



$$H_z = H_0 \cos\left(\frac{m\pi}{a}x\right) \cos\left(\frac{n\pi}{b}y\right) \sin(\beta z)$$

$$H_z|_{z=0} = 0 \rightarrow \sin(\beta d) = 0 \rightarrow \beta d = p\pi$$

$$\therefore \beta = \frac{p\pi}{d}, p = 1, 2, 3, \dots$$

$$\beta = \frac{2\pi}{\lambda}$$

$$\therefore H_z = H_0 \cos\left(\frac{m\pi}{a}x\right) \cos\left(\frac{n\pi}{b}y\right) \sin\left(\frac{\pi p}{d}z\right)$$

Transverse Components:

$$\begin{bmatrix} E_x \\ H_y \\ E_y \\ H_x \end{bmatrix} = \frac{1}{k_c^2} \begin{bmatrix} \partial/\partial z & -jw\epsilon & 0 & 0 \\ -jw\epsilon & \partial/\partial z & 0 & 0 \\ 0 & 0 & \partial/\partial z & jw\mu \\ 0 & 0 & jw\mu & \partial/\partial z \end{bmatrix} \begin{bmatrix} \partial E_x / \partial z \\ \partial H_y / \partial z \\ \partial E_y / \partial z \\ \partial H_x / \partial z \end{bmatrix}$$

$$\therefore k_c^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 \rightarrow \text{same as waveguide}$$

TE<sub>101</sub> mode ~ dominant mode:

$$\rightarrow E_z = 0, H_z = H_0 \cos\left(\frac{\pi}{a}x\right) \sin\left(\frac{\pi}{d}z\right), H_y = 0, E_x = 0, K_c = \frac{\pi}{a}$$

$$\rightarrow E_y = jw\epsilon/k_c^2 \cdot \frac{\partial H_z}{\partial x} = \leftarrow$$

$$\begin{aligned} \rightarrow H_x &= \frac{1}{k_c^2} \frac{\partial}{\partial z} \left[ -H_0 \cdot \frac{\pi}{a} \sin\left(\frac{\pi}{a}x\right) \sin\left(\frac{\pi}{d}z\right) \right] \\ &= -\frac{1}{k_c^2} \cdot H_0 \cdot \frac{\pi}{a} \cdot \frac{\pi}{d} \sin\left(\frac{\pi}{a}x\right) \cos\left(\frac{\pi}{d}z\right) \end{aligned}$$

Quality Factor

$$\rightarrow Q = \frac{\omega_0 W_{av}}{P_L}, W_{av} = \omega_E + \omega_H = 2\omega_E = 2\omega_H$$

Power loss in Quality wall

$$\rightarrow \therefore K_c^2 = \gamma^2 + \kappa^2 \rightarrow \left(\frac{\pi}{a}\right)^2 = -\beta^2 + \left(\frac{\omega}{v}\right)^2$$

$$\therefore \left(\frac{\pi}{a}\right)^2 = -\left(\frac{\pi}{d}\right)^2 + \frac{\omega^2}{v^2}$$

$$\therefore \omega^2 = v^2 \left[ \left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{d}\right)^2 \right] \rightarrow \therefore \omega_0 = \sqrt{\sqrt{\left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{d}\right)^2}}$$

$$\rightarrow W_{av} = 2\omega_E, \omega_E = \frac{C}{4} \iiint |E_t|^2 dv$$

$$\therefore \omega_E = \frac{C}{4} \iint_0^b \int_0^a \left(\frac{\omega_0}{K_c}\right)^2 (H_0)^2 \sin^2\left(\frac{\pi}{a}x\right) \sin^2\left(\frac{\pi}{d}z\right) dx dy dz$$

$$\therefore \omega_E = \frac{C}{4} \left(\frac{\omega_0}{K_c}\right)^2 (H_0)^2 \left(\frac{a}{2}\right)\left(b\right)\left(\frac{d}{2}\right) \checkmark$$

$$\therefore W_{av} = 2\omega_E = \checkmark$$

$$\rightarrow P_L = 2(P_{L1} + P_{L2} + P_{L3})$$

$$\therefore P_{L1} = \frac{1}{2} R_s \iint_0^b \int_0^a (|H_x|^2 + |H_z|^2) dx dy dz$$

$$\therefore P_{L1} = \checkmark$$

$$\therefore P_{L2} = \frac{1}{2} R_s \iint_0^b \int_{x=0}^a (|H_x|^2 + |H_z|^2) dx dy dz = \checkmark$$

$$\therefore P_{L3} = \frac{1}{2} R_s \iint_0^b \int_{z=0}^a (|H_x|^2 + |H_z|^2) dx dy dz = \checkmark$$

$$\therefore Q = \frac{\omega_0 W_{av}}{P_L} = \checkmark \quad \text{Ans: } \checkmark$$

$$\begin{aligned} \rightarrow & P_L \rightarrow \\ & P_L = 2[P_{L1} + P_{L2} + P_{L3}] \end{aligned}$$

