Amplitude Shift Keying (ASK)

We will Study 3 types

BASK

- Binary, two signals with two amplitudes A_1, A_2

- $1: S_1(t) = A_1 \cos(\omega_c t + \theta_c)$
- $0: S_2(t) = A_2 \cos(\omega_c t + \theta_c)$

above two eqs can be written as:

$$\underset{BASK}{x(t)} = \frac{A_1}{2}(1+b)\cos(\omega_c t + \theta_c) + \frac{A_2}{2}(1-b)\cos(\omega_c t + \theta_c)$$

b can only values ± 1:
- if $b=1$ then $x(t) = S_1(t)$
- if $b=-1$ ~~ ~~ $= S_2(t) = 0$

PSD, BW, η_s

$$x(t) = \underbrace{\tfrac{1}{2}(A_1+A_2)\cos(\omega_c t + \theta_c)}_{\text{Carrier @ } f_c} + \underbrace{\tfrac{1}{2}b(A_1-A_2)\cos(\omega_c t+\theta_c)}_{\text{Polar NRZ + modulated @} \pm f_c}$$

Polar NRZ + modulated @ $\pm f_c$
→ same graph as before, just shifted @ $\pm f_c$

→ BW null to null

$\therefore BW_{NN} = f_c + R_b - (f_c - R_b) = 2R_b$

→ BW 3dB

$\therefore BW_{3dB} = \frac{1}{2}(2R_B) = R_B$

$\therefore \eta_s = \frac{R_b}{BW} = \frac{R_B}{R_B} = 1$ b/s/Hz

→ every 1 bit per second occupies 1 Hz

OOK

- Same as BASK but $A_2 = 0$

- $1: S_1(t) = A\cos(\omega_c t + \theta_c)$
- $0: S_2(t) = 0$

above two eqs can be written as:

$$\underset{OOK}{x(t)} = \frac{A}{2}(1+b)\cos(\omega_c t + \theta_c)$$

b can only values ± 1:
- if $b=1$ then $x(t) = S_1(t)$
- if $b=-1$ ~~ ~~ $= S_2(t) = 0$

PSD, BW, η_s

Same as BASK

- $BW_{NN} = 2R_b$
- $BW_{3dB} = R_B$ "default"
- $\eta_s = 1$

MASK

- M'ary, more than two signals $A_1, A_2, \dots A_M$

- $\underset{MASK}{x(t)} = A_c(2m-1-M)\cos(\omega_c t + \theta_c)$

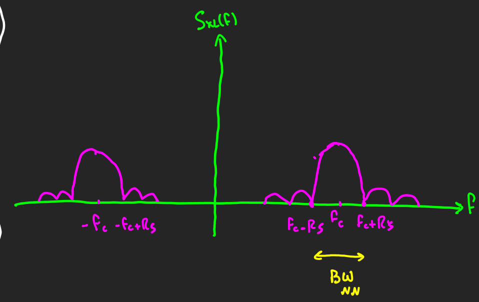
- Amps: $\{\pm A_c, \pm 3A_c, \pm 5A_c, \dots, \pm(M-1)A_c\}$

PSD, BW, η_s

$$\underset{MASK}{x(t)} = \underbrace{A_c(2m-1-M)\cos(\omega_c t + \theta_c)}$$

Polar NRZ + modulated @ $\pm f_c$
→ same graph as before, just shifted @ $\pm f_c$

→ No carrier



→ BW null to null

$\therefore BW_{NN} = f_c + R_s - (f_c - R_s) = 2R_s$

→ BW 3dB

$\therefore BW_{3dB} = \frac{1}{2}(2R_s) = R_s$

$\therefore \eta_s = \frac{R_b}{BW} = \frac{R_b}{R_s}$

$\therefore \eta_s = \frac{R_b}{R_b/K} = K$

→ every K bits per second occupies 1 Hz

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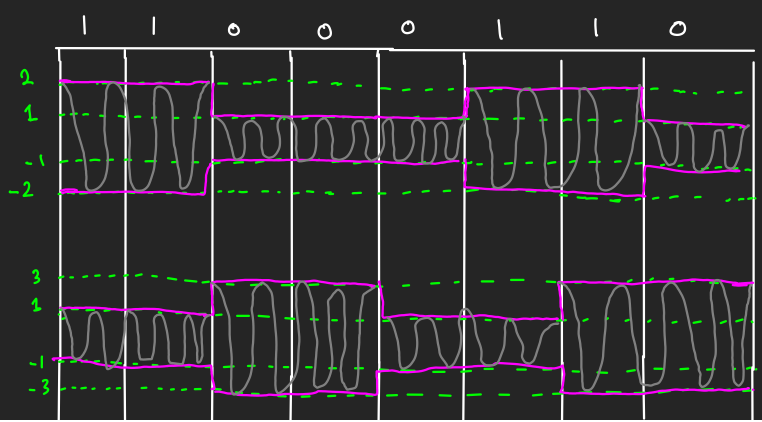
ex:

$I_n = \underline{11000110}$, sketch transmited signal for BASK with $A_1 = 2$, $A_2 = 1$, and for $\underbrace{4ASK}$ with $A_c = 1$:

$M = 4 \rightsquigarrow K = 2$

→ $\{-3, -1, 1, 3\}$

0 0	-3
0 1	-1
1 1	1
1 0	3

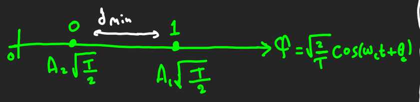


* Constelation diagram:

$$\varphi(t) = \sqrt{\tfrac{2}{T}}\,\cos(\omega_c t + \theta_c)$$

* $E_m, E_{avg}, \rho, d_{min}, P_e$:

BASK



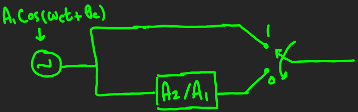
$E_1 = A_1^2\,\frac{T}{2}$, $E_2 = A_2^2\,\frac{T}{2}$

$E_{av} = \frac{E_1+E_2}{2} = \frac{A_1^2+A_2^2}{4}\cdot T$

$\rho_{12} = \cos(0) = 1$

$d_{min} = |A_1 - A_2|\sqrt{\frac{T}{2}}$

* T_x

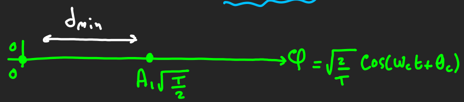


* P_e

$P_e = Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right)$

$$\boxed{P_e = Q\left(\frac{|A_1-A_2|\sqrt{\frac{T_b}{2}}}{\sqrt{2N_0}}\right)}$$

OOK



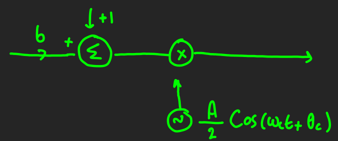
$E_1 = A_1^2\,\frac{T}{2}$, $E_2 = 0$

$E_{av} = \frac{E_1+E_2}{2} = \frac{A_1^2}{4}\cdot T$

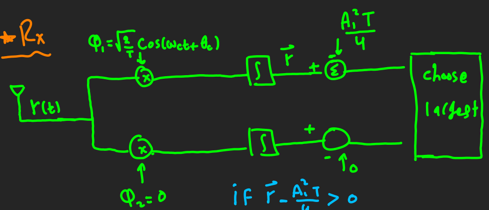
$\rho = 1$

$d_{min} = A_1\sqrt{\frac{T}{2}}$

* T_x



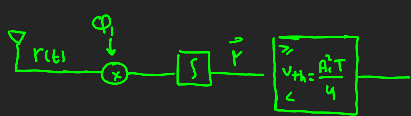
* R_x



if $\vec{r} - \frac{A_1^2 T}{4} > 0$

$\vec{r} > \frac{A_1^2 T}{4}$

Can be simplified :



* P_e

$P_e = Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right)$

$= Q\left(\frac{A_1\sqrt{\frac{T_b}{2}}}{\sqrt{2N_0}}\right) = Q\left(\frac{\sqrt{E_1}}{\sqrt{2N_0}}\right)$, $E_{av} = \frac{E_1}{2}$

$= Q\left(\frac{\sqrt{2E_{av}}}{\sqrt{2N_0}}\right)$

$$\boxed{P_e = Q\left(\sqrt{\frac{E_{av}}{N_0}}\right) = Q\left(\sqrt{\frac{E_b}{N_0}}\right)}$$

* η_p

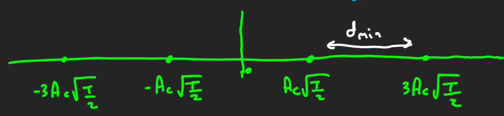
$\eta_p = \underset{P_e = 10^{-5}}{\arg\min}\ \frac{E_b}{N_0}$, $Q\left(\sqrt{\frac{E_b}{N_0}}\right) = 10^{-5}$

$\therefore \frac{E_b}{N_0} = \left[Q^{-1}(10^{-5})\right]^2$

$= 12.59\ dB$

MASK

4ASK



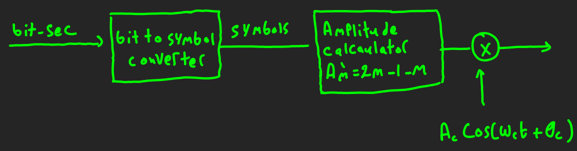
$E_m = A_c^2\,(2m-1-M)^2\,\frac{T_s}{2}$

$E_{av} = \frac{1}{M}\sum_{m=1}^{M} A_c^2\,(2m-1-M)^2\,\frac{T_s}{2} = \frac{A_c^2\,T^s\,(M^2-1)}{6}$

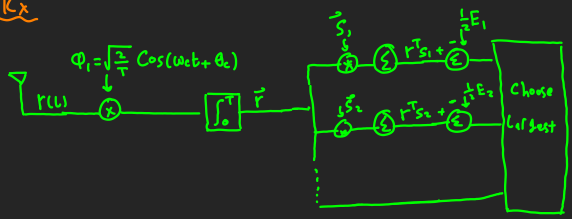
$\rho_{ij} = \begin{cases} 1 & \text{same Polarity} \\ -1 & \text{diff ----} \end{cases}$

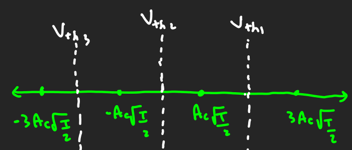
$d_{min} = 2A_c\sqrt{\frac{T_s}{2}}$

* T_x

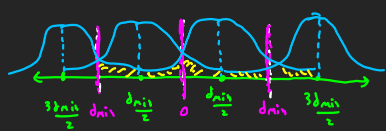


R_x





* P_e



$P_e = \frac{1}{M} *$ total area of tails

$= \frac{1}{M} * [2*1 + (M-2)*2] * P_{tail}$

$= \frac{1}{M} * [2(M-1)]\ P_{tail}$

$P_e = \frac{2(M-1)}{M}\, Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right)$

$\because E_{av} = \frac{A_c^2\,T_s\,(M^2-1)}{6}$, $\because d_{min} = 2A_c\sqrt{\frac{T_s}{2}}$

$\therefore d_{min} = \sqrt{\frac{12\log_2(M)}{M^2-1}\,E_{bav}}$

$$\boxed{\therefore P_e = \frac{2(M-1)}{M}\, Q\left(\sqrt{\frac{6\log_2(M)}{M^2-1}\cdot\frac{E_{bav}}{N_0}}\right)}$$