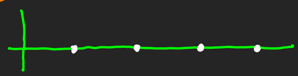
Quadrature Amplitiude Modulation (QAM)

ASK

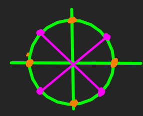
→ single dim Constellation



→ $E_{avg} \propto M^2$

PSK

→ 2D Constellation circle



→ as $M\uparrow \rightsquigarrow P_e\uparrow$ because all symbols are on the same circle.

★ equation:

• $S(t) = \underbrace{A_c}_{\text{carrier amp.}}\left[\underbrace{A_{m_i}}_{\text{in-phase amp.}}\cos(\omega_c t+\theta_c) - \underbrace{A_{m_q}}_{\text{quadrature amp.}}\sin(\omega_c t+\theta_c)\right] \xrightarrow[\text{equivalent}]{\text{Low Pass}} S(t) = A_c A_{m_i} + j A_c A_{m_q}$

→ in MPSK:

$A_{m_i} = \cos\left(\frac{2\pi(m-1)}{M}\right)$

$A_{m_q} = \sin\left(\frac{2\pi(m-1)}{M}\right)$

→ in MASK:

$A_{m_i} = 2m-1-M$

$A_{m_q} = 0$

• $S(t) = A_c\sqrt{A_{m_i}^2 + A_{m_q}^2}\,\cos\left(\omega_c t+\theta_c + \tan^{-1}\left(\frac{A_{m_q}}{A_{m_i}}\right)\right)$

→ General shape of PSD of MASK, MPSK and QAM is the same.

$BW_{3dB} = R_s \rightsquigarrow \eta_s = \frac{R_b}{BW} = \frac{R_b}{R_s} = \frac{R_b}{R_b/K} = K = \log_2(M)$ bits/Hz

★ Constellation diagram:

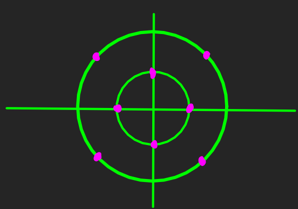
Circular QAM

→ M_1 Phases

→ M_2 Amp.

ex: $M_1 = 4$

$M_2 = 2$



muti-amp PSK

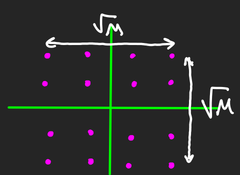
Rectangular QAM

A_{m_i} & $A_{m_q} \in \{(2m-1-M)\}$

$\in \{\pm1, \pm3, \pm5, \pm7, ...\}$

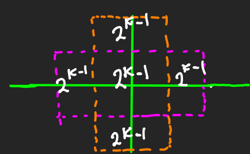
square

→ M is perfect square

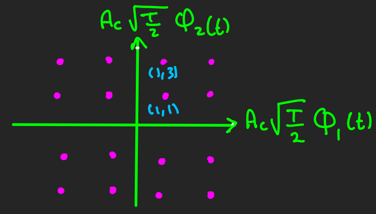


M=16

rect



• E_{avg} & d_{min} for square QAM:



$$\cdot E_{Avg} = \frac{1}{M}\sum_{m=1}^{M} E_m$$

$$= \frac{1}{M/4}\sum_{m=1}^{M/4} E_m$$

$$= \frac{1}{M/4}\cdot\sqrt{M}\cdot A_c^2\frac{T}{2}\cdot\left[1^2+3^2+\dots+(\sqrt{M}-1)^2\right]$$

$$= \frac{1}{\cancel{M/4}}\,\cancel{\sqrt{M}}\cdot A_c^2\,\frac{T}{2}\cdot\frac{\cancel{\sqrt{M}}(M-1)}{6}$$

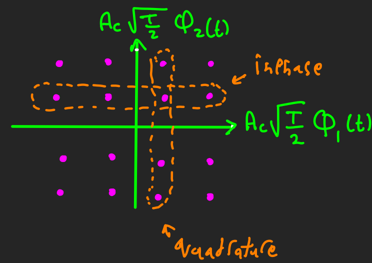
$$\therefore E_{avg} = \frac{A_c^2 T}{3}(M-1) \quad \#$$

, $E_{avg} \propto M$ not M^2 like in MASK

$$\bullet\, d_{min} = 2A_c\sqrt{\frac{T}{2}} = \sqrt{2A_c^2 T} = \sqrt{2\cdot\frac{3E_{avg}}{(M-1)T}\cdot T} = \sqrt{\frac{6E_{avg}}{M-1}} = \sqrt{\frac{6\log(M)}{M-1}E_{b_{avg}}}$$

, $d_{min} \propto \frac{1}{\sqrt{M}}$ not $\frac{1}{M}$ like in MPSK

• Pe:



$$\cdot P_e = 1 - P_c$$

$$= 1 - P\left(\substack{\text{correct}\\ \text{inphase}} \;\&\; \substack{\text{correct}\\ \text{quad}}\right)$$

$$= 1 - P\left(\substack{\text{correct}\\ \text{inphase}}\right)\cdot P\left(\substack{\text{correct}\\ \text{quad}}\right)$$

$$= 1 - \left(1 - P_{e\sqrt{M}ASK}\right)\left(1 - P_{e\sqrt{M}ASK}\right)$$

$$= 1 - \left(1 - P_{e\sqrt{M}ASK}\right)^2 = 1 - \left(1 - 2P_{e\sqrt{M}ASK} + \cancel{P^2_{e\sqrt{M}ASK}}\right) \quad \simeq 0$$

$$\simeq 2P_{e\sqrt{M}ASK}$$

$$\simeq 2\cdot 2\left(1-\frac{1}{\sqrt{M}}\right)Q\left(\sqrt{\frac{3\log(M)}{M-1}\,\frac{E_{bavg}}{N_0}}\right)$$

$$P_{eMASK} = 2\left(1-\frac{1}{M}\right)Q\left(\frac{d_{min}}{\sqrt{N_0}}\right)$$

QAM: $\frac{3\log(M)}{M-1}$

ASK: $\frac{6\log(M)}{M^2-1}$

PSK: $\frac{2\pi^2\log(M)}{M^2}$

$$\therefore P_e \simeq 4\left(1-\frac{1}{\sqrt{M}}\right)Q\left(\sqrt{\frac{3\log(M)}{M-1}\,\frac{E_{bavg}}{N_0}}\right) \quad \#$$

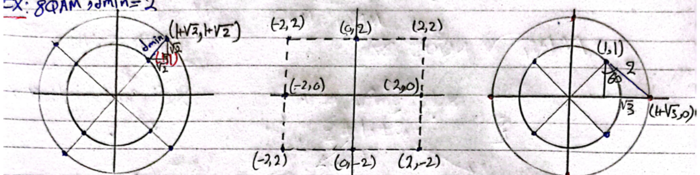
Power Efficiency:

To compare the Power Efficiency of different QAM

⟹ (1) fix dmin (why) ⟹ Pe ≈ same

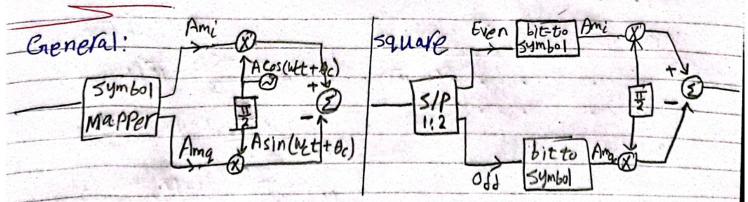
(2) calculate Eav

Ex: 8QAM, dmin = 2



Circular QAM without phase shift	On square	Circular QAM with $\frac{\pi}{4}$-phase shift
$E_{av} = \frac{(1)^2+(1)^2+(1+\sqrt{2})^2+(1+\sqrt{2})^2}{2}$	$E_{av} = \frac{12(2)^2}{8}$	$E_{av} = \frac{1}{2}\left[(1)^2+(1)^2+(1+\sqrt{3})^2\right]$
$= 1+(1+\sqrt{2})^2 = 6{,}83\,J$	$= 6\,J$	$\boxed{= 4{,}73\,J}$

Transmitter:



Receiver: