✩ TM_{mn} modes: "Cuttoff Freq & Cuttoff wavelength"

→ $H_z = 0$, $E_z = E_0 \sin(\frac{m\pi}{a}x)\sin(\frac{n\pi}{b}y)e^{-j\beta z}$

$m = 1, 2, 3, \dots$ $n = 1, 2, 3, \dots$

→ $K_c^2 = A^2 + B^2 = \gamma^2 + K^2$

$\therefore K_c^2 = (\frac{m\pi}{a})^2 + (\frac{n\pi}{b})^2$

- $K = \frac{w}{v}$ ⟶ $K_c = \frac{w_c}{v}$
- K_c: CutoFF wave number
- W_c: ..... Frequency

- $K_c^2 = \gamma^2 + K^2$
- $\therefore \gamma^2 = K_c^2 - K^2$
- $\therefore \gamma = \sqrt{K_c^2 - K^2}$

- $\therefore j\beta = j\sqrt{K^2 - K_c^2}$
- $\therefore \beta = \sqrt{K^2 - K_c^2}$
- $\therefore \beta$ is real

- $\therefore K > K_c$
- $\therefore \frac{w}{v} > \frac{w_c}{v}$

$\boxed{\therefore w > w_c \text{ or } F > F_c}$

$\boxed{\therefore \text{the operating Freq} > \text{CuttoF Freq}}$

• How to calculate CuttoF Freq?

→ $\boxed{K_c = \sqrt{(\frac{\pi m}{a})^2 + (\frac{\pi n}{b})^2}} = \frac{w_c}{v}$

$\therefore V = \frac{1}{\sqrt{\mu\epsilon}}$, For air → $V = C$

$\therefore w_c = 2\pi f_c$

$\therefore \boxed{F_c = \frac{w_c}{2\pi} = \frac{V}{2}\sqrt{(\frac{m}{a})^2 + (\frac{n}{b})^2}}$

$\therefore \lambda_c = V / F_c$

$\therefore \boxed{\lambda_c = \frac{2}{\sqrt{(\frac{m}{a})^2 + (\frac{n}{b})^2}}}$

✩ Transverse Components:

→ it's obtained From the Matrix

→ $E_z = E_0 \sin(\frac{m\pi}{a}x)\sin(\frac{n\pi}{b}y)e^{-j\beta z}$, $H_z = 0$

$$\begin{bmatrix} E_x \\ H_y \\ E_y \\ H_x \end{bmatrix} = \frac{1}{K_c^2}\begin{bmatrix} -j\beta & -jw\mu & 0 & 0 \\ -jw\epsilon & -j\beta & 0 & 0 \\ 0 & 0 & -j\beta & jw\mu \\ 0 & 0 & jw\epsilon & -j\beta \end{bmatrix} \times \begin{bmatrix} \frac{\partial E_z}{\partial x} \\ \cancel{\frac{\partial H_z}{\partial y}} \to 0 \\ \frac{\partial E_z}{\partial y} \\ \cancel{\frac{\partial H_z}{\partial x}} \to 0 \end{bmatrix}$$

- $E_x = -j\beta / K_c^2 \cdot E_0 \cdot \frac{m\pi}{a}\cos(\frac{m\pi}{a}x) \cdot \sin(\frac{n\pi}{b}y) \cdot e^{-j\beta z}$
- $H_y = -jw\epsilon / K_c^2 \cdot \dots$
- $E_y = -j\beta / K_c^2 \cdot E_0 \cdot \frac{n\pi}{b}\cos(\frac{n\pi}{b}y) \cdot \sin(\frac{m\pi}{a}x) \cdot e^{-j\beta z}$
- $H_x = jw\epsilon / K_c^2 \cdot \dots$

✩ Wave Impedance η_{TM}:

$\eta_{TM} = \frac{E_x}{H_y} = -\frac{E_y}{H_x} = \frac{\beta}{w\epsilon}$

$\therefore \beta = \sqrt{K^2 - K_c^2}$

$\therefore \beta = K\sqrt{1 - K_c^2/K^2}$

$\therefore \beta = K\sqrt{1 - (F_c/F)^2}$

$F > F_c$

$\therefore K = \frac{w}{v} = w\sqrt{\mu\epsilon}$

$\therefore \eta_{TM} = \frac{w\sqrt{\mu\epsilon}}{w\epsilon}\sqrt{1 - (F_c/F)^2}$

$\boxed{\therefore \eta_{TM} = \eta\sqrt{1 - (F_c/F)^2} \quad , \eta = \sqrt{\frac{\mu}{\epsilon}}}$