Prob. (9.20)

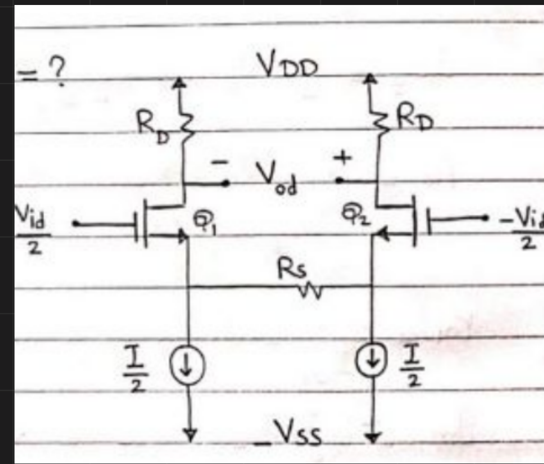
find the diff. half Circuit, derive an expression for A_d

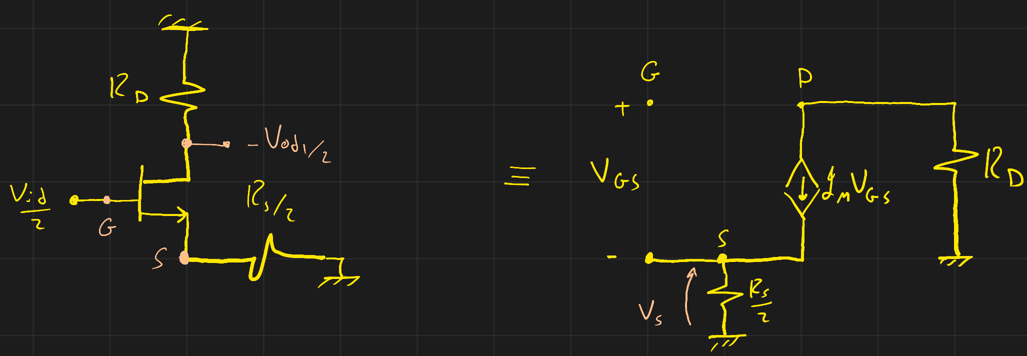
$A_d = \frac{V_{od}}{V_{id}} = f(g_m, R_D, R_S)$, neglect early effect.

no r_o

1 → What is the gain $R_S = 0$ ---→ $A_{d\,max}$

2 → " " " Value of R_S $f(\frac{1}{g_m})$ that reduce gain to half Value





① $A_d = \frac{V_{od1}}{V_{id1}} = \frac{V_{od1}/2}{V_{id}/2} = \frac{V_{od1}}{V_{id}} = \frac{V_D}{V_G} = \frac{V_D}{V_{id}/2}$

→ $V_D = -g_m V_{GS} R_D$ → ⓪

→ $V_{GS} = V_G - V_S = \frac{V_{id}}{2} - g_m V_{GS} \frac{R_s}{2}$

$\therefore V_{GS} + g_m V_{GS} \frac{R_s}{2} = \frac{V_{id}}{2}$

$\therefore V_{GS}\left(1 + g_m \frac{R_s}{2}\right) = \frac{V_{id}}{2}$

$\therefore V_{GS} = \frac{V_{id}/2}{1 + g_m R_s/2}$

$\therefore$ ⓪ → $V_D = -g_m\left(\frac{V_{id}/2}{1+g_m R_s/2}\right) R_D$

$\therefore A_d = \frac{V_D}{V_{id}/2}$

$\therefore \boxed{A_d = -g_m \frac{1}{1+g_m R_s/2} R_D}$

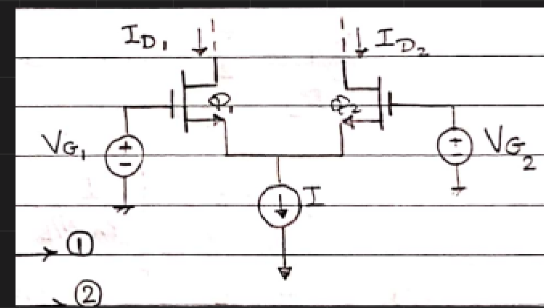
$\therefore \boxed{A_{d\,max}(R_s = 0) = -g_m R_D}$

② $R_s = ?$ @ $A_d = \frac{1}{2} A_{d\,max}$

→ $\therefore \cancel{-g_m} \frac{1}{1+g_m R_s/2} \cancel{R_D} = \frac{1}{2}(\cancel{-g_m R_D})$

$\therefore 1 + g_m R_s/2 = 2$ ⇝ $\therefore \boxed{R_s = \frac{2}{g_m}}$

★ Large signal operation:



$\therefore I_{D_1} = \frac{1}{2}k_n'\left(\frac{W}{L}\right)(V_{GS_1}-V_t)^2 \longrightarrow (1)$

$\therefore I_{D_2} = \frac{1}{2}k_n'\left(\frac{W}{L}\right)(V_{GS_2}-V_t)^2 \longrightarrow (2)$

root of (1), (2):

$\therefore \sqrt{I_{D_1}} = \sqrt{\frac{1}{2}k_n'\left(\frac{W}{L}\right)}\,(V_{GS_1}-V_t) \rightsquigarrow \therefore \sqrt{I_{D_1}} = \sqrt{\frac{I}{2}V_{ov}}\,(V_{GS_1}-V_t) \longrightarrow (3)$

$\therefore \sqrt{I_{D_2}} = \sqrt{\frac{1}{2}k_n'\left(\frac{W}{L}\right)}\,(V_{GS_2}-V_t) \rightsquigarrow \therefore \sqrt{I_{D_2}} = \sqrt{\frac{I}{2}V_{ov}}\,(V_{GS_2}-V_t) \longrightarrow (4)$

$(3)-(4) \rightsquigarrow \therefore \sqrt{I_{D_1}} - \sqrt{I_{D_2}} = \sqrt{\frac{I}{2}V_{ov}}\,(V_{GS_1}-V_{GS_2})$

$\therefore \sqrt{I_{D_1}} - \sqrt{I_{D_2}} = \sqrt{\frac{I}{2}V_{ov}}\,(V_{id}) \longrightarrow (5)$

$(5)^2 \rightsquigarrow I_{D_1} + I_{D_2} - 2\sqrt{I_{D_1}}\sqrt{I_{D_2}} = \frac{I}{2}V_{ov}(V_{id})^2$ $\qquad \therefore I_{D_1} + I_{D_2} = I$

$\therefore I - 2\sqrt{I_{D_1}}\sqrt{I_{D_2}} = \frac{I}{2}V_{ov}(V_{id})^2 \longrightarrow (6)$

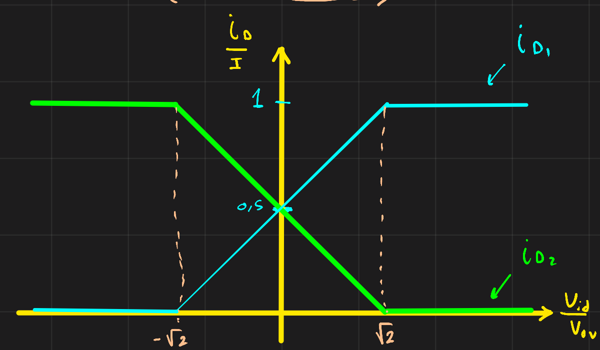
$\therefore$ to get I_{D_1}

$(6) \rightsquigarrow I - 2\sqrt{I_{D_1}}\sqrt{I - I_{D_1}} = \frac{I}{2}V_{ov}(V_{id})^2$

$\therefore I_{D_1} = \frac{I}{2} + \frac{I}{V_{ov}}\left(\frac{V_{id}}{2}\right)\sqrt{1-\left(\frac{V_{id}}{2V_{ov}}\right)^2}$

$\therefore I_{D_2} = \frac{I}{2} - \frac{I}{V_{ov}}\left(\frac{V_{id}}{2}\right)\sqrt{1-\left(\frac{V_{id}}{2V_{ov}}\right)^2}$

region where linear amp is possible



Small signal amp.

→ note:

↳ $V_{id} > 0 \rightsquigarrow V_{GS_1} > V_{GS_2} \rightsquigarrow I_{D_1} = \frac{I}{2} + \cdots$

$I_{D_2} = \frac{I}{2} - \cdots$

↳ $V_{id} < 0 \rightsquigarrow V_{GS_2} > V_{GS_1} \rightsquigarrow I_{D_1} = \frac{I}{2} - \cdots$

$I_{D_2} = \frac{I}{2} + \cdots$

↳ $V_{id} = 0 \rightsquigarrow$ Common mode $\rightsquigarrow I_{D_1} = I_{D_2} = \frac{I}{2}$

$I_{D_1}(v_{id} = \sqrt{2}V_{ov}) = \frac{I}{2} + \frac{I}{\cancel{V_{ov}}}\left(\frac{\sqrt{2}\cancel{V_{ov}}}{2}\right)\sqrt{1-\left(\frac{\sqrt{2}\cancel{V_{ov}}}{2\cancel{V_{ov}}}\right)^2}$

$= \frac{I}{2} + \frac{I}{2}$

$= I$

★ Small signal operation:

$\rightarrow V_{id} < \sqrt{2}\, V_{ov}$

$$\rightarrow i_{D_1} = \frac{I}{2} + \frac{I}{V_{ov}}\left(\frac{V_{id}}{2}\right)\sqrt{1-\left(\frac{V_{id}}{2V_{ov}}\right)^2}$$

$$= \frac{I}{2} + \frac{I}{V_{ov}}\left(\frac{V_{id}}{2}\right)\left(1-\frac{1}{2}\left(\frac{V_{id}}{2V_{ov}}\right)\right)$$

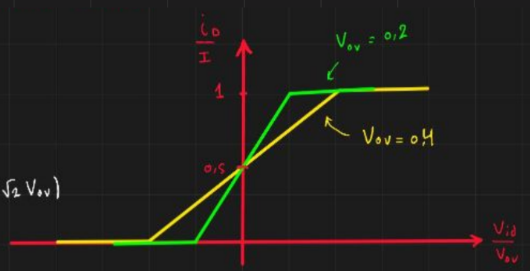
$$= \frac{I}{2} + \left(\frac{I}{2}\right)\left[\frac{V_{id}}{V_{ov}} - \frac{1}{4}\cancel{\left(\frac{V_{id}}{V_{ov}}\right)^2}\right] \rightarrow 0$$

$$= \frac{I}{2} + \frac{I}{V_{ov}}\left(\frac{V_{id}}{2}\right)$$

$$\therefore i_{D_1} = \frac{I}{2} + g_m\left(\frac{V_{id}}{2}\right)$$

$$\therefore i_{D_2} = \frac{I}{2} - g_m\left(\frac{V_{id}}{2}\right)$$

$$\therefore i_d = g_m\left(\frac{v_{id}}{2}\right)$$



→ Notes:

1). increasing V_{ov} extends the linear range $(-\sqrt{2}\,V_{ov} < V_{id} < \sqrt{2}\,V_{ov})$

tradeoff: decrease in gain (g_m) ↓ $\rightarrow g_m = \frac{2I_D}{V_{ov}}$)

2) decrease V_{ov} gain ↑↑ but linear region ↓↓

3) we can increas I (bias current) to increase gain

tradoff: higher power dissipation.

0.18 μm CMOS $\quad$ $\mu_n C_{ox} = 4 \mu_p C_{ox} = 400\ \mu A/V^2$

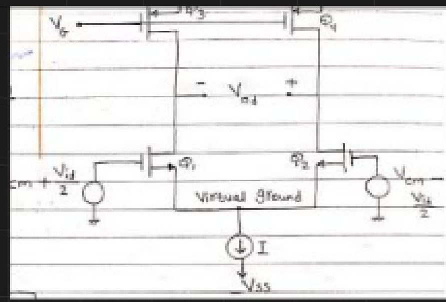
$|V_t| = 0.5\ V$ $\quad$ $|V_A'| = 10\ V/\mu m$

$I = 200\ \mu A$ $\quad$ $L = 2$ minimum $= 2(0.18) = 0.36\ \mu m$

$|V_{ov}| = 0.2\ V$ $\quad$ "diff. Amp. with C.S"

find : * $\frac{W}{L}$ for Q_1, Q_2, Q_3, Q_4

* the differential voltage gain A_d



. $I_{D_1} = I_{D_2} = \frac{I}{2} = \frac{200}{2} = 100\ \mu A$

. $I_D = \frac{1}{2} K_{n_1}' \left(\frac{W}{L}\right)_1 (V_{ov})^2$

$\therefore 100\mu = \frac{1}{2}(400\mu)\left(\frac{W}{L}\right)_1 (0.2)^2$

$\therefore \boxed{\left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = 12.5}$

$I_D = \frac{1}{2} K_{n_3}' \left(\frac{W}{L}\right)_3 (V_{ov})^2$

$100\mu = \frac{1}{2}(100\mu)\left(\frac{W}{L}\right)_3 (0.2)^2$

$\therefore \boxed{\left(\frac{W}{L}\right)_3 = \left(\frac{W}{L}\right)_4 = 50}$

. $A_d = -g_m (r_{o_1} \parallel r_{o_3})$

$\rightarrow r_{o_1} = \frac{V_A}{I_D} = \frac{V_A' L}{I_D} = \frac{(10)(0.36)}{100\mu} = 36\ K\Omega$

$\rightarrow r_{o_3} = - - - - - - - = \frac{(10)(0.36)}{100\mu} = 36\ K\Omega$

$\rightarrow g_{m_1} = \frac{2I_D}{V_{ov}} = \frac{I}{V_{ov}} = \frac{200\mu}{0.2} = 1\ mA/v$

$\therefore A_d = -(1)\left(\frac{1}{2} \times 36\right) \rightarrow \therefore \boxed{A_d = -18}$

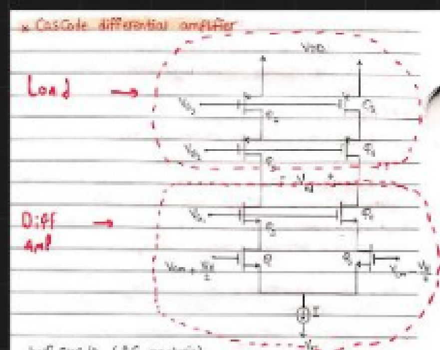
Ex (9.6) $\quad$ CasCode diff. amplifier

$\mu_n C_{ox} = 4 \mu_p C_{ox} = 400\ \mu A/V^2$ $\quad$ $|V_t| = 0.5\ V$

$|V_A'| = 10\ V/\mu m$ $\quad$ $I = 200\ \mu A$ $\quad$ $L = 0.36\ \mu m$

$|V_{ov}| = 0.2\ V$

find : ① $\frac{W}{L}$ for each of $Q_1 \rightarrow Q_8$ $\quad$ ② A_d



. $I_{D_1} = \frac{1}{2} K_{n_1}' \left(\frac{W}{L}\right)_1 (V_{ov})^2$

$100\mu = \frac{1}{2}(400\mu)\left(\frac{W}{L}\right)_1 (0.2)^2$

$\therefore \boxed{\left(\frac{W}{L}\right)_{1,2,3,4} = 12.5}$

. $I_{D_5} = \frac{1}{2} K_{n_5}' \left(\frac{W}{L}\right)_5 (V_{ov})^2$

$100\mu = \frac{1}{2}(100)\left(\frac{W}{L}\right)_5 (0.2)^2$

$\therefore \boxed{\left(\frac{W}{L}\right)_{5,6,7,8} = 50}$

. $A_d = -g_{m_1}(R_{on} \parallel R_{op})$

$R_{on} = g_{m_3} r_{o_3} r_{o_1}$

$R_{op} = g_{m_5} r_{o_5} r_{o_7}$

. $r_o = \frac{V_A}{I_D} = \frac{V_A' L}{I_D} = 36\ K\Omega$

. $g_m = \frac{2I_D}{V_{ov}} = \frac{I}{V_{ov}} = 1\ m\Omega^{-1}$

$\therefore R_{on} = 36^2\ K\Omega$

$\therefore R_{op} = 36^2\ K\Omega$

$\therefore \boxed{A_d = g_{m_1} \frac{36^2}{2} = 648}$