MILP-based approaches for medium-term planning of single-stage continuous multiproduct plants with parallel units

Article *in* Computational Management Science · October 2010

DOI: 10.1007/s10287-009-0096-5

3 authors, including:

Songsong Liu
Harbin Institute of Technology

Jose Pinto
Linde plc

Comput Manag Sci (2010) 7:407–435
DOI 10.1007/s10287-009-0096-5

ORIGINAL PAPER

MILP-based approaches for medium-term planning of single-stage continuous multiproduct plants with parallel units

Songsong Liu · Jose M. Pinto · Lazaros G. Papageorgiou

Received: 7 February 2009 / Accepted: 11 May 2009 / Published online: 10 June 2009

Abstract In this paper, we address the problem of medium-term planning of single-stage continuous multiproduct plants with multiple processing units in parallel. Sequence-dependent changeover times and costs occur when switching from one type of product to another. A traveling salesman problem (TSP)-based mixed-integer linear programming (MILP) model is proposed based on a hybrid discrete/continuous time representation. We develop additional constraints and variables to ensure that subtours do not occur in the solution. The model is successfully applied to an example of a polymer processing plant to illustrate its applicability. In order to solve larger model instances and planning horizons, a rolling horizon approach is developed to reduce the computational expense. Finally, the proposed model is compared to a recently published approach through literature examples, and the results show that the computational performance of the proposed model is superior.

Keywords Scheduling · Mixed-integer linear programming · Hybrid time representation · Traveling salesman problem · Parallel machines · Rolling horizon

S. Liu · L. G. Papageorgiou (✉)
Centre for Process Systems Engineering, Department of Chemical Engineering,
University College London, Torrington Place, London WC1E 7JE, UK
e-mail: l.papageorgiou@ucl.ac.uk

J. M. Pinto
Othmer-Jacobs Department of Chemical and Biological Engineering, Polytechnic University,
6 MetroTech Center, Brooklyn, NY 11201, USA

Present Address:
J. M. Pinto
Praxair Inc., 39 Old Ridgebury Road, Danbury, CT 06810, USA

1 Introduction

Production planning and scheduling improve the performance of multiproduct facilities by tackling rapid-changing demands and various production constraints, so a lot of interest has been shown in the chemical process industry. In the literature, most of the research in planning and scheduling has focused on the area of batch/discrete processes (for example, Pinto and Grossmann 1995; Papageorgiou and Pantelides 1996; Cerdá et al. 1997; Zhu and Majozi 2001; Chen et al. 2002; Leksakul and Techanitisawad 2005; Castro and Grossmann 2006; Pardalos and Shylo 2006; Erdirik-Dogan and Grossmann 2007; Castro et al. 2008). Recent reviews on the planning and scheduling of batch processes can be seen in Burkard and Hatzl (2005) and Méndez et al. (2006). On the other hand, continuous processes are not discussed as much as batch processes, although continuous processes play an important role in the chemical process industry.

Sahinidis and Grossmann (1991) developed a large-scale mixed-integer nonlinear programming (MINLP) model for the problem of cyclic multiproduct production scheduling on continuous parallel lines. A solution method based on generalised Benders decomposition was developed. Kondili et al. (1993) addressed the problem of short-term scheduling of multiproduct energy-intensive continuous plants to minimise the total cost of energy and changeovers, while satisfying customer orders within given deadlines. A mixed-integer linear programming (MILP) model was proposed considering changeover costs and delays when switching a mill from one type of cement to another. Pinto and Grossmann (1994) extended the work of Sahinidis and Grossmann (1991) addressing the problem of optimising cyclic schedules of multiproduct continuous plants with several stages interconnected by intermediate inventory tanks. The proposed large-scale MINLP model was able to handle intermediate storage as well as sequence-dependent changeovers.

Karimi and McDonald (1997) presented two MILP formulations for the detailed short-term scheduling of a single-stage multiproduct facility with multiple parallel semicontinuous processors, based on a continuous time representation to minimise inventory, transition, and shortage costs. Ierapetritou and Floudas (1998) presented a continuous-time MILP formulation based on the state-task network (STN) representation for short-term scheduling for multistage continuous processes, as well as mixed production facilities involving batch and continuous processes. The formulation was proven capable of handling limited storage and cleanup requirements. Mockus and Reklaitis (1999) considered a general MINLP formulation for planning the operation of multiproduct/multipurpose batch and continuous plants with a goal of maximisation of profit, using the STN representation. Lee et al. (2002) addressed scheduling problems in single-stage and continuous multiproduct processes on parallel lines with intermediate due dates and especially restrictions on minimum run lengths. The proposed MILP formulation significantly reduced the model size and computation time compared with previous approaches (Karimi and McDonald 1997; Ierapetritou and Floudas 1998).

Alle and Pinto (2002) proposed an MINLP model for the simultaneous scheduling and optimisation of the operating conditions of continuous multistage multiproduct plants with intermediate storage, which was based on the Traveling Salesman Problem (TSP) formulation. The proposed formulation showed to be faster and able to solve larger problems than the model proposed by Pinto and Grossmann (1994). Also, a linearisation approach was presented to discretise nonlinear variables and compared to the direct solution of the original MINLP, with the results showing that nonlinear restrictions were more effective than linear discrete ones. Alle et al. (2004) extended the models in the work of Pinto and Grossmann (1994) and Alle and Pinto (2002), and proposed an MINLP model for cyclic scheduling of cleaning and production operations in multiproduct multistage plants with performance decay, based on a continuous time representation.

Méndez and Cerdá (2002a) developed an MILP continuous-time short-term scheduling formulation considering sequence-dependent setup times and specific due dates for export orders in a make-and-pack continuous production plant to meet all end-product demands with minimum make-span. In their another work (Méndez and Cerdá 2002b), an MILP mathematical formulation for the short-term scheduling of resource-constrained multiproduct plants with continuous processes is presented, based on a continuous time representation that accounts for sequence-dependent changeover times and storage limitations. The objective is to maximise the revenue from production sales while satisfying specified minimum product requirements. Munawar et al. (2003) considered the cyclic scheduling of continuous multistage multiproduct plants operating in a hybrid flowshop, in which the operation in the plant is a combination of sequential and parallel modes. A generalised simultaneous scheduling and operational optimisation MINLP model for such plants was developed, accounting for sequence- and equipment-dependent transition times.

Shaik and Floudas (2007) presented an MILP model for short-term scheduling of continuous processes using unit-specific event-based continuous-time representation based on the STN representation. The model accounted for various storage requirements such as dedicated, finite, unlimited, and no intermediate storage policies, and allows for unit-dependent variable processing rates, sequence-dependent changeovers, and the option of bypassing storage. Castro and Novais (2007) used a new multiple time grid MINLP formulation based on the resource-task network (RTN) process representation for the periodic scheduling of multistage, multiproduct continuous plants with parallel equipment units that were subject to sequence-dependent changeovers.

Erdirik-Dogan and Grossmann (2006) proposed a bi-level decomposition procedure that allows the optimisation and integration of the planning and scheduling of single-stage single-unit continuous multiproduct plants producing several products that were subject to sequence-dependent changeovers. The decomposition approach reduced the computational cost of the problem by decomposing the original detailed MILP model into an upper level planning problem and a lower level scheduling problem. Chen et al. (2008) proposed a slot-based MILP model for medium-term planning of single-stage single-unit continuous mulitproduct plants based on a hybrid discrete/continuous time representation, in which the total planning horizon was divided into several discrete weeks, and each week was formulated with a continuous time representation. The computational results showed that the proposed model is superior to the model in Erdirik-Dogan and Grossmann (2006). Liu et al. (2008) further improved the work of Chen et al. (2008). They presented an MILP formulation based on the classic TSP formulation using a hybrid discrete/continuous time representation. The proposed model without time slots saved a lot of the computational effort for large problems, compared with literature models.

Erdirik-Dogan and Grossmann (2008) extended their own work from single-unit to parallel units. A detailed slot-based MILP was proposed that accounts for sequence-dependent transition times and costs. An upper-level MILP model was based on a relaxation of the original model to generate a bi-level decomposition scheme to overcome the computational expense for large problems with long time horizons.

In this work, we extend our previous work (Liu et al. 2008) for medium-term planning of a single-stage single-unit multiproduct continuous plant to the case with parallel units. The goal of the work is to develop a compact and efficient MILP formulation for the medium-term planning of single-stage multiproduct continuous plants with parallel processors that are subject to dependent-sequence changeovers based on a classic TSP formulation, using a hybrid discrete/continuous time representation. In the model, a novel set of subtour elimination constraints considering the order of each product in the production sequence is proposed.

The paper is organised as follows. The problem statement is presented in Sect. 2. Then, we introduce the mathematical formulation of the proposed model in Sect. 3. Section 4 describes an illustrative example. A rolling horizon (RH) approach is proposed in Sect. 5. While computational results of the proposed approaches and one recently published approach (Erdirik-Dogan and Grossmann 2008) are presented in Sect. 6. Finally, some concluding remarks are drawn in Sect. 7.

2 Problem statement

In the problem we discuss in this article, a plant with single-stage multiproduct continuous machines that operate in parallel is given (See the example in Fig. 1). Also, given is the total planning horizon, which lasts from several weeks to several months. There are weekly demands of products from customers which are allowed to be delivered only at the end of each week. If the demand is not fulfilled at the desired time, late delivery is allowed. At the same time, backlog penalties are imposed on the plant operation. The limited inventory is also allowed for product storage before sales. Unit backlog penalty and inventory cost are also given, as well as selling prices, and processing rates of each product. Sequence-dependent changeover times and costs occur when switching production between different products.

Due to the weekly demands in the problem, we use a hybrid discrete/continuous time representation, in which the total planning horizon is divided into discrete weeks,

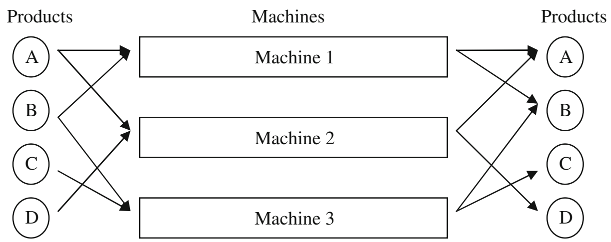

Fig. 1 A multiproduct continuous plant with parallel machines

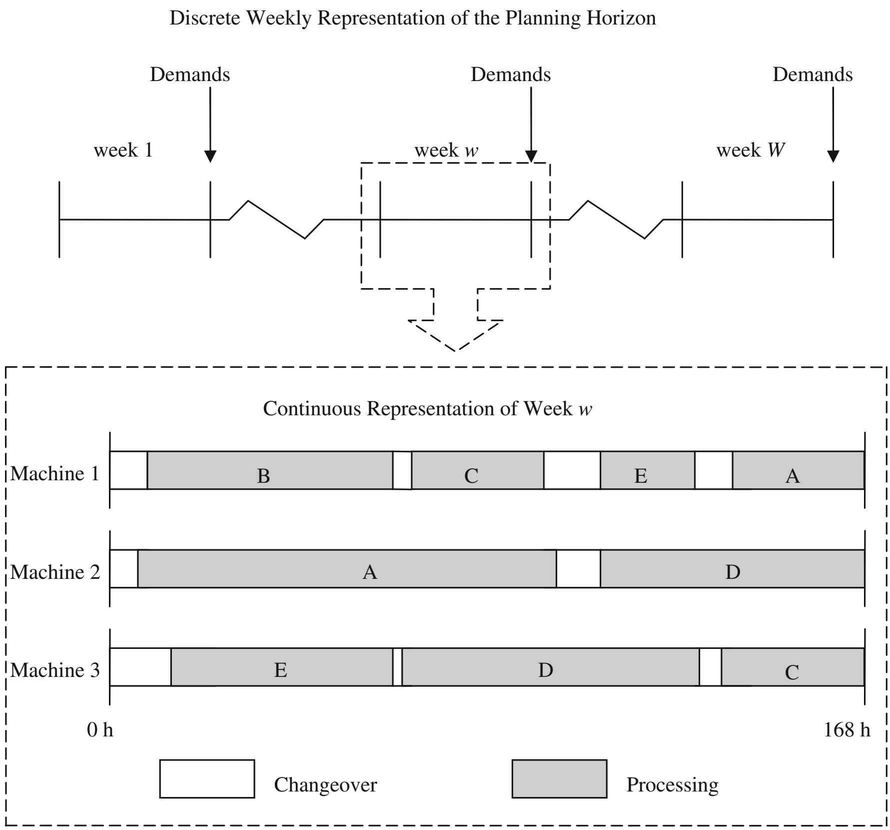

Fig. 2 Hybrid discrete/continuous time representation

and each week is formulated using a continuous time representation (see the example in Fig. 2).

Given the above definitions, the problem is to determine the products to be manufactured during each unit and within each week, production sequence and duration, production amounts, and inventory and backlog levels on each unit and in each week. The objective is to maximise the total profit, involving sales revenue, product inventory, changeover and backlog costs.

3 Mathematical formulation

The proposed TSP-based MILP model for the medium-term planning of single-stage multiproduct continuous plants with parallel units is described in this section. The proposed model is an extension of the single-unit model in Liu et al. (2008). Similarly, here the planning problem can be taken as a TSP problem. In the classic TSP problem, a salesman is required to visit a number of cities in a sequence that minimises the overall costs or time, and in the classic TSP formulation binary variables are used to represent the transition from one city to another (Kallrath and Wilson 1997). Moreover, in a multiproduct plant, a number of products must be produced in a sequence that optimises a performance metric (e.g. profit). So, similar to the binary variables in classic TSP formulation, binary variables Z_{ijmw} and ZF_{ijmw} are introduced to model the changeover from product i to product j in week w and between two consecutive weeks $w-1$ and w, respectively, on unit m.

In addition, product ordering variables O_{imw} are introduced together with additional mathematical constraints in the formulation to eliminate product sequence with subtours in the optimal solution. These constraints consider the order of each product in the production sequence. Subtour elimination constraints have been used in the classic TSP formulation, but are uncommon to scheduling models in process system engineering.

The planning problem is formulated as a MILP optimisation model with the following notation:

Indices

c	Customers
i, j	Products
i^*	Pseudo product
m	Units
w	Weeks

Sets

C	Set of customers
I	Set of products
I_m	Set of products that can be processed in unit m, including pseudo product
$\bar{I}_m$	Set of real products that can be processed in unit m, excluding pseudo product
M	Set of units
M_i	Set of units that can process product i
W	Set of weeks

Parameters

CB_{ic}	Unit backlog penalty cost of product i to customer c
CC_{ijm}	Changeover cost from product i to product j in unit m
CI_i	Unit inventory cost of product i
D_{ciw}	Demand of product i from customer c in week w
N	A large number
PS_{ic}	Unit selling price of product i to customer c
r_{im}	Processing rate of product i in unit m
$V_i^{\max}$	Maximum storage of product i
$V_i^{\min}$	Minimum storage of product i
θ^L	Lower bound for processing time in a week
θ^U	Upper bound for processing time in a week
τ_{ijm}	Changeover time from product i to product j in unit m

Binary variables

E_{imw} 1 if product i is processed in unit m in week w; 0 otherwise
L_{imw} 1 if product i is the last one in unit m in week w; 0 otherwise
F_{imw} 1 if product i is the first one in unit m in week w; 0 otherwise
Z_{ijmw} 1 if product i immediately precedes product j in unit m in week w; 0 otherwise
ZF_{ijmw} 1 if there is a changeover from product i in week $w-1$ to product j in week w in unit m; 0 otherwise

Variables

O_{imw} Order index of product i in unit m in week w
P_{imw} Amount of product i produced in unit m in week w
S_{ciw} Sales volume of product i to customer c in week w
T_{imw} Processing time of product i in unit m in week w
V_{iw} Inventory volume of product i at the end of week w
Δ_{ciw} Backlog of product i to customer c at the end of week w
Π Total profit

3.1 Objective function

The profit of the plant is equal to the sales revenue minus operating costs involving changeover, backlog, and inventory costs. The inventory cost in each week is calculated from the inventory level at the end of each week, multiplied by the unit inventory cost for each product.

$$\begin{aligned}\Pi = &\sum_{c\in C}\sum_{i\in I}\sum_{w\in W} PS_{ic}\cdot S_{ciw} - \sum_{m\in M}\sum_{i\in I_m}\sum_{j\in I_m}\sum_{w\in W} CC_{ijm}\cdot Z_{ijmw}\\ &-\sum_{m\in M}\sum_{i\in I_m}\sum_{j\in I_m}\sum_{w\in W-\{1\}} CC_{ijm}\cdot ZF_{ijmw}\\ &-\sum_{c\in C}\sum_{i\in I}\sum_{w\in W} CB_{ic}\cdot\Delta_{ciw} - \sum_{i\in I}\sum_{w\in W} CI_i\cdot V_{iw}\end{aligned} \tag{1}$$

3.2 Assignment constraints

Assuming that each week comprises the processing of at least one product is assigned to each unit, the first and last products to be processed during each week are assigned:

$$\sum_{i\in I_m} F_{imw} = 1, \quad \forall m\in M, w\in W \tag{2}$$

$$\sum_{i\in I_m} L_{imw} = 1, \quad \forall m\in M, w\in W \tag{3}$$

The above one product per unit assumption can always be valid by introducing a pseudo product i^*, whose changeover times and costs are 0, and fixing Z_{i^*jmw} and Z_{ji^*mw} to 0, for every j, m and w.

A product cannot be assigned as the first or last one in a unit in a week, if the product is not processed in the same week, i.e., if $E_{imw} = 0$, then F_{imw} and L_{imw} should be forced to be 0:

$$F_{imw} \leq E_{imw}, \quad \forall m \in M, i \in I_m, w \in W \tag{4}$$

$$L_{imw} \leq E_{imw}, \quad \forall m \in M, i \in I_m, w \in W \tag{5}$$

3.3 Changeover constraints

For changeovers within a week, if a product is the first one processed in one unit and in a week, then no product is processed precedent to this product in the unit and in the week. Also, if a product is to be processed, but is not the first one, then there is exactly one product precedent to this product in the unit and in the week:

$$\sum_{\substack{i \in I_m \\ i \neq j}} Z_{ijmw} = E_{jmw} - F_{jmw}, \quad \forall m \in M, j \in I_m, w \in W \tag{6}$$

If a product is the last one processed in one unit and in a week, then no product is processed following this product on the unit and in the same week. Also, if a product is to be processed, but is not the last one, then there is exactly one product following this product in the unit and in the week:

$$\sum_{\substack{j \in I_m \\ j \neq i}} Z_{ijmw} = E_{imw} - L_{imw}, \quad \forall m \in M, i \in I_m, w \in W \tag{7}$$

Note that from constraints (6) and (7), there is no changeover from or to a product that is not processed. Figure 3 shows an example of changeover with two products A and B within week w in unit m.

For changeovers between two consecutive weeks, if product j is the first one to be processed in one unit and in week w, there is exactly one changeover from a product at week $w-1$ to product j in the unit. Also, if product i is the last one to be processed in

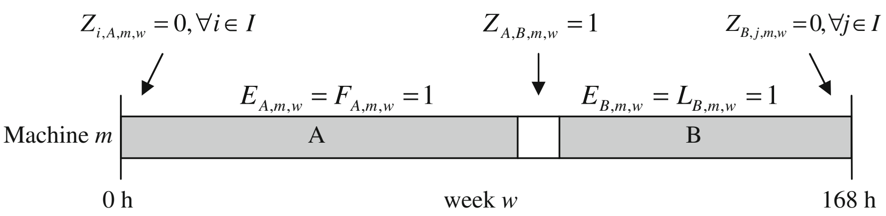

Fig. 3 Assignments and changeovers within 1 week

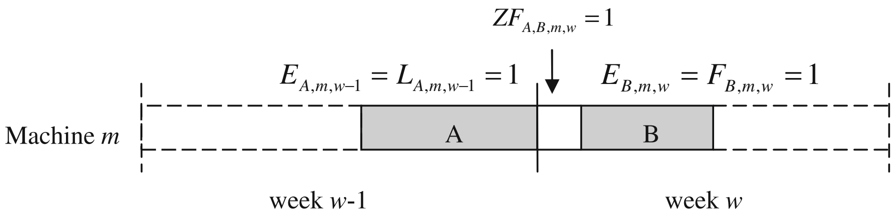

Fig. 4 Assignments and changeovers between 2 weeks

week $w-1$ in one unit, there is exactly one changeover to a product at the beginning of week w in the unit. If a product is not the first or the last one processed in any unit, then there is no changeover involving the products between two weeks in any unit.

$$\sum_{i \in I_m} ZF_{ijmw} = F_{jmw}, \quad \forall m \in M, j \in I_m, w \in W - \{1\} \tag{8}$$

$$\sum_{j \in I_m} ZF_{ijmw} = L_{i,m,w-1}, \quad \forall m \in M, i \in I_m, w \in W - \{1\} \tag{9}$$

Here, we assume the changeover between week $w-1$ and w in each unit occurs at the beginning of week w. Figure 4 is an example of changeover from product A to B between weeks $w-1$ and w in unit m.

It should be noted that the last product processed in week $w-1$ may be the same product as the one processed first in week w in unit m. In such cases, the production process of the product continuously proceeds from week $w-1$ to week w, so no changeover times and costs occur.

It should be added that variables ZF_{ijmw} are treated as continuous, $0 \leq ZF_{ijmw} \leq 1$, as the relevant changeover terms are minimised in the objective function.

3.4 Subtour elimination constraints

The previously described constraints have the potential drawback of generating solutions with subcycles. When a subcycle is present, the solution of the model is an infeasible schedule (See Fig. 5b). So, subtour elimination constraints are needed to generate feasible schedules (See Fig. 5a).

In order to avoid subtours, positive integer variables O_{imw} are introduced to define the order in which each product is processed in a week in the same unit. The later a product is processed, the greater its order index is, as shown in Fig. 6.

Here, we assume that if product i is processed precedent to product j in unit m in week w, the order index of product j is at least one higher than that of product i:

$$O_{jmw} - (O_{imw} + 1) \geq -N \cdot (1 - Z_{ijmw}),$$
$$\forall m \in M, i \in \bar{I}_m, j \in \bar{I}_m, j \neq i, w \in W \tag{10}$$

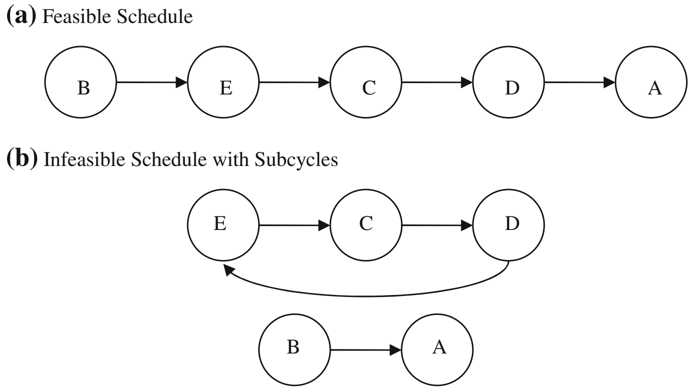

Fig. 5 Feasible schedule and infeasible schedule with subcycle

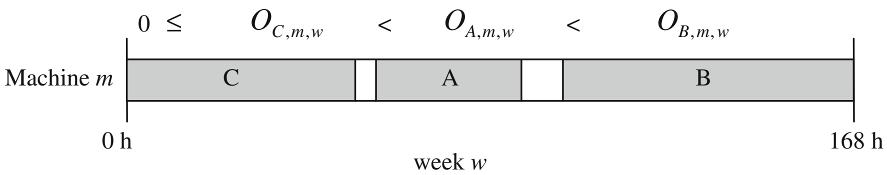

Fig. 6 Order indices within 1 week

Also, when a product is not processed in unit m, its order index is equal to zero:

$$O_{imw} \leq N \cdot E_{imw}, \quad \forall m \in M, i \in \bar{I}_m, w \in W \tag{11}$$

In constraints (10) and (11), N is a large number and is an upper bound of O_{imw}. We can also use $|I|$, the cardinality of set I, as the upper bound of O_{imw}.

Constraints (10) and (11) guarantee that subtours do not exist in any feasible optimal solution, similarly to subtour elimination constraints given in the classic TSP formulation (Kallrath and Wilson 1997). The effect of constraint (10) on subtour elimination is proved by the following theorem.

Theorem *Constraint (10) eliminates subtours in the feasible solutions.*

Proof Assume in a feasible solution, there is a cyclic sequence, consisting of k products $i_1, i_2, \ldots, i_k$, in unit m in week w, where $k \geq 2$.

So, we have

$$Z_{i_1 i_2 mw} = Z_{i_2 i_3 mw} = \cdots = Z_{i_{k-1} i_k mw} = Z_{i_k i_1 mw} = 1.$$

From constraint 11, we obtain

$$\begin{aligned} O_{i_2mw} - O_{i_1mw} &\geq 1, \\ O_{i_3mw} - O_{i_2mw} &\geq 1, \\ &\vdots \\ O_{i_kmw} - O_{i_{k-1}mw} &\geq 1, \\ O_{i_1mw} - O_{i_kmw} &\geq 1. \end{aligned}$$

By adding the above k constraints together, we get

$$O_{i_1mw} - O_{i_1mw} = 0 \geq k,$$

which is a contradiction. So, there are no subtours in the feasible solutions. □

We should notice that the order indices obtained from constraints (10) and (11) do not guarantee values of successive integers. If the latter is required, the following constraints should be included:

$$F_{imw} \leq O_{imw} \leq \sum_{j \in I_m} E_{jmw}, \quad \forall m \in M, i \in \bar{I}_m, w \in W. \tag{12}$$

Note the constraints (11) and (12) force the product order indices to take values between 1 and the total number of products selected for week w.

Alternatively to constraint (11) and (12), the following term can be subtracted by the objective function:

$$\varepsilon \cdot \sum_{m \in M} \sum_{i \in \bar{I}_m} \sum_{w \in W} O_{imw}$$

where ε is a small number.

3.5 Timing constraints

For each product processed in a week, its duration must be restricted between the lower and upper availability bounds (θ^L and θ^U, respectively):

$$\theta^L \cdot E_{imw} \leq T_{imw} \leq \theta^U \cdot E_{imw}, \quad \forall m \in M, i \in \bar{I}_m, w \in W \tag{13}$$

Also, the total processing and changeover time in a unit in a week should not exceed the total available time in each week:

$$\sum_{i \in \bar{I}_m} T_{imw} + \sum_{i \in \bar{I}_m} \sum_{j \in \bar{I}_m} (Z_{ijmw} + ZF_{ijmw}) \cdot \tau_{ijm} \leq \theta^U, \quad \forall m \in M, w \in W - \{1\} \tag{14}$$

$$\sum_{i \in \bar{I}_m} T_{imw} + \sum_{i \in \bar{I}_m} \sum_{j \in \bar{I}_m} Z_{ijmw} \cdot \tau_{ijm} \leq \theta^U, \quad \forall m \in M, w \in \{1\} \tag{15}$$

3.6 Production constraints

The product amount produced in one unit per week is simply given by:

$$P_{imw} = r_{im} \cdot T_{imw}, \quad \forall m \in M, i \in \bar{I}_m, w \in W \tag{16}$$

3.7 Backlog constraints

The backlog of a product to a customer in a week is defined as the backlog at the previous week plus the demand in this week, minus the sales volume to the customer:

$$\Delta_{ciw} = \Delta_{c,i,w-1} + D_{ciw} - S_{ciw}, \quad \forall c \in C, i \in I, w \in W \tag{17}$$

3.8 Inventory constraints

The inventory of a product in a week is defined as the inventory at the previous week plus the total amount produced in all units, minus the total sales volume of the product to all customers:

$$V_{iw} = V_{i,w-1} + \sum_{m \in M_i} P_{imw} - \sum_{c \in C} S_{ciw}, \quad \forall i \in I, w \in W \tag{18}$$

The amounts of products to be stored are limited by minimum and maximum capacities:

$$V_i^{\min} \le V_{iw} \le V_i^{\max}, \quad \forall i \in I, w \in W \tag{19}$$

The planning of single-stage multiproduct plants with parallel continuous processors is formulated as an MILP model that is described by constraints (2)–(11), (13)–(19) with Eq. (1) as the objective function.

4 Illustrative example

To illustrate the applicability of the proposed model, we consider an example of a plant, which is an extension of a real world polymer processing plant discussed in Chen et al. (2008). In this example, the single-stage plant manufactures ten types of products (A–J) on four machines (M1–M4). Each machine can process five of the ten products (see Table 1).

Weekly demands for each product (see Table 2) are ordered from ten customers (C1–C10) for a period of 24 weeks. The processing rate is 110 ton/week for each product in all machines.

The total available processing time in each week is 168 h. The changeover times (in minutes), shown in Table 3, are the same for all machines. The changeover costs

Table 1 Products assignment in each machine

Machines	Products A	B	C	D	E	F	G	H	I	J
M1	√	√	√	√	√					
M2			√	√	√	√	√			
M3					√	√	√	√	√	
M4						√	√	√	√	√

are proportional to the changeover times (in hours) by a factor of 10. For example, the changeover cost from product A to B is $45 \times 10 \div 60 = \$7.5$.

Table 4 shows the product prices for all customers, except for customer C10 whose price is 50% higher. The unit inventory and backlog costs are 10 and 20% of product prices, respectively.

Here, we consider four cases of the example, with planning horizons of 6, 12, 18 and 24 weeks, respectively. The models are implemented in GAMS 22.6 (Brooke et al. 2008) using solver CPLEX 11.0 (ILOG 2007) on a Pentium 4 3.40 GHz, 1.00 GB RAM machine. The optimality gap is set to 0%, and the computational time is limited to 3,600 s. The solution results are shown in Table 5.

We can see from the Table 5 that the proposed MILP model can find the global optimal solution within the specified time limit only for the 6-week case. For the other three cases with planning horizons of 12, 18 and 24 weeks, although the solutions obtained in the specified time limit are not global optimal, the model also provides very good feasible solutions. The gap between the profit given by the proposed MILP model and the global optimal one is within 1% for each case.

The detailed optimal schedule corresponding to the global optimal solution for the 6-week case is shown in Fig. 7. In the optimal schedule for the 6-week case, the pseudo product is not processed, which means that no unit is idle in any week. M4 is the only machine that processes all its assigned five products. Only four products are processed on M1, M2 and M3. Although product E is assigned to M1, M2 and M3, it is only processed on M2 and M3 in the optimal schedule. Also, only M4 processes product G, which is assigned to M2 and M3 as well. Furthermore, product F is processed on all three machines it is assigned to, M2, M3 and M4.

5 Rolling horizon

The proposed single-level MILP model was solved directly and obtained global optimal solutions for horizons of up to 6 weeks. However, because of the exponential growth in the computational effort when planning horizons and model sizes increase, we consider a RH algorithm, which can be used to reduce the computational effort.

5.1 Algorithm description

In the RH algorithm, the problem considered is divided into a set of subproblems with increasing lengths of planning horizon and solved iteratively. The planning horizon of

Table 2 Weekly demands by the customers (ton)

Customers	Products	Weekly demands											
		1	2	3	4	5	6	7	8	9	10	11	12
C1, C5	A	20				20				20			
	C	8	8	8	8	12	12	12	12	12	8	8	8
C2, C6	D	12		12		12		12		12		12	
	E	20		20		20		20		20		20	
	H		48				48		48				48
C3, C7, C9	B	16				16				16			
	G			20									
	J		24		24		24		24		24		24
C4, C8, C10	A	28				28				28			
	B		20		20			20		20			20
	C	20			20			20			20		
	D	40					40			40			
	E	44		44		44		44		44		44	
	F	32			32			32			32		
	G	16		16		16		16		16		16	
	H	4	4	4	12	12	12	4	4	4	12	12	12
	I	20	20	20	20	20	20	20	20	20	20	20	20
	J		12		12	12		12		12	12		12

Customers	Products	Weekly demands											
		13	14	15	16	17	18	19	20	21	22	23	24
C1, C5	A	20				20				20			
	C	8	12	12	12	12	12	8	8	8	8	12	12
C2, C6	D	12		12		12		12		12		12	
	E	20		20		20		20		20		20	
	H		48				48		48				48
C3, C7, C9	B	16				16				16			
	G	20										20	
	J		24		24		24		24		24		24
C4, C8, C10	A	28				28				28			
	B		20		20			20		20			20
	C	20			20			20			20		
	D		40			40					40		
	E	44		44		44		44		44		44	
	F	32			32			32			32		
	G	16		16		16		16		16		16	
	H	4	4	4	12	12	12	4	4	4	12	12	12
	I	20	20	20	20	20	20	20	20	20	20	20	20
	J		12	12		12		12	12		12		12

Table 3 Changeover times (min)

From/to	A	B	C	D	E	F	G	H	I	J
A		45	45	45	60	80	30	25	70	55
B	55		55	40	60	80	80	30	30	55
C	60	100		100	75	60	80	80	75	75
D	60	100	30		45	45	45	60	80	100
E	60	60	55	30		35	30	35	60	90
F	75	75	60	100	75		100	75	100	60
G	80	100	30	60	100	85		60	100	65
H	60	60	60	60	60	60	60		60	60
I	80	80	30	30	60	70	55	85		100
J	100	100	60	80	80	30	45	100	100	

Table 4 Product selling prices ($/ton)

	A	B	C	D	E	F	G	H	I	J
Prices	10	12	13	12	15	10	8	14	7	15

Table 5 Solution results of 6, 12, 18 and 24-week cases

Time horizon (weeks)	6	Sales revenue ($)	36,691
No. of equations	2, 509	Changeover cost ($)	277
No. of continuous variables	2, 581	Backlog cost ($)	2,856
No. of binary variables	912	Inventory cost ($)	8
Computational time (CPU s)	154	Total profit ($)	33,550 (0.00%[a])
Time horizon (weeks)	12	Sales revenue ($)	72,735
No. of equations	5, 065	Changeover cost ($)	547
No. of continuous variables	5, 305	Backlog cost ($)	7,276
No. of binary variables	1, 824	Inventory cost ($)	72
Computational time (CPU s)	3, 600	Total profit ($)	64,841 (0.27%[a])
Time horizon (weeks)	18	Sales revenue ($)	109,597
No. of equations	7, 621	Changeover cost ($)	823
No. of continuous variables	8, 029	Backlog cost ($)	13,680
No. of binary variables	2, 736	Inventory cost ($)	212
Computational time (CPU s)	3, 600	Total profit ($)	94,882 (0.44%[a])
Time horizon (weeks)	24	Sales revenue ($)	145,629
No. of equations	10, 177	Changeover cost ($)	1,089
No. of continuous variables	10, 753	Backlog cost ($)	21,594
No. of binary variables	3, 648	Inventory cost ($)	220
Computational time (CPU s)	3, 600	Total profit ($)	122,725 (0.85%[a])

[a] Gap between current solution and best possible solution

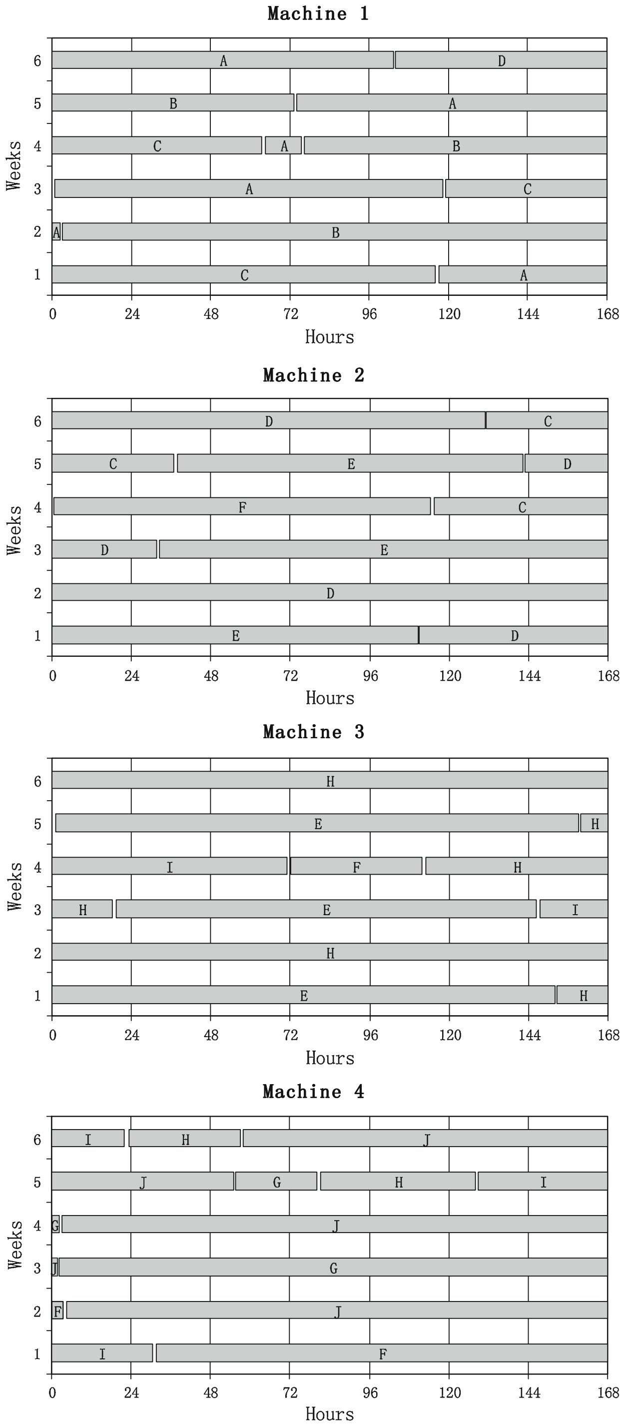

Fig. 7 Optimal schedule for the 6-week case

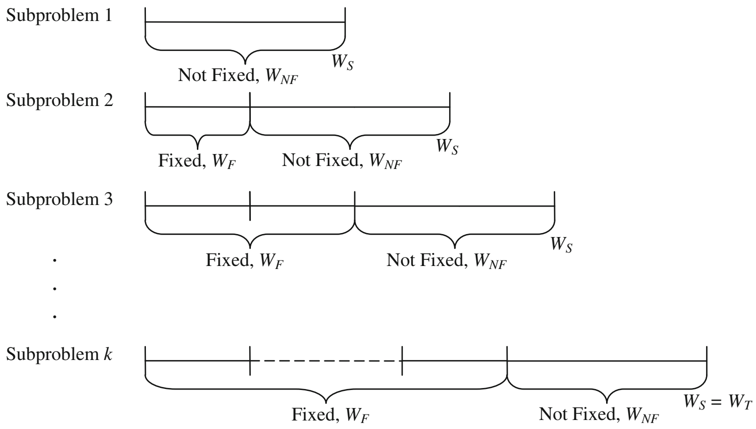

Fig. 8 Rolling horizon approach

each subproblem (W_s) grows successively by a pre-specified number of weeks, while the length of periods (W_F) with fixed binary variables, including E_{iw}, F_{iw}, L_{iw}, Z_{ijw} and ZF_{ijw}, increases by the same time increment. The continuous variables in the fixed time periods (W_F) and all variables in the time periods without fixed variables (W_{NF}) are to be optimised in each subproblem. This iterative scheme stops when the entire planning horizon (W_T) is covered. The solution of the last subproblem is considered as an approximate solution of the full problem. (See Fig. 8)

The proposed RH algorithm procedure can be outlined as follows.

Step 1. Initialise the length of time horizon without fixed binary variables in each subproblem W_{NF}, the length of the time horizon fixed in subproblem 1, $W_F = 0$, the length of planning horizon for subproblem 1, $W_S = W_F + W_{NF}$, and the increment of planning horizon between two subproblems, W_I, such that $W_I < W_S \leq W_T$, where W_T is the length of total planning horizon; Initialise $k = 1$;
Step 2. Fix the binary variables with a planning horizon of W_F weeks to the values obtained in subproblem $k - 1$;
Step 3. Solve subproblem k with a planning horizon of W_S weeks;
Step 4. If $W_S = W_T$, Stop; Otherwise, go to Step 5.
Step 5. Let $k = k$+1, $W_F = W_F + W_I$, $W_S = W_S + W_I$; if $W_S > W_T$, let $W_S = W_T$, then go to Step 2.

From the above procedure, we can see that the proposed RH approach takes the demands in the latter weeks into account, when fixing the initial weeks of each subproblem. So although each subproblem is solved with a shorter horizon, the proposed RH approach is able to foresee some demand information in the latter periods.

When implementing the above RH algorithm, in each iteration, we fix the values of all binary variables within W_F weeks, including E_{imw}, F_{imw}, L_{imw}, Z_{ijmw} and ZF_{ijmw}, as the same as the optimal ones obtained in the previous subproblem. For each subproblem, the continuous variables, especially T_{imw}, V_{iw} and S_{ciw}, within

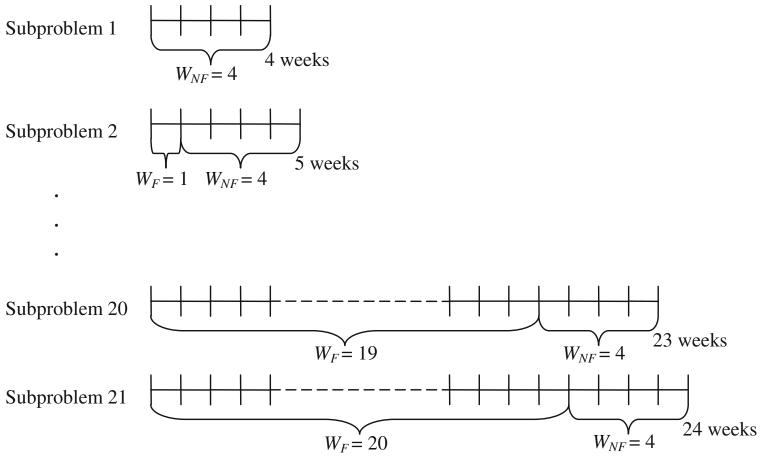

Fig. 9 Rolling horizon approach for the 24-week example

the whole horizon of that subproblem, are to be determined by the model. Thus, the RH approach has more flexibility when encountering unexpected high demands

The performance of the proposed RH algorithm can significantly be affected by the values of W_{NF} and W_I. Usually, there is a tradeoff between the accuracy of the solution and the computational effort of the algorithm. The decision for each problem depends on the computational time limit and the tolerance required.

5.2 Illustrative example revisited

To illustrate the applicability and computational efficiency of the proposed RH approach, we apply the RH approach to the 4 cases discussed in the previous section. In the RH approach, we initialise $W_{NF} = 4$ and $W_I = 1$, then the 6-week case is divided into three subproblems, the 12-week case is divided into nine subproblems, the 18-week case is divided into 15 subproblems, and the 24-week problem is divided into 21 subproblems. See Fig. 9 for the subproblems in the 24-week problem.

The model is also implemented in GAMS 22.6 (Brooke et al. 2008) using solver CPLEX 11.0 (ILOG 2007) on a Pentium 4 3.40 GHz, 1.00 GB RAM machine. The optimality gap and computational time limit are set to be 0.00% and 3,600 s, respectively. The computational results are given in Table 6.

For the 6-week case, the proposed RH approach takes only 77 CPUs to find a feasible solution which is almost the same as the global optimal one given by the single-level MILP model, with only few differences in the schedule on M1 in the first 3 weeks. Although the solution from the proposed RH approach is not as good as the feasible solution from single-level MILP model for the 12-week case, the gap between the two solutions is very small, around 0.01%. Moreover, the RH approach takes 401 CPUs, while the single-level MILP model takes 3,600 CPUs. The superior performance of the proposed RH approach becomes more apparent when the planning horizon of the example increases. For the 18-week case, the profit of the schedule

Table 6 Computational results of single-level MILP and RH

Case	Approach	No. of equations	No. of continuous variables	No. of binary variables	Computational time (CPU s)	Optimality gap (%)	Total profit ($)
6-week	MILP	2,509	2,581	912	154	0.00	33,550
	RH	2,253[a]	2,197[a]	608[a]	77	0.00[b]	33,550
12-week	MILP	5,065	5,305	1,824	3,600	0.27	64,841
	RH	3,897[a]	3,817[a]	608[a]	401	0.00[b]	64,830
18-week	MILP	7,621	8,029	2,736	3,600	0.44	94,882
	RH	5,541[a]	5,437[a]	608[a]	673	0.00[b]	94,903
24-week	MILP	10,177	10,753	3,648	3,600	0.85	122,725
	RH	7,185[a]	7,057[a]	608[a]	892	0.00[b]	123,027

[a] The last subproblem in RH approach
[b] Each subproblem in RH approach

given by RH approach is 94,903, greater than 94,882 obtained from the single-level MILP model. For the 24-week case, the RH approach takes 21 iterations and a total of 892 CPUs to generate a solution with an objective of 123,027, which is better than that of the MILP model, 122,725.

6 Computational results

In this section, the computational efficiencies of the proposed MILP model and the RH approach are compared with that introduced by Erdirik-Dogan and Grossmann (2008), E-D&G for short, in which a bi-level decomposition approach for the scheduling and planning of continuous multiproduct plants with parallel units is proposed. Here, only the first iteration of that approach is used. The details of the upper and lower level problems of the model proposed by Erdirik-Dogan and Grossmann (2008) are described in Appendix A.

Here, we make computational comparisons by using two examples. The proposed MILP model is compared with the E-D&G model using Example 1, which was introduced by Erdirik-Dogan and Grossmann (2008). Example 2 is the one studied in Sect. 4, which will be used to compare the proposed MILP model and the RH approach with the E-D&G model. Also, all models are run in GAMS 22.6 (Brooke et al. 2008) with solver CPLEX 11.0 (ILOG 2007) on a Pentium 4 3.40 GHz, 1.00 GB RAM machine. All models are run with terminating tolerance of 0.00% and the time limit of 3,600 s. For the same representation and a fair comparison of the solution performance of the two models, few modifications are made to the literature model.

6.1 Modifications

We compare the proposed MILP model with the upper and lower level problems of the model proposed by Erdirik-Dogan and Grossmann (2008). There are five differences between the two models. First, the E-D&G model contains processing cost, which is not included in the proposed model. Second, the proposed model considers backlog cost terms in objective function and backlog constraints (constraint 17), while all demands in the E-D&G model must be satisfied (A.1.8 and A.2.18). Third, the E-D&G model does not consider multiple customers, while the proposed model considers the revenue and backlog cost from all of them. Fourth, the proposed model represents the inventory constraints on a weekly basis (constraint 18), while the E-D&G model utilises a linear overestimate of the inventory curve (A.1.4–A.1.7 and A.2.14–A.2.17). Last, the E-D&G model is based on the assumption that at least one product is assigned to each unit without the introduction of a pseudo product.

To make a fair comparison, four modifications are made to both the upper and lower problems of the E-D&G model. First, the operating cost terms are removed from the objective functions. Second, a backlog cost term is added to each objective function, and constraints (A.1.8) and (A.2.18) are replaced by constraint (17). Third, multiple customers are considered in the revenue term in each objective function. Fourth, the inventory constraints (A.1.4–A.1.7) and (A.2.14–A.2.17) are both replaced by constraint (18), and the inventory cost term in the objective function is modified. In addition, pseudo product is not used for both models in the following implementations. Thus, the objective function (A.1.1) of the upper level problem becomes:

$$\begin{aligned}\text{Profit} = &\sum_{c}\sum_{i}\sum_{t} CP_{cit} \cdot S_{cit} - \sum_{i}\sum_{t} CINV_{it} \cdot V_{it} - \sum_{c}\sum_{i}\sum_{t} CB_{ic} \cdot \Delta_{cit} \\ &- \sum_{t}\sum_{m}\sum_{i \in I_m}\sum_{k \in I_m} CTRANS_{ikm} \cdot ZP_{ikmt} \\ &- \sum_{t}\sum_{m}\sum_{i \in I_m}\sum_{k \in I_m} CTRANS_{ikm} \cdot (ZZZ_{ikmt} - ZP_{ikmt}) \end{aligned} \tag{20}$$

and the objective function (A.2.1) of the lower level problem becomes:

$$\begin{aligned}\text{Profit} = &\sum_{c}\sum_{i}\sum_{t} CP_{cit} \cdot S_{cit} - \sum_{i}\sum_{t} CINV_{it} \cdot V_{it} - \sum_{c}\sum_{i}\sum_{t} CB_{ic} \cdot \Delta_{cit} \\ &- \sum_{i \in I_m}\sum_{k \in I_m}\sum_{m}\sum_{t}\sum_{l} (CTRANS_{ikm} \cdot Z_{ikmlt} + CTRANS_{ikm} \cdot TRT_{ikmt}) \end{aligned} \tag{21}$$

Here, we only modify the terms in the objective functions, and the inventory and backlog constraints in the E-D&G model. We do not modify the sequencing and timing constraints, which are the key constraints in the model.

6.2 Example 1

Example 1, which was discussed in the work of Erdirik-Dogan and Grossmann (2008), consists of eight types of products (A–H) and three machines (M1–M3). The original problem considers a total planning horizon of 24 weeks. However, because of the

Table 7 Model and solution statistics of E-D&G and proposed MILP for Example 1

Time horizon (weeks)	4	
Model	E-D&G (Upper/lower level)	Proposed MILP
No. of equations	965/1,490	701
No. of continuous variables	385/1,489	385
No. of binary variables	528/240	288
Total profit ($)	633,851 (633,851/633,851)	633,851
Computational time (CPU s)	1.5 (1.3/0.2)	0.4
Time horizon (weeks)	8	
Model	E-D&G (Upper/lower level)	Proposed MILP
No. of equations	1,953/3,006	1,425
No. of continuous variables	781/3,025	817
No. of binary variables	1,056/480	576
Total profit ($)	1,127,163 (1,127,163/1,127,163)	1,127,163
Computational time (CPU s)	116.2 (116/0.2)	89

limited information provided in the paper, only two cases, with a planning horizon of 4 and 8 weeks, respectively, are considered.

Table 7 shows the solution results of the two models. In both 4 and 8-week cases, we can see that both approaches can find global solutions. However, the E-D&G model takes more CPU time to reach the global optimum than the proposed model. Especially in the 8-week case, the E-D&G model takes 116 CPUs while the proposed model takes 1/4 less time, which is 89 CPUs,

6.3 Example 2

In Example 2, we consider four cases with a planning horizon of 6, 12, 18 and 24 weeks. We initialise $W_{NF} = 4$ and $W_I = 1$, and apply the proposed RH approach to the four cases.

From Table 8, except for the 6-week case, both the E-D&G model and the propose single-level MILP model cannot terminate within the specified time limit. However, the proposed MILP model yields better feasible solutions than those obtained by the E-D&G model, and takes only half of CPU time than the E-D&G model.

For the 6-week case, the proposed MILP model takes only 260 CPUs to get the global optimal solution, while the E-D&G model totally takes over eight times CPU time than the proposed MILP model. Moreover, we can see that for the 6-week case, the solution of E-D&G model is worse than the other two approaches. Because subtours occur in the solution of its upper level problem, the objective given by its upper level problem is an upper bound of the global optimal one, and the solution given by its lower level problem is not an global optimal.

Moreover, for all cases, the RH approach also takes much less CPU time and finds better feasible solutions than those of the E-D&G model. It should be noticed that

Table 8 Model and solution statistics of E-D&G, proposed MILP, and RH for Example 2

Time horizon (weeks)	6		
Model	E-D&G (Upper/lower level)	Proposed MILP	Proposed RH
No. of equations	3,141/4,841	2,373	2,157[a]
No. of continuous variables	2,105/6,201	2,121	2,021[a]
No. of binary variables	1,560/720	840	560[a]
Total profit ($)	33,526 (33,555/33,526)	33,550	33,550
Optimality gap	0.00%/0.00%	0.00%	0.00%[b]
Computational time (CPU s)	2,186 (512/1,674)	260	76
Time horizon (weeks)	12		
Model	E-D&G (Upper/lower level)	Proposed MILP	Proposed RH
No. of equations	6,321/9,725	4,785	3,801[a]
No. of continuous variables	4,229/12,501	4,341	3,641[a]
No. of binary variables	3,120/1,440	1,680	560[a]
Total profit ($)	64,813 (64,850/64,813)	64,833	64,830
Optimality gap	0.28%/0.22%	0.25%	0.00%[b]
Computational time (CPU s)	7,200 (3,600/3,600)	3,600	430
Time horizon (weeks)	18		
Model	E-D&G (Upper/lower level)	Proposed MILP	Proposed RH
No. of equations	9,501/14,609	7,197	5,445[a]
No. of continuous variables	6,353/18,801	6,561	5,261[a]
No. of binary variables	4,680/2,160	2,520	560[a]
Total profit ($)	94,768 (94,875/94,768)	94,807	94,903
Optimality gap	0.48%/0.56 %	0.53%	0.00%[b]
Computational time (CPU s)	7,200 (3,600/3,600)	3,600	621
Time horizon (weeks)	24		
Model	E-D&G (Upper/lower level)	Proposed MILP	Proposed RH
No. of equations	12,681/19,493	9,609	7,089[a]
No. of continuous variables	8,477/25,101	8,781	6,881[a]
No. of binary variables	6,240/2,880	3,360	560[a]
Total profit ($)	122,600 (122,764/122,600)	122,745	123,027
Optimality gap	0.82%/0.52%	0.83%	0.00%[b]
Computational time (CPU s)	7,200 (3,600/3,600)	3,600	808

[a] The last subproblem in RH approach
[b] Each subproblem in RH approach

in the 12-, 18- and 24-week cases, the upper level problem of the E-D&G model terminates when the computation time reaches the time limit, 3,600 s. From Table 8, we can see that there is a gap between the obtained solution and the optimal solution of the upper level problem of the E-D&G model, which is also the upper bound of the problem. So, we can see that the solutions of RH approach are better than those of the upper problem for the 18- and 24-week cases. However, subtours still occur in the obtained solutions of the upper level problem of the E-D&G model, which yield infeasible production sequences of products.

7 Concluding remarks

A novel MILP model for medium-term planning of single-stage continuous multiproduct plants with parallel processing units has been presented in this paper. The model is based on a hybrid discrete/continuous time representation. Because of the similar nature of the problem with the TSP, a formulation similar to the one used to model changeovers in the classic TSP has been proposed. Also, in order to eliminate subtours in the schedule, integer variables representing the ordering of the products and the subtour elimination constraints have been introduced. An example of a polymer processing plant has been used to illustrate the applicability of the proposed model.

In order to overcome the computational expense of solving large problems, we have proposed a RH approach, which significantly reduces the computational time with a good feasible solution. Finally, the proposed MILP model and RH algorithm have been compared favourably with models from recent literature, exhibiting a much improved computational performance for two examples investigated.

Acknowledgments One of the authors (S. Liu) gratefully acknowledges financial support from the Overseas Research Students Awards Scheme (ORSAS), the K.C. Wong Education Foundation, the British Foreign & Commonwealth Office (FCO), and the Centre for Process Systems Engineering (CPSE) at Imperial and University College London.

Appendix A: E-D&G Model

In the bi-level decomposition algorithm proposed by Erdirik-Dogan and Grossmann (2008), the original MILP model of simultaneous planning and scheduling of single-stage multiproduct continuous plants with parallel units is decomposed into an upper level planning and a lower level scheduling problem.

In the decomposition approach, the upper level problem yields a valid upper bound on the profit, while, by excluding the products that were not selected by the upper level problem for each unit at each period, the lower level problem is solved to yield a lower bound on the profit. The two sub-problems are solved iteratively. Integer cuts are used to exclude the current assignment and generate new solutions. Finally, the solution of the lower level problem becomes the final solution after convergence is achieved.

List of symbols

Indices

i, k	Products
l	Slots
$\bar{l}_m$	Last slot of unit m
m	Units
t	Time periods
$\bar{t}$	Last time period

Sets

I	Set of products
I_m	Set of products that can be processed on unit m
L	Set of slots
L_m	Set of slots that belong to unit m
M	Set of units
M_i	Set of units that can process product i

Parameters

$CINV_{it}$	Inventory cost of product i in period t
COP_{it}	Operating cost of product i in period t
CP_{it}	Selling price of product i in period t
$CTRANS_{ikm}$	Transition cost of changing the production from product i to k in unit m
d_{it}	Demand of product i at the end of period t
H_t	Duration of the tth time period
HT_t	Time at the end of the tth time period
$INVI_i$	Initial inventory of product i
MRT_{im}	Minimum run lengths
N_m	Number of slots postulated for unit m
r_{im}	Production rate of product i in unit m
τ_{ikm}	Transition time from product i to product k in unit m

Variables

Area_{it}	Overestimate of the area below the inventory time graph for product i at the end of time period t
INV_{it}	Inventory level of product i at the end of time period t
$INVO_{it}$	Inventory level of product i at the end of time period t after demands are satisfied
NY_{imt}	Total number of slots that are allocated for product i in unit m during time period t
S_{it}	Sales of product i at the end of period t
Te_{mlt}	End time of slot l of unit m during time period t
$TRNP_{mt}$	Total transition time for unit m within each time period
Ts_{mlt}	Start time of slot l of unit m during time period t
X_{imlt}	Amount of product i produced in slot l of unit m during time period t
$\tilde{X}_{imt}$	Amount of product i produced in unit m during time period t
$\tilde{\theta}_{imt}$	Production time of product i in unit m during time period t
Θ_{imlt}	Production time of product i in slot l of unit m during time period t

Binary variables

TRT_{ikmt}	To denote if product i is followed by product k at the end of time period t
W_{imlt}	To denote the assignment of product i to slot l of unit m during time period t

XF_{imt}	To denote if product i the first product in unit m during time period t
XL_{imt}	To denote if product i the last product in unit m during time period t
YOP_{imt}	To denote if product i is assigned to unit m during time period t
YP_{imt}	To denote the assignment of product i to unit m during time period t
Z_{ikmlt}	To denote if product i is followed by product k in slot l of unit m during time period t
ZP_{ikmt}	To denote if product i precedes product k in unit m during time period t
ZZP_{ikmt}	To denote if the link between products i and k is broken
ZZZ_{ikmt}	Transition variable denoting the changeovers across adjacent periods

A.1 Upper level problem

Objective function

$$\begin{aligned}\text{Profit} = &\sum_i \sum_t CP_{it} \cdot S_{it} - \sum_i \sum_t CINV_{it} \cdot \text{Area}_{it} - \sum_t \sum_m \sum_{i \in I_m} COP_{it} \cdot \tilde{X}_{imt} \\ &- \sum_t \sum_m \sum_{i \in I_m} \sum_{k \in I_m} CTRANS_{ikm} \cdot (ZP_{ikmt} - ZZP_{ikmt}) \\ &- \sum_t \sum_m \sum_{i \in I_m} \sum_{k \in I_m} CTRANS_{ikm} \cdot ZZZ_{ikmt} \end{aligned} \tag{A.1.1}$$

Assignment and production constraints

$$\tilde{\theta}_{imt} \leq H_t \cdot YP_{imt} \quad \forall i \in I_m, m, t \tag{A.1.2}$$

$$\tilde{X}_{imt} = r_{im} \cdot \tilde{\theta}_{imt} \quad \forall i \in I_m, m, t \tag{A.1.3}$$

Inventory balance and costs

$$INV_{it} = INVI_i + \sum_{m \in M_i} r_{im} \cdot \tilde{\theta}_{imt} \quad \forall i, t = 1 \tag{A.1.4}$$

$$INV_{it} = INVO_{i,t-1} + \sum_{m \in M_i} r_{im} \cdot \tilde{\theta}_{imt} \quad \forall i, t \neq 1 \tag{A.1.5}$$

$$INVO_{it} = INV_{it} - S_{it} \quad \forall i, t \tag{A.1.6}$$

$$\text{Area}_{it} \geq INVO_{i,t-1} \cdot H_t + \left(\sum_{m \in M_i} r_{im} \cdot \tilde{\theta}_{imt} \right) \cdot H_t \quad \forall i, t \tag{A.1.7}$$

Demand

$$S_{it} \geq d_{it} \quad \forall i, t \tag{A.1.8}$$

Sequencing constraints

$$YP_{imt} = \sum_{k \in I_m} ZP_{ikmt} \quad \forall i \in I_m, m, t \tag{A.1.9}$$

$$YP_{kmt} = \sum_{i \in I_m} ZP_{ikmt} \quad \forall k \in I_m, m, t \tag{A.1.10}$$

$$\sum_{i \in I_m} \sum_{k \in I_m} ZZP_{ikmt} = 1 \quad \forall m, t \tag{A.1.11}$$

$$ZZP_{ikmt} \leq ZP_{ikmt} \quad \forall i \in I_m, k \in I_m, m, t \tag{A.1.12}$$

$$YP_{imt} \geq ZP_{iimt} \quad \forall i \in I_m, m, t \tag{A.1.13}$$

$$ZP_{iimt} + YP_{kmt} \leq 1 \quad \forall i \in I_m, k \in I_m, i \neq k, m, t \tag{A.1.14}$$

$$ZP_{iimt} \geq YP_{imt} - \sum_{k \neq i, k \in I_m} YP_{kw} \quad \forall i \in I_m, m, t \tag{A.1.15}$$

$$TRNP_{mt} = \sum_{i \in I_m} \sum_{k \in I_m} \tau_{ikm} \cdot ZP_{ikmt} - \sum_{i \in I_m} \sum_{k \in I_m} \tau_{ikm} \cdot ZZP_{ikmt} \quad \forall m, t \tag{A.1.16}$$

$$XF_{kmt} \geq \sum_{i \in I_m} ZZP_{ikmt} \quad \forall k \in I_m, m, t \tag{A.1.17}$$

$$XL_{imt} \geq \sum_{k \in I_m} ZZP_{ikmt} \quad \forall i \in I_m, m, t \tag{A.1.18}$$

$$\sum_{i \in I_m} XF_{imt} = 1 \quad \forall m, t \tag{A.1.19}$$

$$\sum_{i \in I_m} XL_{imt} = 1 \quad \forall m, t \tag{A.1.20}$$

$$\sum_{k \in I_m} ZZZ_{ikmt} = XL_{imt} \quad \forall i \in I_m, m, t \tag{A.1.21}$$

$$\sum_{i \in I_m} ZZZ_{ikmt} = XF_{k,m,t+1} \quad \forall k \in I_m, m, t \in T - \{\bar{t}\} \tag{A.1.22}$$

Time balance

$$\sum_{i \in I_m} \tilde{\theta}_{imt} + TRNP_{mt} - \sum_{i \in I_m} \sum_{k \in I_m} (\tau_{ikm} \cdot ZZZ_{ikmt}) \leq H_t \quad \forall m, t \tag{A.1.23}$$

Integer cuts

$$\sum_{(i,t) \in Z_1^r} YP_{imt} - \sum_{(i,t) \in Z_0^r} YP_{imt} \leq \left| Z_0^r \right| - 1 \tag{A.1.24}$$

where $Z_0^r = \{i, t | YP_{imt}^r = 0\}$ and $Z_1^r = \{i, t | YP_{imt}^r = 1\}$.

A.2 Lower level problem

Objective function

$$\begin{aligned}\text{Profit} = &\sum_i \sum_t CP_{it} \cdot S_{it} - \sum_i \sum_t CINV_{it} \cdot \text{Area}_{it}\\ &- \sum_t \sum_m \sum_{i \in I_m} \sum_l COP_{it} \cdot X_{imlt}\\ &- \sum_{i \in I_m} \sum_{k \in I_m} \sum_m \sum_t \sum_l (CTRANS_{ikm} \cdot Z_{ikmlt} + CTRANS_{ikm} \cdot TRT_{ikmt})\end{aligned} \tag{A.2.1}$$

Assignment and processing times

$$\sum_{i \in I_m} W_{imlt} = 1 \quad \forall m, l \in L_m, t \tag{A.2.2}$$

$$\Theta_{imlt} \le H_t \cdot W_{imlt} \quad \forall i \in I_m, m, l \in L_m, t \tag{A.2.3}$$

$$\Theta_{imlt} \ge MRT_{im} \cdot W_{imlt} \quad \forall i \in I_m, m, l \in L_m, t \tag{A.2.4}$$

$$X_{imlt} = r_{im} \cdot \Theta_{imlt} \quad \forall i \in I_m, m, l \in L_m, t \tag{A.2.5}$$

Transitions

$$\sum_{k \in I_m} Z_{ikmlt} = W_{imlt} \quad \forall i \in I_m, m, l \in L_m, t \tag{A.2.6}$$

$$\sum_{i \in I_m} Z_{ikmlt} = W_{k,m,l+1,t} \quad \forall k \in I_m, m, l \in L_m - \{\bar{l}_m\}, t \tag{A.2.7}$$

$$\sum_{k \in I_m} TRT_{ikmt} = W_{imlt} \quad \forall i \in I_m, m, l = \bar{l}_m, t \tag{A.2.8}$$

$$\sum_{i \in I_m} TRT_{ikmt} = W_{kmlt} \quad \forall k \in I_m, m, l = 1, t = T - \{\bar{t}\} \tag{A.2.9}$$

Timing relations

$$Te_{mlt} = Ts_{mlt} + \sum_{i \in I_m} \Theta_{imlt} + \sum_{i \in I_m} \sum_{k \in I_m} \tau_{ikm} \cdot Z_{ikmlt} \quad \forall m, l = L_m, t \tag{A.2.10}$$

$$Ts_{mlt+1} \ge Te_{ml't} + \sum_{i \in I_m} \sum_{k \in I_m} \tau_{ikm} \cdot TRT_{ikmt} \quad \forall m, l = 1, l' = \bar{l}_m, t = T - \{\bar{t}\} \tag{A.2.11}$$

$$Te_{mlt} = Ts_{m,l+1,t} \quad \forall m, l = L - \{\bar{l}_m\}, t \tag{A.2.12}$$

$$Te_{mlt} \le HT_t \quad \forall m, l = \bar{l}_m, t \tag{A.2.13}$$

Inventory balance and costs

$$INV_{it} = INVI_t + \sum_{m \in M_i} r_{im} \cdot \sum_{l \in L_m} \Theta_{imlt} \quad \forall i, t = 1 \tag{A.2.14}$$

$$INV_{it} = INVO_{i,t-1} + \sum_{m \in M_i} r_{im} \cdot \sum_{l \in L_m} \Theta_{imlt} \quad \forall i, t \neq 1 \tag{A.2.15}$$

$$INVO_{it} = INV_{it} - S_{it} \quad \forall i, t \tag{A.2.16}$$

$$\text{Area}_{it} \geq INVO_{i,t-1} \cdot H_t + \left(\sum_{m \in M_i} r_{im} \cdot \sum_{l \in L_m} \Theta_{imlt} \right) \cdot H_t \quad \forall i, t \tag{A.2.17}$$

Demand

$$S_{it} \geq d_{it} \quad \forall i, t \tag{A.2.18}$$

Degeneracy prevention

$$YOP_{imt} \geq W_{imlt} \quad \forall i \in I_m, m, l \in L_m, t \tag{A.2.19}$$

$$YOP_{imt} \leq NY_{imt} \leq N_m \cdot YOP_{imt} \quad \forall i \in I_m, m, t \tag{A.2.20}$$

$$NY_{imt} \geq N_m - \left[\left(\sum_{i \in I_m} YOP_{imt} \right) - 1 \right] - M \cdot (1 - W_{imlt}) \quad \forall i \in I_m, m, t \tag{A.2.21}$$

$$NY_{imt} \leq N_m - \left[\left(\sum_{i \in I_m} YOP_{imt} \right) - 1 \right] + M \cdot (1 + W_{imlt}) \quad \forall i \in I_m, m, t \tag{A.2.22}$$

Subset of products defined by the upper problem

$$YOP_{imt} \leq YP_{imt} \quad \forall i \in I_m, m, t \tag{A.2.23}$$

References

Alle A, Papageorgiou LG, Pinto JM (2004) A mathematical programming approach for cyclic production and cleaning scheduling of multistage continuous plants. Comput Chem Eng 28:3–15

Alle A, Pinto JM (2002) Mixed-integer programming models for the scheduling and operational optimization of multiproduct continuous plants. Ind Eng Chem Res 41:2689–2704

Brooke A, Kendrick D, Meeraus A, Raman R (2008) GAMS: a user's guide. GAMS Development Corporation, Washington

Burkard RE, Hatzl J (2005) Review, extensions and computational comparison of MILP formulations for scheduling of batch processes. Comput Chem Eng 29:1752–1769

Castro PM, Erdirik-Dogan M, Grossmann IE (2008) Simultaneous batching and scheduling of single stage batch plants with parallel units. AIChE J 54:183–193

Castro PM, Grossmann IE (2006) An efficient MILP model for the short-term scheduling of single stage batch plants. Comput Chem Eng 30:1003–1018

Castro PM, Novais AQ (2007) Optimal periodic scheduling of multistage continuous plants with single and multiple time grid formulations. Ind Eng Chem Res 46:3669–3683
Cerdá J, Henning G, Grossmann IE (1997) Mixed-integer linear programming model for short-term scheduling of single-stage multiproduct batch plants with parallel lines. Ind Eng Chem Res 36:1695–1707
Chen C, Liu C, Feng X, Shao H (2002) Optimal short-term scheduling of multiproduct single-stage batch plants with parallel lines. Ind Eng Chem Res 41:1249–1260
Chen P, Papageorgiou LG, Pinto JM (2008) Medium-term planning of single-stage single-unit multiproduct plants using a hybrid discrete/continuous-time MILP model. Ind Eng Chem Res 47:1925–1934
Erdirik-Dogan M, Grossmann IE (2006) A decomposition method for the simultaneous planning and scheduling of single-stage continuous muliproduct plants. Ind Eng Chem Res 45:299–315
Erdirik-Dogan M, Grossmann IE (2007) Planning models for parallel batch reactors with sequence-dependent changeovers. AIChE J 53:2284–2300
Erdirik-Dogana M, Grossmann IE (2008) Simultaneous planning and scheduling of single-stage multi-product continuous plants with parallel lines. Comput Chem Eng 32:2664–2683
Ierapetritou MG, Floudas CA (1998) Effective continuous-time formulation for the short-term scheduling 2 Continuous and semicontinuous processes. Ind Eng Chem Res 37:4360–4374
ILOG (2007) ILOG CPLEX 110 User's Manual. ILOG SA, Gentilly
Kallrath J, Wilson JM (1997) Business optimisation using mathematical programming. Macmillan, Basingstoke, UK
Karimi IA, McDonald CM (1997) Planning and scheduling of parallel semicontinuous processes 2. Short-term scheduling. Ind Eng Chem Res 36:2701–2714
Kondili E, Shah N, Pantelides CC (1993) Production planning for the rational use of energy in multiproduct continuous plants. Comput Chem Eng 17:S123–S128
Lee K, Heo S, Lee H, Lee I (2002) Scheduling of single-stage and continuous processes on parallel lines with intermediate due dates. Ind Eng Chem Res 41:58–66
Leksakul K, Techanitisawad A (2005) An application of the neural network energy function to machine sequencing. Comput Manag Sci 2:309–338
Liu S, Pinto JM, Papageorgiou LG (2008) A TSP-based MILP model for medium-term planning of single-stage continuous multiproduct plants. Ind Eng Chem Res 47:7733–7743
Méndez CA, Cerdá J (2002a) An MILP-based approach to the short-term scheduling of make-and-pack continuous production plants. OR Spectr 24:403–429
Méndez CA, Cerdá J (2002b) An efficient MILP continuous-time formulation for short-term scheduling of multiproduct continuous facilities. Comput Chem Eng 26:687–695
Méndez CA, Cerdá J, Grossmann IE, Harjunkoski I, Fahl M (2006) State-of-the-art review of optimization methods for short-term scheduling of batch processes. Comput Chem Eng 30:913–946
Mockus L, Reklaitis GV (1999) Continuous time representation approach to batch and continuous process scheduling. 1. MINLP formulation. Ind Eng Chem Res 38:197–203
Munawar SA, Bhushan M, Gudi RD, Belliappa AM (2003) Cyclic scheduling of continuous multiproduct plants in a hybrid flowshop facility. Ind Eng Chem Res 42:5861–5882
Papageorgiou LG, Pantelides CC (1996) Optimal campaign planning/scheduling of multipurpose batch/semicontinuous plants. 1. Mathematical formulation. Ind Eng Chem Res 35:488–509
Pardalos PM, Shylo OV (2006) An algorithm for the job shop scheduling problem based on global equilibrium search techniques. Comput Manag Sci 3:331–348
Pinto JM, Grossmann IE (1994) Optimal cyclic scheduling of multistage continuous multiproduct plants. Comput Chem Eng 18:797–816
Pinto JM, Grossmann IE (1995) A continuous-time mixed-integer linear-programming model for short-term scheduling of multistage batch plants. Ind Eng Chem Res 34:3037–3051
Sahinidis NV, Grossmann IE (1991) MINLP model for cyclic multiproduct scheduling on continuous parallel lines. Comput Chem Eng 15:85–103
Shaik MA, Floudas CA (2007) Improved unit-specific event-based continuous-time model for short-term scheduling of continuous processes: rigorous treatment of storage requirements. Ind Eng Chem Res 46:1764–1779
Zhu XX, Majozi T (2001) Novel continuous time MILP formulation for multipurpose batch plants. 2. Integrated planning and scheduling. Ind Eng Chem Res 40:5621–5634