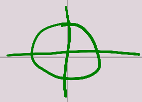
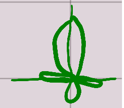
* T_x isotropic antenna: $\rightsquigarrow P_r = \frac{W_T}{4\pi r^2}$ ← transmitted Power (watt)



↗ average Power density (W/m^2)

* T_x Directive with gain G_T: $\rightsquigarrow P_r = \frac{W_T}{4\pi r^2} \cdot G_T \cdot L$



* $W_R = P_r \cdot A_{eff}$ $\rightsquigarrow A_{eff} = \frac{\lambda^2}{4\pi} G_R$ ⟶ $\boxed{\therefore W_R = \frac{W_T}{4\pi r^2} \cdot G_T \cdot L \cdot \frac{\lambda^2}{4\pi} \cdot G_R}$

Recieved Power (W_R), R_x effective Area (A_{eff})

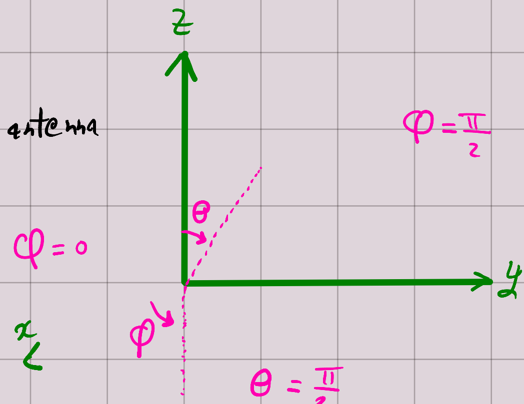
* after Calculating W_R we Can Calculate SNR $\rightsquigarrow SNR = \frac{W_R}{KTB}$, KTB: thermal noize

K: Boltzman Constant, T: Temp, B: R_x BW

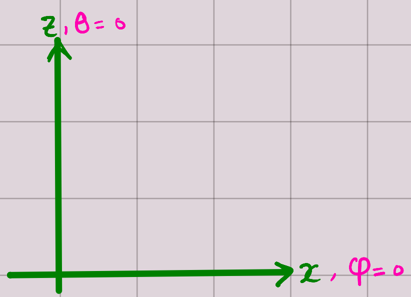
Basic Definitions:

① Antenna radiation Pattern:

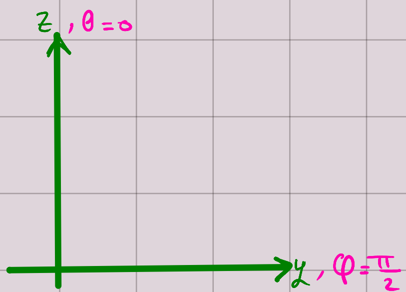
⤳ It's 3-D graphical representation of the radiation from antenna [electric Field or Power density]



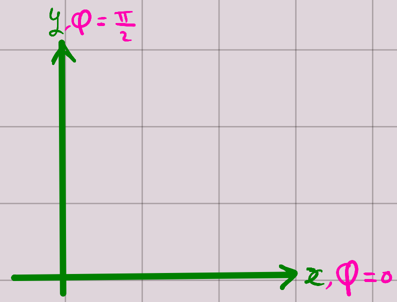
→ practicly pattern is drawn in 2-D [x-y , x-z , y-z]



$\varphi = 0$



$\varphi = \frac{\pi}{2}$



$\theta = \frac{\pi}{2}$

⤳ $0 < \theta < \pi$, $0 < \varphi < 2\pi$

⤳ to draw the Pattern, we get $E_n = f(\theta, \varphi)$ only " no $\hat{r}$ "

→ $E_n = f(\theta, \varphi)$ electric Field Pattern " radiation electric Pattern "

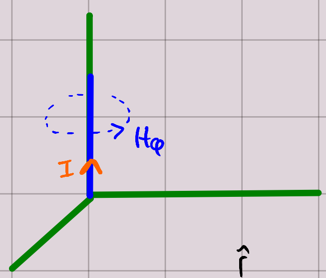
(E_n: normalized)

→ Power Pattern (P_n) = (radiation electric Pattern)^2

② Pointing Vector & Power density

$\rightsquigarrow \vec{P}_{av} = \frac{1}{2} \mathrm{Real}\{\vec{E} \times \vec{H}\} = P(r,\theta,\varphi)\,\hat{r}$ (Direction of Propagation)

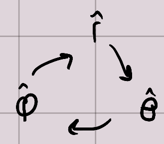
Pointing Vector (w/m²)



$\rightsquigarrow \because \vec{E}(r) = E_r\,\hat{r} + E_\theta\,\hat{\theta}$, $\vec{H}(r) = H_\varphi\,\hat{\varphi}$

$\therefore \vec{P}_{av} = \frac{1}{2}\mathrm{Real}\{(E_r\,\hat{r} + E_\theta\,\hat{\theta}) \times (H_\varphi\,\hat{\varphi})\}$

$= \frac{1}{2}\mathrm{Real}\{-E_r H_\varphi \hat{\theta} + E_\theta H_\varphi\,\hat{r}\}$



$\rightsquigarrow$ in far field E_r is negletble $\simeq 0$

$\therefore \vec{P}_{av} = \frac{1}{2}\mathrm{Real}\{E_\theta H_\varphi\,\hat{r}\}$

$\rightsquigarrow \because E = \eta H$ & $H = E/\eta$, $\eta = \sqrt{\mu/\varepsilon}$, $\eta = 120\pi$ For Free Space

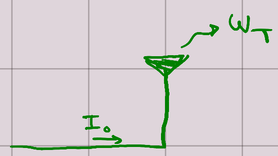
intrinsic impedance

$\therefore \vec{P}_{av}(r,\theta,\varphi) = \frac{1}{2\eta}|E|^2\,\hat{r} = \frac{1}{2}\eta\,|H|^2\,\hat{r}$

③ total transmitted Power

$\rightsquigarrow W_T = \oiint P_{av} \cdot ds$, $ds = r^2 \sin\theta\, d\theta\, d\varphi$ "spharical conditions" , $\theta : 0 \to \pi$, $\varphi : 0 \to 2\pi$

$\rightsquigarrow W_T = \int_{\theta=0}^{\pi}\int_{\varphi=0}^{2\pi} P_{av} \cdot r^2 \sin\theta\, d\theta\, d\varphi = \checkmark$



$\rightsquigarrow W_T = \frac{1}{2} I_o^2 R_r$, I_o: max Current Passes throw antenna

, R_r: radiation resistance

④ Antenna gain:

$\rightsquigarrow G = \dfrac{\text{max Power densty from antenna}}{\text{average Power densty of isotropic antenna}} = \dfrac{P_{max}}{P_{av}} = \dfrac{P_{max}}{W_T/4\pi r^2}$

$\rightsquigarrow G = K \cdot D$, K: Constant represent efficiency of antenna, if not given $K=1 \rightsquigarrow G=D$

$\rightsquigarrow$ total Power loss (dB) $= 10\log\left(\frac{W_T}{W_L}\right)$

Power at Load

⑤ Maxweel's equations:

- $\nabla \times \vec{E} = \frac{-\partial \vec{B}}{\partial t} = -\mu \frac{\partial \vec{H}}{\partial t}$
- $\nabla \times \vec{H} = J + \frac{\partial \vec{D}}{\partial t}$
- $\nabla \cdot \vec{D} = \rho$
- $\nabla \cdot \vec{B} = 0$
- $\vec{D} = \epsilon \vec{E}$
- $B = \mu \vec{H}$

- η wave impedance $\rightarrow \eta = \frac{|\vec{E}|}{|\vec{H}|} = \sqrt{\mu/\epsilon}$

- Lorentz Condition: $\boxed{\nabla \cdot \vec{A} + \mu \epsilon \frac{\partial V}{\partial t} = 0}$

$\vec{A}$: magntic Vector Potential

V: scaler Potential

- $\because \nabla \cdot \vec{B} = 0 \rightarrow \therefore \vec{B} = \nabla \times \vec{A}$
- if $\nabla \times \vec{\text{vector}} = 0 \rightarrow \vec{\text{Vector}} = -\nabla V$

- to transform from spherical to cartezen $\rightarrow \hat{Z} = \cos\theta\, \hat{r} - \sin\theta\, \hat{\theta}$
- $\nabla \times \nabla \times \vec{A} = \nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A}$

- From wave equation (Plane-wave noharmonic time Variation)

$\rightarrow \nabla^2 \vec{E} - \mu \epsilon \frac{\partial^2 \vec{E}}{\partial t} = -\rho/\epsilon$

$\rightarrow \nabla^2 \vec{H} - \mu \epsilon \frac{\partial^2 \vec{H}}{\partial t} = -\mu \vec{J}$

→ We want to drive expressions For $\vec{E}, \vec{H}$ For antenna element, it's easier to drive them not directly by driving $\vec{A}$, V expressions First.

→ we will drive them first For electrostatic and retarted Fields, then Prove them For time Varying Case.

⑥ drive expression For $\vec{A}$ & V For electro & magnto static based on retarted Potential.

$\rightarrow \because \vec{E} = -\nabla V \quad , \because \nabla \cdot \vec{D} = \rho \quad , \because \vec{D} = \epsilon \vec{E} \rightarrow \therefore \nabla \cdot \epsilon \vec{E} = \rho \rightarrow \therefore \nabla \cdot \vec{E} = \rho/\epsilon$

$\because \nabla \cdot \vec{E} = -\nabla^2 V \rightarrow \boxed{\therefore \nabla^2 V = -\rho/\epsilon}$ ← Poission equation

↳ the Solution $\rightarrow \boxed{V(\hat{r}) = \frac{1}{4\pi\epsilon_0} \iiint_{Vol} \frac{\rho(\hat{r})}{R} dv'}$

→ Similarly in magnetostatic:

$\because \vec{B} = \nabla \times \vec{A}$ and $\because \nabla^2 A = -\mu \vec{J}$ → Poission equation in magntism.

→ the Solution gives $\vec{A}(r)$ ⇝ $\therefore \vec{A}(r) = \frac{\mu_0}{4\pi} \oiiint_{Vol} \frac{\vec{J}(\hat{r})}{R} dv'$

→ For time varying Source of current and Voltage $t \to t - \frac{R}{C}$ ← Propagated distance by wave

→ $\vec{A}(\hat{r},t) = \frac{\mu_0}{4\pi} \iiint_{Vol} \frac{\vec{J}(\hat{r}, t-R/c)}{R} dv'$ →$V(\hat{r},t) = \frac{1}{4\pi\epsilon_0} \oiiint_{Vol} \frac{\rho(\hat{r}, t-R/c)}{R} dv'$

$\boxed{Q}$ Prove that both $\vec{A}$ & V satisfy the wave equation on the Condition of Lorentz Condition

★ $\because \nabla \cdot \vec{B} = 0 \longrightarrow \therefore \vec{B} = \nabla \times \vec{A}$

$\because \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \longrightarrow \therefore \nabla \times \vec{E} = -\frac{\partial}{\partial t}(\nabla \times \vec{A}) \longrightarrow \therefore \nabla \times \left(\vec{E} + \frac{\partial \vec{A}}{\partial t}\right) = 0$

$\because$ if $\nabla \times \vec{Vec} = 0 \longrightarrow \therefore \vec{Vec} = -\nabla V$

$\therefore \boxed{\vec{E} + \frac{\partial \vec{A}}{\partial t} = -\nabla V} \to \boxed{1}$

★ • $\because \vec{B} = \nabla \times \vec{A}$, $\because \vec{B} = \mu \vec{H} \longrightarrow \therefore \vec{H} = \frac{1}{\mu} \nabla \times \vec{A}$

$\because \nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$

$\therefore \nabla \times \left(\frac{1}{\mu} \nabla \times \vec{A}\right) = \vec{J} + \epsilon \frac{\partial \vec{E}}{\partial t}$

$\therefore \frac{1}{\mu} \nabla \times \nabla \times \vec{A} = \epsilon \frac{\partial \vec{E}}{\partial t} + \vec{J}$

$\therefore \frac{1}{\mu}[\nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A}] = -\epsilon \frac{\partial}{\partial t}\underbrace{\left(\frac{\partial \vec{A}}{\partial t} + \nabla V\right)}_{\text{From } \boxed{1}} + \vec{J}$

$\therefore \nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A} = -\mu\epsilon \frac{\partial^2 A}{\partial t^2} - \mu\epsilon \frac{\partial \nabla V}{\partial t} + \mu \vec{J}$

$\therefore \nabla^2 \vec{A} - \mu\epsilon \frac{\partial^2 A}{\partial t} = -\mu \vec{J} + \nabla\underbrace{\left(\cancel{\mu\epsilon \frac{\partial V}{\partial t} + \nabla \cdot \vec{A}}\right)}_{=0 \text{ Lorentz Condition}}$

$$\therefore \boxed{\nabla^2 \vec{A} - \mu\epsilon \frac{\partial^2 A}{\partial t^2} = -\mu \vec{J}} \quad \text{I}$$

which satisfies wave equ.

• $\because \nabla \cdot \vec{D} = \rho \longrightarrow \therefore \nabla \cdot \vec{E} = \rho/\epsilon$

From (1) $\longrightarrow \therefore \nabla \cdot \left(-\nabla V - \frac{\partial \vec{A}}{\partial t}\right) = \rho/\epsilon$

$\therefore \nabla^2 V + \frac{\partial}{\partial t} \nabla \cdot \vec{A} = -\rho/\epsilon$

$\because \nabla \cdot \vec{A} = -\mu\epsilon \frac{\partial V}{\partial t}$ From lorentz

$$\therefore \boxed{\nabla^2 V - \mu\epsilon \frac{\partial^2 V}{\partial t^2} = \frac{-\rho}{\epsilon}} \quad \text{II}$$

which satisfies wave equ.

→ For harmonic time variations of the source ($e^{j\omega t}$):

$\therefore \vec{A}(r,t) = \vec{A}(r)e^{j\omega t}$, $V(r,t) = V(r)e^{j\omega t}$, $\vec{J}(r,t) = \vec{J}(r)e^{j\omega t}$, $\rho(r,t) = \rho(r)e^{j\omega t}$

$\because$ (I): $\nabla^2\vec{A} - \mu\epsilon\frac{\partial^2\vec{A}}{\partial t} = -\mu\vec{J}$

$\therefore \nabla^2\vec{A}\cancel{e^{j\omega t}} + \mu\epsilon\omega^2\vec{A}\cancel{e^{j\omega t}} = -\mu\vec{J}\cancel{e^{j\omega t}}$

$\therefore \nabla^2\vec{A} + \beta^2\vec{A} = -\mu\vec{J}$

$\therefore \nabla^2 V + \beta^2 V = \frac{-\rho}{\epsilon}$

→ by solving the two equations we get:

- $\vec{A}(r) = \frac{\mu_0}{4\pi}\oiiint_{vol}\frac{\vec{J}(r)}{R}e^{-j\beta R}\,dv'$
- $V(r) = \frac{1}{4\pi\epsilon_0}\oiiint_{vol}\frac{\rho(r)}{R}e^{-j\beta R}\,dv'$

→ For a wire antenna with harmonic time variation ($\frac{\partial}{\partial t} = j\omega$):

→ the magnetic vector potential $\vec{A}$

- $\vec{A} = \frac{\mu}{4\pi}\int_L\frac{I}{R}e^{-j\beta R}\,dL$, R: distance from element to point of consideration

r: - - - - - - - origin - - - - - - - -

→ to get $\vec{E}$ & $\vec{H}$ after we obtained $\vec{A}$:

→ Far field approximate: r >> point of observation is far.

$\therefore \vec{E} = \nabla V - \frac{\partial\vec{A}}{\partial t} \simeq \frac{-\partial\vec{A}}{\partial t} = -j\omega\vec{A}$

$\therefore \boxed{\vec{E} = -j\omega\vec{A}}$ → $\vec{E}_\theta = -j\omega\vec{A}_\theta$ → $\vec{E}_\phi = -j\omega\vec{A}_\phi$

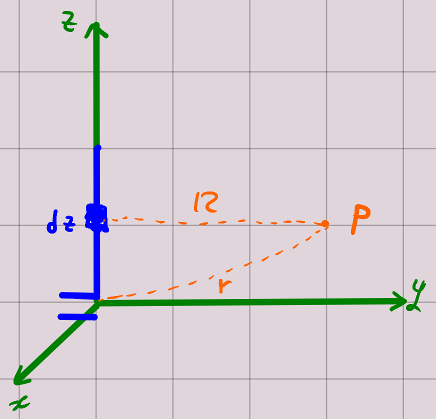
→ $\vec{H}_\phi = \frac{\vec{E}_\theta}{\eta}$ → $\vec{H}_\theta = \frac{-\vec{E}_\phi}{\eta}$

$\because \hat{z} = \cos\theta\,\hat{r} - \sin\theta\,\hat{\theta}$

$\therefore A_z\hat{z} = \underbrace{A_z\cos\theta\,\hat{r}}_{A_r} - \underbrace{A_z\sin\theta\,\hat{\theta}}_{A_\theta}$

$$\boxed{\therefore \vec{E}_\theta = -j\omega\vec{A}_\theta = j\omega A_z\sin\theta\,\hat{\theta}}$$

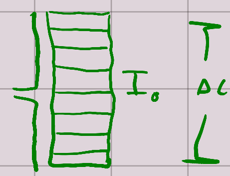
* Drive $\vec{E}$ & $\vec{H}$ For infintismal dipole: $\Delta L \ll \lambda$



$\rightarrow \vec{A}(\hat{r}) = \frac{\mu}{4\pi} \int_{-\Delta L/2}^{\Delta L/2} \frac{I(\grave{z})}{R} e^{-j\beta R} \, dz \, \hat{z}$

$\rightarrow$ For any infinitizmal dipole we have two approx:

- 1) Phase approx $\rightsquigarrow R \simeq r$
- 2) magnitude approx $\rightsquigarrow \frac{1}{R} = \frac{1}{r}$



$\rightarrow$ For infinitizmal dipole $I(z)$ is uniform $\rightsquigarrow I(z) = I_0$

$$\therefore \vec{A}(\hat{r}) = \frac{\mu}{4\pi r} I_0 e^{j\beta r} \int_{-\Delta L/2}^{\Delta L/2} dz \, \hat{z} \rightsquigarrow \therefore \boxed{\vec{A}(\hat{r}) = \frac{\mu I_0}{4\pi r} \Delta L \, e^{-j\beta r} \, \hat{z}}$$

$$\therefore \vec{A}_\theta = -\vec{A}_z \sin\theta \, \hat{\theta} = \frac{-\mu I_0 \Delta L}{4\pi r} \sin\theta \, e^{-j\beta r} \, \hat{\theta}$$

$$\because \vec{E}_\theta = -j\omega A_\theta$$

$$\therefore \boxed{\vec{E}_\theta = j\omega \frac{\mu I_0 \Delta L}{4\pi r} \sin\theta \, e^{-j\beta r} \, \hat{\theta}} \quad , \vec{H}_\phi = \frac{E_\theta}{\eta} = \checkmark$$