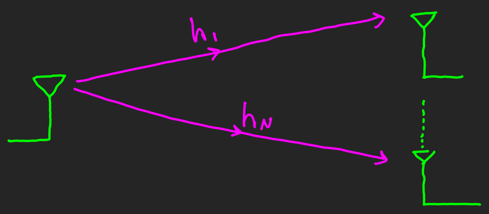
SIMO



Tx (1 Antenna) Rx (N Antennas)

→ we can write this model in two ways:

- $y_i = h_i x + n_i \quad , i = 1, \ldots, N$
- $\begin{bmatrix} y_1 \\ \vdots \\ y_N \end{bmatrix} = \begin{bmatrix} h_1 \\ \vdots \\ h_N \end{bmatrix} x + \begin{bmatrix} n_1 \\ \vdots \\ n_N \end{bmatrix}$

$\bar{y} = \bar{h} x + \bar{n}$

→ $\bar{n}$: Complex random vector has a mean of zero $\mathbb{E}(x_1 x_2) = 0$ because the noise in each path are uncorlated,

it has a covariance matrix $\bar{\bar{R}}_n = \mathbb{E}(\bar{n}\bar{n}^T) = \mathbb{E}(\bar{n}\bar{n}^H) = \begin{bmatrix} N_0 & & 0 \\ & \ddots & \\ 0 & & N_0 \end{bmatrix} = N_0 \bar{\bar{I}}$ $\qquad \underset{N\times 1}{\bar{n}} \cdot \underset{1\times N}{\bar{n}^H} \Rightarrow \underset{N\times N}{\bar{\bar{R}}_n}$

$n_i \sim CN(0, N_0)$, $\bar{n} \sim CN(\bar{0}, N_0 \bar{\bar{I}})$

Mean, Covariance

→ $\bar{h}$: Complex random vector has a mean of zero $\mathbb{E}(x_1 x_2) = 0$ because the channel in each path are uncorlated,

to ensure they are uncorolated the antennas at Rx must not be at the same location (sepration of $\frac{\lambda}{2}$).

it has a covariance matrix $\bar{\bar{R}}_h = \mathbb{E}(\bar{h}\bar{h}^T) = \mathbb{E}(\bar{h}\bar{h}^H) = \begin{bmatrix} \sigma^2 & & 0 \\ & \ddots & \\ 0 & & \sigma^2 \end{bmatrix} = \sigma^2 \bar{\bar{I}}$ $\qquad \underset{N\times 1}{\bar{h}} \cdot \underset{1\times N}{\bar{h}^H} \Rightarrow \underset{N\times N}{\bar{\bar{R}}_h}$

$h_i \sim CN(0, \sigma^2)$, $\bar{h} \sim CN(\bar{0}, \sigma^2 \bar{\bar{I}})$, let $\sigma^2 = 1 \rightsquigarrow \bar{h} \sim CN(\bar{0}, \bar{\bar{I}})$

Mean, Covariance

⋆ Selection Combining

→ We have N antennas at Rx, each has different h_i. Some channels will be in deep fading but some will not and high probability. So we can select the y_i with best h_i (max h_i).

→ max h_i will result in max SNR because h_i acts as a gain to the signal x.

- $SNR_R = (\max |h_i|^2)\, SNR$
- $P_e(\bar{h}) \leq K Q\left(\sqrt{2(\max |h_i|^2)\, SNR}\right)$ For a given h

→ we are intersted in average performance

$\therefore P_{e_{avg}} = \mathbb{E}_{\max|h_i|^2}\left(P_e(\bar{h})\right)$

- Random Varible X : $f_X(x) \to$ PDF

  $F_X(x) \to$ CDF
- $P_r(X \leq x) = F_X(x) = \int_{-\infty}^{x} f_X(x)\, dx$
- $f_X(x) = \frac{d}{dx} F_X(x)$

$$\therefore P_{e_{avg}} = \int K Q(\sqrt{2x\,SNR})\, f_{\max|h_i|^2}(x)\, dx$$

$$F_{\max|h_i|^2}(x) = P_r\{\max|h_i|^2 \le x\}$$

$$= P_r\{|h_1|^2 \le x, |h_2|^2 \le x, \dots, |h_N|^2 \le x\}$$

$$= \underbrace{P_r\{|h_1|^2 \le x\}}\,\underbrace{P_r\{|h_2|^2 \le x\}}\dots\underbrace{P_r\{|h_N|^2 \le x\}}$$

exponitial distribution

$$= \left[F_{exp(h)}(x)\right]^N$$

* Deep Fading :

$$P_{df}\{\text{Recived Power} \le \text{Noise Power}\}$$

$$= P_{df}\{\max|h_i|^2 a^2 \le N_0\}$$

$$= P_{df}\{\max|h_i|^2 \le \frac{1}{SNR}\}$$

$$= \int_0^{1/SNR} PDF(\max|h_i|^2)\, dx = \int_0^{1/SNR} f_{\max|h_i|^2}(x)\, dx = \int_0^{1/SNR} \frac{d}{dx} CDF(\max|h_i|^2)\, dx = \int_0^{1/SNR} \frac{d}{dx}\left[F_{exp(h)}(x)\right]^N dx$$

$$\therefore P_{df} = \left[1 - e^{-\frac{1}{2SNR}}\right]^N$$

@ high SNR : $e^{\frac{-1}{2SNR}} \simeq 1 - \frac{1}{2SNR}$

$$\therefore P_{df} = \frac{1}{2^N}\frac{1}{SNR^N}$$

$$\therefore P_{df} = \text{Const} - \underbrace{N}\, SNR_{dB}$$

Diversity gain : Slope of P_e curve with SNR , as $N\uparrow\uparrow$ the slope$\uparrow\uparrow$ which makes the curve gets closser and closser to the AWGN curve .

* Maximum Ratio Combining (MRC)

→ Key difference we will make use of all antennas .

$$\bar{y} = \bar{h}x + \bar{n}$$

$$r = w_1 y_1 + w_2 y_2 + \dots + w_N y_N = \underset{1\times N}{\bar{w}^H}\ \underset{N\times 1}{\bar{y}} = \begin{bmatrix} w_1 \\ \vdots \\ w_N \end{bmatrix}^H \begin{bmatrix} y_1 \\ \vdots \\ y_N \end{bmatrix}, \quad \underset{N\times 1}{\bar{w}} \longrightarrow \underset{1\times N}{\bar{w}^H}$$

$\bar{w}$: decoding vector

$$\therefore r = (\bar{w}^H \bar{h})x + \underbrace{\bar{w}^H \bar{n}}_{\hat{n}}$$

$$\therefore \text{Recived signal Power}: |\bar{w}^H \bar{h}|^2 a^2$$

$$\therefore \text{Noise signal Power}: N_0 \|\bar{w}\|^2$$

$$\text{Noise Power} = Var(\hat{n}) = \mathbb{E}(\hat{n}^2) \qquad \hat{n} \sim CN(0, Var)$$

$$= \mathbb{E}[(\bar{w}^H \bar{n})^2]$$

$$= \mathbb{E}[\bar{w}^H \bar{n}\, \bar{n}^H w] \qquad |\bar{w}^H \bar{n}| = |\bar{w}\bar{n}^H|$$

$$= \bar{w}^H\, \mathbb{E}(\bar{n}\bar{n}^H)\, \bar{w}$$

$$= \bar{w}^H N_0 \bar{\bar{I}}\, \bar{w}$$

$$= N_0 \bar{w}^H \bar{w}$$

$$= N_0 \|\bar{w}\|^2 \quad \#$$

$$\therefore SNR_R = \frac{|\bar{w}^H \bar{h}|^2 a^2}{N_0 \|\bar{w}\|^2} = \frac{|\bar{w}^H \bar{h}|^2}{\|\bar{w}\|^2} SNR$$

Let $w = P\hat{w}$, P: magntiude , $\hat{w}$: unite vector

$$\therefore \|\bar{w}\|^2 = \|P\hat{w}\|^2 = P^2$$

$$\therefore |\bar{w}^H \bar{h}|^2 = |P\hat{w}^H \bar{h}|^2 = P^2 |\hat{w}^H \bar{h}|^2$$

$$\therefore SNR_R = \frac{|\bar{\omega}^H \bar{h}|^2}{\|\bar{\omega}\|^2} SNR = \frac{\cancel{p^2} |\hat{\omega}^H \bar{h}|^2}{\cancel{p^2}} SNR$$

$$\therefore SNR_R = |\hat{\omega}^H \bar{h}|^2 SNR$$

$\therefore$ to obtain high SNR_R, $\hat{\omega}^H \bar{h}$ need to be maxmium, $\hat{\omega}^H \bar{h}$ is a dot product between two Vectors and to have max Value of a dot product the two Vectors need to be in the Same direction. And hence we don't have Control over $\bar{h}$, we will Chosse $\hat{\omega}^H$ so that it's at the Same direction as $\bar{h}$

$$\therefore \hat{\omega}^H = \frac{\bar{h}}{\|\bar{h}\|}$$

$$\therefore SNR_R = \left|\frac{\bar{h}^H}{\|\bar{h}\|} \bar{h}\right|^2 SNR = \left|\frac{\|\bar{h}\|^2}{\|\bar{h}\|}\right|^2 SNR = \|\bar{h}\|^2 SNR = \left(\sum_{i=1}^{N} |h_i|^2\right) SNR$$

$$\therefore P_e = K\,Q\left(\sqrt{2(\|\bar{h}\|^2 SNR)}\right)$$

$$\therefore P_{e_{avg}} = \mathbb{E}_{\|\bar{h}\|^2}(P_e) = \int K\,Q\left(\sqrt{2(x\,SNR)}\right) f_{\|\bar{h}\|^2}(x)\, dx$$

* Deep Fading:

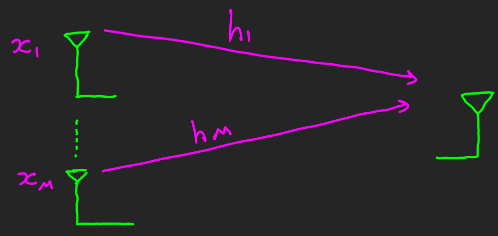
$P\{$Recived Power $\leq$ Noise Power$\}$

$$= P_{dF}\left\{ \|\bar{h}\|^2 \leq \frac{1}{SNR} \right\} = \int_0^{1/SNR} f_{\|\bar{h}\|^2}(x)\, dx$$

$$\therefore P_{dF_{MRC}} = \frac{1}{N!} \frac{1}{SNR^N} \qquad P_{dF_{SC}} = \frac{N!}{2^N} \frac{1}{SNR^N}$$

→ MRC & SC have the Same diversity gain N (array gain), MRC is more Power effecient (Power gain) at Low SNR ($P_{dF_{MRC}} < P_{dF_{SC}}$ at low SNR)

MISO



Tx(M antennas) Rx (1 antenna)

$\rightarrow y = \sum_{i=1}^{M} h_i x_i + n$

$\rightarrow y = \bar{h}^H \bar{x} + n$

$\bar{x}$: transmitted signal

$\bar{x} = \underbrace{\bar{\omega}}_{\text{Precoding Vector}} \underbrace{x}_{\text{Modulated Symbol}}$

↳ Determines x_i for each antenna at Tx.

$\therefore y = \bar{h}^H \bar{\omega} x + n$

→ Recived signal Power : $|\bar{h}^H \bar{\omega}|^2 a^2$

→ Noise Power : N_0

$\therefore SNR_R = \frac{|\bar{h}^H \bar{\omega}|^2 a^2}{N_0} = |\bar{h}^H \bar{\omega}|^2 \, SNR$

∴ we need to maxmize $\bar{h}^H \bar{\omega}$, ∴ make $\bar{\omega}$ at the same direction as $\bar{h}^H$, but in order to know $\bar{h}^H$ the receiver needs first to send it. if we don't know $\bar{h}$ then $\bar{\omega}$ can be ones.

→ note that here it's $\bar{\omega}$ not $\hat{\omega}$, meaning theortcly we can increase the SNR_R by increasing the magnitude of $\bar{\omega}$ too, but it's limitted to transmitting device Power.