

Lab2

# Control

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**Section:** 7

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## Step A, Closed-loop Transfer Function

Theoretical:

$$\begin{aligned} J\ddot{\theta} + B\dot{\theta} &= T_c, \quad \therefore T_c = K(\theta_r - \theta) \\ \therefore J\ddot{\theta} + B\dot{\theta} &= K(\theta_r - \theta) \\ \therefore J\theta(s)S^2 + B\theta(s)S &= K\theta_r - K\theta(s) \\ \therefore \underset{\substack{\uparrow \\ \text{o/p}}}{\theta(s)} [JS^2 + BS + K] &= \underset{\substack{\uparrow \\ \text{i/p}}}{K\theta_r}, \quad \text{we need } \theta \text{ to track } \theta_r \\ \therefore \boxed{\text{T.F} = \frac{\theta(s)}{\theta_r} = \frac{K}{JS^2 + BS + K}} &\neq \text{Second order system.} \end{aligned}$$

Matlab:

```
1 %-----a)-----%
2 j = 600000;
3 b = 20000;
4 k = 1;
5
6 TF = tf(k, [j b k])
7
8
```

TF =

$$\frac{1}{600000 s^2 + 20000 s + 1}$$

Continuous-time transfer function.

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## Step B, state-space representation

Matlab:

```
9 %-----b)-----%
10 k=1;
11 State_Space = ss(TF)
12
```

---

State\_Space =

A =

	x1	x2
x1	-0.03333	-0.001707
x2	0.0009766	0

B =

	u1
x1	0.03125
x2	0

C =

	x1	x2
y1	0	0.05461

D =

	u1
y1	0

Continuous-time state-space model.

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### Step C, Max value of K to have a stable system

Theoretical:

C)

$$J = 600 \times 10^3, B = 20 \times 10^3$$

$$\therefore 600 \times 10^3 s^2 + 20 \times 10^3 s + K = 0$$

$s^2$	$600 \times 10^3$	$K$
$s^1$	$20 \times 10^3$	$0$
$s^0$	$K$	

$\therefore$  System is stable for all values of  $K > 0$   
since there's no upper limit,  $\therefore K_{\max} = +\infty$

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### Step D, Max value of K to have $M_p < 10\%$

Theoretical:

D)

$K_{\max}$  for  $M_p < 0.1$

$\therefore J s^2 + B s + K = 0$

$\therefore s^2 + \frac{B}{J} s + \frac{K}{J} = 0$

$\therefore 2\gamma \omega_n = \frac{B}{J}, \therefore \omega_n = \sqrt{\frac{K}{J}}$

$\therefore \frac{B}{2J\gamma} = \sqrt{\frac{K}{J}}$

$\therefore K = J \left( \frac{B}{2J\gamma} \right)^2$

$\therefore M_p = e^{\frac{-\pi\gamma}{\sqrt{1-\gamma^2}}} = 0.1$

$\therefore \gamma = 0.59$

$\therefore K = 600 \times 10^3 \left( \frac{20}{2 \times 600 \times 0.59} \right)^2$

$\therefore K_{\max} = 479$

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### Step E, Max value of K to have rise time < 80 sec

Theoretical:

Q)

Values of K to provide rise time < 80 sec

$$\therefore t_r = \frac{\pi - \cos^{-1}(\eta)}{\omega_n \sqrt{1 - \eta^2}} < 80$$
$$\therefore \frac{\pi - \cos^{-1}(\eta)}{\frac{B}{2J\eta} \sqrt{1 - \eta^2}} < 80$$
$$\therefore \eta = 0.53$$
$$\therefore K > 592$$

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## Step F, Plot step response for K=200,400,1000,2000

### Matlab code

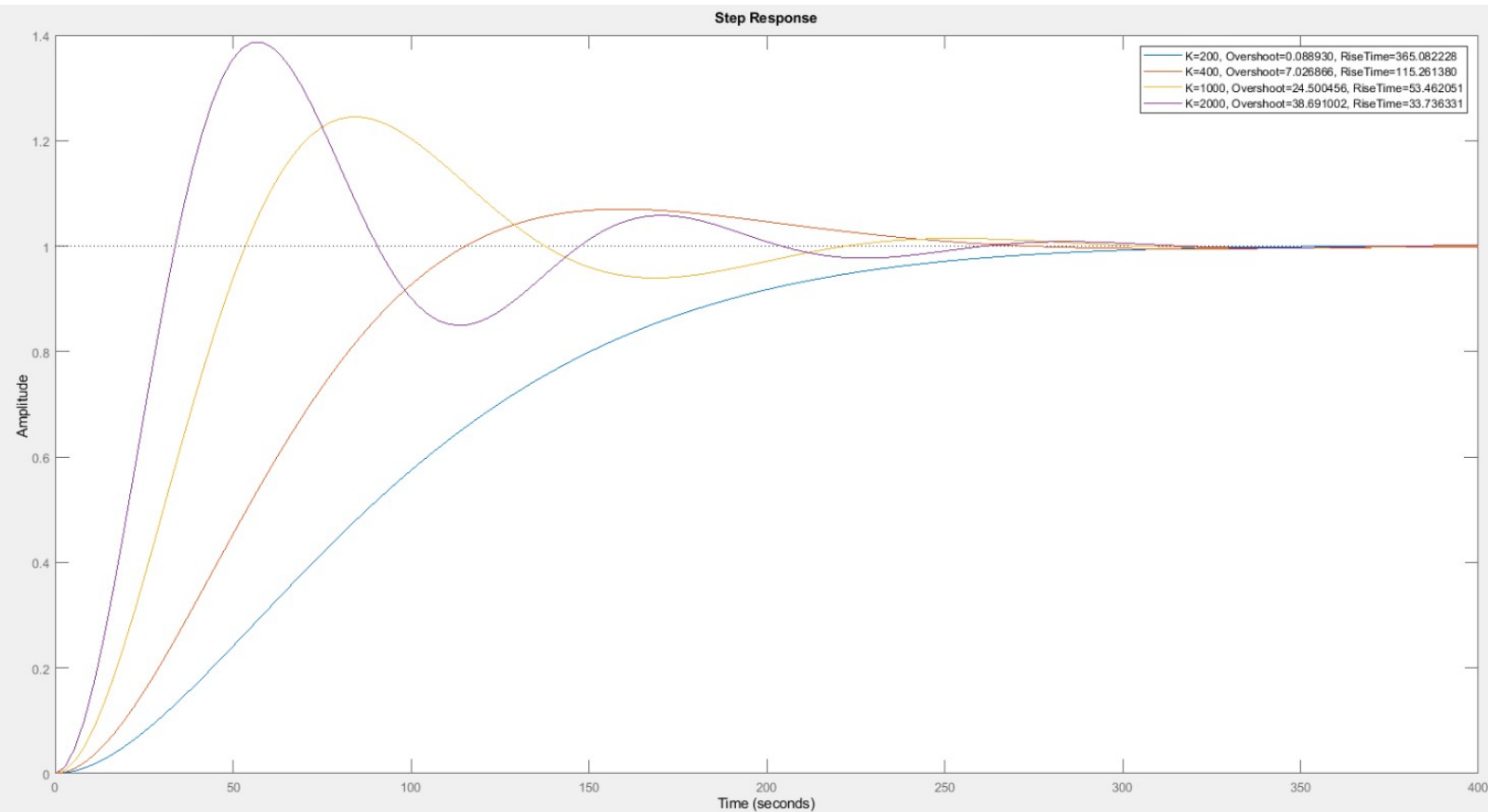
```
15 %-----f)-----%
16 % as K increases, overshoot increses and rise time decreases
17 - k = 200;
18 - TF1 = tf(k, [j b k]);
19 - info1 = stepinfo(TF1 , 'RiseTimeThreshold', [0 1]);
20 - legend1 = compose('K=200, Overshoot=%f, RiseTime=%f', info1.Overshoot, info1.RiseTime);
21
22 - k = 400;
23 - TF2 = tf(k, [j b k]);
24 - info2 = stepinfo(TF2, 'RiseTimeThreshold', [0 1]);
25 - legend2 = compose('K=400, Overshoot=%f, RiseTime=%f', info2.Overshoot, info2.RiseTime);
26
27 - k = 1000;
28 - TF3 = tf(k, [j b k]);
29 - info3 = stepinfo(TF3, 'RiseTimeThreshold', [0 1]);
30 - legend3 = compose('K=1000, Overshoot=%f, RiseTime=%f', info3.Overshoot, info3.RiseTime);
31
32 - k = 2000;
33 - TF4 = tf(k, [j b k]);
34 - info4 = stepinfo(TF4 , 'RiseTimeThreshold', [0 1]);
35 - legend4 = compose('K=2000, Overshoot=%f, RiseTime=%f', info4.Overshoot, info4.RiseTime);
36
37 - figure(1);
38 - step(TF1, TF2, TF3 , TF4);
39 - legend(string(legend1) , string(legend2), string(legend3) , string(legend4));
```







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## Step F, Plot step response for $K=200,400,1000,2000$

Matlab output image



	$K=200$ , Overshoot=0.088930, RiseTime=365.082228
	$K=400$ , Overshoot=7.026866, RiseTime=115.261380
	$K=1000$ , Overshoot=24.500456, RiseTime=53.462051
	$K=2000$ , Overshoot=38.691002, RiseTime=33.736331

Do the plots to confirm your calculations in previous parts ?  
-Yes, we see that as  $K$  increases Rise time decreases confirming our theoretical solution in step e.



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## Step G, Plot poles and zeros for K=200,400,1000,2000

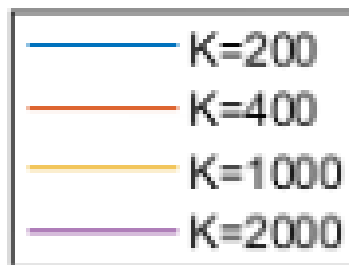
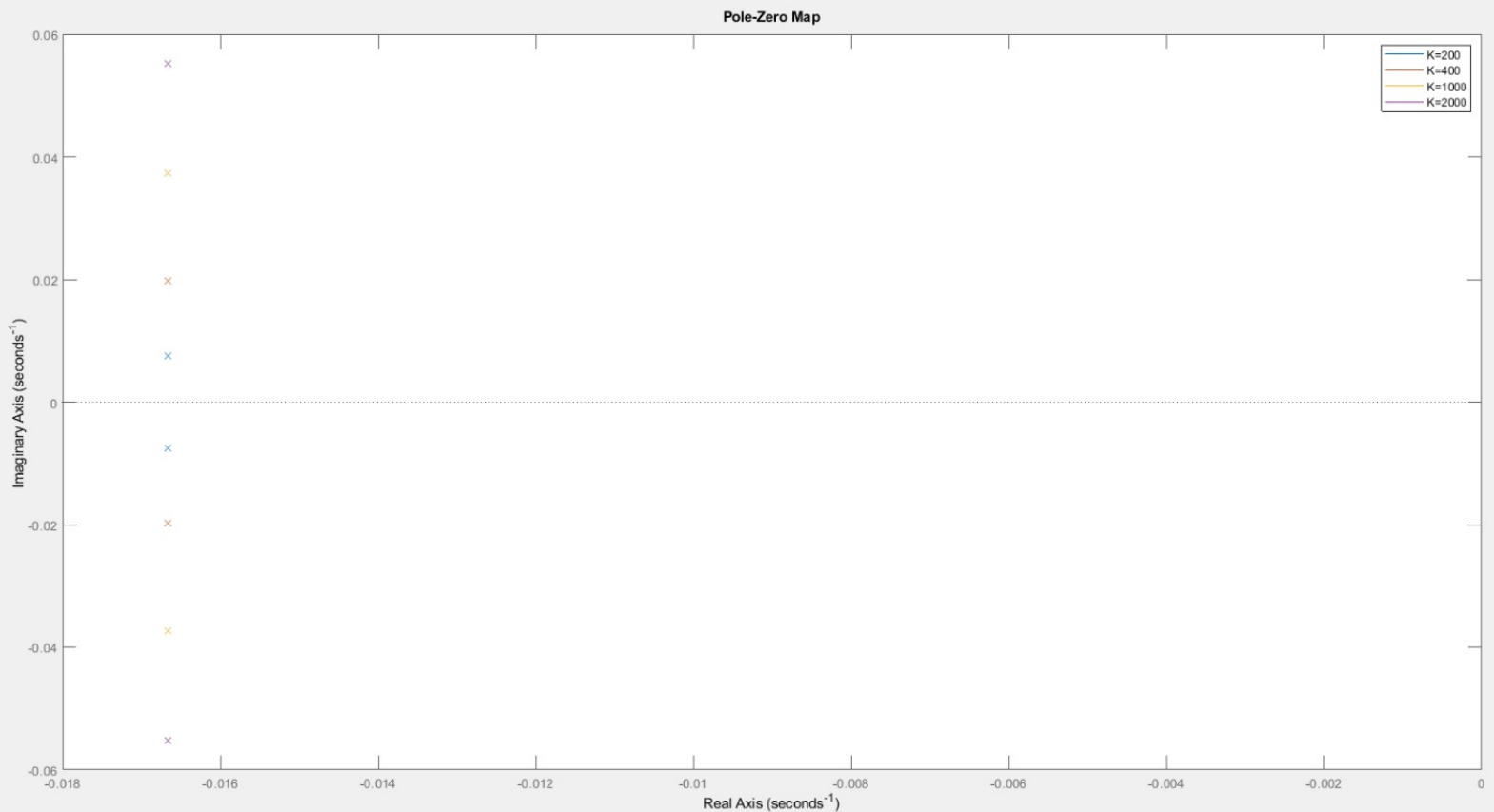
Matlab code

```
43 %-----g) -----%
44 % as we increase K, the poles location increases verticlly
45 - figure(2);
46 - pzplot(TF1, TF2, TF3 , TF4)
47 - legend('K=200', 'K=400', 'K=1000', 'K=2000')
48 %grid on
49
```

---

## Step G, Plot step response for $K=200,400,1000,2000$

Matlab output image



Effect of  $K$  on the closed loop zeros and poles?

- As  $K$  increases, system poles location increases vertically, Making the system have more oscillating behavior.

## Step H, find the steady state error for K=200,400,1000,2000

Theoretical:

h)  
Find steady state error for each K

- Step input

$$\rightarrow \theta(s) = T.F. \cdot \frac{1}{s}$$
$$\therefore \theta(s) = \frac{K}{s(JS^2 + BS + K)}$$
$$\begin{cases} \text{error} = \theta(\infty) - 1 \\ \theta(\infty) = \lim_{s \rightarrow 0} s \theta(s) \end{cases}$$
$$\therefore \theta(\infty) = \lim_{s \rightarrow 0} \frac{K}{JS^2 + BS + K} = \frac{K}{K} = 1$$
$$\therefore \text{error} = 1 - 1 = 0$$

Matlab:

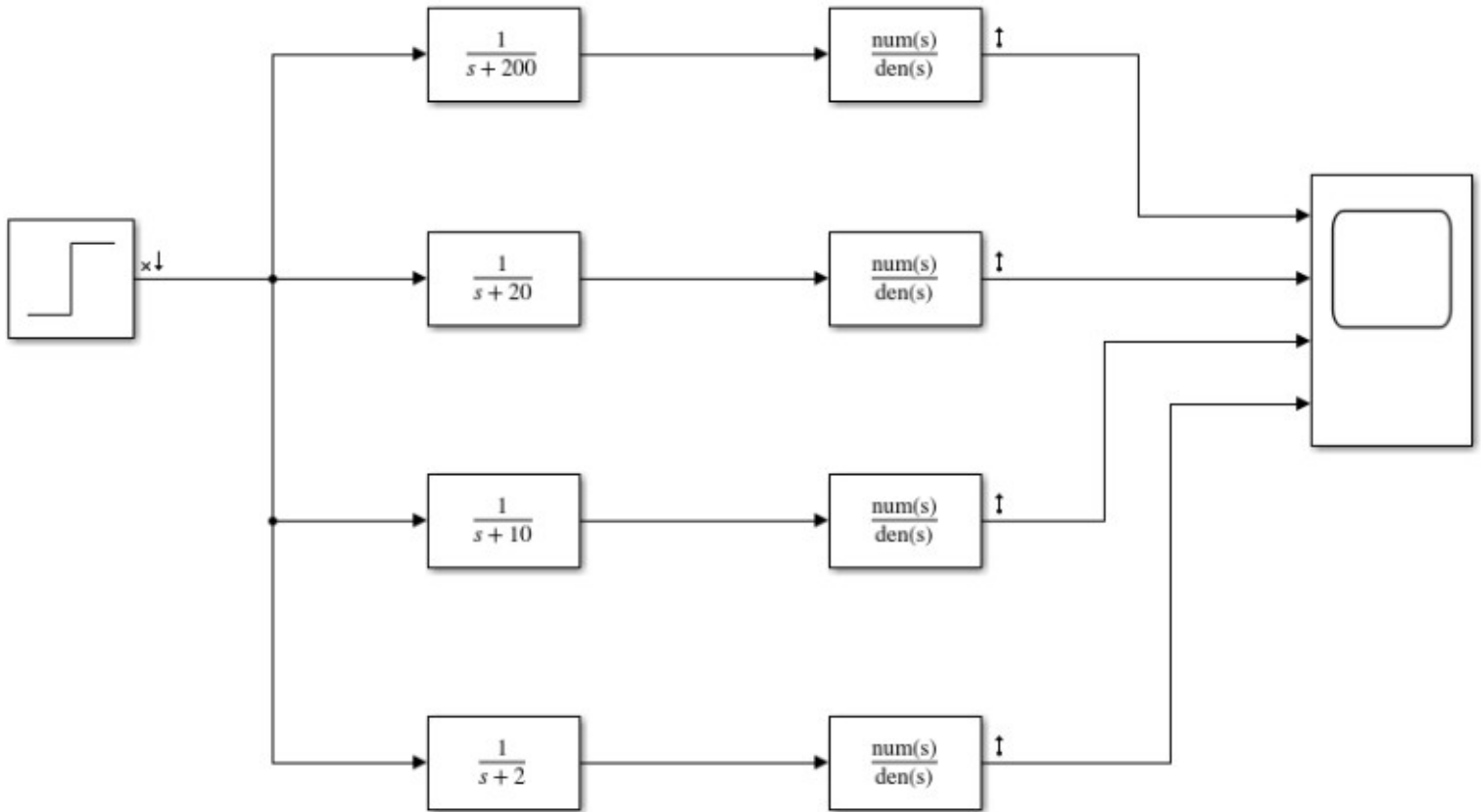
```
52 %-----h)-----%
53 % error is very close to zero, confirming the theoretical solution
54 - [theta1 t] = step(TF1);
55 - ess1 = 1 - theta1(end);
56
57 - [theta2 t] = step(TF2);
58 - ess2 = 1 - theta2(end);
59
60 - [theta3 t] = step(TF3);
61 - ess3 = 1 - theta3(end);
62
63 - [theta4 t] = step(TF4);
64 - ess4 = 1 - theta4(end);
```

ess1 =	ess3 =
-5.1400e-04	0.0033
ess2 =	ess4 =
0.0049	-8.7153e-04

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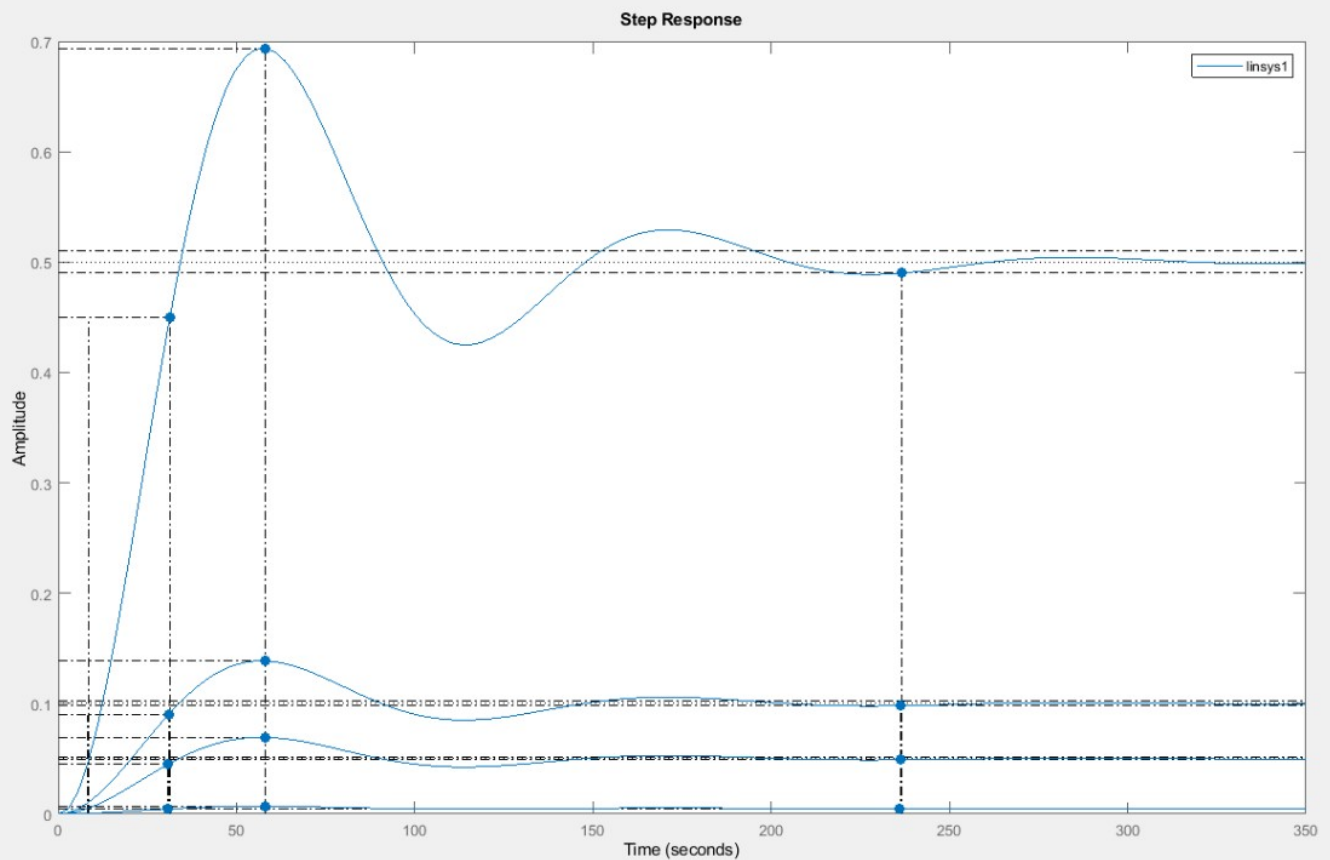
## Step I, Using Simulink add poles at -200, -20, -10, -2

Simulink Blocks Image



## Step I, Using Simulink add poles at -200, -20, -10, -2

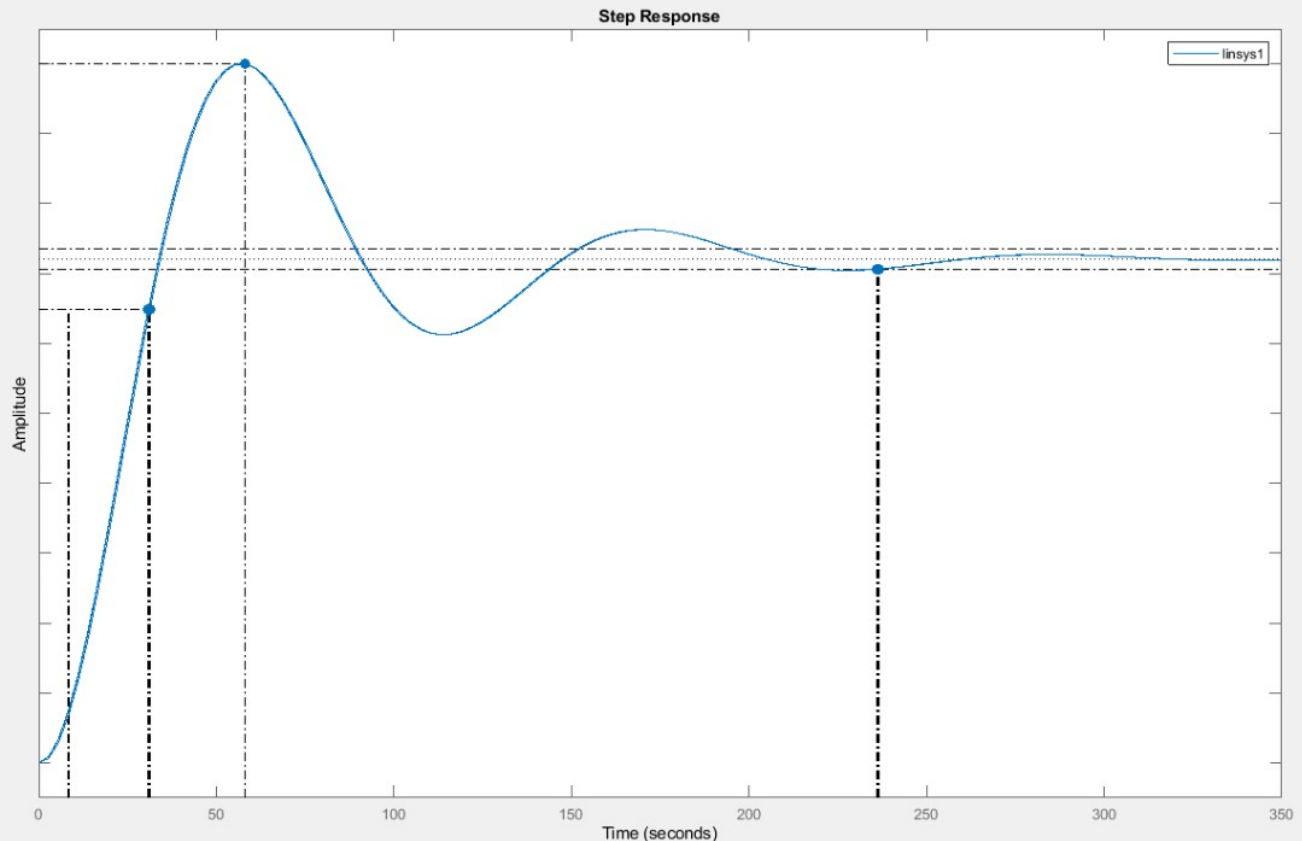
Step response of the 4 responses with rise time, overshoot, settling time and peak time.



The linearization result "linsys1" is created in the Linear Analysis Workspace.

## Step I, Using Simulink add poles at -200, -20, -10, -2

Step response of the 4 responses normalized with rise time, overshoot, settling time and peak time.



The linearization result "linsys1" is created in the Linear Analysis Workspace.

## **Step J, Discuss the effect upon the transient response of the proximity of a higher-order pole to the second-order system**

In general the effect of adding poles is makes the step response slower, in our case the original system poles are at -0.017 on the real axis, hence making them the dominant poles, so the response nearly doesn't change because the smallest added pole is 117 time larger than the original system poles.