Problem 1

Consider the following 8-bit bit sequence $c(n)$.

	$c(0)$	$c(1)$	$c(2)$	$c(3)$	$c(4)$	$c(5)$	$c(6)$	$c(7)$
	0	1	1	0	0	0	1	0
$m=2$	1	0	0	1	1	0	0	0
$m=-1$	1	1	0	0	0	1	0	0

We wish to compute the autocorrelation function $r(m)$ for the sequence.

- Compute $r(m)$ for $m = 0, 2, -1$.
- What is the range of $r(m)$, i.e., how many values of m do we need to compute all possible points of $r(m)$?

• $M = 0$

$\rightarrow 0+1+1+0+0+0+1+0 = 3$

$M = 2$

$\rightarrow = 0$

$m = -1$

$= 1$

• We need number of points equal to the length of input sequence

↳ $N = 8$

Problem 2

Calculate the location of a GPS receiver and the timing error between the receiver clock and the GPS satellite given the following information received using the GPS system

- Satellite 1 location at time of transmission: (15600,7540,20140) km
- Satellite 1 time of transmission: 348.14824 s
- Satellite 1 signal received at time: 348.21898
- Satellite 2 location at time of transmission: (18760,2750,18610) km
- Satellite 2 time of transmission: 345.16475 s
- Satellite 2 signal received at time: 345.23695
- Satellite 3 location at time of transmission: (17610,14630,13480) km
- Satellite 3 time of transmission: 339.36534
- Satellite 3 signal received at time: 339.44224
- Satellite 4 location at time of transmission: (19170,610,18390) km
- Satellite 4 time of transmission: 344.58665
- Satellite 4 signal received at time: 344.65907

	Sat 1	Sat 2	Sat3	Sat3
Location: km	15600, 7540, 20140	18760, 2750, 18610	17610, 14630, 13480	19170, 610, 18390
t sec :	348.14824	345.16475	339.36534	344.58665
$t' + \Delta t_{error}$:	348.21898	345.23695	339.44224	344.65907
$\Delta t = t - (t' + \Delta t_{error})$:	0.07074	0.0722	0.0769	0.07242

$$\rightarrow d_i = \sqrt{(u_x - x_i)^2 + (u_y - y_i)^2 + (u_z - z_i)^2} + \Delta t_{error} * C$$

$$\rightarrow d_i = \Delta t * C$$

$$\therefore (0.07074) * C * 10^{-3} = \sqrt{(u_x - 15600)^2 + (u_y - 7540)^2 + (u_z - 20140)^2} + \Delta t_{error} * C * 10^{-3} \longrightarrow 1$$

$$\therefore (0.0722) * C * 10^{-3} = \sqrt{(u_x - 18760)^2 + (u_y - 2750)^2 + (u_z - 18610)^2} + \Delta t_{error} * C * 10^{-3} \longrightarrow 2$$

$$\therefore (0.0769) * C * 10^{-3} = \sqrt{(u_x - 17610)^2 + (u_y - 14630)^2 + (u_z - 13480)^2} + \Delta t_{error} * C * 10^{-3} \longrightarrow 3$$

$$\therefore (0.07242) * C * 10^{-3} = \sqrt{(u_x - 19170)^2 + (u_y - 610)^2 + (u_z - 18390)^2} + \Delta t_{error} * C * 10^{-3} \longrightarrow 4$$

→ 4 equations & 4 unknowns

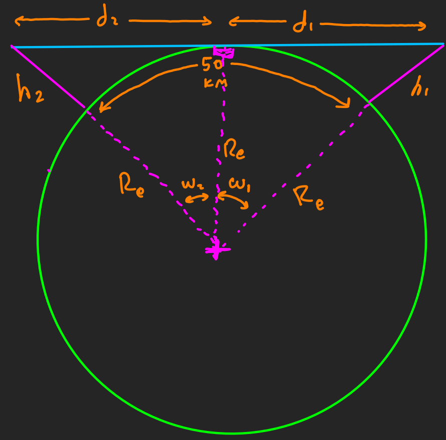
$\rightarrow u_x = -251.56$, $u_y = -104.52$, $u_z = 6282.46$, $\Delta t = -0.0039253$ Sec

Problem 3

We would like to establish a microwave link between two locations, where the distance between them across the surface of the earth (i.e., the length of the earth arc connecting them) is 50 km. Find the minimum antenna heights needed at one point if the antenna height on the other point is 10 m or 100 m.

$\because$ arc length $= r \cdot \theta_{rad}$, $R_e = 6371$ km

$\therefore R_e \cdot (w_1 + w_2)_{rad} = 50$ km $\rightarrow$ ①



For $h_1 = 10$ m

$\rightarrow \cos(w_1) = \dfrac{R_e}{R_e + h_1} = \dfrac{6371}{6371 + 10 * 10^{-3}} \rightsquigarrow \therefore w_1 = 0.1015°$

① $\rightarrow 6371\left(0.1015 * \dfrac{\pi}{180} + w_2\right) = 50 \rightsquigarrow \therefore w_2 = 6.077 * 10^{-3}$ rad

$\therefore w_2 = 0.3483°$

$\rightarrow \cos(w_2) = \dfrac{R_e}{R_e + h_2}$, $\therefore \cos(0.3483) = \dfrac{6371}{6371 + h_2} \rightsquigarrow \therefore h_2 = 0.117$ km

$\boxed{h_2 = 117.71 \text{ m}}$

if it's required to find the loss distance between the two antennas:

$d_{LOS} = d_T + d_R \approx \sqrt{2rh_T} + \sqrt{2rh_R}$

For $h_1 = 100$ m

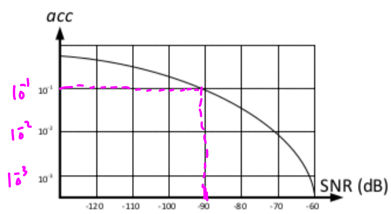
$w_1 = 0.321°$, ① $\rightarrow w_2 = 2.248 * 10^{-3}$ rad $\rightsquigarrow \therefore w_2 = 0.128°$

$\rightarrow \therefore h_2 = 0.0158$ km

$\boxed{\therefore h_2 = 15.89 \text{ m}}$

Problem 4

Perform a link budget analysis for a GPS system. The performance of the system is measured based on the accuracy in determining the distance between the GPS receiver and one GPS satellite. The following is a performance curve for the GPS system, where the performance metric *acc* is defined as the ratio between the error in measuring the distance and the actual distance.



Perform the link budget analysis with the following assumptions

- The target performance level is 10% $acc = 0.1 = \frac{10}{100} = 10^{-1}$
- Signal propagation is assumed to occur in free space
- Atmospheric absorption is measured as 0.5 dB
- Beam misalignment effect is measured as 1.2 dB
- Transmitter and receiver beamforming gains are measured as 15 dB and 20 dB respectively
- GPS carrier frequency is 1.575 GHz
- Distance between the user and the satellite ranges between 20,000 km to 20,020 km.
- Noise power is 10 dBm

@ $acc = 0.1 \rightsquigarrow SNR = -90$ dB

$\cdot P_T = P_R + P.L - G + Loss$

$\rightarrow SNR_{dB} = P_{R\,dBm} - N_{0\,dBm}$

$\therefore -90 = P_R - 10 \rightsquigarrow \therefore P_R = -80$ dBm #

$\rightarrow P.L = 20\log(r) + 20\log(f) + 20\log\left(\frac{4\pi}{c}\right)$

$\therefore P.L = 20\log(20020 * 10^3) + 20\log(1.575 * 10^9) + 20\log\left(\frac{4\pi}{c}\right)$

$\therefore P.L = 182.4$ dB #

$\therefore P_{T\,dBm} = -80 + 182.4 - (15 + 20) + (0.5 + 1.2)$

$\therefore \boxed{P_T = 69.1 \text{ dBm}}$ #