

* TE Mode

$$\rightarrow E_z = 0$$

$$\rightarrow H_z = [C_1 \sin(Ax) + C_2 \cos(Ax)] [C_3 \sin(By) + C_4 \cos(By)] C_5 e^{-j\beta z}$$

. Boundary Conditions :

→ Boundary Condition won't be applied to H_z because it doesn't equal zero at the boundary.

→ But E_x, E_y does

$$\therefore E_x|_{y=0} = 0, E_x|_{y=b} = 0$$

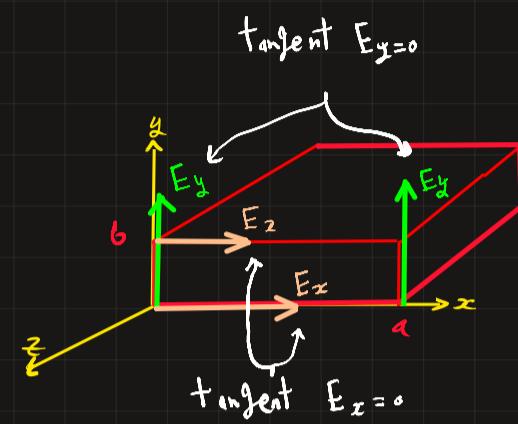
$$, E_y|_{x=0} = 0, E_y|_{x=a} = 0$$

→ From matrix :

$$\cdot E_x = -j\omega\mu/k_c^2 \cdot \frac{\partial H_z}{\partial y} = 0$$

$$\cdot E_y = j\omega\mu/k_c^2 \cdot \frac{\partial H_z}{\partial x} = 0$$

$$\begin{vmatrix} E_x \\ H_y \\ E_y \\ H_x \end{vmatrix} = \frac{1}{k_c^2} \begin{vmatrix} -j\beta & -j\omega\mu & 0 & 0 \\ -j\omega\epsilon & -j\beta & 0 & 0 \\ 0 & 0 & -j\beta & j\omega\mu \\ 0 & 0 & j\omega\epsilon & -j\beta \end{vmatrix} \times \begin{vmatrix} \frac{\partial E_z}{\partial z} \\ \frac{\partial H_z}{\partial y} \\ \frac{\partial E_z}{\partial y} \\ \frac{\partial H_z}{\partial x} \end{vmatrix}$$



$$1. E_y|_{y=0} \Rightarrow \frac{\partial H_z}{\partial x}|_{y=0} = 0$$

$$0 = (AC_1 \cos Ax - C_2 A \sin Ax)(\dots) e^{-j\beta z} \quad \boxed{C_1 = 0}$$

$$H_z = C_2 \cos Ax (\dots) e^{-j\beta z}$$

$$2. E_x|_{y=0} \Rightarrow \frac{\partial H_z}{\partial y}|_{y=0} = 0$$

$$0 = C_2 \cos Ax (\dots) e^{-j\beta z} \quad \boxed{C_2 = 0}$$

$$3. E_y|_{x=a} = 0 \Rightarrow \frac{\partial H_z}{\partial x}|_{x=a} = 0$$

$$0 = -H_z A \sin Ax \cos By \quad \boxed{A = \frac{m\pi}{a}, m=0, 1, 2, 3}$$

$$\sin Ax = 0$$

$$Ax = n\pi$$

$$4. E_x|_{x=a} \Rightarrow \frac{\partial H_z}{\partial y}|_{x=a} = 0$$

$$0 = H_z \cos(\frac{m\pi}{a}x) (-B \sin By) \quad \boxed{B = \frac{n\pi}{b}, n=0, 1, 2, 3}$$

$$\therefore H_z = H_0 \underbrace{C_3 C_4 C_5}_{\text{Cos}(Ax) \cos(By)} \bar{e}^{-j\beta z}$$

$$\therefore H_z = H_0 \cos(\frac{m\pi}{a}x) \cos(\frac{n\pi}{b}y) \bar{e}^{-j\beta z}$$

, $n = 0, 1, 2, 3, \dots$

, $m = 0, 1, 2, 3, \dots$

$$\therefore K_c = \left(\frac{m\pi}{a} \right)^2 + \left(\frac{n\pi}{b} \right)^2$$

as TM

$$\therefore f_c = \frac{v}{2} \sqrt{\left(\frac{m}{a} \right)^2 + \left(\frac{n}{b} \right)^2}$$

$\therefore F > F_c$, $K > K_c$

$$\therefore \lambda_c = \frac{2}{\sqrt{\left(\frac{m}{a} \right)^2 + \left(\frac{n}{b} \right)^2}}$$

$\therefore \lambda < \lambda_c$

* Transverse Components :

$$\therefore k_c^2 = \left(\frac{m\pi}{a} \right)^2 + \left(\frac{n\pi}{b} \right)^2$$

$$f_c = \frac{v}{2} \sqrt{\left(\frac{m}{a} \right)^2 + \left(\frac{n}{b} \right)^2} \text{ as TM}_{mn}$$

$$\lambda_c = \frac{2}{\sqrt{\left(\frac{m}{a} \right)^2 + \left(\frac{n}{b} \right)^2}} \text{ as TM}_{mn}$$

Transverse Components:

$$E_z = 0, H_z = H_0 \cos(\frac{m\pi}{a}x) \cos(\frac{n\pi}{b}y) \bar{e}^{-j\beta z}$$

$$E_x = -j\omega\mu \frac{\partial H_z}{\partial y} = \frac{\omega\mu}{k_c^2} H_0 \cos(\frac{m\pi}{a}x) \sin(\frac{n\pi}{b}y) \bar{e}^{-j\beta z}$$

$$H_y = -j\beta \frac{\partial H_z}{\partial y} = \frac{j\beta}{k_c^2} H_0 \cos(\frac{m\pi}{a}x) \sin(\frac{n\pi}{b}y) \bar{e}^{-j\beta z}$$

$$E_y = \frac{\omega\mu}{k_c^2} \frac{\partial H_z}{\partial x} = \frac{\omega\mu}{k_c^2} H_0 \left(\frac{m\pi}{a} \right) \sin(\frac{m\pi}{a}x) \cos(\frac{n\pi}{b}y) \bar{e}^{-j\beta z}$$

$$H_x = \frac{-j\beta}{k_c^2} \frac{\partial H_z}{\partial x} = \frac{-j\beta}{k_c^2} H_0 \left(\frac{m\pi}{a} \right) \sin(\frac{m\pi}{a}x) \cos(\frac{n\pi}{b}y) \bar{e}^{-j\beta z}$$

Wave Impedance η_{TE} :

$$\eta_{TE} = \frac{E_x}{H_y} = -\frac{E_y}{H_x} = \frac{\omega\mu}{\beta}$$

$$\therefore \beta = k \sqrt{1 - \left(\frac{f_c}{f} \right)^2} = \omega \sqrt{\mu\epsilon} \sqrt{1 - \left(\frac{f_c}{f} \right)^2}$$

$$\therefore \eta_{TE} = \frac{\eta}{\sqrt{1 - \left(\frac{f_c}{f} \right)^2}}$$

→ Wave Impedance :

$$\therefore \gamma_{TE} = \frac{E_x}{H_x} = -\frac{E_y}{H_z} = \frac{\omega\mu}{\beta} \quad , \beta = k \sqrt{1 - \left(\frac{f_c}{f} \right)^2}$$

$$\beta = \omega \sqrt{\mu\epsilon} \sqrt{\dots}$$