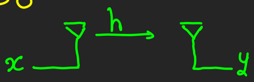
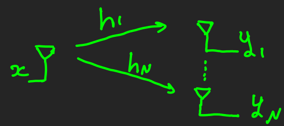
Recall

SISO



- $SNR_R = |h|^2\,SNR$
- $P_e(h) = KQ\left(\sqrt{2|h|^2 SNR}\right)$
- Diversity order $=1$

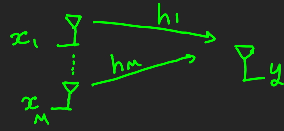
SIMO



- $SNR_R = \|\bar{h}\|^2 SNR$
- $P_e(h) = KQ\left(\sqrt{2\|\bar{h}\|^2 SNR}\right)$
- Diversity order $= N$
- Diversity achiving technique:

MRC : $r = \bar{\omega}^H \bar{y}$ " Linear Combination of $\bar{y}$ "

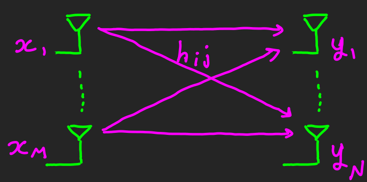
MISO



- $SNR_R = \|\bar{h}\|^2 SNR$
- $P_e(h) = KQ\left(\sqrt{2\|\bar{h}\|^2 SNR}\right)$
- Diversity order $= M$
- Diversity achiving technique:

$\bar{x} = \bar{\omega}\, x$

MIMO



$$\bar{y} = \bar{\bar{H}}_{N\times M}\,\bar{x} + \vec{n}$$

$$\underset{N\times 1}{\begin{bmatrix} y_1 \\ \vdots \\ y_N \end{bmatrix}} = \underset{N\times M}{\begin{bmatrix} h_{11} & \cdots & h_{1M} \\ \vdots & h_{ij} & \vdots \\ h_{N1} & \cdots & h_{NM} \end{bmatrix}} \underset{M\times 1}{\begin{bmatrix} x_1 \\ \vdots \\ x_M \end{bmatrix}} + \underset{N\times 1}{\begin{bmatrix} n_1 \\ \vdots \\ n_N \end{bmatrix}}$$

Singular Value decomposition:

$$\underset{N\times M}{\bar{\bar{H}}} = \underset{N\times N}{\bar{\bar{U}}}\ \underset{N\times M}{\bar{\bar{\Sigma}}}\ \underset{M\times M}{\bar{\bar{V}}^H}$$

→ $\bar{\bar{U}}$, $\bar{\bar{V}}$: are unitary matrices ($\bar{\bar{U}}^H \bar{\bar{U}} = \bar{\bar{V}}^H \bar{\bar{V}} = I = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & \ddots & & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}$)

→ each vector in $\bar{\bar{U}}$ & $\bar{\bar{V}}$ are unitary vectors (there norm $=1$), and they are orthognal to one another.

→ Σ : diagonal matrix

if $N<M$ $\begin{bmatrix} \alpha_1 & & & \\ & \alpha_2 & & \\ & & \ddots & \\ & & & \alpha_N \end{bmatrix}$

if $N>M$ $\begin{bmatrix} \alpha_1 & & \\ & \alpha_2 & \\ & & \ddots \\ & & & \alpha_N \end{bmatrix}$

α_i : Singular Values $\in \mathbb{R}^{++}$ $(\alpha_i > 0)$

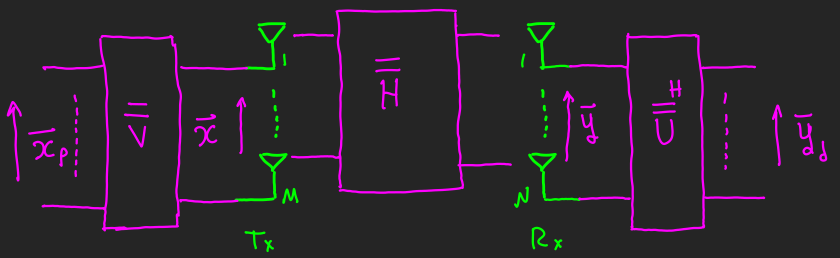
→ $\operatorname{rank}(\bar{\bar{H}})$: # non-zero α_i's $\leq \min(N,M)$

→ $\|\bar{h}\|^2 = \sum_{i=1}^{N} |h_i|^2$

→ $\|\bar{\bar{H}}\|_F^2 = \sum_{j=1}^{N}\sum_{i=1}^{M} |h_{ij}|^2 = \sum_{i=1}^{r} \alpha_i^2$

• $\bar{y} = \bar{\bar{H}}\,\bar{x} + \bar{n}$, $\bar{\bar{H}} = \bar{\bar{U}}\,\bar{\bar{\Sigma}}\,\bar{\bar{V}}^H$

→ for the following setup:



• $\vec{x} = \bar{\bar{V}}\,\vec{x}_p$

• $\bar{y}_d = \bar{\bar{U}}^H \bar{y}$

∴ $\bar{y}_d = \bar{\bar{U}}^H[\bar{\bar{H}}\vec{x} + \vec{n}] = \bar{\bar{U}}^H[\bar{\bar{U}}\bar{\bar{\Sigma}}\bar{\bar{V}}^H\vec{x} + \vec{n}]$

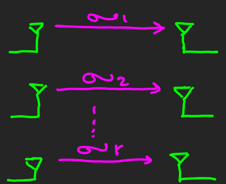
$= \underbrace{\bar{\bar{U}}^H\bar{\bar{U}}}_{I}\,\bar{\bar{\Sigma}}\,\underbrace{\bar{\bar{V}}^H\bar{\bar{V}}}_{I}\,\vec{\omega}\,x + \bar{\bar{U}}^H\vec{n}$, $\vec{x}_p = \vec{\omega}\,x$ ← symbol

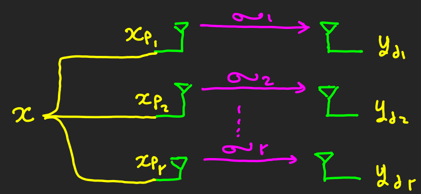
∴ $\bar{y}_d = \bar{\bar{\Sigma}}\,\vec{x}_p + \underbrace{\bar{\bar{U}}^H\vec{n}}_{\tilde{\vec{n}}}$ → $\sim CN(\vec{0}, N_0 I_{N\times N})$

$\tilde{\vec{n}}$ → $\sim CN(\vec{0}, N_0\|\bar{\bar{U}}\|^2) \equiv \sim CN(\vec{0}, N_0 I_{N\times N})$

∴ $\bar{y}_d = \begin{bmatrix}\alpha_1 & & 0\\ & \ddots & \\ 0 & & \alpha_r\end{bmatrix}\vec{x}_p + \tilde{\vec{n}}$

∴ $y_{d_i} = \alpha_i\,x_{p_i} + \underbrace{\tilde{n}_i}_{\sim CN(0, N_0)}$ $i = 1, \dots, r$





→ $\bar{\omega}$ is all ones vector and needs to be normalized $\frac{1}{\sqrt{M}}$

∴ $x_{p_i} = \frac{1}{\sqrt{M}}\,x$

∴ $y_{d_i} = \alpha_i\,\frac{1}{\sqrt{M}}\,x + n_i$ $i = 1, 2, \dots, r$

∴ $\bar{y}_d = \underbrace{\frac{1}{\sqrt{M}}\begin{bmatrix}\alpha_1\\ \vdots\\ \alpha_r\end{bmatrix}}_{\vec{h}}\,x + \vec{n}$

∴ $\bar{y}_d = \vec{h}\,x + \vec{n}$ "Like SIMO"

∴ $SNR_R = \|\vec{h}\|^2 SNR = (\frac{1}{M}\sum_{i=1}^{r}\alpha_i^2)\,SNR$

∴ $SNR_R = \frac{1}{M}\|\bar{\bar{H}}\|_F^2\,SNR$ #

→ maxmium diversity "N×M" , but loss in power by factor $\frac{1}{M}$

∴ $P_e(\bar{\bar{H}}) \le K\,Q\left(\sqrt{2\,\frac{\|\bar{\bar{H}}\|_F^2}{M}\,SNR}\right)$

∴ $P_{e_{avg}}(\bar{\bar{H}}) = \mathbb{E}_{\|\bar{\bar{H}}\|^2}\,P_e(\bar{\bar{H}}) \le \text{Const.}\,\frac{1}{SNR^{NM}}$