

Track Two: Financial Data Engineers

Question 1: Financial Structural Product Designer

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1.1 Macro Economy

US Economy: Short-term (3 months)

- The US economy is expected to remain robust, supported by strong manufacturing PMI and non-farm payroll data.
 - **GDP Growth:** Expected to maintain a quarterly growth rate of around 1.5-2.0%.
 - **Inflation:** Likely to stay around 4.0-4.5%, influenced by energy prices and supply chain issues.
 - **Unemployment:** Expected to remain low, around 3.5-3.7%, supported by strong non-farm payrolls data.
- Geopolitical tensions and hawkish Federal Reserve statements may introduce volatility but are unlikely to derail economic momentum.

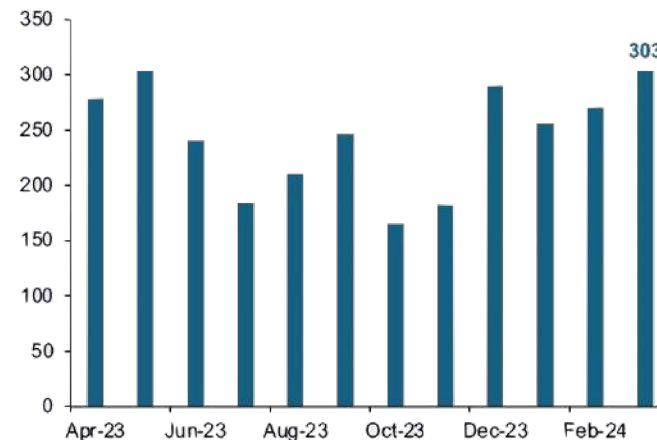
US Economy: Medium-term (1 year)

- The medium-term outlook suggests stable but cautious economic expansion.
 - **GDP Growth:** Projected annual growth rate of 2.0-2.5%.
 - **Inflation:** Forecasted to moderate to 3.0-3.5% as supply chain issues resolve.
 - **Unemployment:** Expected to stabilize around 3.6-3.8%.
- The Federal Reserve may adopt a more patient approach to rate adjustments, balancing inflation control with economic growth.

Fig. 1: ISM Manufacturing PMI – back in expansion



Fig. 2: Nonfarm payroll – reacceleration



1.1 Macro Economy

Eurozone: Short-term (3 months)

- The Eurozone might see moderate economic improvement with easing inflation and potential ECB rate cuts.
 - **GDP Growth:** Predicted to increase modestly by 0.5-1.0% quarter-over-quarter.
 - **Inflation:** Expected to ease to around 3.0-3.5%, with potential ECB rate cuts to stimulate growth.
 - **Business Activity:** Composite PMI anticipated to stay above 50, indicating expansion.
- Business activity likely to improve, as indicated by the rising Composite PMI.

Eurozone: Medium-term (1 year)

- Economic activity in the Eurozone is expected to continue improving with controlled inflation and supportive ECB measures.
 - **GDP Growth:** Predicted annual growth rate of 1.5-2.0%.
 - **Inflation:** Likely to stabilize around 2.5-3.0% with ECB policy support.
 - **Business Activity:** Composite PMI expected to maintain levels above 50, suggesting sustained recovery.
- Potential for significant recovery if inflation remains under control and ECB implements supportive measures.

Fig. 3: Eurozone inflation drops to 2.4% in March

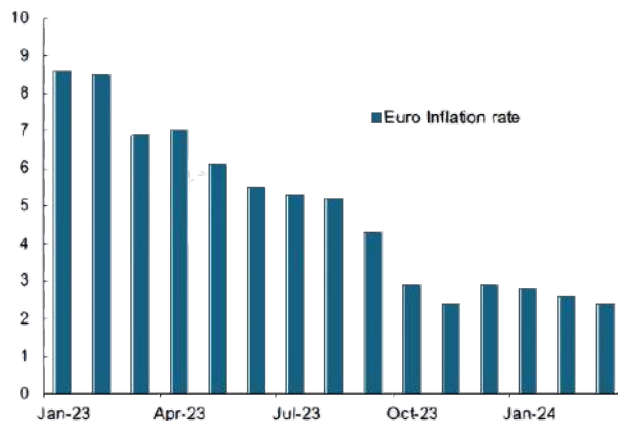
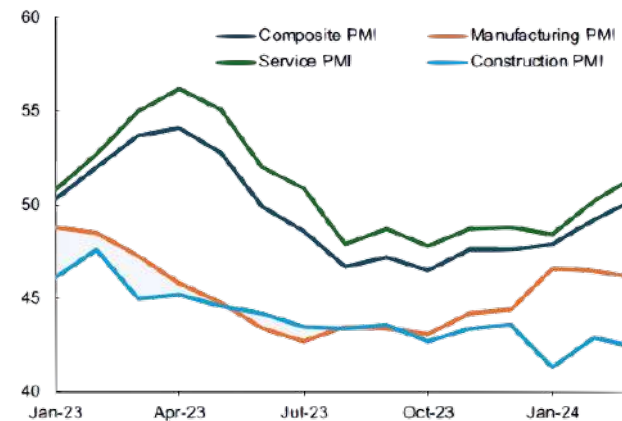


Fig. 4: Eurozone economic activity improved



1.1 Macro Economy

China: Short-term (3 months)

- China's economy likely to continue its recovery with solid manufacturing expansion despite potential external challenges.
 - **GDP Growth:** Likely to see a quarterly increase of 1.0-1.5%.
 - **Manufacturing PMI:** Expected to remain above 50, signaling continued expansion.
 - **Exchange Rates:** The RMB might appreciate slightly against the USD, assuming stable global trade conditions.
- The sustainability of this recovery depends on external factors, including the US dollar strength and global trade conditions.

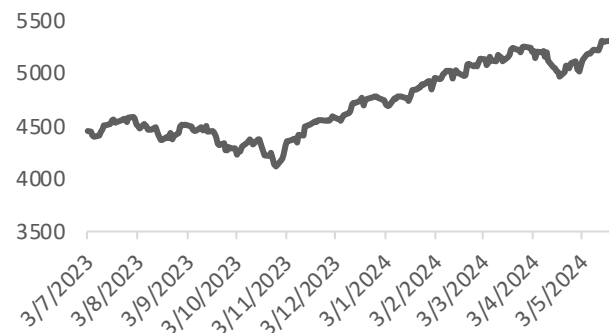
China: Medium-term (1 year)

- China's economic outlook remains optimistic with strong GDP growth and consistent manufacturing expansion.
 - **GDP Growth:** Expected annual growth rate of 5.0-5.5%.
 - **Manufacturing PMI:** Anticipated to remain consistently above 50.
 - **Exchange Rates:** The RMB might continue to appreciate modestly against the USD.
- Assumes effective management of exchange rates and continuous policy support from the central bank.

1.2 S&P 500

Short-term (3 months)	Medium-term (1 year)
<p>Bullish Sentiment:</p> <ul style="list-style-type: none"> Several firms, including BMO Capital Markets and Deutsche Bank, have increased their targets to 5,600 and 5,500, respectively. The overall average year-end target among top strategists now sits at 5,289, suggesting a stable or slightly declining trend from current levels. Driven by strong performances from megacap technology stocks. <p>Bearish Concerns:</p> <ul style="list-style-type: none"> Some analysts, such as those from Stifel, warn of a possible sharp correction. Stifel predicts a potential 10% decline, anticipating the S&P 500 could fall to 4,750 due to persistent inflation and delayed Fed rate cuts. Potential for heightened volatility and a mid-year correction. <p>Outlook:</p> <ul style="list-style-type: none"> Mixed signals with potential for continued rally, but significant volatility and potential pullback as the market adjusts to macroeconomic factors and interest rate policies. 	<p>Continued Growth:</p> <ul style="list-style-type: none"> Many strategists are optimistic about the S&P 500's performance over the next 12 months. <ul style="list-style-type: none"> Mike Wilson of Morgan Stanley projects the index to reach 5,400 by Q2 2025. Bottom-up estimates predict an 11% increase, with a target of around 5,856. <p>Sector Performance:</p> <ul style="list-style-type: none"> Growth expected to be driven by consumer discretionary and energy sectors. The utilities sector may lag, indicating varied performance within the index. <p>Economic Influences:</p> <ul style="list-style-type: none"> The trajectory will heavily depend on inflation trends and the Federal Reserve's actions regarding interest rates. Persistently high inflation could delay anticipated rate cuts, impacting market sentiment and valuations.

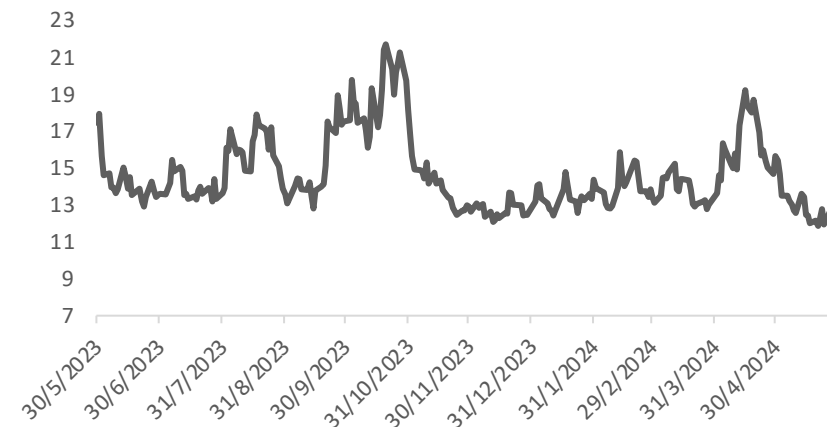
S&P 500 - Continued Growth



1.3 VIX

Short-term (3 months)	Medium-term (1 year)
<p>Key Events:</p> <ul style="list-style-type: none"> • FOMC Decision: Upcoming Federal Open Market Committee meeting could spike the VIX by 5-10%. • NFP Report: Non-Farm Payroll report can cause significant market reactions, leading to VIX movements of 3-7%. • Current Volatility Levels: Recent 32% decline in the VIX suggests a calmer market, but temporary spikes back to 20-25 are possible during high-impact news events. <p>Outlook:</p> <ul style="list-style-type: none"> • VIX likely to hover around 16-18 in quieter periods, with potential spikes to 20-25 around significant economic data releases and Fed meetings. 	<p>Influencing Factors:</p> <ul style="list-style-type: none"> • Economic Data and Fed Policy: Ongoing tracking of inflation and employment data will keep markets vigilant. Policy shifts can drive the VIX up by 10-15%. • Geopolitical Events: Tensions, such as the Israel-Iran situation, could escalate, leading to VIX surges of 10-20%. • Market Sentiment and Risk Assets: Performance of major stocks and commodities will also affect the VIX. Significant corrections could push the VIX from 16-18 to 30-35 during major sell-offs. <p>Outlook:</p> <ul style="list-style-type: none"> • VIX may stay relatively subdued between 15-20 in stable periods, with periodic spikes to 25-35 in response to major economic reports, Fed policy changes, and geopolitical events over the next year.

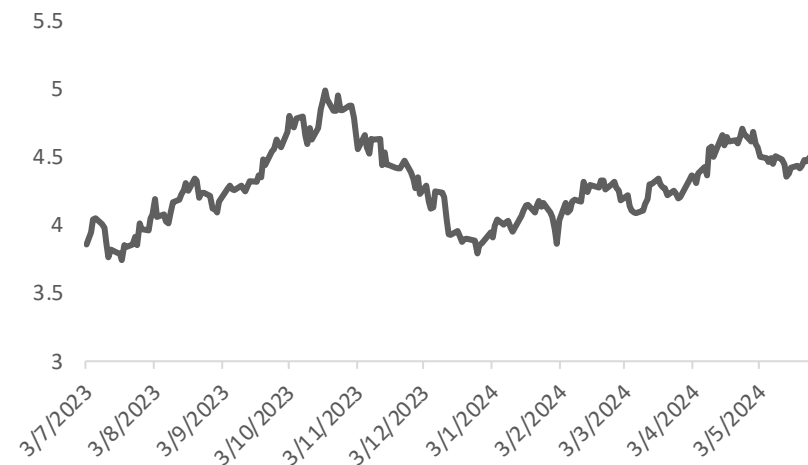
VIX Index



1.4 US 10-Year Treasury

Short-term (3 months)	Medium-term (1 year)
<p>Yield Expectations:</p> <ul style="list-style-type: none"> Expected to experience modest fluctuations influenced by economic data releases and Federal Reserve communications. Current Yield: Approximately 4.4% Expected Yield Range: Around 4.4%, with potential minor downward adjustments. <p>Influencing Factors:</p> <ul style="list-style-type: none"> Recent trend of yields inching lower amid softer economic data and moderated inflation. Federal Reserve's emphasis on caution regarding inflation suggests interest rate cuts are unlikely in the immediate future. 	<p>Yield Expectations:</p> <ul style="list-style-type: none"> Potential gradual decline as the Federal Reserve is expected to start easing its monetary policy. Expected Yield Range: 4.2% - 4.4% <p>Economic Forecasts:</p> <ul style="list-style-type: none"> Continued moderation in inflation and potential softening in economic growth expected to lead to rate cuts in the latter half of 2024. Despite current inflation stickiness, economic resilience might prompt only a couple of rate cuts in 2024, with more significant easing anticipated in 2025.

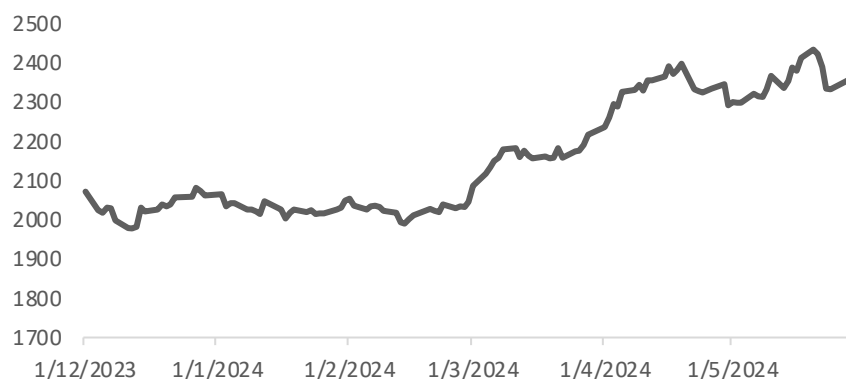
US 10 Year Treasury – modest fluctuations



1.5 Gold

Short-term (3 months)	Medium-term (1 year)
<p>Price Expectations:</p> <ul style="list-style-type: none"> Expected to remain strong, supported by speculative buying, central bank purchases, and interest rate expectations. Current Price: \$2324.15 per ounce (as of April 5th) Expected Price Range: \$2300 - \$2400 per ounce <p>Supporting Factors:</p> <ul style="list-style-type: none"> Speculative buying as investors bet on further price increases. Central banks globally increasing their gold reserves. Anticipation of US interest rate cuts bolsters gold's appeal as a non-yielding asset. 	<p>Price Expectations:</p> <ul style="list-style-type: none"> Ongoing economic uncertainties and potential recession fears drive safe-haven demand. Expected Price Range: \$2400 - \$2600 per ounce <p>Influencing Factors:</p> <ul style="list-style-type: none"> Economic conditions and potential recession fears support safe-haven demand. Interest rate cuts by major economies could weaken the US dollar, enhancing gold's attractiveness. Geopolitical risks and economic instability may further enhance gold's allure. ETF holdings declining but strong demand from central banks and retail investors persists.

Gold Price - Increasing



1.6 Copper

Short-term (3 months)	Medium-term (1 year)
<p>Price Expectations:</p> <ul style="list-style-type: none"> Expected to remain volatile but generally positive due to China's economic data and rate cut hopes. Current Price: \$9397.5 per metric ton (London copper price, as of April 4th) Expected Price Range: \$9000 - \$9600 per metric ton <p>Influencing Factors:</p> <ul style="list-style-type: none"> Better-than-expected manufacturing activity in China boosts demand expectations. Hopes for rate cuts in the US and Europe support industrial metal prices. Fluctuations in the US dollar impact copper prices; a stronger dollar may suppress gains, while a weaker dollar could enhance them. 	<p>Price Expectations:</p> <ul style="list-style-type: none"> Moderately optimistic outlook contingent on global economic growth and potential supply issues. Expected Price Range: \$9500 - \$10,500 per metric ton <p>Influencing Factors:</p> <ul style="list-style-type: none"> Global Economic Growth: Sustained growth in key economies, particularly China, will bolster demand. Supply Concerns: Potential disruptions in supply chains or mining output could push prices higher. Monetary Policies: Easing policies in major economies might boost industrial activity and copper demand. While short-term volatility is expected, the overall trend could be upward, driven by fundamental demand and potential supply constraints.

Copper Price - falls at recent months



1.7 Japanese Equities

Short-term (3 months)	Medium-term (1 year)
<p>Key Factors:</p> <ul style="list-style-type: none"> • Continued Foreign Investment: Foreign investors have shown significant interest in Japanese stocks, with net foreign purchases reaching a record high. • Share Buybacks: Japanese companies have been actively engaging in share buybacks, enhancing shareholder value and supporting stock prices. • Supportive Monetary Policy: The Bank of Japan (BoJ) is expected to maintain its accommodative monetary stance, supporting stable Japanese Government Bond (JGB) yields and avoiding a yield curve inversion. <p>Economic Indicators:</p> <ul style="list-style-type: none"> • BoJ's goal of maintaining inflation above 2% encourages investment in equities. • Fiscal policies and corporate reforms provide a favorable environment for equity markets. 	<p>Positive Outlook:</p> <ul style="list-style-type: none"> • Driven by macroeconomic recovery and structural reforms: <ul style="list-style-type: none"> • Economic Recovery and Inflation: Real GDP growing by 1.9% year-on-year; inflation above the BoJ's 2% target supports consumer spending and economic expansion. • Corporate Governance Reforms: Improved capital efficiency and increased returns on equity (ROE), boosting investor confidence. <p>Market Performance:</p> <ul style="list-style-type: none"> • Japanese equities outperformed the FTSE All-World index over 12 months with an 18.5% return in US dollar terms. • Retail Investor Participation: Revamped Nippon Individual Savings Account (NISA) program aims to drive more retail investment into equities. • Competitive Valuations: Japanese equities' forward price-to-earnings (P/E) ratio remains attractive compared to other markets. <p>Key Indicators:</p> <ul style="list-style-type: none"> • Fiscal year 2024 wage increases of 5.28% among large companies to support consumer spending. • Domestic and international investors find Japanese stocks appealing due to their competitive valuations.

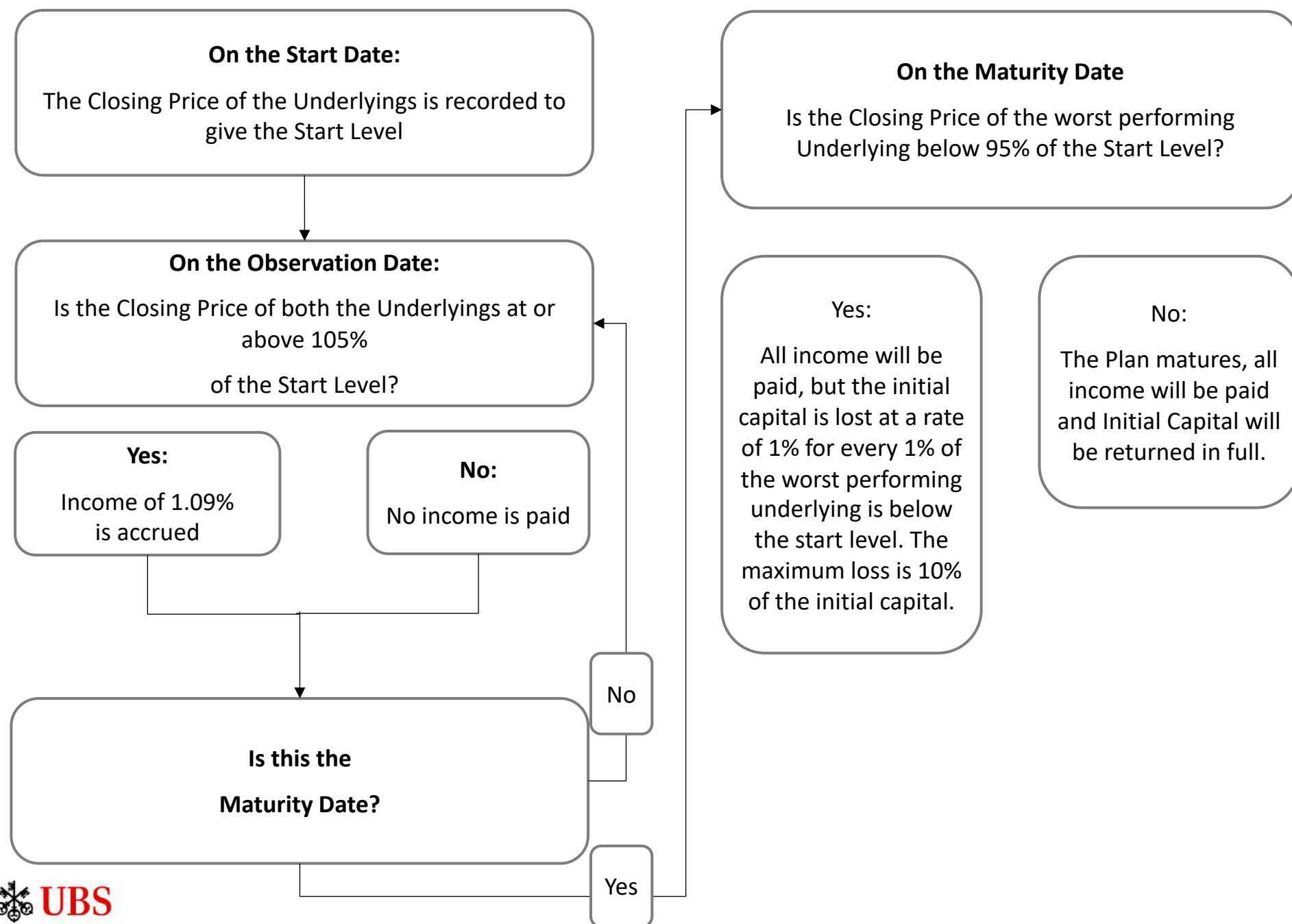
2.1 Investment Proposal for Client A

Investment Preference	Goal and Risk Tolerance
<p>Investment Amount:</p> <ul style="list-style-type: none">• \$10 Million USD <p>Investment Horizon:</p> <ul style="list-style-type: none">• Medium-term (1-2 years) <p>Preferred Investment Sectors:</p> <ul style="list-style-type: none">• Artificial Intelligence (AI)• Japanese Stocks• Gold• Copper	<p>Investment Goals:</p> <ul style="list-style-type: none">• High-yield structured product with returns exceeding typical deposit rates• Targeted potential return: 12%+ <p>Risk Tolerance:</p> <ul style="list-style-type: none">• Desire for 100% principal protection• Willing to accept up to 10% potential loss of principal• Principal should remain intact if the targeted assets do not decline significantly

2.2 Product Design

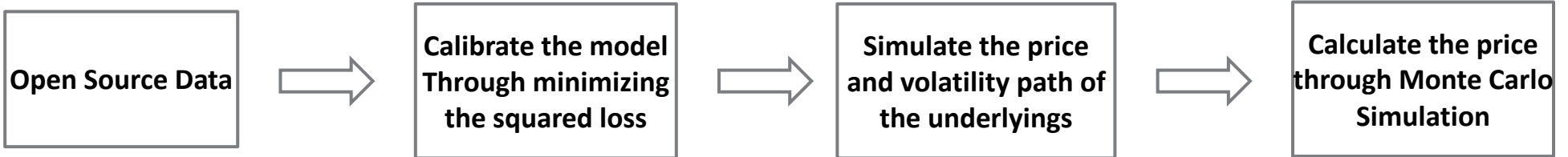
Product Type	Monthly Range Accrual Note with a Capital Protected Feature
Underlyings	Gold Futures and Copper Futures from CME
Investment Term	18 months
Potential Income	1.09% for each month the Plan runs (13.08% p.a.); income is paid if the Closing Price of both the Underlyings is at or above the Income Trigger Level on a monthly Observation Date.
Potential Loss	If the finish level of the worst performing underlying is below the Capital Protected Level, your initial capital is lost at a rate of 1% for every 1% of the worst performing underlying is below the start level. The maximum loss is 10% of the Initial Capital.
Start Date	The first trading day of a month.
Observation Dates	The last trading days of each month as the Plan is running.
Income Trigger Level	105% of the start level
Capital Loss Level	95% of the start level
Capital Protected Level	90% of the start level

2.2 Product Design – Flow Chart



2.3 Pricing Model

We Calibrated a Heston Stochastic Volatility Model to price the product



We searched for open data to help building the model:

- Gold future and Copper future price historical data from Yahoo Finance
- Gold option and Copper option price historical data from barchart.com
- Risk-free rate from 2Y Daily Treasury Par Yield Curve Rates of US government
- Other information that may be helpful in determining the parameters

We used squared loss with penalty as loss function to calibrate the Heston Model:

- We excluded the observations whose trading volume is below 10 in the dataset, to get more precise option prices
- We added a penalty in the loss function, to guarantee the Feller condition and to avoid the volatility being negative in the simulation
- We use minimize function with bounds to restrict parameter values during iterations

Based on the parameters we got from model calibration, we run a simulation to get the volatility and price path of gold future and copper future:

- We used the correlation coefficient of future prices, along with model parameters to build the correlation relationship among prices and volatilities.
- Quadratic-Exponential method is used for volatility process simulation to get higher precision
- Exact representation instead of simple Euler discretization method is used for price process simulation.

Based on the price paths generated during the last step, we can calculate the price of sophisticated path-dependent products:

- We tried products with different features, such as auto call, range accrual with different observation frequencies, and best-of and worst-of settings.
- To get the proper coupon rate, we set the price of the product to zero by adjusting the coupon rate and keep all other parameters unchanged

Key assumptions:

- Risk free rate stays constant during the plan
- Gold future price and copper future price are correlated, with its correlation coefficient stays constant during the plan.

2.3 Pricing Model

Here are key steps we used to calculate the price of the product:

1. Use market data to calibrate Heston parameters. By implementing the loss function below, we get the parameters for gold and copper that minimize the function value

$$Loss = \sum_{i=0}^N (P_{Heston} - P_{Market})^2 * (1 + (\sigma^2 - 2\kappa\theta)_+)$$

2. Use Quadratic-Exponential method and exact representation method by Andersen, Leif. (2007) to simulate the volatility and price path. These methods are discretization schemes that perform better than Euler scheme on S and v process.

$$dS_{gold}(t) = \mu S dt + \sqrt{v(t)} S dW_{gold,1}(t)$$

$$dS_{copper}(t) = \mu S dt + \sqrt{v(t)} S dW_{copper,1}(t)$$

$$dv_{gold}(t) = \kappa(\theta - v(t))dt + \sigma\sqrt{v(t)}dW_{gold,2}(t)$$

$$dv_{copper}(t) = \kappa(\theta - v(t))dt + \sigma\sqrt{v(t)}dW_{copper,2}(t)$$

$$\rho_{gold} = Corr(W_{gold,1}(t), W_{gold,2}(t))$$

$$\rho_{copper} = Corr(W_{copper,1}(t), W_{copper,2}(t))$$

During the simulation, we also used historical data to construct the correlation relationship between gold and copper future prices, by setting

$$\rho_{historical} = Corr(W_{gold,1}(t), W_{copper,1}(t))$$

3. Based on the price paths generated above, we are now able to calculate the price of the product:

$$S(t) = \min(S_{gold}(t), S_{copper}(t))$$

$$price = coupon/12 * \sum_{i=1}^{18} I(S(\tau_{obs,i}) \geq 1.05S_0)$$

$$- principal * I(S(\tau_{obs,18}) < 0.95S_0) * \max\left(\frac{S(\tau_{obs,18})}{S_0} - 1, -0.10\right)$$

Where τ_{obs} refer to the 18 observation dates of the plan, and coupon refers to the annualized coupon rate of return of the plan.

4. By setting the *price* to zero and solve the coupon, we can get the “fair” coupon for the plan. In our case the annual fair coupon is 13.08%, which is 1.09% monthly.

2.4 Model Verification

In order to show the validity of the model, we will take a look at the parameters we get from the calibration of the model, as well as the paths we used to calculate the prices.

1. Model Parameters

Name	Bounds	Gold	Copper
σ_0^2	[0.01, 0.5]	0.010608	0.061359
κ	[0.30, 5.0]	3.591278	3.000034
θ	[0.01, 0.5]	0.102742	0.047566
σ_{vol}	[0.05, 1.0]	0.859042	0.300067
ρ	[-1.0, 1.0]	0.366126	-0.09997

We can see from the table that all parameters are within the bounds, and none of them is close to the boundaries, meaning no extreme value.

2.4 Model Verification

1. Model Parameters

Gold Options

Type	Strike	Volume	Market	heston	Diff %
call	2350	460	42	49.75713	0.184694
call	2360	27	41.4	44.4942	0.074739
call	2365	28	38.2	41.53243	0.087237
call	2375	28	34	37.06989	0.090291
put	2355	143	38.1	36.4104	-0.04435
put	2360	161	40.7	38.89654	-0.04431
put	2365	31	43.5	41.49292	-0.04614
put	2390	52	58.5	56.04437	-0.04198

Copper Options

Type	Strike	Volume	Market	heston	Diff %
call	4.7	43	0.1305	0.134417	0.003917
call	4.72	12	0.119	0.12237	0.00337
call	4.76	23	0.0985	0.100277	0.001777
call	4.77	31	0.094	0.095175	0.001175
call	4.79	12	0.0855	0.085477	-2.3E-05
put	4.7	140	0.0645	0.063591	-0.00091
put	4.74	57	0.0825	0.08012	-0.00238
put	4.75	140	0.0875	0.08467	-0.00283

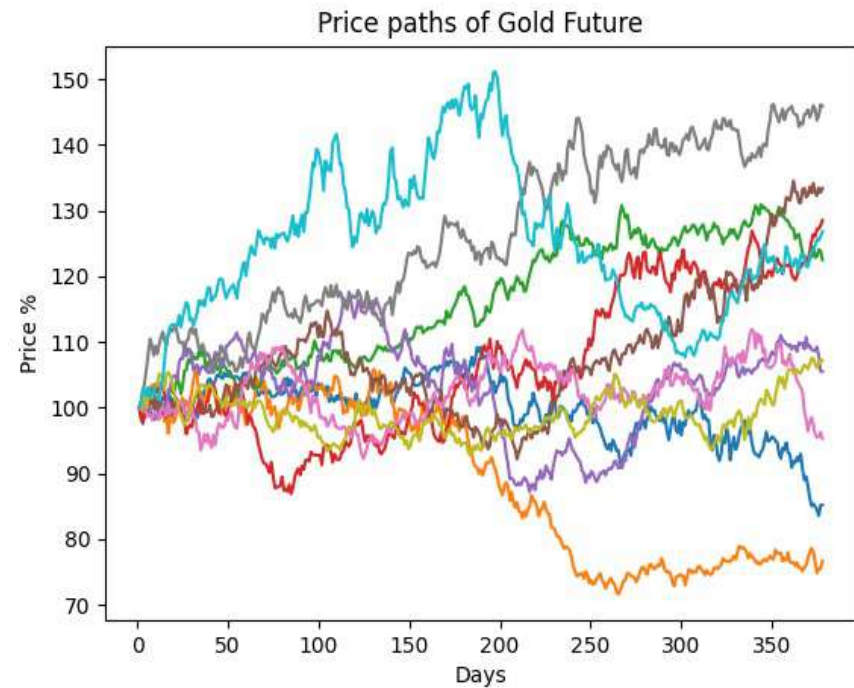
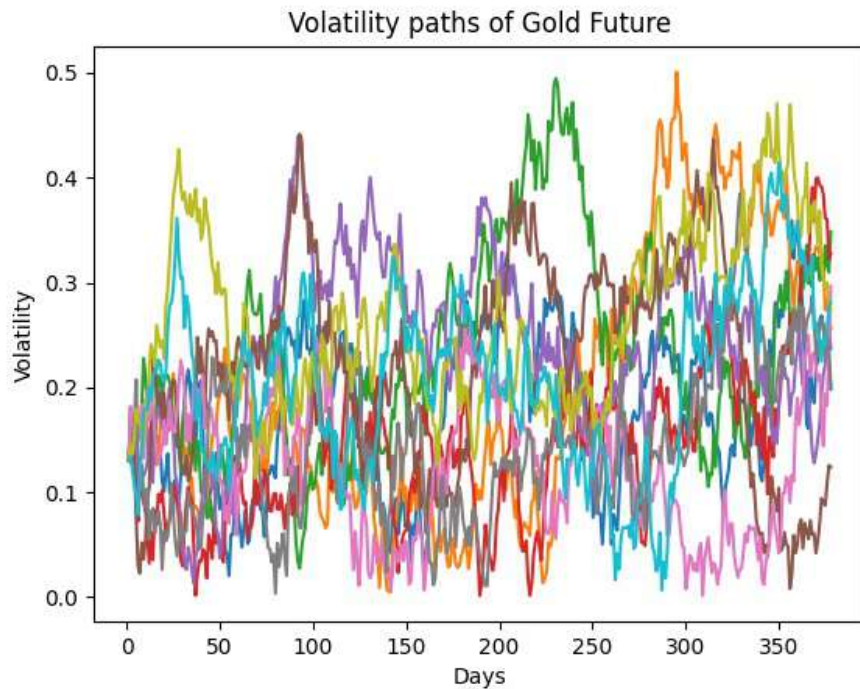
We used the pricing model to calculate the price of the vanilla option for gold and copper future and compare the results to the market price. We choose some near the money.

From the tables above, we can find that copper option pricing model performs well, with pricing error less than 0.5%. However, gold option pricing model shows bigger error. The error may become smaller if we can have the access to more option price data, by considering the volatility surface.

2.4 Model Verification

2. Generated Paths

We randomly selected 10 paths to illustrate our simulation



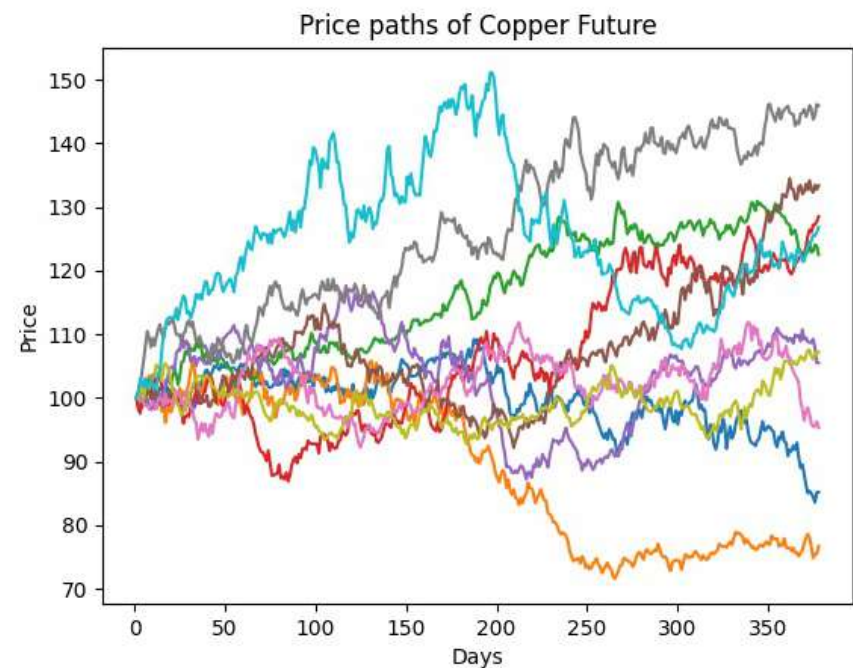
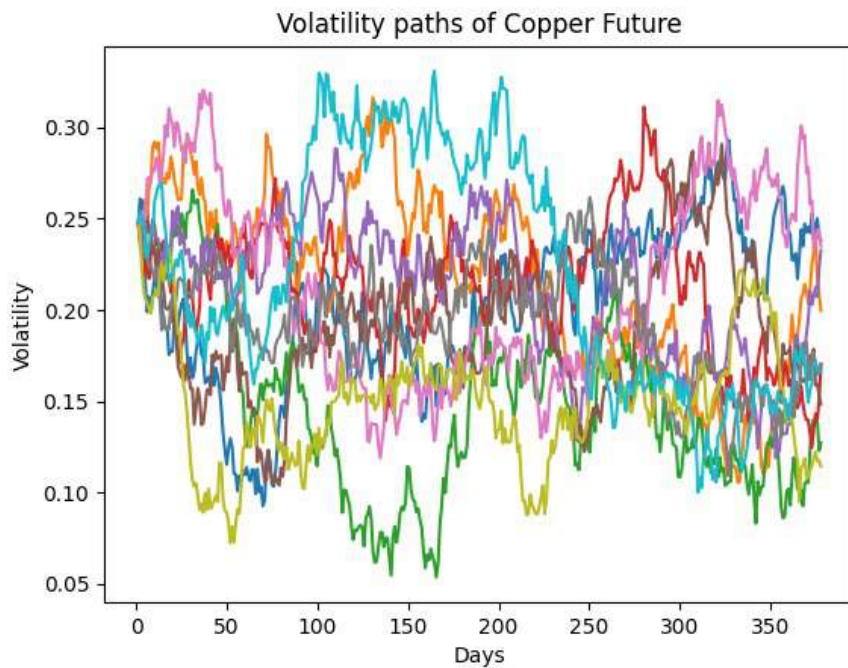
The volatility moves around initial volatility, which is in line with the model assumption

There are different levels of fluctuations in price paths, which reflects the randomness of volatility.

2.4 Model Verification

2. Generated Paths

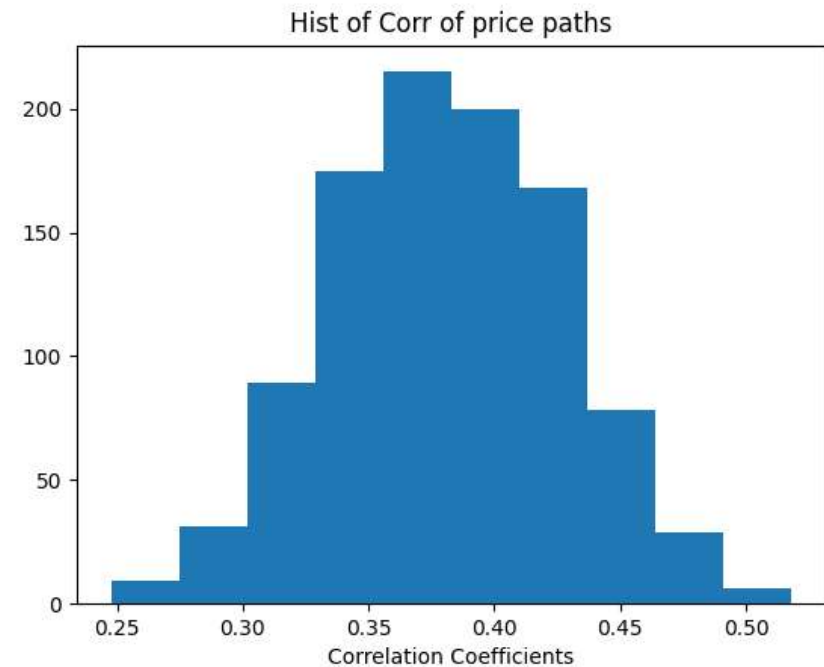
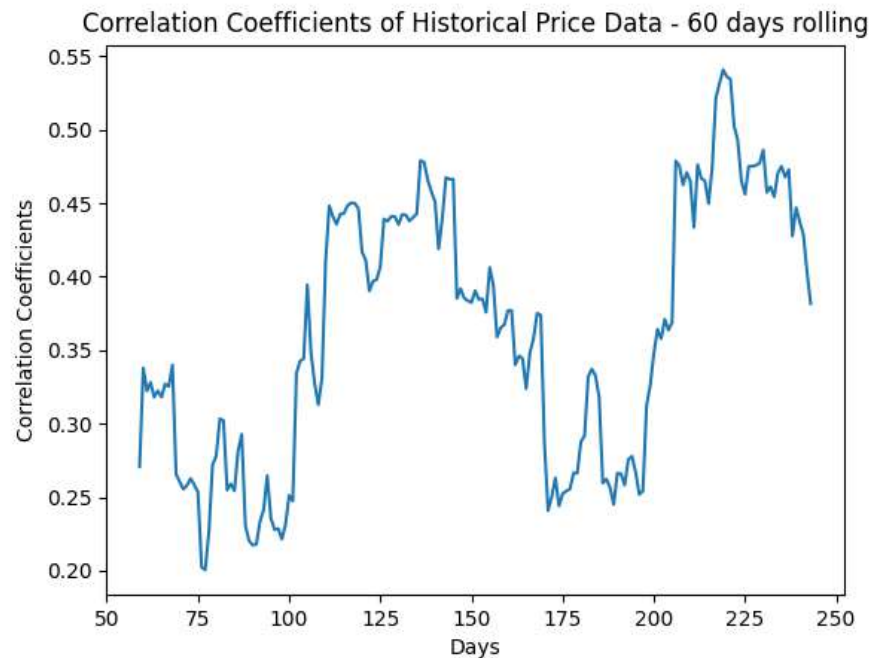
We randomly selected 10 paths to illustrate our simulation



For the paths of copper future prices and volatilities, the graphs show similar results.

2.4 Model Verification

3. Correlation Coefficients



We calculated the correlation coefficients of gold prices and copper prices based on historical data and generated paths. Left graph shows the correlation fluctuates around 0.35, which is in line with the peak around 0.35 in the right graph

Also, in the simulation process, we used 1 year correlation in historical price data in generating price paths. The correlation coefficients is 0.3475, which is also in line with our simulation results.

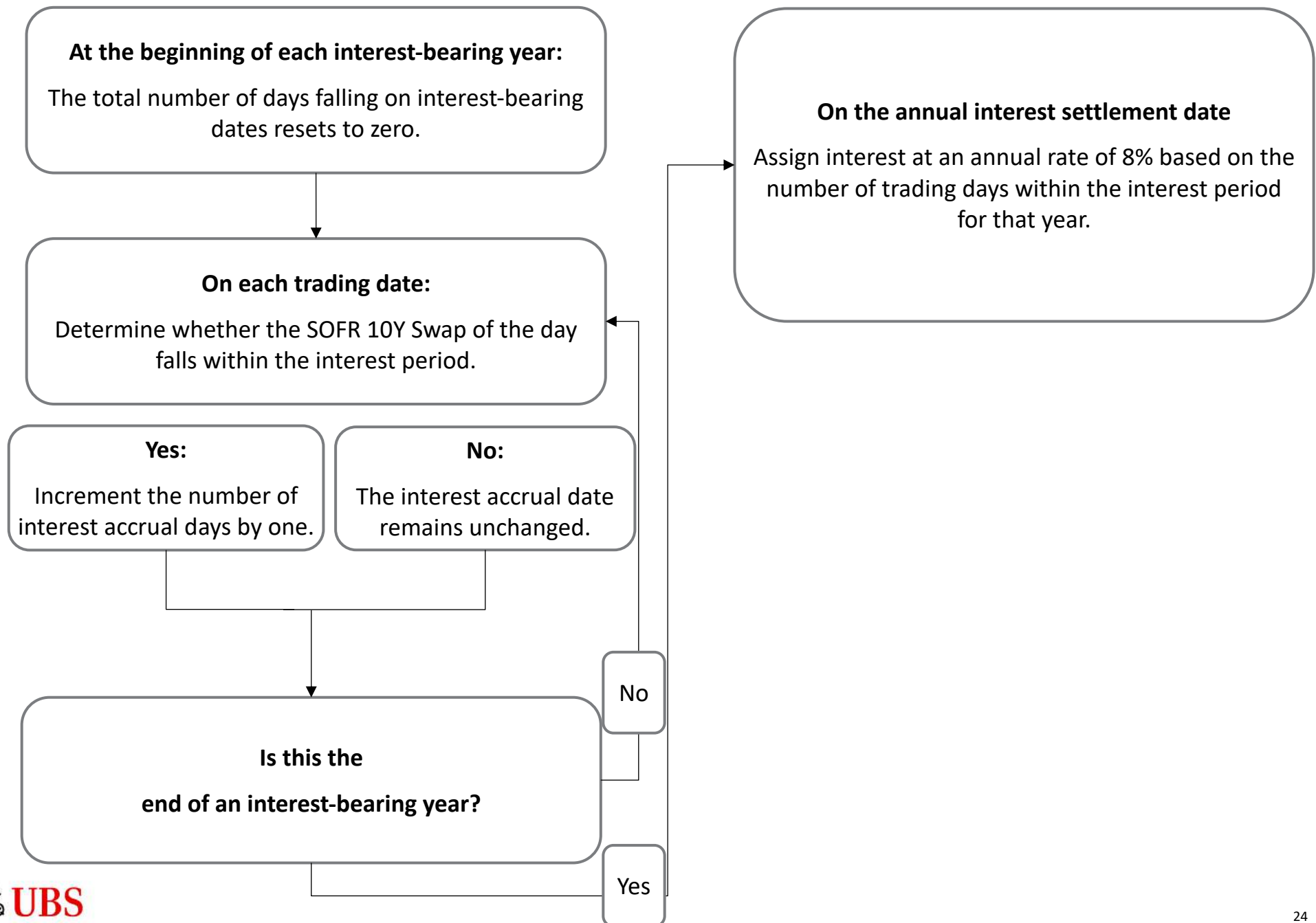
3.1 Investment Proposal for Client C

Investment Preference	Goal and Risk Tolerance
<p>Investment Amount:</p> <ul style="list-style-type: none">• \$20 Million USD <p>Investment Horizon:</p> <ul style="list-style-type: none">• Long-term (10+ years) <p>Investment Preferences:</p> <ul style="list-style-type: none">• Neutral on USD interest rates• Believes USD SOFR CMS 10Y will fluctuate within a certain range	<p>Investment Goals:</p> <ul style="list-style-type: none">• Achieve relatively high yields• Principal protection required <p>Risk Tolerance:</p> <ul style="list-style-type: none">• Desires principal protection• Willing to seek high yields within the range of interest rate fluctuations

3.2 Product Design

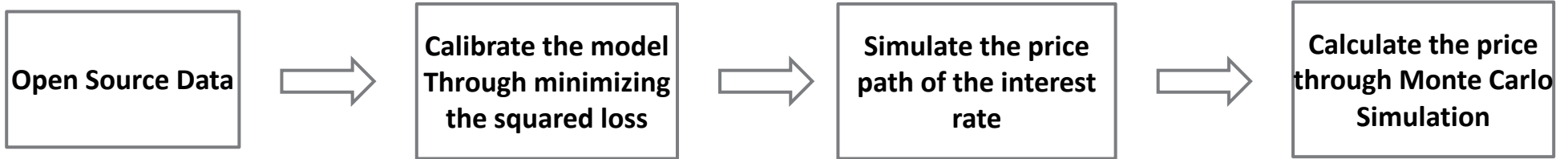
Product Type	Range Accrual Linked to SOFR CMS 10Y
Underlying	SOFR CMS 10Y
Investment Term	10 years
Potential Income	Interest generated when the linked rate (SOFR CMS 10Y) is within the accrual range and paid yearly.
Potential Loss	No interest will be generated when the linked rate (SOFR CMS 10Y) is outside the accrual range.
Start Date	The first trading day of the year.
Interest Rate	Assign interest at an annual rate of 8% based on the number of trading days within the interest period for that year.
Range Upper Bound	4.5%
Range Lower Bound	2.5%
Capital Protected Level	100% of the start level

3.2 Product Design – Flow Chart



3.3 Pricing Model

We Calibrated a Heston Stochastic Volatility Model to price the product



We searched for open data to help building the model:

- The trading data of the daily fixing leg of 10Y interest rate swap linked to SOFR, along with its forward rate data for calibration. From Bloomberg.

- Initialize Parameters: Select initial guesses for the model parameters such as mean reversion rate, volatility, and the time-dependent mean.
- Define Loss Function: Set up the objective function as the sum of squared differences between the model's outputs and observed market prices.
- Optimize Parameters: Use an optimization algorithm to minimize the least squares loss, iteratively adjusting the parameters until the model fits the market data.

Based on the parameters we got from model calibration, we run a simulation to get the price path of the interest rate:

- Generate Rate Paths: Use the Hull-White model's stochastic differential equation to simulate multiple potential future paths of the interest rate, incorporating random fluctuations.

- Accumulate Interest: Generate multiple interest rate paths via Monte Carlo simulation and calculate the total accrued interest for each path over time.
- Discount and Average: Discount the total accrued interest for each path back to the present day and average these values to determine the product's current price.

Key assumptions:

- Mean-reverting process: The short-term interest rate reverts to a time-dependent mean.
- Normal distribution of changes: Interest rate changes are normally distributed, following a continuous process.
- No arbitrage: The model assumes no arbitrage opportunities in the financial markets.

3.3 Pricing Model

Here are key steps we used to calculate the price of the product:

1. Use market data to calibrate Hull-White model parameters. Hull-White model is a stochastic differential equation (SDE) that describes the dynamics of the interest rate $r(t)$:

$$dr(t) = [\theta(t) - ar(t)] dt + \sigma dW(t)$$

2. The mean reversion rate is often calibrated by fitting the model to the historical interest rate data. One common approach is to use the method of moments or maximum likelihood estimation (MLE).

$$a = \frac{\sum_{i=1}^N (r(t_i) - \bar{r})(r(t_{i-1}) - \bar{r})}{\sum_{i=1}^N (r(t_{i-1}) - \bar{r})^2}$$

3. The volatility can be estimated from the residuals of the interest rate changes:

$$\sigma = \sqrt{\frac{1}{N-1} \sum_{i=1}^N \left(\frac{r(t_i) - r(t_{i-1})}{\Delta t} - ar(t_{i-1}) \right)^2}$$

4. The time-dependent drift term is calibrated to fit the initial term structure of interest rates. It can be derived from the market prices of zero-coupon bonds (where f is the forward rate):

$$\theta(t) = \frac{\partial f(t, T)}{\partial t} + af(t, T) + \frac{\sigma^2}{2a}(1 - e^{-2a(T-t)})$$

3.4 Model Verification

In order to show the validity of the model, we will take a look at the parameters we get from the calibration of the model, as well as the paths we used to calculate the prices.

1. Model Parameters

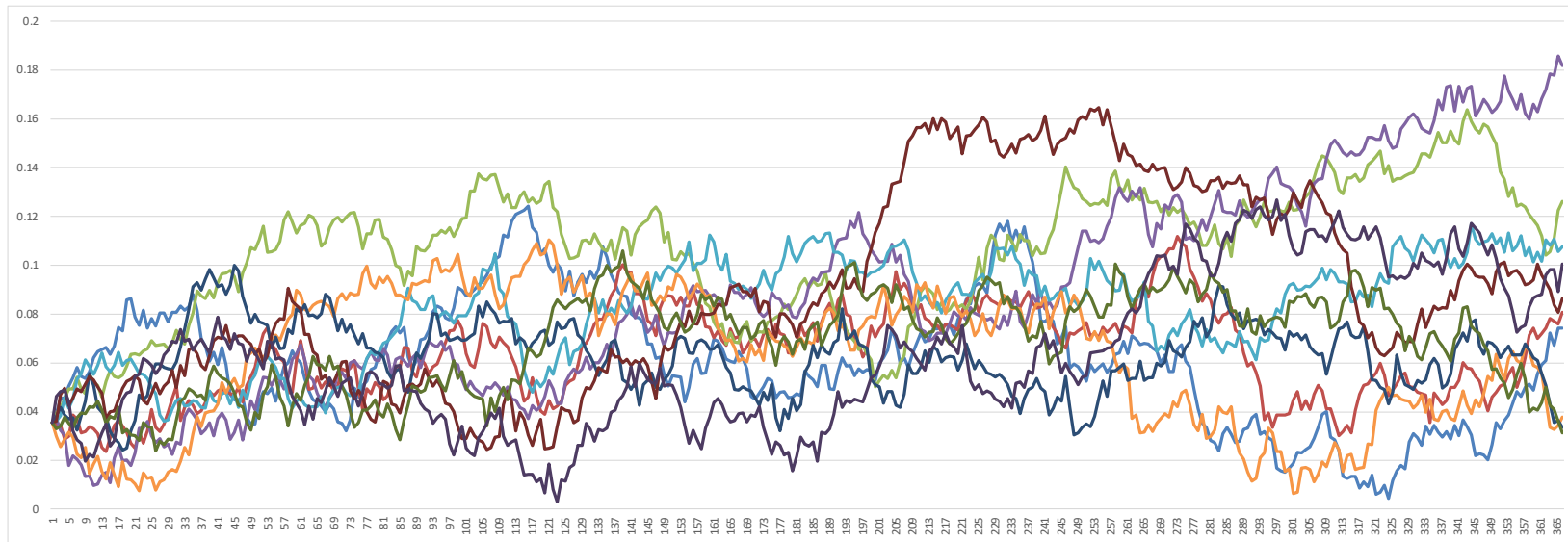
Name	Bounds	Calibration
α	[0.01, 0.2]	0.132542
σ	[0.01, 0.3]	0.029412

We can see from the table that all parameters are within the bounds, and none of them is close to the boundaries, meaning no extreme value.

3 Model Verification

2. Generated Paths

We randomly selected 10 paths to illustrate our simulation



The future path of interest rates remains at an appropriate level, the number of days within the interest calculation period is suitable, and it conforms to the volatility and mean-reversion properties preset by the model.