Multi-Objective Stochastic Programming Approaches for Supply Chain Management

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Abstract A multi-objective stochastic programming model is developed to design robust supply chain configuration networks. Demands, supplies, processing, and transportation costs are all considered as the uncertain parameters, which will be revealed after building the sites at the strategic level. The decisions about the optimal flows are made at the tactical level depending upon the actual values of uncertain parameters. It is also assumed that the suppliers are unreliable. To develop a robust model, two additional objective functions are added into the traditional supply chain design problem. So, the proposed model accounts for the minimization of the expected total cost and the risk, reflected by the variance of the total cost and the downside risk or the risk of loss. Finally, different simple and interactive multi-objective techniques such as goal attainment, surrogate worth trade-off (SWT), and STEM methods are used to solve the proposed multi-objective model.

1 Introduction

A supply chain (SC) is a network of suppliers, manufacturing plants, warehouses, and distribution channels organized to acquire raw materials, convert these raw materials to finished products, and distribute these products to customers. The concept of supply chain management (SCM), which appeared in the early 1990s, has recently raised a lot of interest since the opportunity of an integrated management of the supply chain can reduce the propagation of unexpected/undesirable events through the network and can affect decisively the profitability of all the members.

A crucial component of the planning activities of a manufacturing firm is the efficient design and operation of its supply chain. Strategic level supply chain planning

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involves deciding the configuration of the network, i.e., the number, location, capacity, and technology of the facilities. The tactical level planning of supply chain operations involves deciding the aggregate quantities and material flows for purchasing, processing, and distribution of products. The strategic configuration of the supply chain is a key factor influencing efficient tactical operations, and therefore has a long lasting impact on the firm. Furthermore, the fact that the supply chain configuration involves the commitment of substantial capital resources over long periods of time makes the supply chain design problem an extremely important one.

Many attempts have been made to model and optimize supply chain design, most of which are based on deterministic approaches, see for example Bok et al. (2000), Timpe and Kallrath (2000), Gjerdrum et al. (2000), and many others. However, most real supply chain design problems are characterized by numerous sources of technical and commercial uncertainty, and so the assumption that all model parameters, such as cost coefficients, supplies, demands, etc., are known with certainty is not realistic.

In order to take into account the effects of the uncertainty in the production scenario, a two-stage stochastic model is proposed in this paper. Decision variables which characterize the network configuration, namely those binary variables which represent the existence and the location of plants and warehouses of the supply chain are considered as first-stage variables – it is assumed that they have to be taken at the strategic level before the realization of the uncertainty. On the other hand, decision variables related to the amount of products to be produced and stored in the nodes of the supply chain and the flows of materials transported among the entities of the network are considered as second-stage variables, corresponding to decisions taken at the tactical level after the uncertain parameters have been revealed.

There are a few research works addressing comprehensive (strategic and tactical issues simultaneously) design of supply chain networks using two-stage stochastic models. MirHassani et al. (2000) considered a two-stage model for multi-period capacity planning of supply chain networks, and used Benders decomposition to solve the resulting stochastic integer program. Tsiakis et al. (2001) considered a two-stage stochastic programming model for supply chain network design under demand uncertainty, and developed a large-scale mixed-integer linear programming model for this problem. Alonso-Ayuso et al. (2003) proposed a branch-and-fix heuristic for solving two-stage stochastic supply chain design problems. Santoso et al. (2005) integrated a sampling strategy with an accelerated Benders decomposition to solve supply chain design problems with continuous distributions for the uncertain parameters. However, the robustness of decision to uncertain parameters is not considered in above studies.

Azaron et al. (2008) developed a multi-objective stochastic programming approach for designing robust supply chains. The objective functions of this model are (1) the minimization of the sum of current investment costs and the expected future processing, transportation, shortage, and capacity expansion costs, (2) the minimization of the variance of the total cost, and (3) the minimization of the financial risk or the probability of not meeting a certain budget. Then, they used goal attainment technique, see Hwang and Masud (1979) for details, to solve the resulting multi-objective problem.

This method has the same disadvantages as those of goal programming; namely, the preferred solution is sensitive to the goal vector and the weighting vector given by the decision maker. To overcome this drawback, we also use interactive multi-objective techniques with explicit or implicit trade-off information given such as SWT and STEM methods, see Hwang and Masud (1979) for details, to solve the problem. The other advantage of this paper over (Azaron et al. 2008) is that we minimize downside risk or risk of loss instead of financial risk. By applying this concept, we avoid the use of binary variables to determine the financial risk, which significantly reduces the computational time to solve the final large scale mixed-integer nonlinear programming problem.

To the best of our knowledge, only ε-constraint method (Guillen et al. 2005), fuzzy optimization (Chen and Lee 2004), and goal attainment method (Azaron et al. 2008) have been used to solve existing multi-objective supply chain design models. In this paper, we use interactive multi-objective techniques to solve the problem.

The paper is organized as follows. In Sect. 2, we describe the supply chain design problem. In Sect. 3, we explain the details of multi-objective techniques to solve the problem. Section 4 presents the computational experiments. Finally, we draw the conclusion of the paper in Sect. 5.

2 Problem Description

We first describe a deterministic mathematical formulation for the supply chain design problem. Consider a supply chain network $G = (N, A)$, where N is the set of nodes and A is the set of arcs. The set N consists of the set of suppliers S, the set of possible processing facilities P, and the set of customer centers C, i.e., $N = S \cup P \cup C$. The processing facilities include manufacturing centers M and warehouses W, i.e., $P = M \cup W$. Let K be the set of products flowing through the supply chain.

The supply chain configuration decisions consist of deciding which of the processing centers to build. We associate a binary variable y_i to these decisions: $y_i = 1$ if processing facility i is built, and 0 otherwise. The tactical decisions consist of routing the flow of each product $k \in K$ from the suppliers to the customers. We let x_{ij}^k denote the flow of product k from a node i to a node j of the network where $(ij) \in A$, and z_j^k denote shortfall of product k at customer centre j, when it is impossible to meet demand. A deterministic mathematical model for this supply chain design problem is formulated as follows (see Santoso et al. (2005) for more details):

$$\text{Min} \sum_{i \in P} c_i y_i + \sum_{k \in K} \sum_{(ij) \in A} q_{ij}^k x_{ij}^k + \sum_{k \in K} \sum_{j \in C} h_j^k z_j^k \tag{1a}$$

s.t.

$$y \in Y \subseteq \{0,1\}^{|P|} \tag{1b}$$

$$\sum_{i \in N} x_{ij}^k - \sum_{l \in N} x_{jl}^k = 0 \qquad \forall j \in P, \quad \forall k \in K \tag{1c}$$

$$\sum_{i \in N} x_{ij}^k + z_j^k \geq d_j^k \qquad \forall j \in C, \quad \forall k \in K \tag{1d}$$

$$\sum_{j \in N} x_{ij}^k \leq s_i^k \qquad \forall i \in S, \quad \forall k \in K \tag{1e}$$

$$\sum_{k \in K} r_j^k \left(\sum_{i \in N} x_{ij}^k \right) \leq m_j y_j \qquad \forall j \in P \tag{1f}$$

$$x_{ij}^k \geq 0 \qquad \forall (ij) \in A, \quad \forall k \in K \tag{1g}$$

$$z_j^k \geq 0 \qquad \forall j \in C, \quad \forall k \in K \tag{1h}$$

The objective function (1a) consists of minimizing the total investment, production/transportation, and shortage costs. Constraint (1b) enforces the binary nature of the configuration decisions for the processing facilities. Constraint (1c) enforces the flow conservation of product k across each processing node j. Constraint (1d) requires that the total flow of product k to a customer node j plus shortfall should exceed the demand d_j^k at that node. Constraint (1e) requires that the total flow of product k from a supplier node i should be less than the supply s_i^k at that node. Constraint (1f) enforces capacity constraints of the processing nodes. Here, r_j^k and m_j denote per-unit processing requirement for product k at node j and capacity of facility j, respectively.

We now propose a stochastic programming approach based on a recourse model with two stages to incorporate the uncertainty associated with demands, supplies, processing/transportation, shortage, and capacity expansion costs. It is also assumed that we have the option of expanding the capacities of sites after the realization of uncertain parameters. Considering $\xi = (d, s, q, h, f)$ as the corresponding random vector, the two-stage stochastic model, in Matrix form, is formulated as follows (see Azaron et al. (2008) for details):

$$\text{Min } c^T y + E[G(y,\xi)] \quad \text{[Expected Total Cost]} \tag{2a}$$

s.t.

$$y \in Y \subseteq \{0,1\}^{|P|} \quad \text{[Binary Variables]} \tag{2b}$$

where $G(y,\xi)$ is the optimal value of the following problem:

$$\text{Min } q^T x + h^T z + f^T e \tag{2c}$$

s.t.

$$Bx = 0 \quad \text{[Flow Conservation]} \tag{2d}$$

$$Dx + z \geq d \quad \text{[Meeting Demand]} \tag{2e}$$

$$Sx \leq s \quad \text{[Supply Limit]} \tag{2f}$$

$$Rx \leq My + e \quad \text{[Capacity Constraint]} \tag{2g}$$

$$e \leq Oy \quad \text{[Capacity Expansion Limit]} \tag{2h}$$

$$x \in R_{+}^{|A|\times|K|},\ z \in R_{+}^{|C|\times|K|}\ e \in R_{+}^{|P|} \quad \text{[Continuous Variables]} \tag{2i}$$

Above vectors c, q, h, f, d, and s correspond to investment costs, processing/transportation costs, shortfall costs, expansion costs, demands, and supplies, respectively. The matrices B, D, and S are appropriate matrices corresponding to the summations on the left-hand-side of the expressions (1c)–(1e), respectively. The notation R corresponds to a matrix of r_j^k, and the notation M corresponds to a matrix with m_j along the diagonal. e and O correspond to capacity expansions and expansion limits, respectively.

Note that the optimal value $G(y, \xi)$ of the second-stage problem (2c)–(2i) is a function of the first stage decision variable y and a realization $\xi = (d, s, q, h, f)$ of the uncertain parameters. The expectation in (2a) is taken with respect to the joint probability distribution of uncertain parameters.

In this paper, the uncertainty is represented by a set of discrete scenarios with given probability of occurrence. It is also assumed that suppliers are unreliable and their reliabilities are known in advance. The role of unreliable suppliers is implicitly considered in the model by properly way of generating scenarios. It means that in case of having an unreliable supplier, its supply value is set to zero in the corresponding scenarios, see Azaron et al. (2008) for more details.

3 Multi-Objective Techniques

As explained, to develop a robust model, two additional objective functions are added into the traditional supply chain design problem. The first is the minimization of the variance of the total cost, and the second is the minimization of the downside risk or the risk of loss. The definition of downside risk or the expected total loss is:

$$DRisk = \sum_{l=1}^{L} p_l Max\,(Cost_l - \Omega, 0) \tag{3}$$

where p_l, Ω, and $Cost_l$ represent the occurrence probability of the lth scenario, available budget, and total cost when the lth scenario is realized, respectively. The downside risk can be calculated as follows:

$$
\begin{aligned}
& DRisk = \sum_{l=1}^{L} p_l DR_l \\
& DR_l \geq Cost_l - \Omega \qquad \forall l \\
& DR_l \geq 0 \qquad\qquad\quad \forall l
\end{aligned} \tag{4}
$$

The proper multi-objective stochastic model for our supply chain design problem will be:

$$\text{Min } f_1(x) = c^T y + \sum_{l=1}^{L} p_l \left(q_l^T x_l + h_l^T z_l + f_l^T e_l \right) \text{ [Expected Total Cost]} \tag{5a}$$

$$\text{Min } f_2(x) = \sum_{l=1}^{L} p_l \left(q_l^T x_l + h_l^T z_l + f_l^T e_l - \sum_{l=1}^{L} p_l \left(q_l^T x_l + h_l^T z_l + f_l^T e_l \right) \right)^2 \text{ [Variance]} \tag{5b}$$

$$\text{Min } f_3(x) = \sum_{l=1}^{L} p_l DR_l \quad \text{[Downside Risk]} \tag{5c}$$

s.t.

$$Bx_l = 0 \quad l = 1, \ldots, L \tag{5d}$$

$$Dx_l + z_l \geq d_l \quad l = 1, \ldots, L \tag{5e}$$

$$Sx_l \leq s_l \quad l = 1, \ldots, L \tag{5f}$$

$$Rx_l \leq My + e_l \quad l = 1, \ldots, L \tag{5g}$$

$$e_l \leq Oy \quad l = 1, \ldots, L \tag{5h}$$

$$c^T y + q_l^T x_l + h_l^T z_l + f_l^T e_l - \Omega \leq DR_l \quad l = 1, \ldots, L \tag{5i}$$

$$y \in Y \subseteq \{0, 1\}^{|P|} \tag{5j}$$

$$x \in R_+^{|A| \times |K| \times L}, \quad z \in R_+^{|C| \times |K| \times L}, \quad e \in R_+^{|P| \times L}, \quad DR \in R_+^L \tag{5k}$$

3.1 Goal Attainment Technique

Goal attainment method is one of the multi-objective techniques with priori articulation of preference information given. In this method, the preferred solution is sensitive to the goal vector and the weighting vector given by the decision maker; the same as goal programming technique. However, goal attainment method has fewer variables to work with and is a one-stage method, unlike interactive multi-objective techniques, so it will be computationally faster.

This method requires setting up a goal and weight, b_j and $g_j (g_j \geq 0)$ for $j = 1, 2, 3$, for the three mentioned objective functions. The g_j relates the relative under-attainment of the b_j. For under-attainment of the goals, a smaller g_j is associated with the more important objectives. When g_j approaches 0, then the associated objectivefunction should be fully satisfied or the corresponding objective function value should be less than or equal its goal b_j. g_j, $j = 1, 2, 3$, are generally normalized so that $\sum_{j=1}^{3} g_j = 1$. The proper goal attainment formulation for our problem is:

$$\text{Min } w \tag{6a}$$

s.t.

$$c^T y + \sum_{l=1}^{L} p_l \left(q_l^T x_l + h_l^T z_l + f_l^T e_l\right) - g_1 w \le b_1 \tag{6b}$$

$$\sum_{l=1}^{L} p_l \left(q_l^T x_l + h_l^T z_l + f_l^T e_l - \sum_{l=1}^{L} p_l \left(q_l^T x_l + h_l^T z_l + f_l^T e_l\right)\right)^2$$
$$-g_2 w \le b_2 \tag{6c}$$

$$\sum_{l=1}^{L} p_l DR_l - g_3 w \le b_3 \tag{6d}$$

$$Bx_l = 0 \quad l = 1, \ldots, L \tag{6e}$$

$$Dx_l + z_l \ge d_l \quad l = 1, \ldots, L \tag{6f}$$

$$Sx_l \le s_l \quad l = 1, \ldots, L \tag{6g}$$

$$Rx_l \le My + e_l \quad l = 1, \ldots, L \tag{6h}$$

$$e_l \le Oy \quad l = 1, \ldots, L \tag{6i}$$

$$c^T y + q_l^T x_l + h_l^T z_l + f_l^T e_l - \Omega \le DR_l \quad l = 1, \ldots, L \tag{6j}$$

$$y \in Y \subseteq \{0, 1\}^{|P|} \tag{6k}$$

$$x \in R_+^{|A|\times|K|\times L}, \quad z \in R_+^{|C|\times|K|\times L}, \quad e \in R_+^{|P|\times L}, \quad DR \in R_+^{L} \tag{6l}$$

Lemma 1. *If* (y^*, x^*, z^*, e^*) *is Pareto-optimal, then there exists a b and g pair such that* (y^*, x^*, z^*, e^*) *is an optimal solution to the optimization problem (6).*

The optimal solution using this formulation is sensitive to b and g. Depending upon the values for b, it is possible that g does not appreciably influence the optimal solution. Instead, the optimal solution can be determined by the nearest Pareto-optimal solution from b. This might require that g be varied parametrically to generate a set of Pareto-optimal solutions.

3.2 STEM Method

The main drawback of the goal attainment technique to solve ***(5) is that the prefered solution extremely depends on the goals and weights. To overcome this drawback, we resort to STEM and SWT methods, which are two main interactive multi-objective techniques, to solve the multi-objective model.

In this subsection, we explain the details of the STEM method, which is an interactive approach with implicit trade-off information given. STEM allows the decision maker (DM) to learn to recognize good solutions and the relative importance of the objectives. In this method, phases of computation alternate (interactively) with phases of decision. The major steps of the STEM method to solve the multi-objective problem are:

Step 0. Construction of a pay-off table:
A pay-off table is constructed before the first interactive cycle. Let f_j^*, $j = 1, 2, 3$, be feasible ideal solutions of the following three problems:

Min $f_j(x), \quad j = 1, 2, 3$

s.t.

$$x \in S \ \text{(feasible region of (5))} \tag{7}$$

In the pay-off table, row j corresponds to the solution vector x^*, which optimizes the objective function f_j. A z_{ij} is the value taken on by the ith objective f_i when the jth objective f_j reaches its optimum f_j^*.
Step 1. Calculation phase:
At the mth cycle, the feasible solution to the problem ***(8) is sought, which is the "nearest", in the MINIMAX sense, to the ideal solution f_j^*:

Min γ

s.t.

$$\begin{aligned} &\gamma \geq (f_j(x) - f_j^*)\pi_j, \quad j = 1, 2, 3 \\ &x \in X^m \\ &\gamma \geq 0 \end{aligned} \tag{8}$$

where X^m includes S plus any constraint added in the previous $(m - 1)$ cycles; π_j gives the relative importance of the distances to the optima. Let us consider the jth column of the pay-off table. Let $f_j^{\max}$ and $f_j^{\min}$ be the maximum and minimum values; then π_j, $j = 1, 2, 3$, are chosen such that $\pi_j = \frac{\alpha_j}{\sum_i \alpha_i}$, where $\alpha_j = \frac{f_j^{\max} - f_j^{\min}}{f_j^{\max}}$.

From the above equations, we conclude that if the value of f_j does not vary much from the optimum solution by varying x, the corresponding objective is not sensitive to a variation in the weighting values; therefore, a small weight π_j can be assigned to this objective function.
Step 2. Decision phase:
The compromise solution x^m is presented to the DM. If some of the objectives are satisfactory and others are not, the DM relaxes a satisfactory objective f_j^m enough to allow an improvement of the unsatisfactory objectives in the next iterative cycle.

The DM gives Δf_j as the amount of acceptable relaxation. Then, for the next cycle the feasible region is modified as:

$$X^{m+1} = \begin{cases} X^m \\ f_j(x) \leq f_j(x^m) + \Delta f_j,\ if \quad j = 1, 2, 3 \\ f_i(x) \leq f_i(x^m),\ if \quad i = 1, 2, 3, i \neq j \end{cases} \tag{9}$$

The weight π_j is set to zero and the calculation phase of cycle $m + 1$ begins.

3.3 Surrogate Worth Trade-Off (SWT) Method

In this subsection, we explain the details of the SWT method, which is an interactive approach with explicit trade-off information given. It is a virtue that all the alternatives during the solution process are non-dominated. Thus the decision maker is not bothered with any other kind of solutions. The major steps in the SWT method to solve the multi-objective problem (5) are:

Step 1. Determine the ideal solution for each of the objectives in problem (5). Then set up the multi-objective problem in the form of (10).

Min $f_1\ (x)$

s.t.

$$\begin{aligned} & f_2(x) \leq \varepsilon_2 \\ & f_3(x) \leq \varepsilon_3 \\ & x \in S \ \text{ (Feasible region of problem (5))} \end{aligned} \tag{10}$$

Step 2. Identify and generate a set of non-dominated solutions by varying ε 's parametrically in problem (10). Assuming $\mu_j,\ j = 2, 3$, as the Lagrange multipliers corresponding with the first set of constraints of problem (10), the non-dominated solutions are the ones, which have non-zero values for μ_j.

Step 3. Interact with the DM to assess the surrogate worth function w_j, or the DM's assessment of how much (from -10 to 10) he prefers trading μ_j marginal units of the first objective for one marginal unit of the jth objective $f_j(x)$, given the other objectives remaining at their current values.

Step 4. Isolate the indifference solutions. The solutions, which have $w_j = 0$ for all j, are said to be indifference solutions. If there exists no indifference solution, develop approximate relations for all worth functions $w_j = \widehat{w}_j(f_j, j = 2, 3)$, by multiple regressions. Solve the simultaneous equations $\widehat{w}_j(f) = 0$ for all j to obtain f^* (f^* does not include $f_1{}^*$). Then, solve problem (11). Present this solution to the DM, and ask if this is an indifference solution. If yes, it is a preferred solution; proceed to Step 5. Otherwise, repeat the process of generating more non-dominated solutions around $\widehat{w}_j(f) = 0$ and refining the estimated f^* until it results in an indifference solution.

Min $f_1(x)$

s.t.

$$\begin{aligned} f_2(x) &\le f_2^*(x) \\ f_3(x) &\le f_3^*(x) \\ x &\in S \end{aligned} \tag{11}$$

Step 5. The optimal solution $f_1{}^*$ along with f^* and x^* would be the optimal solution to the multi-objective problem (5).

4 Numerical Experiments

Consider the supply chain network design problem depicted in Fig. 1. A wine company is willing to design its supply chain. This company owns three customer centers located in three different cities L, M, and N, respectively. Uniform-quality wine in bulk (raw material) is supplied from four wineries located in A, B, C, and D. There are four possible locations E, F, G, and H for building the bottling plants.

For simplicity, without considering other market behaviors (e.g., novel promotion, marketing strategies of competitors, and market-share effect in different markets), each market demand merely depends on the local economic conditions. Assume that the future economy is either boom, good, fair, or poor, i.e., four situations with associated probabilities of 0.13, 0.25, 0.45, or 0.17, respectively. The unit production costs and market demands under each scenario are shown in Table 1.

The supplies, transportation costs, and shortage costs are considered as deterministic parameters. (475,000, 425,000, 500,000, 450,000) are investment costs for

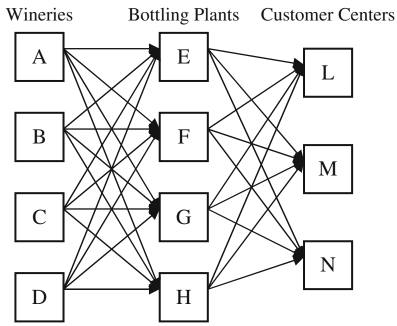

Fig. 1 The supply chain design problem of the wine company

Table 1 Characteristics of the problem

Future economy	Demands			Unit production costs				Probabilities
	L	M	N	E	F	G	H	
Boom	400	188	200	755	650	700	800	0.13
Good	350	161	185	700	600	650	750	0.25
Fair	280	150	160	675	580	620	720	0.45
Poor	240	143	130	650	570	600	700	0.17

Table 2 Pay-off table

Mean	1856986	307077100000	119113
Var	6165288	0	3985288
DRisk	2179694	1495374000	4467.3

building each bottling plant E, F, G, and H, respectively. (65.6, 155.5, 64.3, 175.3, 62, 150.5, 59.1, 175.2, 84, 174.5, 87.5, 208.9, 110.5, 100.5, 109, 97.8) are the unit costs of transporting bulk wine from each winery A, B, C, and D to each bottling plant E, F, G, and H, respectively. (200.5, 300.5, 699.5, 693, 533, 362, 163.8, 307, 594.8, 625, 613.6, 335.5) are the unit costs of transporting bottled wine from each bottling plant E, F, G, and H to each distribution center L, M, and N, respectively. (10,000, 13,000, 12,000) are the unit shortage costs at each distribution center L, M, and N, respectively. (375, 187, 250, 150) are the maximum amount of bulk wine that can be shipped from each winery A, B, C, and D, respectively, if it is reliable. (315, 260, 340, 280) are the capacities of each bottling plant E, F, G, and H, respectively, if it is built.

We also have the option of expanding the capacity of bottling plant F, if it is built. (100, 80, 60, 50) are the unit capacity expansion costs, when the future economy is boom, good, fair or poor, respectively. In addition, we cannot expand the capacity of this plant more than 40 units in any situation. Moreover, winery D is an unreliable supplier and may lose its ability to supply the bottling plants. The reliability of this winery is estimated as 0.9. So, the total number of scenarios for this supply chain design problem is equal to $4 \times 2 = 8$.

The problem attempts to minimize the expected total cost, the variance of the total cost and the downside risk in a multi-objective scheme while making the following determinations:

(a) Which of the bottling plants to build (first-stage variables)?
(b) Flows of materials transported among the entities of the network (second-stage variables)?

First, goal attainment technique is used to solve this multi-objective supply chain design problem (refer to Azaron et al. (2008) to see some results). Then, we use STEM and SWT methods to solve the problem.

In the beginning, we construct the pay-off table, using STEM method, which is shown in Table 2.

Then, we go to the calculation phase and solve the problem (8) using LINGO 10 on a PC Pentium IV 2.1-GHz processor. The compromise solution for the location (strategic) variables is $[1, 1, 1, 0]$. Then, we go to the decision phase and the compromise solution is presented to the DM, who compares its objective vector $f^1 = \left(f_1{}^1, f_2{}^1, f_3{}^1\right) = (2219887, 253346, 39887)$ with the ideal one, $f^* = \left(f_1^*, f_2^*, f_3^*\right) = (1856986, 0, 4467.3)$. If $f_2{}^1$ is satisfactory, but the other objectives are not, the DM must relax the satisfactory objective $f_2{}^1$ enough to allow an improvement of the unsatisfactory objectives in the first cycle. Then, $\Delta f_2 = 999746654$ is considered as the acceptable amount of relaxation, and the feasible region is modified as following in the next iteration cycle:

$$X^2 = \begin{cases} X^1 \\ f_1(x) \leq 2219887 \\ f_2(x) \leq f_2(x^1) + 999746654 = 1000000000 \\ f_3(x) \leq 39887 \end{cases} \tag{12}$$

In this case, three times of the acceptable level of the standard deviation is almost equal to 1,00,000, and comparing this with the expected total cost implies an acceptable amount of relaxation for the variance. In the second cycle, the compromise solution for the location variables is still $[1, 1, 1, 0]$. This compromise solution is again presented to the DM, who compares its objective vector $f^2 = \left(f_1{}^2, f_2{}^2, f_3{}^2\right) = (2132615, 1000000000, 4467.3)$ with the ideal one. If all objectives of the vector f^2 are satisfactory, f^2 is the final solution and the optimal vector including the strategic and tactical variables would be x^2.

The total computational time to solve the problem using STEM method is equal to 18:47 (mm:ss), comparing to 02:26:37 (hh:mm:ss) in generating 55 Pareto-optimal solutions using goal attainment technique, see Azaron et al. (2008) for details.

We also use the SWT method to solve the same problem. Using this method, the single objective optimization problem for generating a set of non-dominated solutions is formulated according to (10). Then, $\varepsilon_j,\ j = 2, 3$, are varied to obtain several non-dominated solutions.

For example, by considering $\varepsilon_2 = 100000$ and $\varepsilon_3 = 1000$, we obtain a non-dominated solution for the location variables as $[1, 1, 1, 0]$, which can also be considered as an indifference solution by the DM. This indifference solution has the same structure as the STEM compromise solutions, but certainly with different second-stage variables. The corresponding objective vector is $f^* = (f_1{}^*, f_2{}^*, f_3{}^*) = (2143678, 748031500, 4467.3)$. The computational time to get this solution is equal to 06:39 (mm:ss).

Comparing this solution with the final STEM solution shows that however the risk has been reduced in the SWT method, but the expected total cost has been increased, and none of these solutions can dominate the other one.

5 Conclusion

Determining the optimal supply chain configuration is a difficult problem since a lot of factors and objectives must be taken into account when designing the network under uncertainty. The proposed model in this paper accounts for the minimization of the expected total cost, the variance of the total cost, and the downside risk in a multi-objective scheme to design a robust supply chain network. By using this methodology, the trade-off between the expected total cost and risk terms can be obtained. The interaction between the design objectives has also been shown. Therefore, this approach seems to be a good way of capturing the high complexity of the problem.

We used goal attainment, which is a simple multi-objective technique, and STEM and SWT methods, which are two main interactive multi-objective techniques, to solve the problem. The main advantage of these interactive techniques is that the prefered solution does not depend on the goal and weight vectors, unlike traditional goal programming technique. We also avoided using several more binary variables in defining financial risk by introducing downside risk in this paper, which significantly reduced the computational times.

Since the final mathematical model is a large-scale mixed-integer nonlinear program, developing a meta-heuristic approach such as genetic algorithm or simulated annealing will be helpful in terms of computational time.

In case the random data vector follows a known continuous joint distribution, one should resort to a sampling procedure, for example Santoso et al. (2005), to solve the problem. In this case, an integration of sampling strategy along with Benders decomposition technique would be suitable to solve the resulting stochastic mixed-integer program.

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