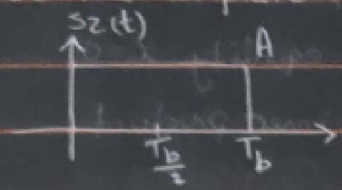
For example 9 MF proplem, find E_{12}

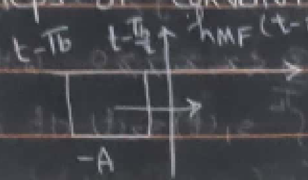
$$E_{12} = \int_0^{T_b/2} 0 * A\, dt + \int_{T_b/2}^{T_b} A^2\, dt = \left[T_b - \frac{T_b}{2}\right] A^2 = \frac{A^2 T_b}{2}$$

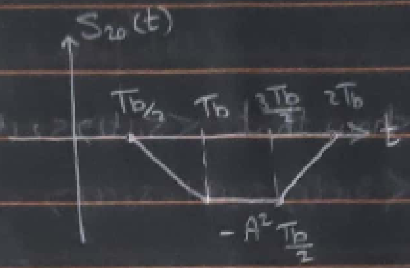
$$S_{10}(T) = E_1 - E_{12} = 0 \;,\; E_{20} = \frac{A^2 T_b}{2} - A^2 T_b = \frac{-A^2 T_b}{2}$$

$$V_{th}^* = \frac{S_{01}(T) + S_{20}(T)}{2} = -\frac{A^2 T_b}{4}$$

do the Complete steps of Convolution.







$t - \frac{T_b}{2} = 0 \Rightarrow t = \frac{T_b}{2}$ amp $= 0$

$t - \frac{T_b}{2} = \frac{T_b}{2} \Rightarrow t = T_b$ amp $= -\frac{A^2 T_b}{2}$

$t - \frac{T_b}{2} = T_b \Rightarrow t = \frac{3T_b}{2}$ amp $= -\frac{A^2 T_b}{2}$

$t - \frac{T_b}{2} = \frac{3T_b}{2} \Rightarrow t = 2T_b$ amp $= 0$

---

note ④ ch1 (last note in ch 1)

lec 4 Why signals $x(t)$ has infinite dimensions?

since x can take any value.

Can we represent any digital signal using finite dimension space? How?

yes, using M dimensions for M'ary signaling

show that $S_{ij} = \int_0^{T_b} s_i(t)\, Q_j(t)\, dt$

$$s_i(t) = \sum_{K=1}^{N} s_{ik} Q_K(t)$$

$$s_i(t)\, Q_j(t) = \sum_{K=1}^{N} s_{ik}\, Q_K(t)\, Q_j(t) \qquad \int \text{ both sides}$$

$$\int_0^{T_b} s_i(t)\, Q_j(t)\, dt = \sum_{K=1}^{N} s_{ik}\, Q_K(t) \int_0^{T_b} Q_j(t)\, dt$$

at $K = j$ only has a value

$$= s_{ij} * \underbrace{\int_0^{T_b} Q_j(t)\, Q_j(t)\, dt}_{1}$$

#

Prove relation Eg & Distance

$$E_g = \int_0^{T_b} |s_1(t) - s_2(t)|^2 dt \qquad s_1(t) - s_2(t) = g(t)$$

$$= \int_0^{T_b} |g(t)|^2 dt = \int_0^{T_b} g(t).g(t)\, dt = \sum_{j=1}^{N} g_i^2 = \|g\|^2 = \|s_1 - s_2\|^2$$

a vector space with inner product ① $\langle x, y+z \rangle = \langle x,y \rangle + \langle x,z \rangle$

② $\langle \alpha x, y \rangle = \alpha \langle x, y \rangle$

③ $\langle x, x \rangle \geq 0$ with equality $x = 0$

show that $\langle s_1(t), s_2(t) \rangle = \int_0^{T_b} s_1(t) s_2(t)\, dt$ is inner product

$$\langle s_1(t), s_2(t) + s_3(t) \rangle = \int_0^{T_b} s_1(t)(s_2(t) + s_3(t))\, dt$$

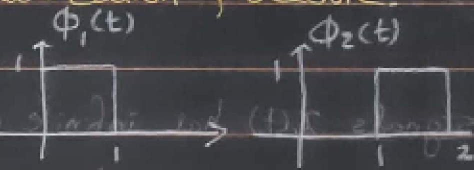
$$= \int_0^{T_b} s_1(t) s_2(t)\, dt + \int_0^{T_b} s_1(t) s_3(t)\, dt$$

$$= \langle s_1(t), s_2(t) \rangle + \langle s_1(t), s_3(t) \rangle$$

How can we systematically find $\{\phi_j(t)\}_{j=1}^{N}$ for any signal set $\{s_i(t)\}_{i=0}^{M}$

Using Gram_shmidt orthogonalization procedure.

verify they are orthogonal



$$\int_0^2 \phi_1(t).\phi_2(t)\, dt = \int 0\, dt = \text{zero}$$

$\therefore$ orthogonal

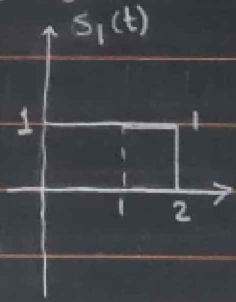
verify that from $\underline{s_i}$ you can get $s_i(t)$

$$s_i = \int_0^{T_b} s_i(t)\, \phi_j(t)\, dt$$

$$s_i(t) = \sum_{j=1}^{N} s_{ij}\, \phi_j(t)$$

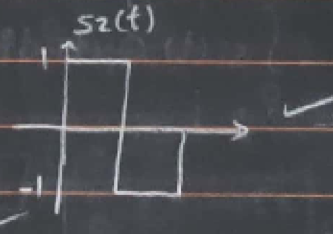
$$s_1(t) = \sum_{j=1}^{N} s_{1j}\, \phi_j(t) = s_{11}\phi_1(t) + s_{12}\phi_2(t) + s_{13}\phi_3(t)$$

$$= 1^* \phi_1(t) + 1^* \phi_2(t)$$



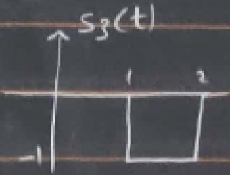
$$s_2(t) = s_{21}\phi_1(t) + s_{22}\phi_2(t)$$

$$= 1^*\phi_1(t) - 1^*\phi_2(t)$$

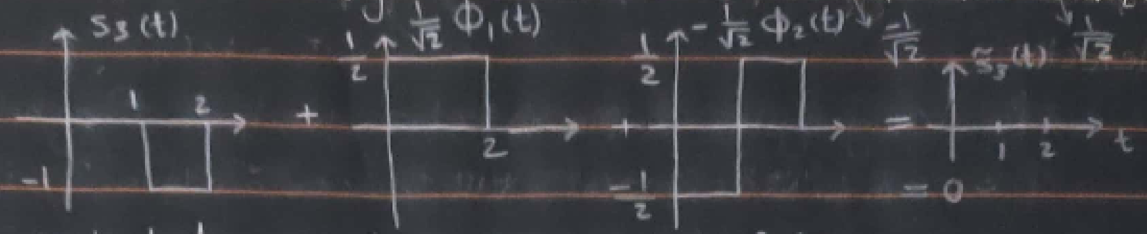


$$s_3(t) = s_{31}\phi_1(t) + s_{32}\phi_2(t)$$

$$= 0^*\phi_1(t) - \phi_2(t)$$



in Example (6) p 72 verify that $\tilde{s}_3(t) = s_3(t) - s_{31}\phi_1(t) - s_{32}\phi_2(t) = 0$



what is the angle between s_1 & s_2 in example 5 & 6?

in example ⑤ $\theta_{12} = \cos^{-1}\left(\frac{1}{\sqrt{E_1 E_2}}\int s_1(t)\, s_2(t)\, dt\right)$

$= \cos^{-1}\left(\frac{1}{\sqrt{2*(1+1)}}\int_0^1 1\, dt + \int_1^2 -1\, dt\right)$

$= 90^\circ$

in example ⑥ from graph $\Rightarrow 90^\circ$.

Compare synthesis & analysis eq with FS T eqn

$s_i(t) = \sum_{j=1}^{N} s_{ij}\, \phi_j(t) \qquad \phi_n(t) = e^{j2\pi n f_0 t}$

what are statistical properties of r & n?

noise component of the correlator o/ps are indep. & identically distributed (i.i.d)

noise → zero mean & G R.V.

what if noise is not white?

noise will be distributed in random shapes.

what is the best mapping g(.)?

MAP "maximum aposteriori probability"

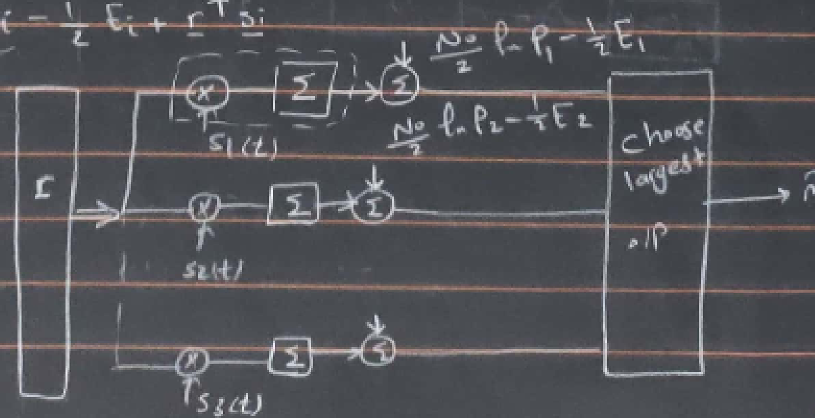
Threshold is a special case of decision regions in case of binary system why:

since in binary system we divide the space only into 2 regions

if (r) lies on space of zero signal ∴ r = 0.

Draw a block diagram for optimal Map Rx

$\hat{m} = \arg\max_i \frac{N_0}{2} \ln P_i - \frac{1}{2} E_i + r^T s_i$



Compare $\hat{m} = \arg\max_i P(\underline{r} \mid s_i)$ with equiprobable case simple detector

for binary system simple detector, we have threshold

$\hat{m} = 1$ if $s(t) > V_{th}$

$\hat{m} = 0$ if $s(t) < V_{th}$

Complete the proof details

$$\hat{m} = \arg\max_i p_i \frac{1}{(\pi N_0)^{N/2}} e^{\frac{-\|\underline{r} - s_i\|^2}{N_0}}$$

in case equiprobable is const.

$$\hat{m} = \arg\max_i p_i \, e^{\frac{-\|\underline{r} - s_i\|^2}{N_0}} = \arg\max_i \ln p_i - \frac{\|\underline{r} - s_i\|^2}{N_0}$$

$$= \arg\max_i - \frac{\|\underline{r} - s_i\|^2}{N_0}$$

$$= \arg\min_i \|\underline{r} - s_i\|^2$$

const. for equal energy

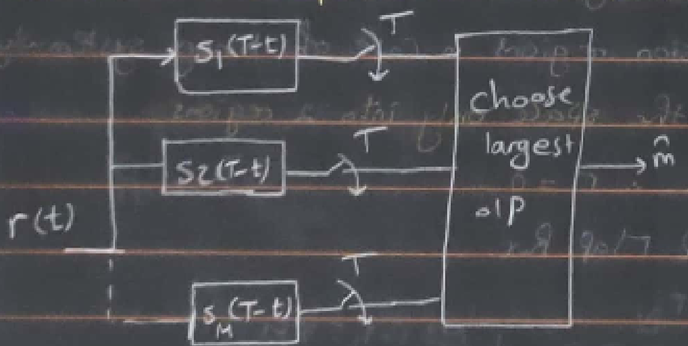
$$\hat{m} = \arg\min_i \left(E_i - 2\underline{r}^T \underline{s_i}\right)$$

$$\boxed{\hat{m} = \arg\max_i \underline{r}^T s_i}$$

How can we change block diagram to be MAP

before choosing max, add $\frac{N_0}{2} \ln p_i$ to each signal.

Draw the complete Rx with MF



For example 7 Page 87

why $P_2 = 1-P$?

Since system is binary so we have only two probabilities

if one of them $= P$ ∴ other one must equal $(1-P)$

since sum of probabilities must equal "one".

what is the value of threshold for equiprobable signals

$$\frac{N_o}{4\sqrt{E}} \ln \frac{1-P}{P} \quad , \quad P = \frac{1}{2} \Rightarrow V_{th} = \frac{N_o}{4\sqrt{E}} \ln 1 = 0$$

Compare threshold with simple detector $s_1(T) = \sqrt{E}$, $s_2(T) = -\sqrt{E}$

$$V_{th}^* = \left(\frac{S_1(T) + S_2(T)}{2} \right) = \frac{\sqrt{E} - \sqrt{E}}{2} = 0$$

the same in case equiprobable

given a detector $\hat{m} = g(r)$, how to calculate its P_e using signal space?

graphically using the concept of decision regions.

why this set $-3\alpha \quad -\alpha \quad \alpha \quad 3\alpha$ preferred ?

less energy than this $2\alpha \quad 4\alpha \quad 6\alpha$

why $P(\text{any bit in error in the symbol}) \leq \sum_{i=1}^{\log M} P(i\text{th bit is in error})$?

Since P_e is small, so probability of mistaking in symbol with its nearest neighbors is dominant.

How upper bound on P_b is proved ?

$$P_b = \frac{P_e}{\log_2 M}$$

what is the dimensionality of the integral in th ??

dimensionality = M No. of basis function.

what is Monte-Carlo simulations?

is a mathematical technique that generates random variables for modelling risk or uncertainty of a certain system.

why $P_{ik} < P_e$ (lower bound)?

because in case P_{ik} we assume two points only, $\therefore$ Prob of error of them ... note ① ch2

Calculate l_{ant} of voice Transmission without modulation; if GSM at 900 MHZ

$$l_{ant} = \frac{c}{4f_c} = \frac{3*10^8}{4*900M} = 0.083\ m = 83\ mm.$$

prove that $E_{c(t)} = E$

$$E = \frac{\left(\sqrt{\frac{2E}{T}}\right)^2}{2} \cdot T = \frac{\cancel{2}E}{\cancel{2}\cancel{T}}\cancel{T} = E$$

if repetition condition holds, show that $\int_0^{T_b} \cos(2\pi f_c t)dt = 0$

$$\int_0^{mT_c} \cos(2\pi f_c t)\, dt$$

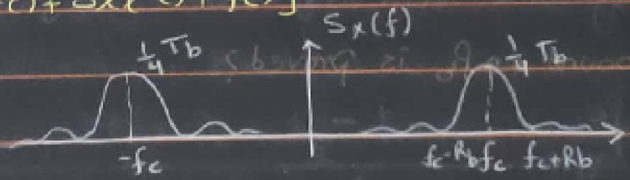
$$\text{period} = \frac{1}{\frac{1}{T_c}} = T_c \quad \text{within period} \quad \therefore \text{area} = 0$$

why PSK cannot be applied to non coherent.?

in PSK, information in phase, and non coherent Detection doesn't need phase.

what is the PSD of modulated signal?

$$S_x(f) = \frac{1}{4}\left[S_{xe}(f-f_c) + S_{xe}(f+f_c)\right]$$



why $BW|_{3dB} = 0.44\, R_b$?

$$T_b \operatorname{sinc}^2(T_b f) = \frac{1}{2}\frac{(T_b)^2}{2} \qquad E = 2 * T_b \int_0^{\infty} \frac{\sin^2(T_b f)}{(T_b f)^2}\, df$$

With modulation $BW = \frac{1}{T}$?

$(3dB) \Rightarrow BW = \frac{1}{2} * 2 * R_b$

$= R_b = \frac{1}{T}$

Can we increase R_b with the same BW?

with M-ary modulation every wave form carries multiple bits

what if equiprobable, equal energy?

① $E_{av} = \frac{1}{2} \sum_{i=1}^{M} E_i$

② $E_{av} = E \sum_{i=1}^{M} P_i$

①&② $E_v = \frac{M}{2} E$

$\Phi_1(t)$ & $\Phi_2(t)$ in p 17 verify they are orthonormal

$$\int_0^{T_b} \frac{2}{T_b} \sin(2\pi f_c t) \cos(2\pi f_c t)\, dt$$

$$= \int_0^{T_b} \frac{1}{T_b} \sin(2\pi(2f_c)t)\, dt$$

$1/2/T_b$

$= \frac{1}{2} T_b$ within period $= 0$ orthogonal

$E_1 = \frac{1}{T_b}$, $E_2 = \frac{1}{T_b}$ "orthonormal", for no repetition condition not necessary they are orthogonal

verify that E_b/N_o is dimensionless.

$E_b = \frac{E_{av}}{K}$, $E_{av} = \sum_{i=1}^{M} P_i E_i$

since dimensionality changes, this affects E_{av} & K

$\therefore E_b$ is dimensionless.

why in curve $\max = \frac{1}{2}$

$P_b = \frac{P_e}{K}$ "gray code signal"

Min M-ary system $\Rightarrow M = 4 \therefore K = 2$

$\text{Max } P_e = 1 \therefore \text{Max } P_b = \frac{1}{2}$ ✓

What is the optimal receiver? equiprobable & AWGN note ② ch 2

$\hat{m} = \arg\max_i \left(-\frac{1}{2}E_i + \underline{r}^T \underline{s_i}\right)$ ML receiver

↳ special case from MAP receiver

Is ASK a fixed energy mod. scheme?

No.

verify $x_{BASK}(t)$ $\quad x_{BASK}(t)\Big|_{b=1} = \frac{A_2}{2}\cancel{(1-1)}^{0} + \frac{A_1}{2}(1+1)\cos(\omega_c t + \theta_c)$

$= A_1 \cos(\omega_c t + \theta_c)$ ✓

How to transform binary to Gray Code?

make each sequence differs from the previous in one bit only

Why gray code is more used?

probability of error ↓↓

How? example of $g(t)$? what is the effect on energy & B.W?

by multiply with $g(t)$, $g(t)$ can be rectangular or pulse shaped

why ignore time shift? in $S_{OOK}(f)$

because of mag. $|e^{j\theta}| = 1$

repeat for general BASK ...

$$S_{BASK}(f) = \frac{|G(f)|^2}{T_b} \mathcal{F}[R_I(K)] \Rightarrow R_I(K) \triangleq E[I_n I_{n+K}] \begin{cases} E[I_n^2] & K=0 \\ E[I_n] & K \neq 0 \end{cases}$$

$\hookrightarrow T_b \operatorname{sinc}^2(\pi T_b f)$

$$x(t) = \frac{A_1 + A_2}{2} \cos(\omega_c t + \theta_c) + \frac{A_1 - A_2}{2} b \cos(\omega_c t + \theta_c)$$

$$E[I_n^2] = \frac{(A_1 + A_2)^2}{8} + \frac{(A_1 - A_2)^2}{8}$$

$E[I_n] =$

How to do this? double the spectral effeciency in OOK

by using only half of spectral

Why for MASK $E[I_n] = 0$

for $E[I_n]$ @ $M=4 \Rightarrow x(t) \Rightarrow \pm 3A_c, \pm 1 A_c$

$(3A_c) - 3A_c + A_c - A_c = 0$

Verify orthonormal $\Rightarrow \phi_1(t) = \sqrt{\frac{2}{T}} \cos(\omega_c t + \theta_c)$

$E_1 = \underbrace{\frac{(\frac{2}{T})}{2}}_{\text{Power}} \cdot T = 1$ normal , $\phi_2 = \sqrt{\frac{2}{T}} \sin(\omega_c t + \theta_c)$

$$\int_0^T \frac{1}{T} \sin(2(\omega_c t + \theta)) \, dt = \text{zero}$$

$\therefore$ orthonormal.

For same A_1, which has lower P_e by observing Constellation?

d_{min} for BASK $= (A_1 - A_2)\sqrt{\frac{T_b}{2}}$

d_{min} for OOK $= A_1\sqrt{\frac{T_b}{2}} > d_{min}$ for BASK

$\therefore P_e$ for OOK lower than P_e for BASK

why $E_{av} = \frac{1}{M}\sum_{m=1}^{M} E_m$ for Many ASK

since No. of energies $= M$

How these values change for general $g(t)$ pulse shaper?

??

prove $\sum_{i=1}^{n}(2i+1)^2 = \frac{n(2n+1)(2n-1)^2}{3}$ using $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

$4i^2 + 1 + 4i$

$$2\frac{n(n+1)(2n+1)}{3} + 1 + 4n = \frac{2n(n+1)(2n+1)+3+12n}{3}$$

$$= \frac{(2n^2+2n)(2n+1) + 12n + 3}{3}$$

$$= \frac{4n^3+2n^2+4n^2+2n+12n+3}{3}$$

why transmitted signal in ASK can be written as? $\sqrt{\frac{2E_m(t)}{T}}\cos(\omega_c t+\theta_c)$

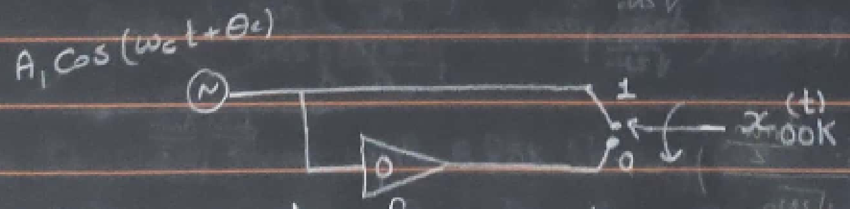
since $E_m(t) = \frac{\left(\sqrt{\frac{2E_m(t)}{T}}\right)^2}{2} T$

$= E_m(t)$ ✓

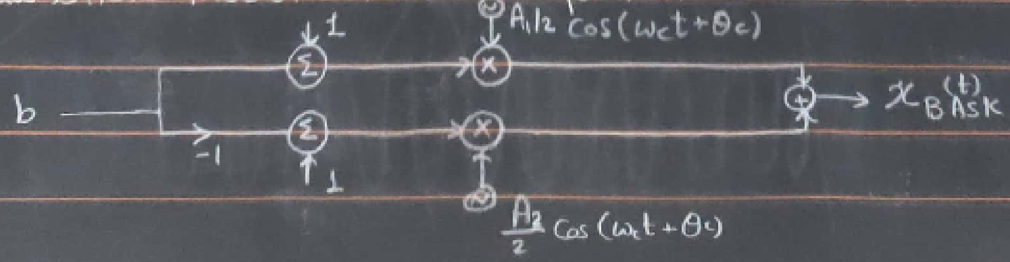
for BASK Transmitter, How?

Can be realized using Transistor switching between Cut off & saturation

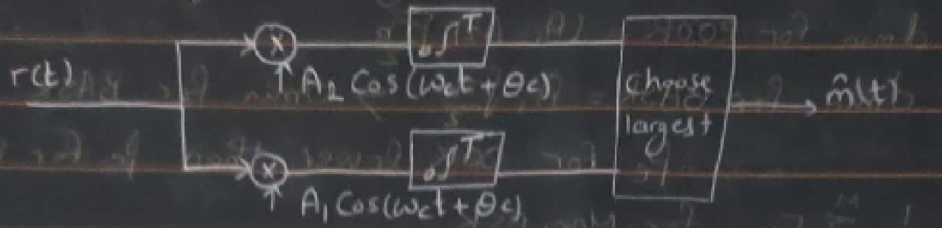
Draw Tx Block diagram of OOK as shift keying

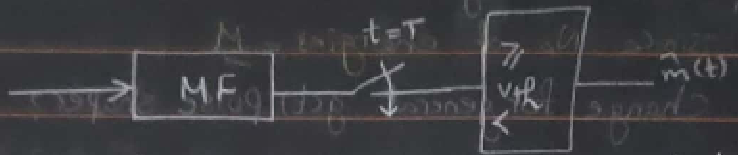


Draw BASK modulator from equation as in OOK



Design Rx for BASK? What about MF implementation?





By noting that noise can be written inphase & quadrature comp. find $r_o(t)$

$$x_{OOK}(t)+n(t) = A_1\cos(\omega_c t+\theta_c) + n_c(t)\cos(\omega_c t+\theta_c) - n_s(t)\sin(\omega_c t+\theta_c)$$

$$r_o(T) = \int_0^T A_1\cos^2(\omega_c t+\theta_c) + A_1 n_c(t)\cos^2(\omega_c t+\theta_c) - n_s(t)\cdot\frac{A_1}{2}\sin(2(\omega_c t+\theta_c))\,dt$$

$$A_1\left(\tfrac{1}{2}+\tfrac{1}{2}\cos(\sim)\right)$$

$$r_o(T) = \frac{A_1}{2}T + \frac{A_1}{2}\left(\frac{1}{\omega_c}\right)\left[\sin(\omega_c T+\theta_c)-\sin(\theta_c)\right]$$

$$+\frac{A_1 n_c}{2}T + \frac{A_1}{2}\left(\frac{n_c}{\omega_c}\right)\left[\sin(\omega_c T+\theta_c)-\sin(\theta_c)\right]$$

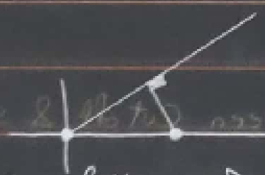
$$-\frac{A_1 n_s}{2}T + \frac{-A_1}{2}\left(\frac{n_s}{2}\right)\left[\cos(\omega_c T+\theta)-\cos(\theta_c)\right]$$

Why $\int_0^T (s_1(t)-s_2(t))^2\,dt = \|s_1-s_2\|^2$

$$\int_0^T (s_1(t))^2+(s_2(t))^2-2s_1(t)s_2(t)\,dt$$

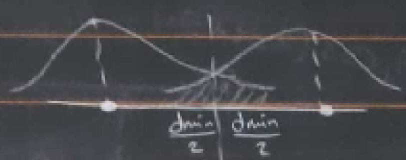
repeat for OOK "phase effect"

$$P_e = Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right) = Q\left(\frac{\sqrt{E_1}\cos\phi_e}{\sqrt{2N_0}}\right)$$

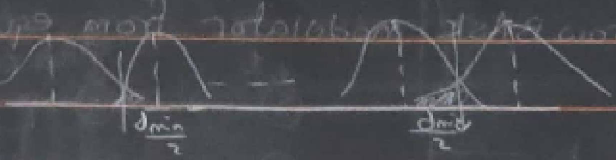


Show carefully $P_e = Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right)$ for outer points

This area ⇒ $P_e = Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right)$



$$\therefore P_e = Q\left(\frac{2\cdot\frac{d_{min}}{2}}{\sqrt{2N_0}}\right) = ✓$$



Verify d_{min} & P_e

$$E_{bav} = \frac{A_c^2 T_s (M^2-1)}{6 \log_2 M}, \quad d_{min} = 2A_c\sqrt{\frac{T_s}{2}} = \sqrt{2T_s}\,A_c$$

$$A_c = \sqrt{\frac{6\log_2 M \, E_{bav}}{(M^2-1)T_s}} \Rightarrow d_{min} = \sqrt{\frac{12\log_2 M \, E_{bav}}{(M^2-1)}}$$

$$\therefore P_e = \frac{2(M-1)}{M} Q\left(\sqrt{\frac{6\log_2 M}{M^2-1}\frac{E_{bav}}{N_o}}\right)$$

find limit

$$\lim_{M\to\infty} \frac{6\log_2 M}{M^2-1} \; \frac{\infty}{\infty}, \quad \lim_{M\to\infty} \frac{6}{M\ln 2}\left(\frac{1}{2M}\right) = \frac{6}{M^2 2\ln 2} = 0$$

Draw Constellation For MASK $M=2$

$$x_{MASK} = A_c(2m-1-M)\cos(\omega_c t + \theta_c) \quad \left[m=\frac{1}{2}, M=2\right]$$

$$= A_c(-1)\cos(\omega_c t + \theta_c)$$

or , $A_c(1)\cos(\omega_c t + \theta_c)$

$-A_c\sqrt{\frac{T_b}{2}}$ $\qquad$ $A_c\sqrt{\frac{T_b}{2}}$

note③ ch2

Verify That $E_1 = E_2 = \frac{A^2 T_b}{2}$

$$S_1(t) = A\cos(\omega_c t + \theta_c), \quad S_2(t) = A\cos(\omega_c t + \theta_c + \pi)$$

$$E_1 = \frac{A^2}{2} T_b \qquad E_2 = \frac{A^2}{2} T_b$$

Power

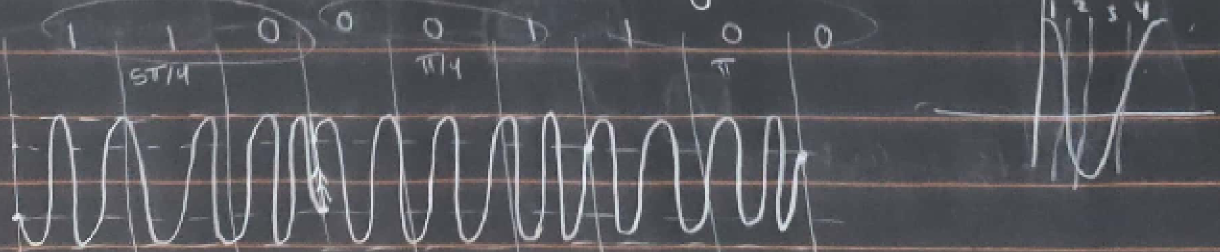
verify Complex envelope for MPSK

$$x_{MPSK_\ell}(t) = A\cos\left(\frac{2\pi(m-1)}{M}\right) + jA\sin\left(\frac{2\pi(m-1)}{M}\right)$$

$$r(t) = \sqrt{A^2\cos^2 + A^2\sin^2} = \sqrt{A^2(\cancel{\cos^2+\sin^2})} = A$$

$$\theta = \tan^{-1}\frac{\sin\left(\frac{2\pi(m-1)}{M}\right)}{\cos\left(\frac{2\pi(m-1)}{M}\right)} = \tan^{-1}\tan\frac{2\pi(m-1)}{M} = \frac{2\pi(m-1)}{M}$$

Draw Tx wave Form with BPSK (binary Coded)



what are advantage & dis advantage

① No Carrier component ⇒ advantage

② Higher PSD mag ⇒ dis advantage

add psd in case of indep signals -- why?

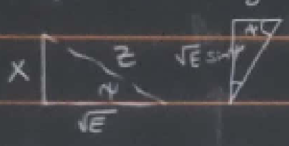
Cross Correlation = 0

show formally $\eta_s|_{MPSK} = \log_2 M$

$$\eta_s = \frac{Rb}{BW}, \quad BW|_{MPSK} = \frac{Rb}{\log_2 M} \quad \text{sinc } BW = \frac{1}{Tb \log_2 M} = \frac{1}{Ts}$$

$$\therefore \eta_s = \log_2 M$$

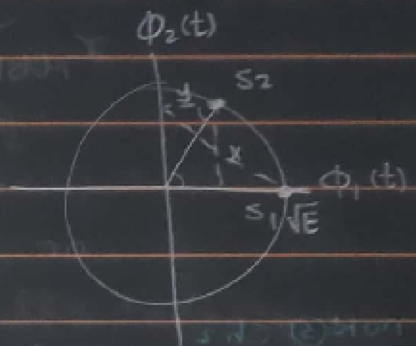
why For BPSK $S_1 = \begin{bmatrix} \sqrt{E} \\ 0 \end{bmatrix}$, $S_2 = \begin{bmatrix} \sqrt{E} \cos\varphi \\ \sqrt{E} \sin\varphi \end{bmatrix}$



$X = \sqrt{E} \sin\varphi$

$y = \sqrt{E} \sin\varphi \; \frac{1}{\tan\varphi}$

$\therefore y = \sqrt{E} \cos\varphi$



for PRK verify polar $d(t)*2-1 = b$, $b = \pm 1$

$d(t) \begin{cases} \to 0 \Rightarrow 2*0-1=-1 \\ \to 1 \Rightarrow 2*1-1=1 \end{cases}$

for PRK Rx why $r_o(t) = \begin{cases} \sqrt{Eb} + N \\ -\sqrt{Eb} + N \end{cases}$

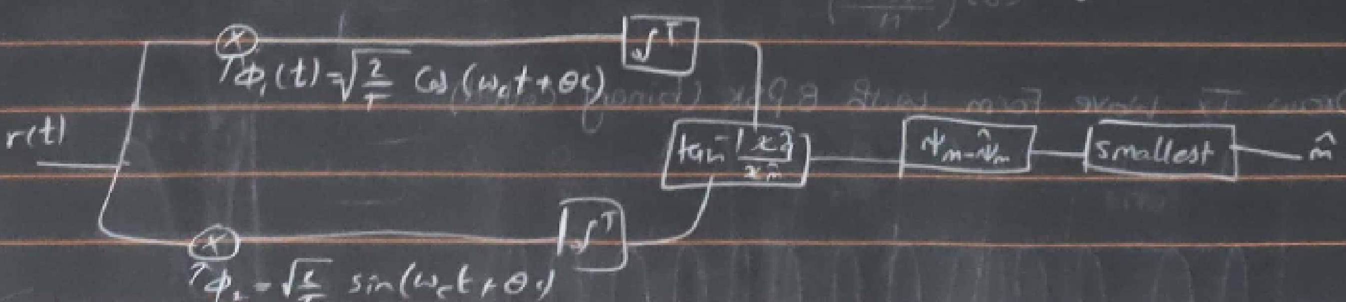
$$x_{PRK} = Ab \cos(w_c t + \theta_2) * \sqrt{\frac{2}{Tb}} \cos$$

$Eb = \frac{A^2}{2} Tb$

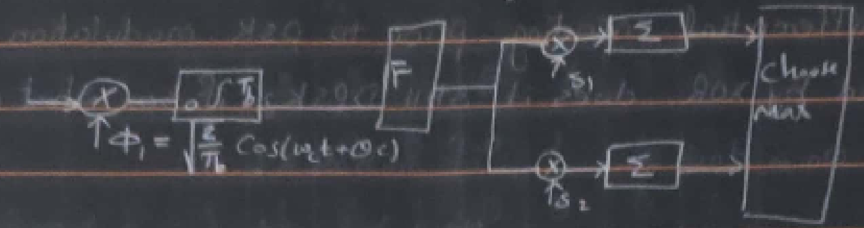
$$\frac{1}{2} Ab \sqrt{\frac{2}{Tb}} + \frac{1}{2} \cos^2$$

$$A\sqrt{\frac{1}{2Tb}} \quad A, -A\sqrt{\frac{1}{2Tb}}$$

for 8PSK Draw optimal receiver include all signals value



Draw the optimal receiver for general BPSK



what is the value of φ $Pe|_{OOK} = Pe|_{BPSK}$?

$\varphi = 180$

verify $p(r_1, r_2) = \frac{1}{\pi N_0} e^{-((r_1-\sqrt{E})^2 + r_2^2)/N_0}$

$r_1 = \frac{1}{(\pi N_0)^{\frac{1}{2}}} e^{-(r_1-\sqrt{E})^2/N_0}$, $r_2 = \frac{1}{(\pi N_0)^{\frac{1}{2}}} e^{-\frac{r_2^2}{N_0}}$

indep $\therefore p(r_1, r_2) = \frac{1}{\pi N_0} e^{-\frac{(r_1-\sqrt{E})^2 - r_2^2}{N_0}}$ ✓

why $M \to \infty$ $Pe \to 1$

$Pe \simeq 2Q(\sqrt{\cdot \; -} \;)$, Max $Q = \frac{1}{2}$

$\therefore$ Max $Pe = 1$

why QPSK most commonly used

has no trade off, same η_p of PRK twice η_s

why MPSK with $M \geq 16$ is rarely used?

since $M \uparrow\uparrow$, $Pe \uparrow\uparrow$

Compare ASK & PSK

	ASK	PSK	Prefered
η_s	$\eta_s = 1$	$\eta_s = 1$	same
η_p	$\eta_p|_{10^{-15}} = 15, \ldots$ OOK	$\eta_p|_{10^{-15}} = 8 \ldots$ PRK	PSK $\equiv$ PRK
TX Complex	Simple	slight Complex	ASK
synch.	no synch.	Synchro.	ASK

lec 9 note (4) Ch 2

Can we Implement a non-coherent version of psk?

Yes, by using differential encoding prior to psk modulation

If XNOR is Replaced by XOR, does it still DPSK? if so, what is difference

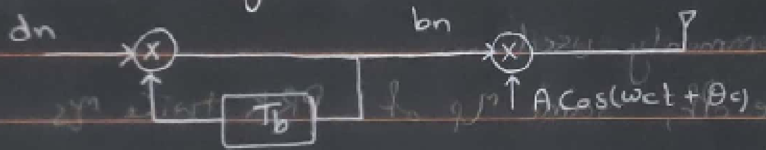
Yes , $a_n = d_n \oplus a_{n-1}$, if initial = 1

New bit = 1 ∴ toggle

New bit = 0 ∴ No Change

Find PSD & BW, η_s for DPSK ? Compare with BPSK

Draw the block diagram



assume A=1, what are I, Q Components in this example?

m = 1, 2, 3 ∴ $x_{i_1}(t) = \sqrt{E}\cos\left(\frac{2\pi(0)}{M}\right) = \sqrt{E}$

$x_{q_1}(t) = \sqrt{E}\sin(0) = 0$

$x_{i_2}(t) = \sqrt{E}\cos\left(\frac{2\pi}{8}\right)$

$x_{q_2}(t) = \sqrt{E}\sin\left(\frac{2\pi}{8}\right)$

$x_{i_3}(t) = \sqrt{E}\cos\left(\frac{2\pi(2)}{8}\right)$, $x_{q_3}(t) = \sqrt{E}\sin\left(\frac{2\pi(2)}{8}\right)$

Can we decrease phase transitions in QPSK to avoid large side lobes & inst. amp Change when filtered?

Solution ⇒ use OQPSK or $\frac{\pi}{4}$-PSK

How $\frac{\pi}{4}$ QSPK Can be detected using FM receiver & integrator?

Does it need differential encoding?

yes, because it's not synchronizing modulation

yes it need diff. encoding

repeat example 7 with $\frac{\pi}{4}$ - DQPSK

	0	1	2	3	4	5	6	7	8
d_n		1	1	0	0	0	1	1	0
a_n	1	1	1	0	1	0	0	0	1
b_n	1	1	1	-1	1	-1	-1	-1	1
Φ_n	0	0	0	π	0	π	π	π	0

why in PSK points placed on 2-D circle?

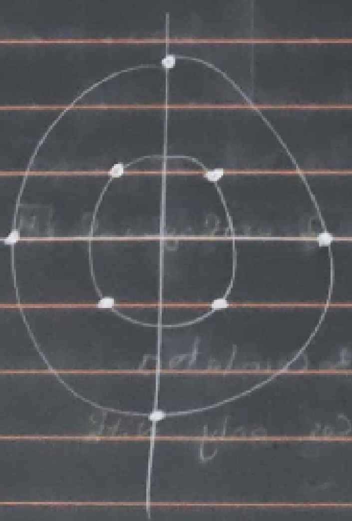
as each two points, between them phase in 2d Const.

why the envelope needs to be constant?

for M'Ary PSK, amp must be const. since phase represent information not amplitude

starting from $S(f)$ $\frac{|G(f)|^2}{T}$ $\mathcal{F}\{R_I(k)\}$ find PSD 16QAM

assume in 8 QAM $A_1 = 1$, $A_2 = 2$ find in-phase & quad. components & what is the average symbol energy?



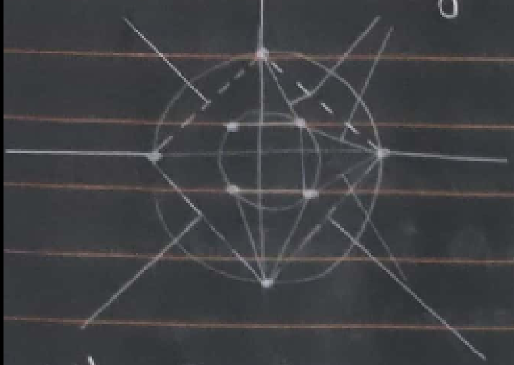
$E_1 = \frac{T_b}{2}$

$E_2 = T_b \times 2$

$\therefore E_{av} = \frac{\frac{T_b}{2} + 2T_b}{2} = \frac{5}{4} T_b$

Draw Decision regions of ML for 8QAM, 16 QAm

very difficult !!



Why 32 QAM cons. is not 8*4 rectangle

this way increases power

so $2^{K-1} = 2^4 \Rightarrow 16$ poinst at middle

then $2^{K-3} = 2^2$ for each side

to avoid corners (high Energy)

why E_{bav} for QAM $= \frac{Ac^2 T (M-1)}{3 \log_2 M}$

Since $E_{av} = \frac{Ac^2 T (M-1)}{3}$

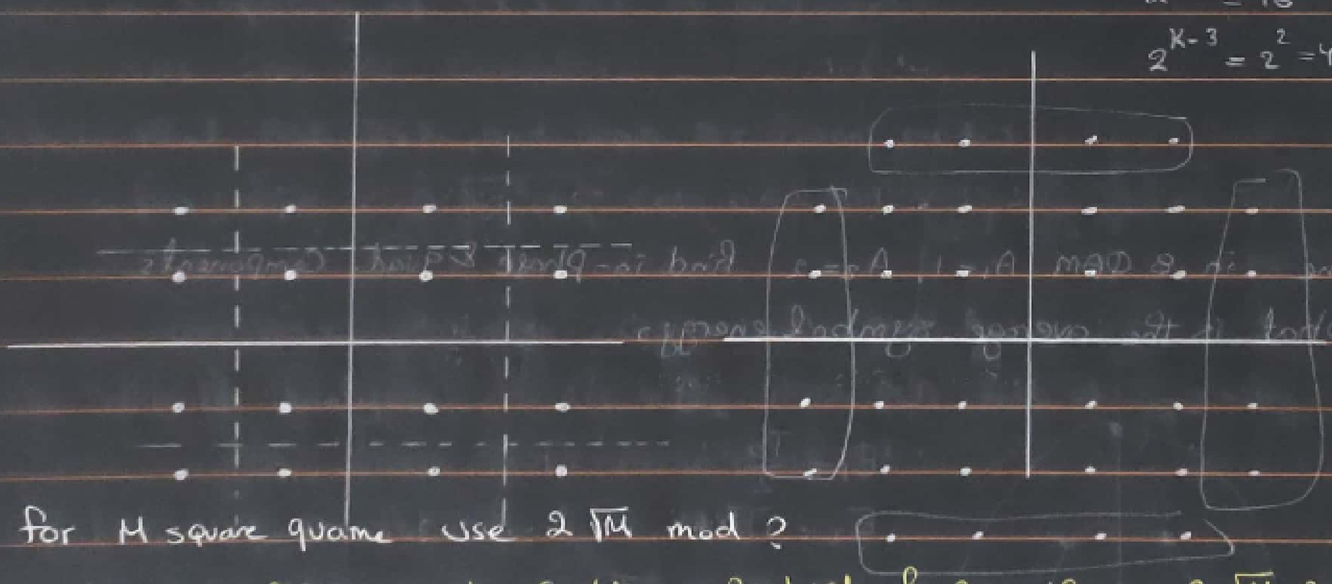
$\therefore E_{bav} = \frac{E_{av}}{3 \log_2 M}$

what is the optimal decision region for 16 QAM & 32 QAM

$K = 5$

$2^{K-1} = 16$

$2^{K-3} = 2^2 = 4$



for M square quame use $2\sqrt{M}$ mod ?

because QAm can be Cartisean product of 2 orthogonal $\sqrt{M}$ Ask. mod. & demod.

The optimal receiver for general QAm use 2 Correlators.

because all bits represented using sin & Cos only with The same phase.

FSK

For BFSK for min P_e why ρ_{12} must be as negative as possible?

as -ve as possible ρ_{12} means that diff. between two signals is large $\Rightarrow$ P_e decreases.

what happens if $\theta_1 \neq \theta_2$?

$$\rho_{12} = \frac{1}{T_b}\int_0^{T_b} \cos(2\pi(f_1-f_2)t + \theta_1 - \theta_2)\, dt$$

$$\frac{1}{T_b} \left. \frac{\sin(2\pi(f_1-f_2)t + (\theta_1-\theta_2))}{2\pi(f_1-f_2)} \right|_0^{T_b}$$

$$= \frac{1}{T_b} \frac{\sin(2\pi\Delta f T_b)\cos(\theta_1-\theta_2) + \cos(2\pi\Delta f T_b)\sin(\theta_1-\theta_2)}{2\pi\Delta f}$$

sinc funct. added to cos

what is the corr. P_e $\quad \Delta f^* = \frac{0.715}{T_b}$

$$P_e = Q\left(\sqrt{\frac{E_1+E_2-2(-0.2172)E_1E_2}{2N_o}}\right)$$

why $\Rightarrow$ simplicity of Tx & Rx of orthognal?

We can use correlator.

show that ASK, PSK, and Qam are linear mod.?

for PSK $\frac{A}{\sqrt{2}}\sqrt{b_{E_1}^2 + b_{o_1}^2}\cos\left(\omega_c t + \tan^{-1}\frac{b_{o_1}}{b_{E_1}}\right) + \frac{A}{\sqrt{2}}\sqrt{b_{E_2}^2+b_{o_2}^2}\cos\left(\omega_c t + \tan^{-1}\frac{b_{o_2}}{b_{E_2}}\right)$

for ASK $\sqrt{\left(\frac{A_1}{2}\right)^2 + \left(\frac{A_2}{2}\right)^2}\cos\left(\omega_c t + \theta_c + \tan^{-1}\frac{A_2}{A_1}\right) + \ldots$

The above analysis, if $\theta_1 \neq \theta_2$ does PSD fall as $\frac{1}{f^4}$? why

yes, phase shift doesn't effect

repeat analysis for $\Delta f = \frac{1}{2T_b}$

$$S(t) = A\cos\left(\frac{2\pi t}{T_b}\right)\cos(\omega_c t) \mp A\sin\left(\frac{\pi t}{T_b}\right)\sin(\omega_c t)$$

$$\mathcal{F}\{g(t)\} = \mathcal{F}\left\{A\sin\left(\frac{2\pi t}{T_b}\right)\mathrm{rect}\left(\frac{t}{T_b}\right)\right\}$$

For MFSK, as $M \to \infty$ $\eta_s \to ?$

$\eta_s \to 0$

why $d_{min}|_{MFSK} = \sqrt{2E}$

amp is Constant as will as phase

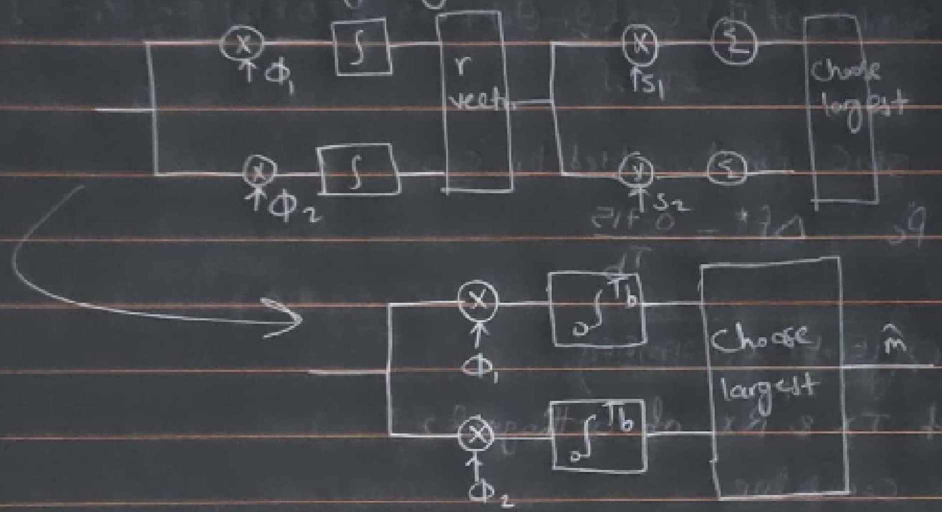
So dmin is Const. also

why Cannot draw Constellation for $M > 3$?

We Cannot draw more than 3 orthogonal $\phi(t)$

Starting from ML, how can reduce to parallel realization?

for Binary system $M = 2$ $K = 1$ $2 \Rightarrow \phi(t)$



why For MFSK $\forall th|_{opt} = 0$

Optimal design $\Rightarrow$ min $P_e \Rightarrow \xi_{ij}$ -ve as possible $\therefore \underline{0}$ is $\forall th^*$

in MFSK we don't need inner product? show Carefully

r_s for all signals are the same since freq

doesn't affect inner product as an amplitude

why $P_{n_1}(n) = \frac{1}{\sqrt{\pi N_0}} e^{-n^2/N_0}$? gaussian distribution

what is the min $\frac{E_b}{N_0}$ even for BER=0?

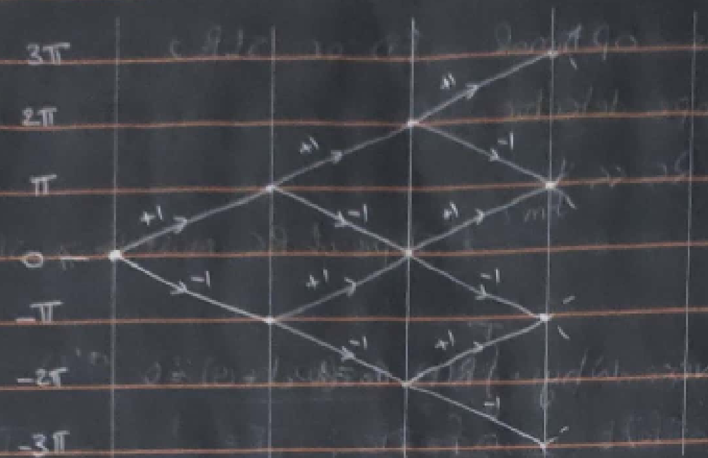
$\frac{E_b}{N_0} > 2 \ln 2 = 1.42$ dB

$\boxed{MSK}$

Draw tree diagram for CPFSK with $h=1$ is that a memory? why?

$$\theta((K+1)T_b) - \theta(KT_b) = b_K \pi h \begin{cases} +\pi & \text{one} \\ -\pi & \text{zero} \end{cases}$$

Yes memory modulation



verify $A\sin(\theta(0)) = 0$ for in phase Comp.

$\theta(0) = \pi \quad -T_b < t < 0 \qquad \sin(\pi) = 0$

$\theta(0) = 0 \quad 0 < t < T_b \qquad \sin(0) = 0$ ✓

verify $\overset{\cos(\theta(T_b))}{A_k \sin\left(\frac{\pm\pi(t-T_b)}{2T_b}\right)}$ for quad Comp

t is multiples of T_b "integers" nT_b

$\sin\left(\frac{\pm\pi(n-1)}{2}\right) = ✓ \neq 0$

$\cos(\theta(T_b)) = \cos(-\frac{\pi}{2})$ or $\cos(\frac{\pi}{2}) =$ Zero ✓

prove That $|G_1(f)|^2 = \frac{16A^2T_b^2}{\pi^2}\left[\frac{\cos(2\pi f T_b)}{16T_b^2f^2-1}\right]^2$

Same as FSK

why $2T_b$ for MSK?

$s(t)$ defined in $-T_b < t < 0$, $0 < t < T_b \Rightarrow 2T_b$

what about BPSK psd?

drops to $\frac{1}{4}$ its max value

Chapter 3

$P_e = Q\left(\sqrt{\frac{\alpha E_b}{N_o}}\cos\phi\right)$ What is α for OOK, BFSK, BPSK?

$\alpha = 1$ for OOK & BFSK

$\alpha = 2$ for BPSK

without proof which is optimal ED or SLR?

I think it's envelope detector.

why for ED $\frac{1}{f_c} \ll RC \ll \frac{1}{f_m}$

to reduce ripples

Slope of RC must be $\geq$ slope of m(t)

for square law Receiver why $\int_0^T R(t)\cos(2\omega_c t+\theta) = 0$

$2(2\pi f_c)t \quad \therefore f = 2f_c \quad T = \frac{1}{2f_c} = \frac{1}{2}T$ within period

area = zero

Show details.

$$z_2 = 2\int_0^T r(t)\sin(\omega_c t)\,dt$$

$$= 2\int_0^T R(t)\sin(\omega_c t)\cos(\omega_c t+\theta_c)\,dt$$

$$= \frac{2}{2}\int_0^T R(t)[-\sin(\theta_c) + \underbrace{\sin(2\omega_c t+\theta_c)}_{0}]\,dt$$

$$= R(t)\left[-R(t)\sin(\theta_c)\,T_b\right]$$

Draw block diagram for ~~MK~~ SLR with MF

How many Correlators needed for 4 FSK?

2 Correlators

why n_s & n_c $p(n_c, n_s) = \frac{1}{2\pi\sigma_{n_o}^2} e^{-\frac{(n_c^2+n_s^2)}{2\sigma_o^2}}$

$p(n_c) = \frac{1}{\sqrt{2\pi}\,\sigma_{n_o}} e^{-\frac{n_c^2}{2\sigma_o^2}}$, $p(n_s) = \frac{1}{\sqrt{2\pi}\,\sigma_{n_o}} e^{-\frac{n_s^2}{2\sigma_o^2}}$

$p(n_c, n_s) = p(n_c)\,p(n_s)$

Draw quadratic Rx for OOK?

Write distributions for o/p of ED for BASK with A_1 & A_2?

$$p(r_0|m=1) = \frac{r_0}{\sigma_{n_0}^2} e^{-(r^2+A_1^2)/2\sigma_{n_0}^2} \; I_0\left(\frac{r_0 A_1}{\sigma_{n_0}^2}\right)$$

$$p(r_0|m=0) = \frac{r_0}{\sigma_{n_0}^2} e^{-(r^2+A_2^2)/2\sigma_{n_0}^2} \; I_0\left(\frac{r_0 A_2}{\sigma_{n_0}^2}\right)$$

Using approx $I_0(z) \cong \frac{e^z}{\sqrt{2\pi z}}$ show that $A_2 \cong Q\left(\frac{A}{2\sigma_{n_0}}\right)$

$$z = \frac{V_{th}^2}{2\sigma_{n_0}^2}$$

$$\int_{-\infty}^{V_{th}} \frac{r_0}{\sigma_{n_0}^2} e^{-r_0^2/2\sigma_{n_0}^2} I_0\left(\frac{r_0 A_1}{\sigma_{n_0}^2}\right) dr$$

$$\int_{-\infty}^{V_{th}^2/2\sigma_{n_0}^2} \cancel{e^{-z}} \frac{\cancel{e^z}}{\sqrt{2\pi z}} dz = \int_{-\infty}^{V_{th}^2/2\sigma_{n_0}^2} \frac{1}{\sqrt{2\pi z}} dz = Q(z)$$

$$= Q\left(\frac{A}{2\sigma_{n_0}^2}\right)$$

Why $E_b|_{OOK} = \frac{A^2 T_b}{4}$

$$\frac{1}{2}\frac{A^2 T_b}{2} + \frac{1}{2} * 0 = \frac{A^2 T_b}{4}$$

$$\rho_{12} = \frac{1}{A^2 T_b}\left[\int_0^{T_b} s_1^1(t)\, s_2^1(t)\, dt + \int_{T_b}^{2T_b} s_1^2(t)\, s_2^2(t)\, dt\right]$$

$$\int_0^{T_b} x_1(t)\, x_1(t)\, dt + \int_{T_b}^{2T_b} x_1(t)\, x_2(t)\, dt = \int |x_1(t)|^2 - |x_1(t)|^2 = 0$$

$$\int_0^{T_b} x_2(t)\, x_2(t)\, dt + \int_{T_b}^{2T_b} x_2(t) * x_1(t)\, dt = \int |x_2(t)|^2 - |x_2(t)|^2 = 0$$

$$\int_0^{T_b} x_1 x_2\, dt + \int_{T_b}^{2T_b} x_1 x_1\, dt = -|x_1|^2 + |x_1|^2 = 0$$

$$\int_0^{T_b} x_2 x_1\, dt + \int_{T_b}^{2T_b} x_2 x_2 = -|x_2|^2 + |x_2|^2 = 0$$