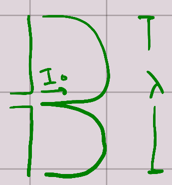
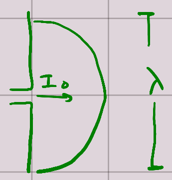
Some Current distributions For dipoles:

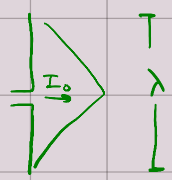
→ 1) λ dipole "$L=\lambda$"



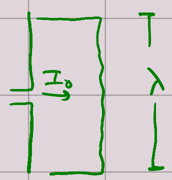
→ 2) $\lambda/2$ dipole



→ 2) Short dipole

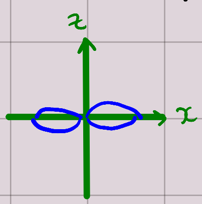


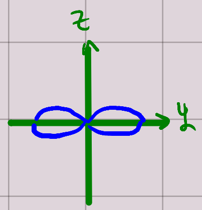
→ 2) infintizmal dipole

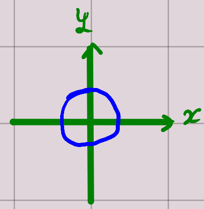


Radiation Pattern of infintizmal dipole:

$$\therefore \vec{E}_\theta(r) = j\omega \frac{\mu I_0 \Delta L}{4\pi r} e^{j\beta r} \sin\theta \ \hat{\theta} \longrightarrow \therefore |E_n| = \sin\theta$$







NNBW = FNBW :

→ $E_n = \sin\theta = 0$

→ $\theta_n = 0, \pi, 2\pi, \ldots$

$\therefore \Delta\theta_n = \pi = 180^\circ$

HPBW :

→ $E_n = \sin\theta = \frac{1}{\sqrt{2}}$

→ $\theta_n = 45, 135, \ldots$

$\therefore \Delta\theta_h = 135 - 45 = 90^\circ$

Transmitted Power:

$$\therefore W_T = \iint P \cdot ds \quad , P = \frac{1}{2\eta}|\vec{E}|^2 = \frac{1}{2\eta}\cdot\left(\frac{\omega\mu I_0 \Delta L}{4\pi r}\right)^2 \sin^2\theta \quad , ds = r^2 \sin\theta \, d\theta \, d\varphi$$

$$\therefore W_T = \frac{1}{2\eta}\cdot\left(\frac{\omega\mu I_0 \Delta L}{4\pi \cancel{r}}\right)^2 \int_0^{\pi}\int_0^{2\pi} \sin^2\theta \cdot \cancel{r^2} \sin\theta \, d\theta \, d\varphi$$

$$\therefore W_T = \frac{1}{2\eta}\left(\frac{\omega\mu I_0 \Delta L}{4\pi}\right)^2 \cancelto{4/3}{\int_0^{\pi} \sin^3\theta \, d\theta} \cdot \cancelto{2\pi}{\int_0^{2\pi} d\varphi}$$

$$\int_0^{\pi} \sin^3\theta \, d\theta = 2\int_0^{\pi/2} \sin^3\theta \, d\theta = 2\left(\frac{2}{3}\right)$$

$$\therefore \boxed{W_T = \frac{1}{2\eta}\left(\frac{\omega\mu I_0 \Delta L}{4\pi}\right)^2 \cdot \frac{8\pi}{5}}$$

★ Radiation resistance:

$$\because W_T = \frac{1}{2} I_0^2 R_r \rightsquigarrow \therefore R_r = \frac{2W_T}{I_0^2} = \frac{\omega^2 \mu^2 \cancel{I_0^2} \Delta L^2}{\eta \cdot 16\pi^2 \cdot \cancel{I_0^2}} \cdot \frac{8\pi}{5}$$

$$\because \eta = 120\pi \;,\; \omega = 2\pi F = \frac{2\pi \cdot C}{\lambda} \;,\; \mu_0 = 4\pi \times 10^{-7}$$

$$\therefore R_r = 80\pi^2 \left(\frac{\Delta L}{\lambda}\right)^2$$

★ Directivity, gain:

$$\rightarrow D = G = \frac{P_{max}}{W_T / 4\pi r^2} \;,\; \because P = \frac{1}{2\eta}|\vec{E}|^2 = \frac{1}{2\eta} \cdot \left(\frac{\omega\mu I_0 \Delta L}{4\pi r}\right)^2 \sin^2\theta \rightsquigarrow \therefore P_{max} = \frac{1}{2\eta} \cdot \left(\frac{\omega\mu I_0 \Delta L}{4\pi r}\right)^2$$

$$\therefore D = \frac{\cancel{\frac{1}{2\eta}} \cdot \left(\cancel{\frac{\omega\mu I_0 \Delta L}{4\pi r}}\right)^2}{\cancel{\frac{1}{2\eta}} \cdot \left(\cancel{\frac{\omega\mu I_0 \Delta L}{4\pi}}\right)^2 \cdot \frac{8\pi}{5} \cdot \frac{1}{4\pi r^2}} = \frac{3}{2}$$

★ Effective Area:

$$\rightarrow A_{eff} = \frac{\lambda^2}{4\pi} G = \frac{\lambda^2}{4\pi} \cdot \frac{3}{2} \rightsquigarrow \therefore A_{eff} = \frac{3\lambda^2}{8\pi}$$

★ Effective Length:

$$L_{eff} = \frac{1}{I_0} \int_{-\Delta L/2}^{\Delta L/2} I(z)\, dz$$

$$= \frac{1}{\cancel{I_0}} \cdot \cancel{I_0} \cdot \Delta L$$

$$= \Delta L$$

★ effective Current:

$$I_{eff} = \frac{1}{L} \int_{-\Delta L/2}^{\Delta L/2} I(z)\, dz$$

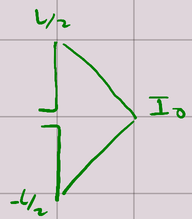
$$= \frac{1}{\cancel{\Delta L}} \cdot I_0 \cdot \cancel{\Delta L}$$

$$= I_0$$

* **Short dipole : $L \le \frac{\lambda}{10}$**

$\rightarrow$ we can apply magnitude approx of far field $\rightsquigarrow \frac{1}{R} = \frac{1}{r}$

$\rightarrow$ - - - - - - - - - Phase approx $\rightsquigarrow R = r - \cancel{z}^{\approx 0}\cos\theta \rightsquigarrow R = r$



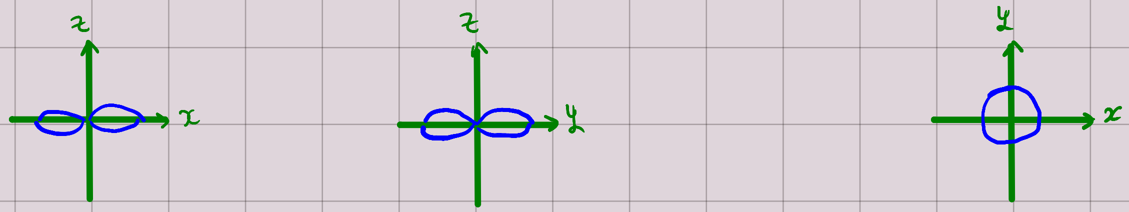
$$\rightarrow \vec{A}_z = \frac{\mu}{4\pi r} e^{-j\beta r} \int_{-L/2}^{L/2} \cancel{I(z)\, dz}^{\frac{1}{2} I_o L}$$

$$\therefore \vec{A}_z = \frac{\mu I_o L}{8\pi r} e^{-j\beta r} \rightarrow \vec{A}_\theta = -\vec{A}_z \sin\theta\, \hat{\theta} \quad , \vec{E}_\theta = -j\omega A_\theta$$

$$\therefore \vec{E}_\theta = \frac{j\omega\mu I_o L}{8\pi r} e^{-j\beta r} \sin(\theta)\, \hat{\theta} \quad , \vec{H}_\varphi = \frac{\vec{E}_\theta}{\eta} = \checkmark$$

* **Radiation Pattern of infinitizmal dipole:**

$$\therefore \vec{E}_\theta(r) = \frac{j\omega\mu I_o L}{8\pi r} e^{-j\beta r} \sin\theta\, \hat{\theta} \longrightarrow \therefore |E_n| = \sin\theta$$



* **NNBW = FNBW :**

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$\rightarrow \theta_n = 0, \pi, 2\pi, ---$

$\therefore \Delta\theta_n = \pi = 180°$

* **HPBW :**

$\rightarrow E_n = \sin\theta = \frac{1}{\sqrt{2}}$

$\rightarrow \theta_n = 45, 135, - -$

$\therefore \Delta\theta_h = 135 - 45 = 90°$

$\boxed{3}$ Transmitted power (WT) & radiation resistance (Rr):

$$WT = \iint_s P\, dA \quad , P = \frac{1}{2\eta} |E|^2$$

$$\therefore P = \frac{1}{2\eta} \frac{\omega^2\mu^2 I_o^2 \ell^2}{64\pi^2 r^2} \sin^2\theta$$

$$\therefore WT = \int_0^{\pi}\int_0^{2\pi} P r^2 \sin\theta\, d\theta\, d\phi$$

$$\therefore WT = \int_0^{\pi}\int_0^{2\pi} \frac{1}{2\eta} \frac{\omega^2\mu^2 I_o^2 \ell^2}{64\pi^2 r^2} \sin^2\theta\, r^2 \sin\theta\, d\theta\, d\phi$$

$$\therefore WT = \frac{1}{2\eta} \frac{\omega^2\mu^2 I_o^2 \ell^2}{64\pi^2} \underbrace{\int_0^{2\pi} d\varphi}_{2\pi} \underbrace{\int_0^{\pi} \sin^3\theta\, d\theta}_{2\int_0^{\pi/2} \sin^3\theta\, d\theta = 2 \cdot \frac{2}{3} \cdot 1 = \frac{4}{3}}$$

$$\therefore WT = \frac{1}{2\eta} \frac{\omega^2\mu^2 I_o^2 \ell^2}{64\pi^2} \cdot \frac{8\pi}{3}$$

$$\rightarrow WT = \frac{1}{2} I_o^2 R_r \longrightarrow R_r = \frac{2WT}{I_o^2}$$

$$\therefore R_r = \frac{\omega^2\mu^2}{\eta} \frac{\ell^2}{8\pi^2} \frac{\pi}{3}$$

$, \omega = 2\pi f = \frac{2\pi c}{\lambda} = \frac{2\pi * 3*10^8}{\lambda}$

$\eta = 120\pi$ air

$\mu = \mu_o = 4\pi * 10^{-7}$

$$\therefore R_r = 20\pi^2 \left(\frac{\ell}{\lambda}\right)^2$$

4 Directivity = gain:

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$$D = \frac{P_{max}}{\frac{W_T}{4\pi r^2}} \quad , \quad P_{max} = \frac{1}{2\eta}\,\frac{\omega^2\mu^2 I_o^2 \ell^2}{64\pi^2 r^2}$$

$$\therefore D = \frac{\frac{1}{2\eta}\,\cancel{\frac{\omega^2\mu^2 I_o^2 \ell^2}{64\pi^2 r^2}}}{\frac{1}{2\eta}\,\cancel{\frac{\omega^2\mu^2 I_o^2 \ell^2}{64\pi^2}} \cdot \frac{8\pi}{3} \cdot \frac{1}{4\pi r^2}}$$

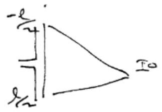
$$\therefore D = \frac{3}{2}$$

5 effective area:

$$A_{eff} = \frac{\lambda^2}{4\pi} G = \frac{\lambda^2}{4\pi} \cdot \frac{3}{2} = \frac{3}{8\pi}\lambda^2$$

6 effective length:

$$\ell_{eff} = \frac{1}{I_o}\int_{wire} I(z)\, dz$$



area $= \frac{\ell I_o}{2}$

$$\ell_{eff} = \frac{1}{I_o} * \frac{\ell I_o}{2} = \frac{\ell}{2}$$

7 effective Current:

$$I_{eff} = \frac{1}{\ell}\int_{wire} I(z)\, dz = \frac{1}{\ell}\,\frac{I_o \ell}{2}$$

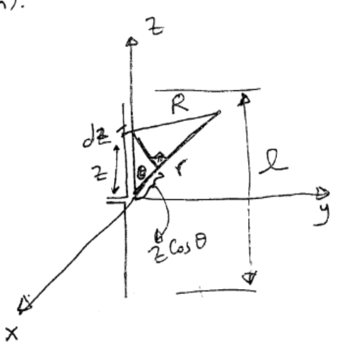
$$I_{eff} = \frac{I_o}{2}$$

* Prove that $\vec{A} \propto F\{I(z)\}$ For general Length wire Antenna.

$$\therefore \vec{A} = \frac{\mu}{4\pi}\int \frac{I(z)}{R} e^{-j\beta R}\, dz$$

$$\therefore \frac{1}{R} = \frac{1}{r} \quad , \quad \therefore R = r - z\cos\theta$$

$$\therefore \vec{A} = \frac{\mu}{4\pi r} e^{-j\beta r}\int_{-l/2}^{l/2} I(z) e^{j\beta z\cos\theta}\, dz$$



$$\therefore \vec{A} = \frac{\mu}{4\pi r} e^{-j\beta r}\int_{-\infty}^{\infty} I(z)\, e^{j\beta z\cos\theta} \cdot \text{rect}\left(\frac{z}{L}\right) dz$$

→ let $\beta\cos\theta = u$

$$\therefore \vec{A} = \frac{\mu}{4\pi r} e^{-j\beta r}\int_{-\infty}^{\infty} \text{rect}\left(\frac{z}{L}\right) . I(z)\, e^{juz}\, dz$$

$$\therefore \vec{A} = \frac{\mu}{4\pi r} e^{-j\beta r} . F\left\{\text{rect}\left(\frac{z}{L}\right) . I(z)\right\}$$

$$\therefore \vec{A} = \frac{\mu}{4\pi r} e^{-j\beta r} . I(u)$$

For infinitzmal dipole:

$$I(u) = I_o \, F\{ \text{rect}(\tfrac{z}{L}) \}$$

$$= I_o \cdot L \, Sa(\tfrac{Lu}{2}) \qquad , u = \beta \cos\theta$$

$$\therefore A_z = \frac{\mu}{4\pi r} e^{-j\beta r} \cdot I_o L \, Sa(\tfrac{L\beta}{2}\cos\theta)$$

$$\therefore \vec{E}_\theta = j\omega A_z \sin\theta$$

$$\therefore \vec{E}_\theta = \frac{j\omega\mu I_o L}{4\pi r} e^{-j\beta r} \sin\theta \, Sa(\tfrac{L\beta}{2}\cos\theta) \, \hat{\theta}$$

For infintizmal dipole $L \lll \rightsquigarrow \therefore Sa \simeq 1$

$$\therefore \vec{E}_\theta = \frac{j\omega\mu I_o L}{4\pi r} e^{-j\beta r} \sin\theta \quad \#$$

For Short dipole:

$$\rightarrow I(u) = F\{ I_o \, \text{tri}(\tfrac{z}{L/2}) \}$$

$$= I_o \cdot \frac{L}{2} Sa^2(\tfrac{Lu}{2\,2})$$

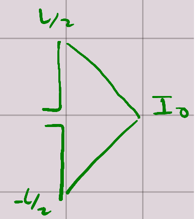
$$\therefore A_z = \frac{\mu}{4\pi r} e^{-j\beta r} \cdot \frac{L I_o}{2} Sa^2(\tfrac{Lu}{2\,2})$$

$$\therefore \vec{E}_\theta = j\omega A_z \sin\theta$$

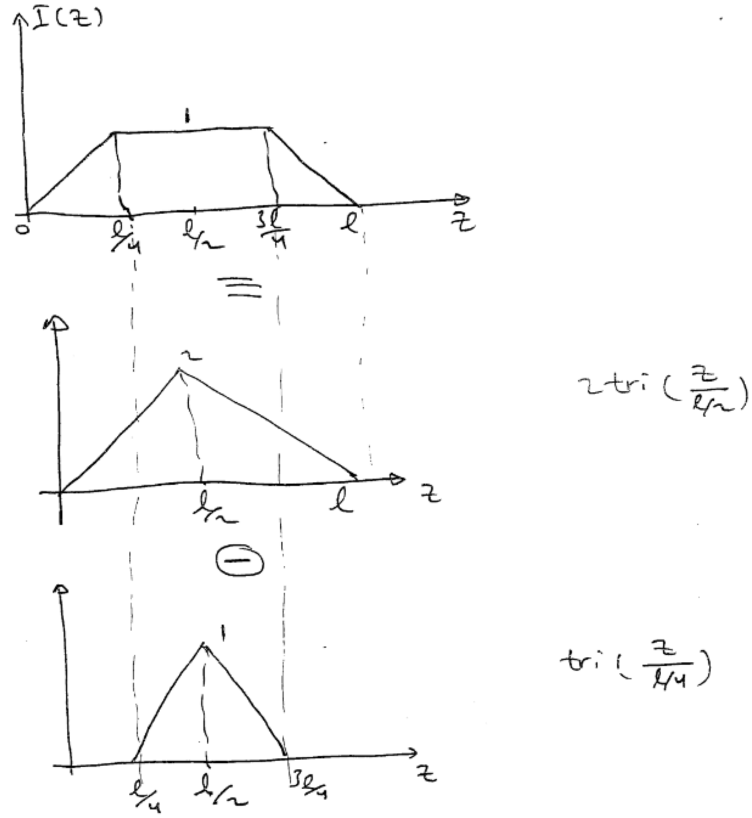
$$\therefore \vec{E}_\theta = \frac{j\omega\mu I_o L}{8\pi r} e^{-j\beta r} \sin\theta \, Sa^2(\tfrac{L\beta \cos\theta}{2\,2})$$

$\therefore L \lll \rightsquigarrow \therefore Sa^2(\cdot) \simeq 1$

$$\therefore \vec{E}_\theta = \frac{j\omega\mu I_o L}{8\pi r} e^{j\beta r} \sin\theta$$



$\Rightarrow$ Recall from Signals:



$$\therefore I(z) = 2\,\text{tri}(\tfrac{z}{\ell/2}) - \text{tri}(z/\ell_4)$$