Phase Shift Keying (PSK)

→ We will start by studying 4 types:

1) General BPSK and then a special case where $\rho = -1$ called PRK.

2) MPSK and then a special case where $M = 4$ called QPSK

⋆ MPSK:

→ M symbols each will be assigned a phase $\psi \in \{0, \underbrace{\overset{A\cos(\omega_c t + \psi_1)}{\frac{2\pi}{M}}, \overset{A\cos(\omega_c t + \psi_2)}{\frac{4\pi}{M}}, \frac{6\pi}{M}, \ldots, (M-1)\frac{2\pi}{M}}\}$, → 2π is divided into M angles

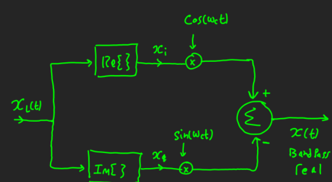
→ Amplitude is constant for all symbols.

- $x(t) = A\cos\left(\omega_c t + \theta_c + \frac{2\pi(m-1)}{M}\right)$, $m = 1, 2, 3, \ldots, M$

↳ Above expression can be written as: $\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$

$$x(t) = \underbrace{A\cos\left(\frac{2\pi(m-1)}{M}\right)}_{\text{In-Phase Comp.}} \cos(\omega_c t + \theta_c) - \underbrace{A\sin\left(\frac{2\pi(m-1)}{M}\right)}_{\text{Quadrature Comp.}} \sin(\omega_c t + \theta_c) \qquad \text{I}$$

- Above expression means that we can use the inphase & quadrature components implementation at Tx & Rx instead of directly changing the ocilator phase for each symbol "Simplifies design complexity"



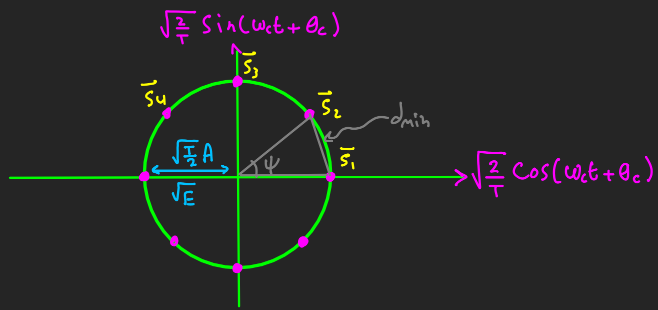
- I can also be written as: $x(t) = A\cos\left(\frac{2\pi(m-1)}{M}\right) + j\sin\left(\frac{2\pi(m-1)}{M}\right)$

⋆ Constellation diagram:

→ orthogonal basis functions:

$\varphi_1 = \sqrt{\frac{2}{T}}\cos(\omega_c t + \theta_c)$ $\quad 0 \le t \le T$

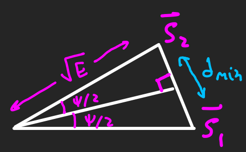
$\varphi_2 = \sqrt{\frac{2}{T}}\sin(\omega_c t + \theta_c)$ $\quad 0 \le t \le T$



→ each vector $(\vec{S}_1, \vec{S}_2, \ldots, \vec{S}_M)$ has a length of $\sqrt{\frac{T}{2}}A$ and energy $A^2\frac{T}{2}$

- $E_m = A^2\frac{T}{2}$, $\qquad \rho_{ij} = \cos\left(\frac{2\pi(m_i - m_j)}{M}\right)$

- $d_{min} = 2\sqrt{E}\sin\left(\frac{\pi}{M}\right) \simeq \frac{2\pi\sqrt{E}}{M}$ $\quad$ when $M >>$



⋆ Probability of error:

→ Assume that error can occure between only adjacent symbols:

$\therefore P_e \simeq 2P_e\{\text{Falls in one of it's adjacent}\}$

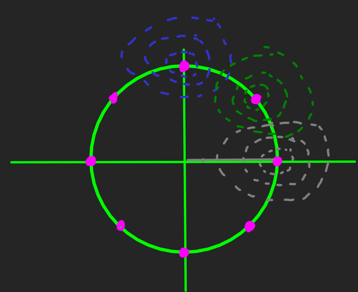
$\therefore \simeq 2Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right)$

$\simeq 2Q\left(\frac{2\sqrt{E}}{\sqrt{2N_0}}\sin\left(\frac{\pi}{M}\right)\right)$

$\therefore P_e \simeq 2Q\left(\sqrt{\frac{E}{N_0}2\sin^2\left(\frac{\pi}{M}\right)}\right)$

$\simeq 2Q\left(\sqrt{\frac{E_b \cdot K}{N_0}2\sin^2\left(\frac{\pi}{M}\right)}\right)$

$\simeq 2Q\left(\sqrt{\frac{E_b \cdot K}{N_0}2\left(\frac{\pi}{M}\right)^2}\right)$ #



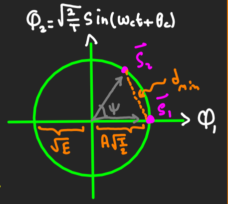
General BPSK

- $b=1 : S_1(t) = A\cos(\omega_c t+\theta_c)$

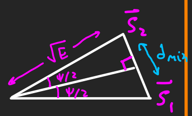
  $b=-1 : S_2(t) = A\cos(\omega_c t+\theta_c+\Psi)$

- $\rho_{12} = \frac{1}{\sqrt{E_1E_2}}\int_0^{T_b} S_1(t)S_2(t)\,dt = \cos(\Psi)$

E_m, ρ_{12}, d_{min}:



- $E_1=E_2=A^2\frac{T}{2}$
- $E_{avg}=\frac{1}{2}(E_1+E_2)=A^2\frac{T}{2}$
- $\rho_{12}=\cos(\Psi)$
- $d_{min}=2\sqrt{E}\sin(\frac{\Psi}{2})$

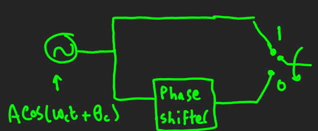


Pe:

- $P_e = Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right) = Q\left(\frac{2\sqrt{E_b}\sin(\frac{\Psi}{2})}{\sqrt{2N_0}}\right)$

  $= Q\left(\sqrt{\frac{2E_b}{N_0}\sin^2(\frac{\Psi}{2})}\right)$ #

Tx:



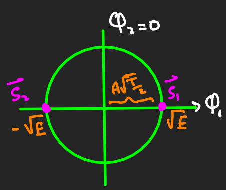
PRK

- Same as BPSK but $\Psi=\pi \rightsquigarrow \rho_{12}=-1$ "antipodal"
- $b=1: S_1(t)=A\cos(\omega_c t+\theta_c)$

  $b=-1: S_2(t)=A\cos(\omega_c t+\theta_c+\pi)$

  $= -A\cos(\omega_c t+\theta_c)$
- $x(t)=Ab\cos(\omega_c t+\theta_c)$

E_m, ρ_{12}, d_{min}:



- $E_1=E_2=A^2\frac{T}{2}$
- $E_{avg}=A^2\frac{T}{2}$
- $\rho_{12}=\cos(180)=-1$ $\qquad \vec{S}_1=\sqrt{E}$
- $d_{min}=2\sqrt{E}=2A\sqrt{\frac{T}{2}}$ $\qquad \vec{S}_2=-\sqrt{E}$

Pe:

- $P_e = Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right) = Q\left(\frac{2\sqrt{E_b}}{\sqrt{2N_0}}\right)$

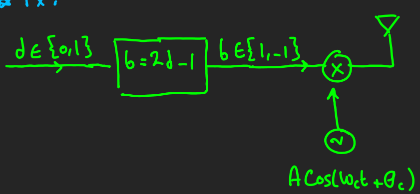
  $= Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$ # $\qquad P_{e_{OOK}} = Q\left(\sqrt{\frac{E}{N_0}}\right)$

- $\gamma_p = \min\frac{E_b}{N_0} = \frac{1}{2}\left[Q^{-1}(10^{-5})\right]^2$

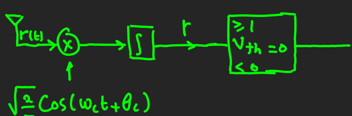
  $\gamma_p = 9.58$ $\qquad \gamma_{p_{OOK}} = 12.59$

  better "Same data, less Power"

Tx:



Rx:



QPSK

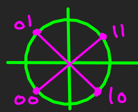
- Special case of MPSK "$M=4$, $\Psi_{ij}=\frac{\pi}{4}$, gray coded"
- $x(t)=\frac{A}{\sqrt{2}}b_E\cos(\omega_c t+\theta_c) - \frac{A}{\sqrt{2}}b_O\sin(\omega_c t+\theta_c)$

We could've used equation (I) but this is more effiecient.

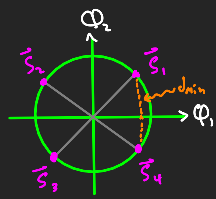
- $x(b)=\frac{A}{\sqrt{2}}b_E + j\frac{A}{\sqrt{2}}b_O \in \frac{A}{\sqrt{2}}\{1+j, 1-j, -1+j, -1-j\}$

  gray$\{11, 10, 01, 00\}$

  $\Psi=\tan^{-1}(\frac{b_O}{b_E})\{\frac{\pi}{4}, \frac{7\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}\}$



E_m, ρ_{12}, d_{min}:



- $\vec{S}_m=<\pm\frac{A}{\sqrt{2}}\sqrt{\frac{T}{2}}, \pm\frac{A}{\sqrt{2}}\sqrt{\frac{T}{2}}>$
- $E_m=\frac{A^2}{2}\frac{T}{2}+\frac{A^2}{2}\frac{T}{2}=A^2\frac{T}{2}=E_{avg}$
- $\rho_{ij}=\left\{\begin{matrix}0\\1\end{matrix}\right\}$
- $\vec{d}_{min}=\vec{S}_1-\vec{S}_4=<0, 2\frac{A}{\sqrt{2}}\sqrt{\frac{T}{2}}> \rightsquigarrow \therefore d_{min}=A\sqrt{T}=\sqrt{2E}$

Pe:

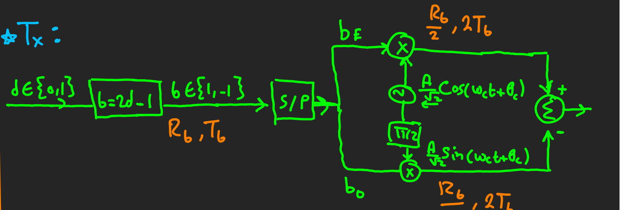
- Assume that error can occure between only adjacent symbols:

$\therefore P_e \simeq 2P\{\text{falls in one of its adjacent}\}$

$\simeq 2P_{e_{PRK}}$ "each two adjacent symbols are PRK"

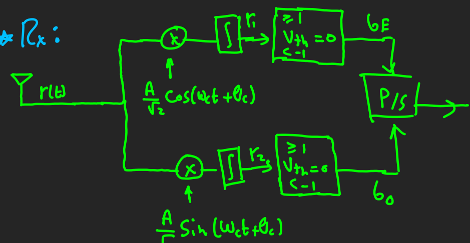
$\simeq 2Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$

Tx:

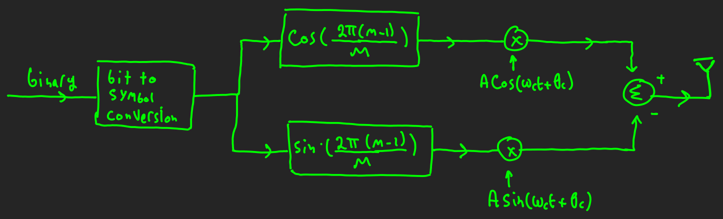


two indepedant PRK modulators each operates @ $\frac{R_b}{2}$

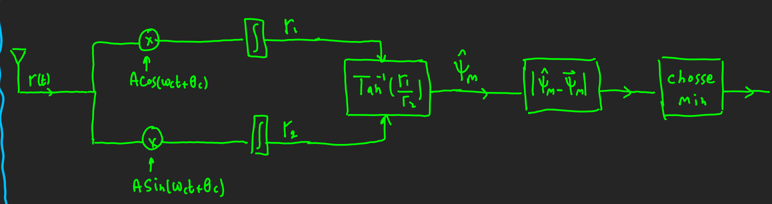
Rx:



★ Tx for MPSK:



★ Rx for MPSK:



other Variants of PSK

→ Non-Coherent detection Can't be used with ordinary PSK because we Put info in the Phase.

→ Also a Phase shif of 180° Can happen in ordinary PSK which makes the sidelobes in the spectrum bigger causing inteferance.

→ So a number of Variants was developed to overcome these Problems.

DPSK

→ Differential encoding Prior to PSK facilites NC detection, messages is in Phase difference between two sucssive symbols.

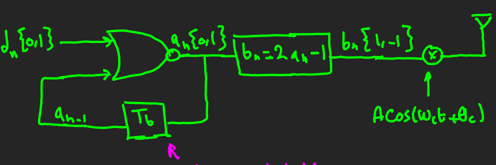
. $x(t) = A\cos(\omega_c t+\theta_c+\psi_n)$, $\psi_n = \psi_{n-1} + \Delta\psi$ (π when binary)

→ DBPSK:

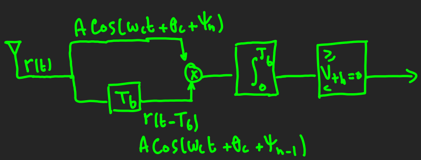
encode binary seq $\{d_n\}_{n=1}^{\infty}$ using differintal encoding

$a_n = \overline{d_n \oplus a_{n-1}}$

→ Tx:



→ Rx: detection of Absence of noise, and asume θ_c is slowly Varing between 2 succive symbols.



ex: $b \angle a$ xor(a,b)

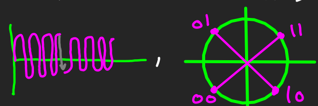
n	0	1	2	3	4	5	6	7
d_n		1	0	0	1	0	1	1
a_n	1	1	0	1	1	0	0	0
b_n	1	1	-1	1	1	-1	-1	-1
ψ_n	0	0	π	0	0	π	π	π
	NC	NC	toggle	toggle	NC	toggle	NC	NC

↑ initial condition

OQPSK

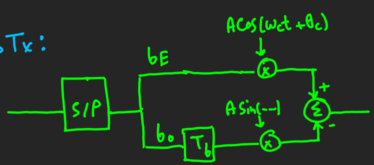
→ offest QPSK solves the Problem of 180° Phase difference when both even & odd bits toggles

00 → 11 , 01 → 10

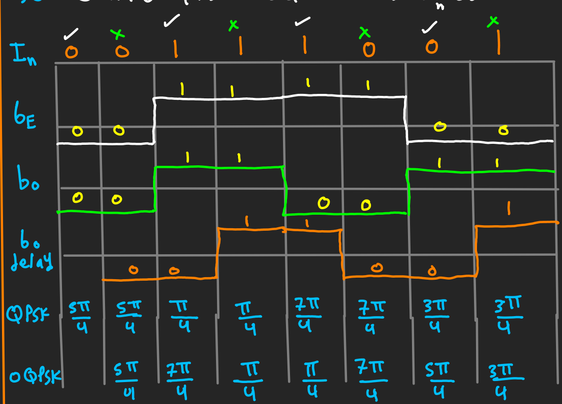


→ Prevents odd & even bits from flipping together by shifting the odd stream one bit (half symbol)

→ Tx:



→ ex: Compare QPSK & OQPSK Phase I_n = 00111001



gray{ 11 , 10 , 01 , 00 }

$\psi = \tan^{-1}\left(\frac{b_o}{b_E}\right)\left\{ \frac{\pi}{4}, \frac{7\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4} \right\}$

→ by shifting b_o, we enssure that one bit at most is changing ⤳ $\Delta\psi_{max} = \pm\frac{\pi}{2}$

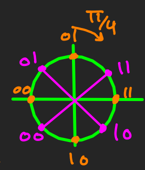
$\frac{\pi}{4}$ QPSK

→ Limit Phase difference to ±135°

→ better Performance in Fading chanel

→ non-Coherent reciver.

→ take symbol from X and next symbol from + and so on .....



→ ex: I_n = 11 00 01 10

$\frac{\pi}{4}$, π , $\frac{3\pi}{4}$, $\frac{3\pi}{2}$