Amplitude Shift Keying (ASK)

We will study 3 types

BASK

- Binary, two signals with two amplitudes A_1, A_2

- 1: $S_1(t) = A_1 \cos(\omega_c t + \theta_c)$
- 0: $S_2(t) = A_2 \cos(\omega_c t + \theta_c)$

above two eqs can be written as:

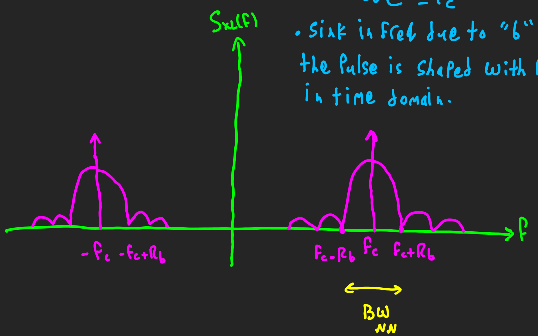
$$\underset{BASK}{x(t)} = \frac{A_1}{2}(1+b)\cos(\omega_c t + \theta_c) + \frac{A_2}{2}(1-b)\cos(\omega_c t + \theta_c)$$

b can only take values ± 1:
- ↳ if $b=1$ then $x(t) = S_1(t)$
- ↳ if $b=-1$ ~~~ ~~~ $= S_2(t)$

PSD, BW, η_s

$$x(t) = \underbrace{\frac{1}{2}(A_1+A_2)\cos(\omega_c t + \theta_c)}_{\text{Carrier @ } \pm f_c} + \underbrace{\frac{1}{2}b(A_1 - A_2)\cos(\omega_c t + \theta_c)}$$

- Polar NRZ + modulated @ $\pm f_c$
  ↳ same graph as before, just shifted @ $\pm f_c$
- Sinc in freq due to "b"
  the pulse is shaped with rect in time domain.



→ BW null to null

$\therefore BW_{NN} = f_c + R_b - (f_c - R_b)$
$= 2R_b$

→ BW 3dB

$\therefore BW_{3dB} = \frac{1}{2}(2R_B)$
$= R_B$

$\therefore \eta_s = \frac{R_b}{BW} = \frac{R_B}{R_B} = 1$ b/s/Hz

↳ every 1 bit per second occupies 1 Hz

OOK

- Same as BASK but $A_2 = 0$

- 1: $S_1(t) = A\cos(\omega_c t + \theta_c)$
- 0: $S_2(t) = 0$

above two eqs can be written as:

$$\underset{OOK}{x(t)} = \frac{A}{2}(1+b)\cos(\omega_c t + \theta_c)$$

b can only take values ± 1:
- ↳ if $b=1$ then $x(t) = S_1(t)$
- ↳ if $b=-1$ ~~~ ~~~ $= S_2(t) = 0$

PSD, BW, η_s

Same as BASK

- $BW_{NN} = 2R_b$
- $BW_{3dB} = R_B$ "default"
- $\eta_s = 1$ b/s/Hz

MASK

- M'ary, more than two signals $A_1, A_2, \dots A_M$

- $\underset{MASK}{x(t)} = A_c(2m-1-M)\cos(\omega_c t + \theta_c)$

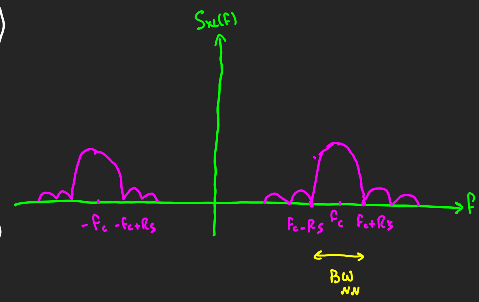
- Amps: $\{\pm A_c, \pm 3A_c, \pm 5A_c, \dots, \pm(M-1)A_c\}$

PSD, BW, η_s

$$\underset{MASK}{x(t)} = A_c\underbrace{(2m-1-M)\cos(\omega_c t + \theta_c)}$$

Polar NRZ + modulated @ $\pm f_c$
↳ same graph as before, just shifted @ $\pm f_c$

→ No carrier



→ BW null to null

$\therefore BW_{NN} = f_c + R_s - (f_c - R_s)$
$= 2R_s$

→ BW 3dB

$\therefore BW_{3dB} = \frac{1}{2}(2R_s)$
$= R_s$

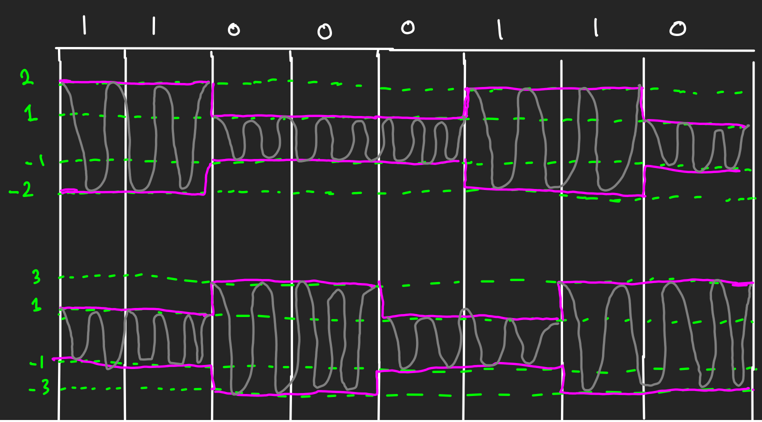
$\therefore \eta_s = \frac{R_b}{BW} = \frac{R_b}{R_s}$

$\therefore \eta_s = \frac{R_b}{R_b/K} = K$

- every K bits per second occupies 1 Hz
- More spectrum effecient but at the cost of high power at Tx

ex:

$I_n = \underline{11000110}$, sketch transmited signal for BASK with $A_1 = 2$, $A_2 = 1$, and for $\underline{4ASK}$ with $A_c = 1$:



$M = 4 \rightsquigarrow K = 2$
↳ $\{-3, -1, 1, 3\}$

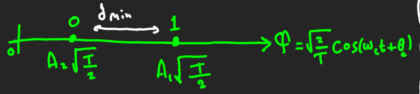
00	-3
01	-1
11	1
10	3

Constellation diagram:

$$\varphi(t) = \sqrt{\frac{2}{T}} \cos(\omega_c t + \theta_c)$$

$E_m, E_{avg}, \rho, d_{min}, P_e$:

BASK



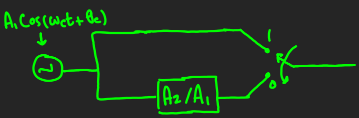
$$E_1 = A_1^2 \frac{T}{2} \quad , E_2 = A_2^2 \frac{T}{2}$$

$$E_{av} = \frac{E_1+E_2}{2} = \frac{A_1^2+A_2^2}{4} . T$$

$$\rho_{12} = \cos(0) = 1$$

$$d_{min} = |A_1 - A_2| \sqrt{\frac{T}{2}}$$

Tx

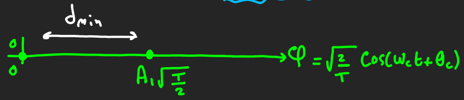


Pe

$$P_e = Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right)$$

$$\boxed{P_e = Q\left(\frac{|A_1 - A_2|\sqrt{\frac{T_b}{2}}}{\sqrt{2N_0}}\right)}$$

OOK



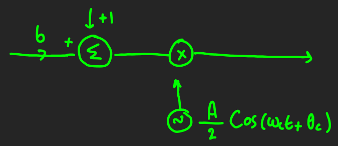
$$E_1 = A_1^2 \frac{T}{2} \quad , \quad E_2 = 0$$

$$E_{av} = \frac{E_1+E_2}{2} = \frac{A_1^2}{4} . T$$

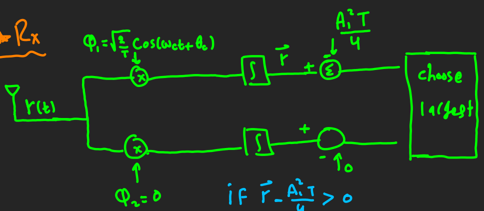
$$\rho = 1$$

$$d_{min} = A_1 \sqrt{\frac{T}{2}}$$

Tx



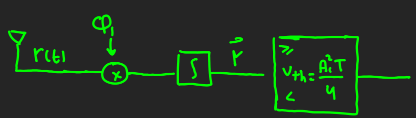
Rx



$$\text{if } \vec{r} - \frac{A_1^2 T}{4} > 0$$

$$\vec{r} > \frac{A_1^2 T}{4}$$

Can be simplified:



Pe

$$P_e = Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right)$$

$$= Q\left(\frac{A_1\sqrt{\frac{T_b}{2}}}{\sqrt{2N_0}}\right) = Q\left(\frac{\sqrt{E_1}}{\sqrt{2N_0}}\right) \quad , E_{av} = \frac{E_1}{2}$$

$$= Q\left(\frac{\sqrt{2E_{av}}}{\sqrt{2N_0}}\right)$$

$$\boxed{P_e = Q\left(\sqrt{\frac{E_{av}}{N_0}}\right) = Q\left(\sqrt{\frac{E_b}{N_0}}\right)}$$

η_p

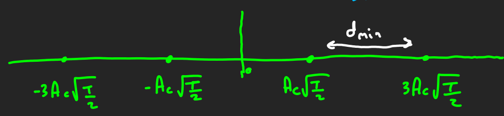
$$\eta_p = \underset{P_e = 10^{-5}}{\arg\min} \frac{E_b}{N_0} \quad , Q\left(\sqrt{\frac{E_b}{N_0}}\right) = 10^{-5}$$

$$\therefore \frac{E_b}{N_0} = \left[Q^{-1}(10^{-5})\right]^2$$

$$= 12.59 \text{ dB}$$

MASK

4ASK



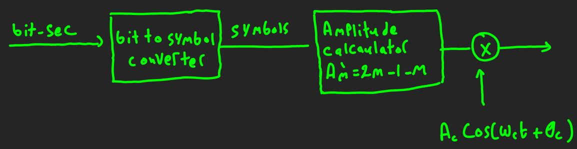
$$E_m = A_c^2 (2m-1-M)^2 \frac{T_s}{2}$$

$$E_{av} = \frac{1}{M}\sum_{m=1}^{M} A_c^2 (2m-1-M)^2 \frac{T_s}{2} = \frac{A_c^2 T^s (M^2-1)}{6}$$

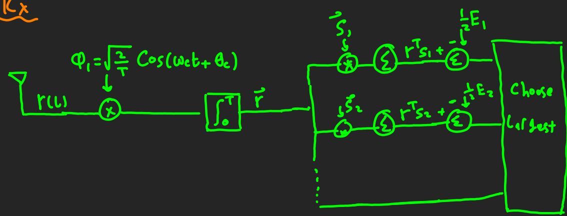
$$\rho_{ij} = \begin{cases} 1 & \text{Same Polarity} \\ -1 & \text{diff ----} \end{cases}$$

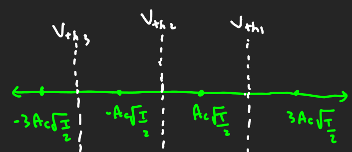
$$d_{min} = 2A_c\sqrt{\frac{T_s}{2}}$$

Tx

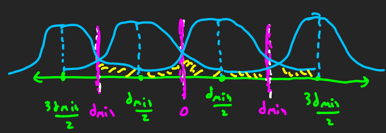


Rx





Pe



$$P_e = \frac{1}{M} * \text{total area of tails}$$

$$= \frac{1}{M} * [2*1 + (M-2)*2] * P_{tail}$$

$$= \frac{1}{M} * [2(M-1)] \, P_{tail}$$

$$P_e = \frac{2(M-1)}{M} Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right)$$

$$\because E_{av} = \frac{A_c^2 T_s (M^2-1)}{6} \quad , \because d_{min} = 2A_c\sqrt{\frac{T_s}{2}}$$

$$\therefore d_{min} = \sqrt{\frac{12\log_2(M)}{M^2-1} E_{bav}}$$

$$\boxed{\therefore P_e = \frac{2(M-1)}{M} Q\left(\sqrt{\frac{6\log_2(M)}{M^2-1} \cdot \frac{E_{bav}}{N_0}}\right)}$$