$$

• $P_z = \frac{1}{2} Re \left\{ \vec{E} \times \vec{H^*} \right\} = \frac{1}{2} Re \left\{ E_x H_y^* - E_y H_x^* \right\}$ watt / m²

$\therefore P_z = \frac{1}{2} \eta \left(|H_y|^2 + |H_x|^2 \right)$

where η is η_{TM} or η_{TE} $\rightarrow \therefore W_T = \frac{1}{2} \eta \int_0^b \int_0^a \left(H_y^2 + H_x^2 \right) \cdot dx\,dy$

• Ex: Find the transmitted Power in a rectangular waveguide For a TM_{mn} mode:

• $H_z = 0$, $E_z = E_0 \sin\left(\frac{m\pi}{a} x\right) \sin\left(\frac{n\pi}{b} y\right) e^{-j\beta z}$

• $W_T = \frac{1}{2} \eta_{TM} \int_0^b \int_0^a \left(H_x^2 + H_y^2 \right) \cdot dx\,dy$

$$\begin{bmatrix} E_x \\ H_y \\ E_y \\ H_x \end{bmatrix} = \frac{1}{k_c^2} \begin{bmatrix} -j\beta & -j\omega\mu & 0 & 0 \\ -j\omega\epsilon & -j\beta & 0 & 0 \\ 0 & 0 & -j\beta & j\omega\mu \\ 0 & 0 & j\omega\epsilon & -j\beta \end{bmatrix} \times \begin{bmatrix} \frac{\partial E_z}{\partial x} \\ \frac{\partial H_z}{\partial y} \\ \frac{\partial E_z}{\partial y} \\ \frac{\partial H_z}{\partial x} \end{bmatrix}$$

$\therefore H_x = j\omega\epsilon / k_c^2 \cdot \frac{\partial E_z}{\partial y} = j\omega\epsilon / k_c^2 \cdot E_0 \frac{n\pi}{b} \cdot \sin\left(\frac{m\pi}{a} x\right) \cos\left(\frac{n\pi}{b} y\right) e^{-j\beta z}$

$\therefore H_y = -j\omega\epsilon / k_c^2 \cdot \frac{\partial E_z}{\partial x} = -j\omega\epsilon / k_c^2 \cdot E_0 \frac{m\pi}{a} \cdot \cos\left(\frac{m\pi}{a} x\right) \sin\left(\frac{n\pi}{b} y\right) e^{-j\beta z}$

$\therefore W_T = \frac{1}{2} \eta_{TM} \int_0^b \int_0^a \left[\left(\frac{\omega\epsilon}{k_c^2}\right)^2 (E_0)^2 \left(\frac{n\pi}{b}\right)^2 \sin^2\left(\frac{m\pi}{a} x\right) \cos^2\left(\frac{n\pi}{b} y\right) dx\,dy \right]$

$+ \left[\left(\frac{\omega\epsilon}{k_c^2}\right)^2 (E_0)^2 \left(\frac{m\pi}{a}\right)^2 \cos^2\left(\frac{m\pi}{a} x\right) \sin^2\left(\frac{n\pi}{b} y\right) dx\,dy \right]$

* $\int_0^a \sin^2(\cdots x)\, dx = \frac{a}{2}$
* $\int_0^b \cos^2(\cdots y)\, dy = \frac{b}{2}$

$\therefore W_T = \frac{1}{2} \eta_{TM} \left(\frac{\omega\epsilon}{k_c^2}\right)^2 (E_0)^2 \left[\left(\frac{n\pi}{b}\right)^2 \left(\frac{a}{2}\right)\left(\frac{b}{2}\right) + \left(\frac{m\pi}{a}\right)^2 \left(\frac{a}{2}\right)\left(\frac{b}{2}\right) \right]$

$\therefore W_T = \frac{1}{2} \eta_{TM} \left(\frac{\omega\epsilon}{k_c^2}\right)^2 (E_0)^2 \left(\frac{a}{2}\right)\left(\frac{b}{2}\right) \left[\left(\frac{n\pi}{b}\right)^2 + \left(\frac{m\pi}{a}\right)^2 \right]$

$\therefore W_T = \frac{ab}{8} \eta_{TM} \left(\frac{\omega\epsilon}{k_c^2}\right)^2 (E_0)^2 (k_c^2)$

$\therefore \boxed{W_T = \frac{ab}{8} \eta_{TM} \left(\frac{\omega\epsilon}{k_c}\right)^2 (E_0)^2}$ watt

Ex: Find W_T For a TE_{mn} mode:

TE_{mn}: $H_z = H_0 \cos\left(\frac{m\pi}{a} x\right) \cos\left(\frac{n\pi}{b} y\right) e^{-j\beta z}$, $E_z = 0$

From the matrix: $H_x = \frac{j\beta}{k_c^2} H_0 \left(\frac{m\pi}{a}\right) \sin\left(\frac{m\pi}{a} x\right) \cos\left(\frac{n\pi}{b} y\right) e^{-j\beta z}$

$H_y = \frac{j\beta}{k_c^2} H_0 \left(\frac{n\pi}{b}\right) \cos\left(\frac{m\pi}{a} x\right) \sin\left(\frac{n\pi}{b} y\right) e^{-j\beta z}$

$W_T = \frac{1}{2} \eta_{TE} \int_0^a \int_0^b \left[\left(\frac{\beta}{k_c^2}\right)^2 |H_0|^2 \left(\frac{m\pi}{a}\right)^2 \underbrace{\sin^2\left(\frac{m\pi}{a} x\right)}_{a/2} \underbrace{\cos^2\left(\frac{n\pi}{b} y\right)}_{b/2} dx\,dy \right.$

$\left. + \left(\frac{\beta}{k_c^2}\right)^2 |H_0|^2 \left(\frac{n\pi}{b}\right)^2 \underbrace{\cos^2\left(\frac{m\pi}{a} x\right)}_{a/2} \underbrace{\sin^2\left(\frac{n\pi}{b} y\right)}_{b/2} dx\,dy \right]$

$W_T = \frac{1}{2} \eta_{TE} \left(\frac{\beta}{k_c^2}\right)^2 |H_0|^2 \cdot \frac{ab}{4} \underbrace{\left(\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right)}_{k_c^2}$

$W_T = \frac{ab}{8} \eta_{TE} \left(\frac{\beta}{k_c}\right)^2 |H_0|^2$ watt

$\therefore \boxed{W_T = \frac{ab}{8} \eta_{TE} \left(\frac{\beta}{k_c}\right)^2 (H_0)^2}$ watt

• Ex: Find W_T For dominate Mode TE_{10}: $m = 1$, $n = 0$

• $E_z = 0$, $H_z = H_0 \cos\left(\frac{\pi}{a} x\right) e^{-j\beta z}$, $H_y = 0$, $k_c^2 = \frac{\pi^2}{a^2}$

$\therefore W_T = \frac{1}{2} \eta \iint \left(H_x^2 + H_y^2 \right) \cdot dx\,dy$

$\therefore H_x = -j\beta / k_c^2 \cdot \frac{\partial H_z}{\partial x} = +j\beta / k_c^2 \cdot H_0 \cdot \frac{\pi}{a} \cdot \sin\left(\frac{\pi}{a} x\right) e^{-j\beta z}$

$\therefore W_T = \frac{1}{2} \eta \cdot \left(\frac{\beta}{k_c^2}\right)^2 (H_0)^2 \left(\frac{\pi}{a}\right)^2 \int_0^b \int_0^a \sin^2\left(\frac{\pi}{a} x\right) dx\,dy$

$\therefore W_T = \frac{1}{2} \eta \left(\frac{\beta}{k_c^2}\right)^2 (H_0)^2 \left(\frac{\pi}{a}\right)^2 \left(\frac{a}{2}\right) (b)$

$\therefore W_T = \frac{ab}{4} \eta_{TE} (H_0)^2 \left(\frac{\beta}{k_c^2}\right)^2 k_c^2$

$\therefore \boxed{W_T = \frac{ab}{4} \eta_{TE} (H_0)^2 \left(\frac{\beta}{k_c}\right)^2}$

• Waveguide losses:

• they are due to:

→ 1) Dielectric losses $(\sigma_d \neq 0)$ "not a Perfect dielectric" → Small and Can be neglected

→ 2) Wall Losses $(\sigma_{wall} \neq \infty)$ "not a Perfect Conductor"

↳ Considered as an attenuation Factor α_w and Can be obtained.