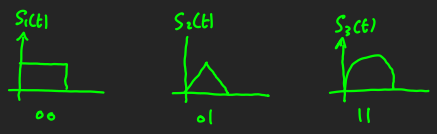
* Probabity of error ;

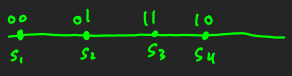
- Generally For any dector $P_e = 1 - \sum_{i=1}^{M} P_i \int_{D_i} P(\vec{r}|\vec{s_i})\, d\vec{r}$, we never use it , Complex .
- P_e doesn't change under rotation or transiation ?
  - ↳ Same decision region area.
  - ↳ Same distance between Constellation Points.
  - ↳ Noise is Spherically Symetric.

* Bit vs Symbol Prob. error :



- P_e is the average Symbol error , it doesn't give info about bit error.
  - ↳ if we sent s_1 and recived s_2, Symbol error = 1 & bit error = 1
  - ↳ if we sent s_1 and recived s_3 , Symbol error = 1 & bit error = 2
- Symbol error makes sence when send Symbols info, like english letters.
- In Geneal there's no relation between Symbol error & bit error, except two cases :

Gray Coded Constellation



- $P_b \simeq \frac{P_e}{\log_2(M)}$ ← Symbol error , # bits $= K = \log_2(M)$
- assumes error happens between s_i and s_{i+1}

M'ary $M = 2^K$

- $P_b = \frac{(M/2)}{M-1} P_e$ ← Symbol error

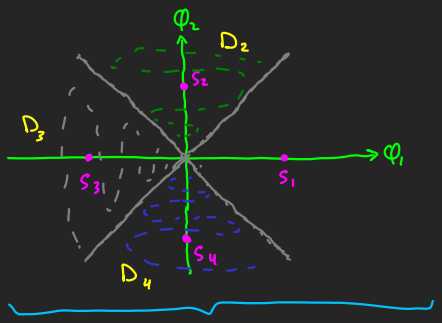
* Bounds on P_e ;

- As we saw, P_e equation is hard to solve. We also don't care about the exact value when designing.

we can take the upper bound "worst case".

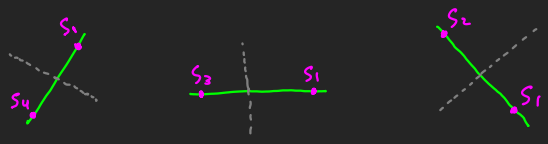
$P_e = P(\vec{r}$ Lies in the shaded region$)$

$P_e = P[\vec{r} \in (D_2 \cup D_3 \cup D_4)]$



- we can approximate to 3 separate binary systems.
- we know the P_e in case of binary system $Q\left(\frac{d}{\sqrt{2N_0}}\right)$
- we can add the 3 P_e of the 3 separate binary systems and that will be an upper bound.

$\therefore P_e \leq \sum_{\substack{k=1 \\ i \neq k}}^{M} P_{ik} \quad \equiv > \quad P_e \leq P_{12} + P_{13} + P_{14}$



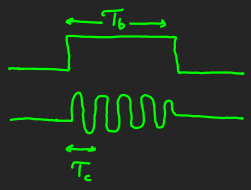
$\therefore P_e \leq \sum_{\substack{k=1 \\ i \neq k}}^{M} Q\left(\frac{d_{ik}}{\sqrt{2N_0}}\right)$. this approx. gives relativily high worst case
. higher than the actual system.

- We can approximate even more by taking only the smallest $d_{ik_{min}}$ and assuming all other d_{ik} is $d_{ik_{min}}$

$\therefore P_e \leq (M-1) Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right)$

Important Concepts in digital modulation:

choice of Carrier Freq



$T_b = m T_c$

$\frac{1}{T_b} = \frac{1}{m} \cdot \frac{1}{T_c} \rightsquigarrow \therefore R_b = \frac{f_c}{m}$

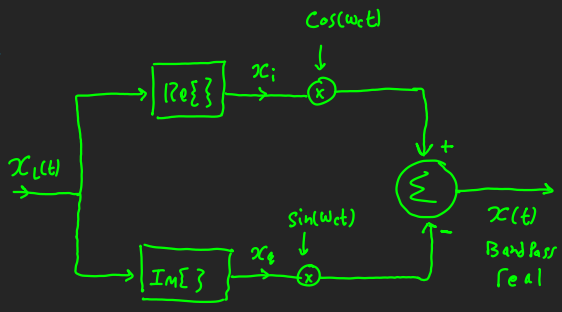
$\therefore f_c = m R_b$ carrier freq has to be multible of the bit rate.

In Phase & Quadrature Components:

- any real, narrowband, bandPass signal $x(t)$ can be represented with these two components.

$x(t) = \underbrace{x_i(t)}_{\text{in-Phase component, Real}} + j \underbrace{x_q(t)}_{\text{Quadrature component, Real}}$, $x(t) = Re\{ \underbrace{x_L(t)}_{\text{complex}} \underbrace{e^{j\omega_c t}}_{\text{separate effect of high freq}} \}$

$x(t) = x_i(t)\cos(\omega_c t) - x_q(t)\sin(\omega_c t)$



PSD & BW:

- For Linearly modulated signals "Ask, Psk, QAM", each signal is composed of an amplitude I_n and a shape $g(t)$ $S(t) = I_n g(t)$, they can be expressed as:

$X_L(t) = \sum_{n=-\infty}^{\infty} I(n) g(t - nT)$

- in this expression PSD is given by $S_{x_L}(f) = \frac{|G(f)|}{T} \cdot \mathcal{F}\{R_I(k)\}$

($\mathcal{F}\{g(t)\}^2 \Rightarrow$ ESD; Autocorrelation of amplitudes; T: symbol duration; DFT of ACF)

- We will work with the special case that I_n are i.i.d

↳ $R_I(k) = E\{I_n I_{n+k}\} = \begin{cases} E\{I_n^2\} & k=0 \\ \mu_I^2 & o.w \end{cases}$

- PSD of Modulated Signal

↳ $S_x(f) = \frac{1}{4}\left[S_{x_L}(f - f_c) + S_{x_L}(f + f_c)\right]$

- example:

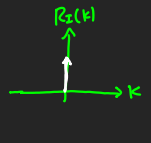
$I_n = \pm 1$, $g(t) = rect(\frac{t}{T_b})$ Polar NRZ, find B.W?

- $g(t) \xrightarrow{\mathcal{F}} G(f)$
- $rect(\frac{t}{T_b}) \xrightarrow{\mathcal{F}} T_b sinc(T_b f)$
- $R_I(k) = \begin{cases} E\{I_n^2\} & k=0 \\ \mu_I & o.w \end{cases}$

↳ $E\{I_n^2\} = E\{1,1,1,\dots,N\} = \frac{1 \cdot N}{N} = 1$

↳ $\mu_I = \frac{1-1+1-1+\dots}{N} = \frac{0}{N} = 0$

$\therefore R_I(k) = \begin{cases} 1 & k=0 \\ \mu_I & o.T \end{cases} = \delta(k)$



$\therefore S_{x_L}(f) = \frac{G(f)^2}{T} \cdot \mathcal{F}\{R_I(k)\}$

$\therefore S_{x_L}(f) = \frac{T_b^2 sinc^2(T_b f)}{T_b} \cdot \mathcal{F}\{\delta(k)\}$

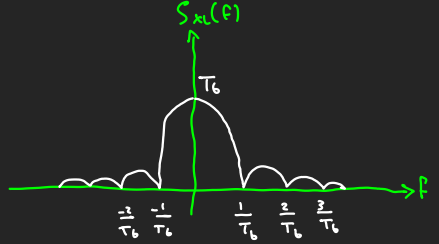
$\therefore S_{x_L}(f) = T_b sinc^2(T_b f)$

$\therefore$ BW null to null

↳ $BW = \frac{1}{T_b} = R_b$

$\therefore$ BW 3 dB

↳ $BW = \frac{R_b}{2}$



* M'ary modulation :

bit stream → [bit-to-symbol conversion] → symbols → [modulation] → M wave forms

take K bits and convert them to a symbol

- $R_s = \frac{R_b}{K} = \frac{R_b}{\log_2(M)}$ symbols/sec , $E_{b_{avg}} = \frac{E_{sym}}{K}$

- example: 4 PSK

$I_n = 10\,11\,01\,00\,11\,10$

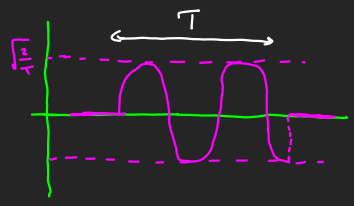
$M = 4 = 2^K \rightarrow K = 2$

bits	symbol	θ
00	0	0
01	1	$\pi/2$
11	3	π
10	2	$3\pi/2$

- why M'ary ?
  - ↳ less BW for the same rate $BW_{M'ary} = \frac{BW_b}{K}$
  - ↳ more information for same BW

* Signal Space :

- orthogonal basis functions $\varphi_1(t) = \sqrt{\frac{2}{T}}\sin(\omega_c t + \theta_c)$, $\varphi_2(t) = \sqrt{\frac{2}{T}}\cos(\omega_c t + \theta_c)$ $0 < t < T$



- $E_m = \|\vec{S}_m\|^2$, $\rho_{ij} = \cos(\theta_{ij})$

- $P_e = Q\left(\frac{d}{\sqrt{2N_0}}\right)$ (binary) $\leq (M-1)Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right)$

- Two Performance metrics :
  - ↳ Symbol efficiency $\eta_s = \frac{R_b}{BW}$ b/s/Hz , it is a measure of how well we used the BW avalible Required $\eta_s = \frac{R_b}{BW} \uparrow\uparrow$ biggest bit rate possible for smallest BW possible
  - ↳ Power efficiency $\eta_p = \min_{@P_e = 10^{-5}} \frac{E_b}{N_0}$ ↓↓ Lowest energy possible for worst Noise.