CH1 High frequency transmission line

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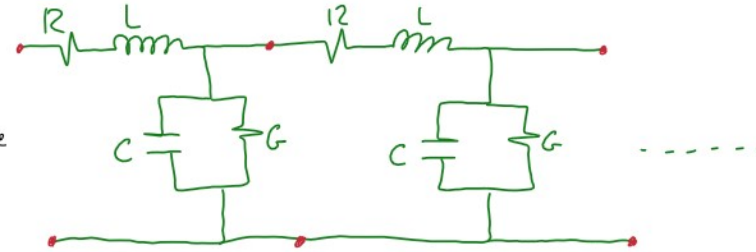
→ at high freq, the wavelength is small compaired with line length.

→ if the T L is uniform, it can be represented as cascaded distributed sections each containing R, L, C, G



→ therfor the series Impedance Z, and shunt admittance Y, permeter are given as: ($e^{j\omega t}$ time function)

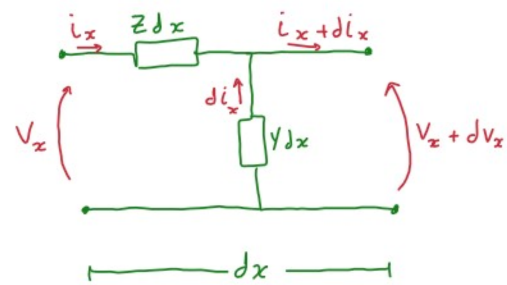
$$Z = R + j\omega L \quad \Omega/m$$
$$Y = G + j\omega C \quad S/m$$

* Voltage and Current Differential Equations:

· Consider a length dx of the T.L

· i_x, V_x in → $i_x + di_x, V_x + dV_x$ out

· we expect dV_x to be negative, because voltage decreases with distance.



→ $\cancel{V_x} - (\cancel{V_x} + dV_x) = i_x Z dx \rightsquigarrow \therefore \frac{dV_x}{dx} = -i_x Z \longrightarrow ①$

→ $di_x = -(V_x + dV_x) Y dx$

$= -V_x Y dx - Y \cancel{dV_x dx}^{\,0} \rightsquigarrow \therefore \frac{di_x}{dx} = -V_x Y \longrightarrow ②$

- we have two diff eq.

→ $\frac{\partial ①}{\partial x}$: $\frac{\partial^2 V_x}{\partial x^2} = -\frac{\partial i_x}{\partial x} Z$

$\frac{\partial^2 V_x}{\partial x^2} = -(-V_x Y) Z$

$\therefore \boxed{\frac{\partial^2 V_x}{\partial x^2} = Z Y V_x}$

· $\sqrt{ZY} = \sqrt{(R + j\omega L)(G + j\omega C)}$ is called the Propogation Constant and its units is per meter.

· $\gamma = \sqrt{ZY}$

$\therefore \boxed{\frac{\partial^2 V}{\partial x^2} = \gamma^2 V_x} \longrightarrow ③$

- Solving ③

· $\boxed{V_x = V_+ e^{-\gamma x} + V_- e^{\gamma x}}$ ....④

$\underrightarrow{\text{Incident}}$ $\underleftarrow{\text{Reflected}}$

· $\boxed{I_x = I_+ e^{-\gamma x} + I_- e^{\gamma x}}$ ...... ⑤

· Relations between V_+, I_+ and V_-, I_-

→ substituting ④, ⑤ in ①

$$\therefore -\gamma V_+ e^{-\gamma x} + \gamma V_- e^{\gamma x} = -(I_+ e^{-\gamma x} + I_- e^{\gamma x}) Z$$

$\therefore -\gamma V_+ = -I_+ Z$ $\qquad$ $\therefore \gamma V_- = -I_- Z$

$\therefore I_+ = \frac{\gamma V_+}{Z} = \frac{V_+}{Z/\gamma}$ $\qquad$ $\therefore I_- = \frac{-\gamma V_-}{Z} = \frac{-V_-}{Z/\gamma}$

$\therefore I_+ = \frac{V_+}{Z_o}$ $\qquad$ $\therefore I_- = \frac{-V_-}{Z_o}$

$\frac{Z}{\gamma}$ represents an Impedance Z_o called the characterstics Impedance

$\cdot Z_o = \frac{Z}{\gamma} = \frac{Z}{\sqrt{ZY}} = \sqrt{\frac{Z}{Y}}\ \Omega$

$\therefore V_x = V_+ e^{-\gamma x} + V_- e^{\gamma x}$ $\qquad$ $\therefore I_x = \frac{V_+}{Z_o} e^{-\gamma x} - \frac{V_-}{Z_o} e^{\gamma x}$

Note:

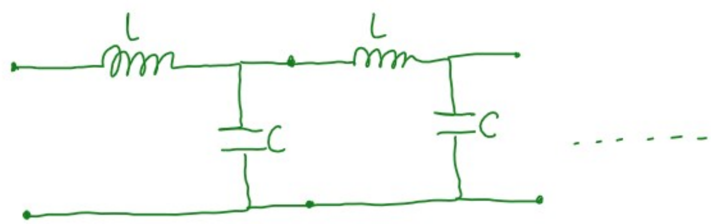
$\cdot \gamma = \sqrt{ZY} = \sqrt{(R+J\omega L)(G+J\omega C)} = \alpha + J\beta$

→ α is called the attenuation Constant and β is the Phase shift.

- we have two types of T.Ls

① Ideal "Lossless" T.L

. $R=0$, $G=0$, T.L Composed of LC circuit



$\cdot Z = J\omega L$, $Y = J\omega C$

$\cdot \gamma = \sqrt{ZY} = \sqrt{J^2\omega^2 LC} = J\omega\sqrt{LC} = \alpha + J\beta$

$\therefore \alpha = 0$, $\beta = \omega\sqrt{LC}$ "Linear with Freq"

$\cdot Z_o = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{J\omega L}{J\omega C}} = \sqrt{\frac{L}{C}} = R_o$ "Pure resistive"

② Distortionless T.L

. $R \neq 0$, $G \neq 0$

$\cdot \gamma = \sqrt{ZY} = \sqrt{(R+J\omega L)(G+J\omega C)}$

$= \sqrt{R(1+J\frac{\omega L}{R}) \cdot G(1+J\frac{\omega C}{G})}$

$\cdot$ let $\frac{L}{R} = \frac{C}{G}$ or $RC = LG$

$\therefore \gamma = \sqrt{RG(1+J\frac{\omega L}{R})^2} = \sqrt{RG}\,(1+J\omega\frac{L}{R})$

$\frac{L^2 G}{R} = \frac{L\cancel{R}C}{\cancel{R}}$

$= \sqrt{RG} + jwL\sqrt{\frac{G}{R}} = \sqrt{RC} + jw\sqrt{LC}$

$\therefore \alpha = \sqrt{RC}$ Const. $\therefore \beta = w\sqrt{LC}$ Linear with freq.

- we have Control over R, C and we design the T.L with R, C ↓↓ so that α ↓↓
- β is Linear Like in ideal. but we need to Make $\frac{L}{R} = \frac{C}{G}$ to maintain the Linearty in β.

- $Z_o = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{R + jwL}{G + jwC}} = \sqrt{\frac{R\cancel{(1 + jwL/R)}}{G\cancel{(1 + jwC/G)}}}$

$\therefore Z_o = \sqrt{\frac{R}{G}} = \sqrt{\frac{L}{C}} = R_o$ "Pure resestive"