EE 746 : Neuromorphic Engineering

Assignment 1: Spiking Neuron Models

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Note: The codes for all the Neuron models are available *here*.

Problem 1: Leaky Integrate and Fire Model

(a) The dynamics of the membrane potential $V(t)$, in the LIF neuron model is given by the equation

$$C\frac{dV(t)}{dt} = -g_L(V(t) - E_L) + I_{app}(t) \tag{1}$$

The steady state value of the membrane potential, for a constant current I_0 applied to the neuron, is ($i.e.$ $\frac{dV}{dt} = 0$)

$$\boxed{V_{ss} = E_L + \frac{I_0}{g_L}}$$

Given that $C = 300\ p\text{F},\ g_L = 30\ n\text{S},\ V_T = 20\ m\text{V}$ and $E_L = -70\ m\text{V}$,
In order to initiate a spike, we must have

$$\begin{aligned} V_{ss} &\geq V_T \\ \therefore \qquad \frac{I_0}{g_L} + E_L &\geq V_T \\ I_0 &\geq\ g_L \times (V_T - E_L) \end{aligned} \tag{2}$$

Thus, the minimum value of the steady state current, I_C necessary to initiate a spike is

$$\boxed{I_C = g_L \times (V_T - E_L) = 2.70\ n\text{A}}$$

(b) The equivalent difference equation for (1) is written using Range-Kutta second-order method.

$$\text{Let} \quad \frac{dV(t)}{dt} = f(t, V) := C^{-1} \times (-g_L(V - E_L) + I_{app}),\ \text{given } V(0) = V_{ss}$$

$$\begin{aligned} k_1 &= hf(t_n, V_n) \\ k_2 &= hf(t_n + h, V_n + k_1) \\ \therefore V_{n+1} &= V_n + 0.5 \times (k_1 + k_2) \end{aligned}$$

Numerically solving the equation for a set of N neurons, for ith instant in time, the input and output matrices are I and V, wherein the columns I_t, V_t denote the values of currents and membrane potentials of N neurons at $'t'$th instant:

$$I_t = \begin{pmatrix} I_{1t} & I_{2t} & I_{2t} & I_{3t} & \dots & I_{Nt} \end{pmatrix}^T$$
$$V_t = \begin{pmatrix} V_{1t} & V_{2t} & V_{2t} & V_{3t} & \dots & V_{Nt} \end{pmatrix}^T$$

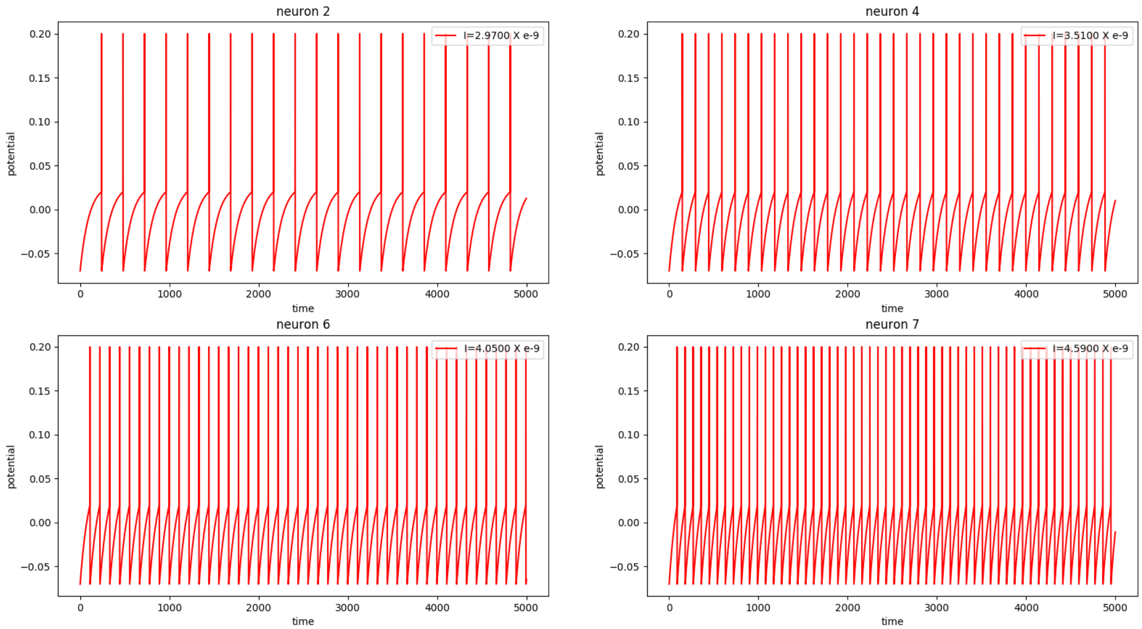
(c) Let the magnitude of the input current for the k^{th} neuron be given by the expression ($\alpha = 0.1$)

$$I_{app,k} = (1 + k\alpha)I_c$$

The required plots are listed below.

(d) Plot of the average time interval between spikes from (c) as a function of $I_{app,k}$

Figure 1: The membrane potential for neurons 2, 4, 6 and 8 from $t = 0$ to $500ms$



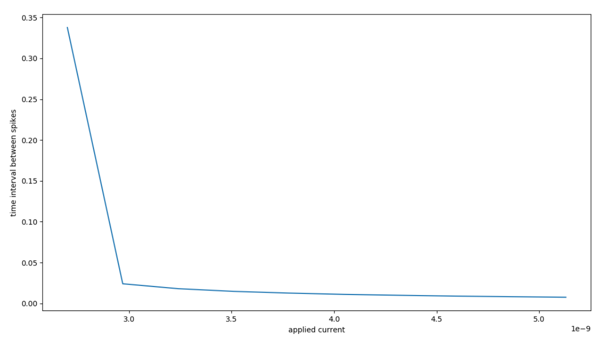


Figure 2: The average time interval between spikes from (c) as a function of $I_{app,k}$

Problem 2: Izhikevich Model

(a) The dynamics of the membrane potential $V(t)$, in the Izhikevich neuron model is given by the equations

$$C\frac{dV(t)}{dt} = k_z(V(t) - E_r)(V(t) - E_t) - U(t) + I_{app}(t) \tag{3}$$

$$\frac{dU(t)}{dt} = a[b(V(t) - E_r) - U(t)]$$

When $V(t) \geq v_{peak}$, $V(t) \to c$ and $U(t) \to U(t) + d$.
Thus, the steady state values of V and U for $I_{app} = 0$ are given by the following expressions:

$$\boxed{V_{ss} = \frac{b}{k_z} + E_t}$$

$$\boxed{U_{ss} = b\left(\frac{b}{k_z} + E_t - E_r\right)}$$

Neuron type	V$_{ss}$(mV)	U$_{ss}$(pA)
RS	-42.8571	-34.2858
IB	-40.8333	170.8335
CH	-39.3333	20.6667

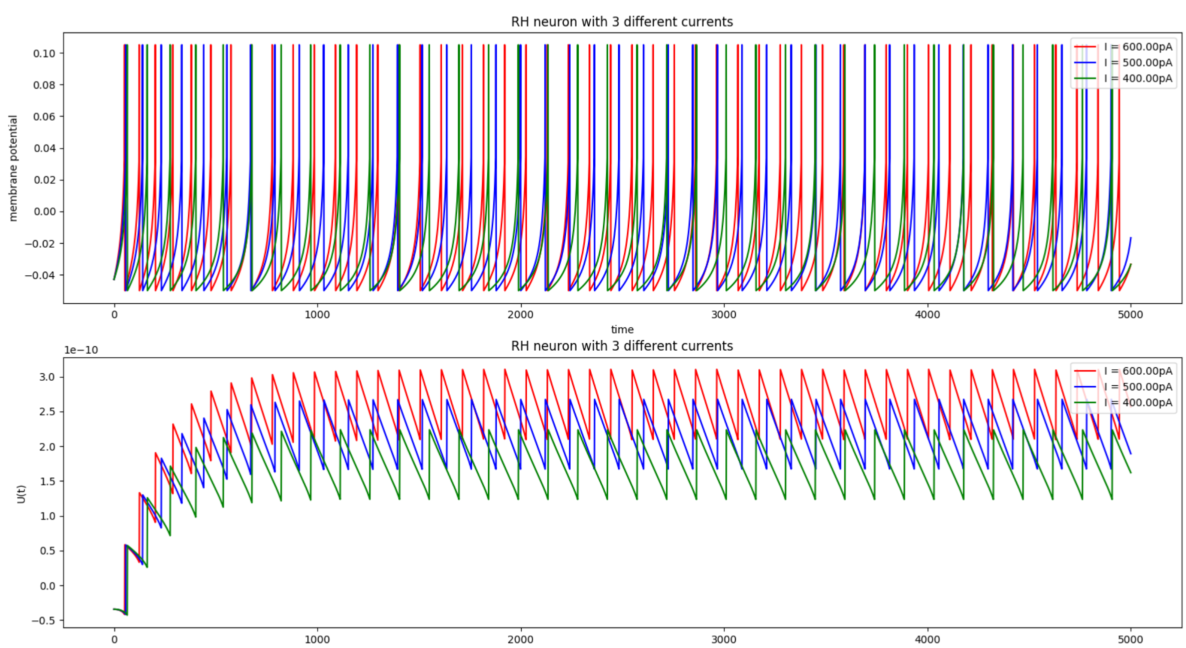
(b) The equivalent difference equation for (3) is written using Range-Kutta fourth-order method.

$$\text{Let} \quad \frac{dV(t)}{dt} = f(t, V) := C^{-1}(k_z(V(t) - E_r)(V(t) - E_t) - U(t) + I_{app}(t)), \text{ given } V(0) = V_{ss}$$

$$\begin{aligned} V_{n+1} &= V_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \\ t_{n+1} &= t_n + h \\ k_1 &= hf(t_n, V_n) \\ k_2 &= hf(t_n + \frac{1}{2}h, V_n + \frac{1}{2}k_1) \\ k_2 &= hf(t_n + \frac{1}{2}h, V_n + \frac{1}{2}k_2) \\ k_2 &= hf(t_n + h, V_n + k_3) \end{aligned}$$

(c) The required plots are listed below.[1] (Note: $= 500ms$)

Figure 3: The membrane potential for RH neuron for $t = 0$ to $500ms$ and $I_{app} = 400, 500, 600\ p$A



[1]The setup of the problem remains same as in problem 1.

Figure 4: The membrane potential for IB neuron for $t = 0$ to $500ms$ and $I_{app} = 400, 500, 600$ pA

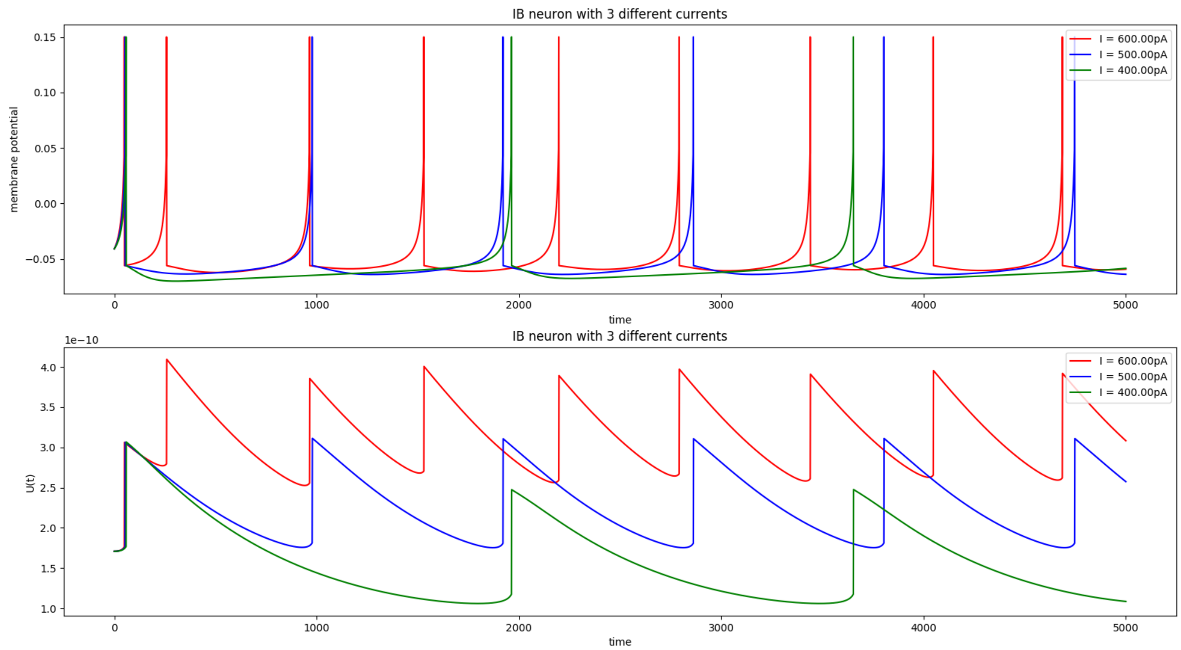
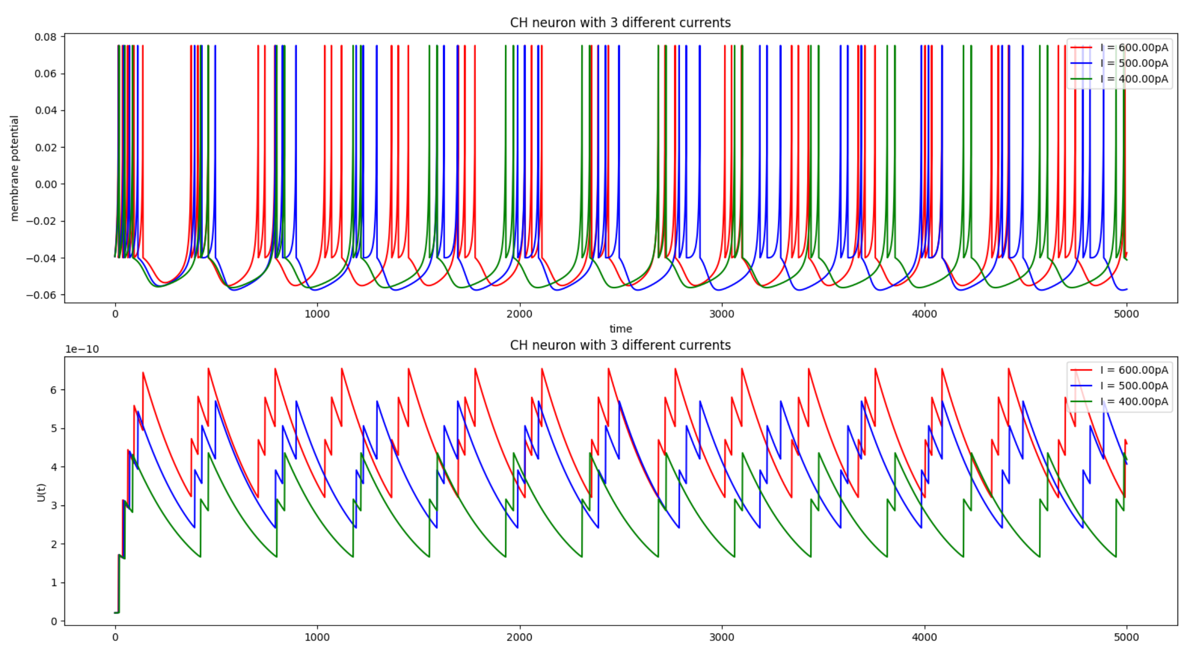


Figure 5: The membrane potential for CH neuron for $t = 0$ to $500ms$ and $I_{app} = 400, 500, 600$ pA



Problem 3: Adaptive Exponential Integrate-and-Fire Model

(a) The dynamics of the membrane potential $V(t)$, in the AEF neuron model is given by the equations

$$C\frac{dV(t)}{dt} = g_L(V(t) - E_L) + g_L\Delta_T e^{\frac{V(t)-V_T}{\Delta_T}} - U(t) + I_{app}(t) \tag{4}$$

$$\tau_w \frac{dU(t)}{dt} = a[V(t) - E_L] - U(t)$$

When $V(t) \geq 0$, $V(t) \rightarrow V_r$ and $U(t) \rightarrow U(t) + d$.

The equivalent difference equation for (3) is written using Euler method.

$$\text{Let} \quad \frac{dV(t)}{dt} = f(t, V) := C^{-1}(g_L(V(t) - E_L) + g_L\Delta_T e^{\frac{V(t)-V_T}{\Delta_T}} - U(t) + I_{app}(t)), \text{ given } V(0) = V_{ss}$$

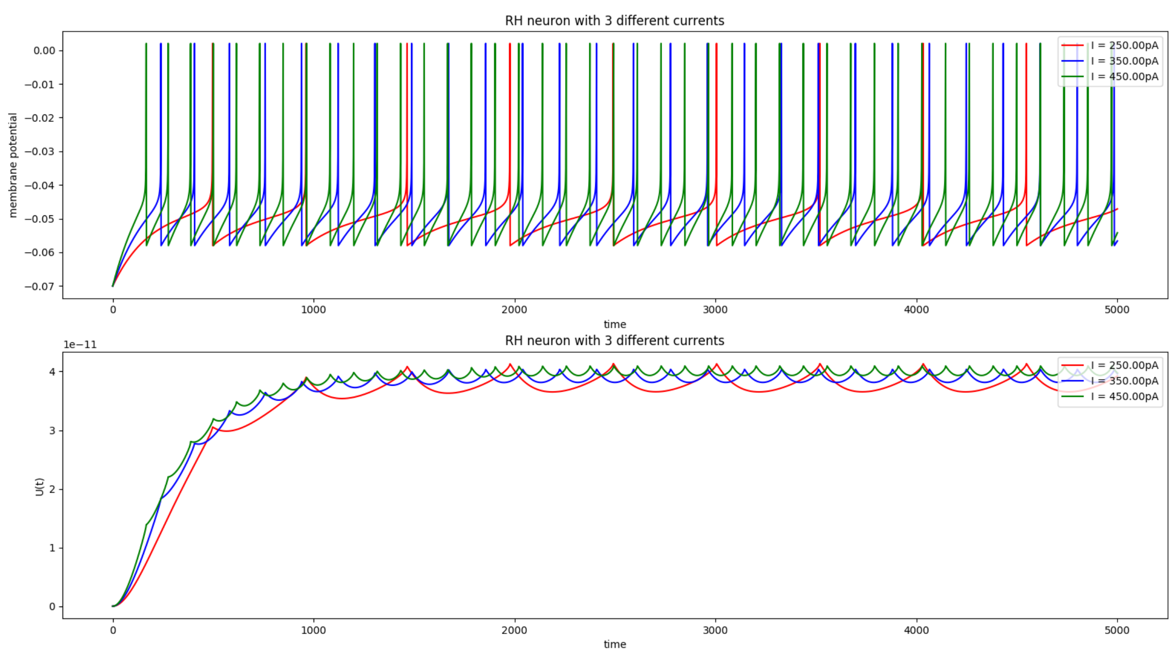
$$V_{n+1} = V_n + hf(V_n, t_n)$$
$$t_{n+1} = t_n + h$$

(b) Solving numercially, the steady state values of V and U for $I_{app} = 0$ are

Neuron type	$V_{ss}(m\text{V})$	$U_{ss}(p\text{A})$
RS	-69.9999	0.0002
IB	-57.9696	0.1216
CH	-57.9690	0.0620

(c) The required plots are listed below.[2]

Figure 6: The membrane potential for RH neuron for $t = 0$ to $500ms$ and $I_{app} = 250, 350, 450$ pA



[2]The setup of the problem remains same as in problem 1.

Figure 7: The membrane potential for IB neuron for $t = 0$ to $500ms$ and $I_{app} = 250, 350, 450\ pA$

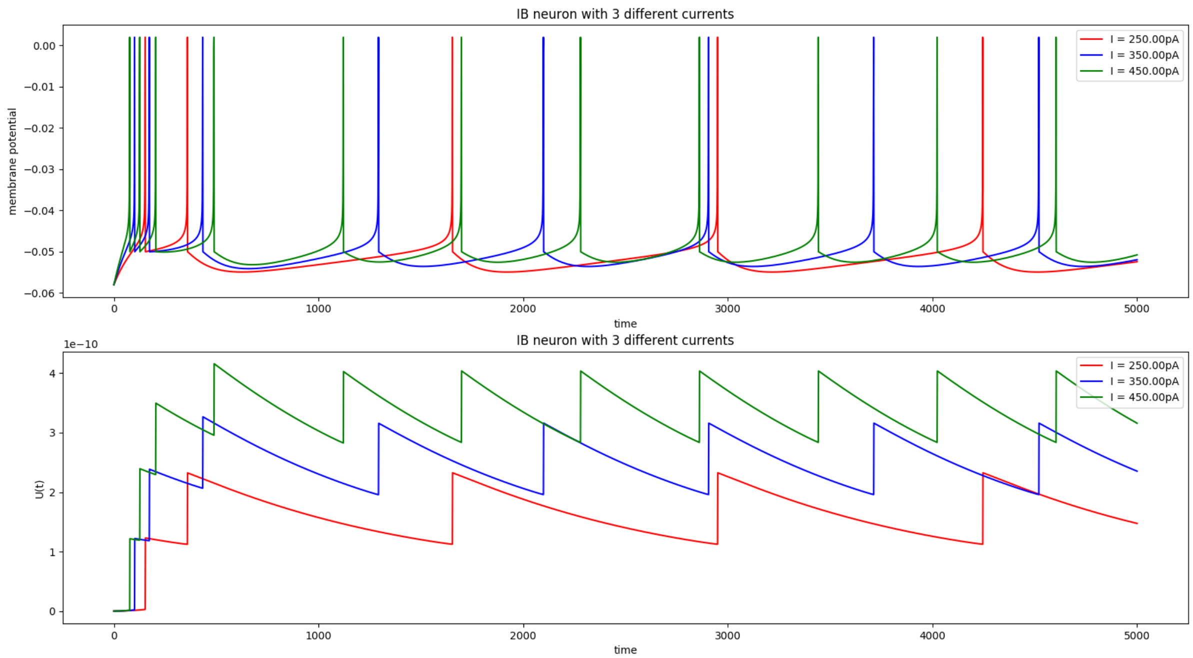
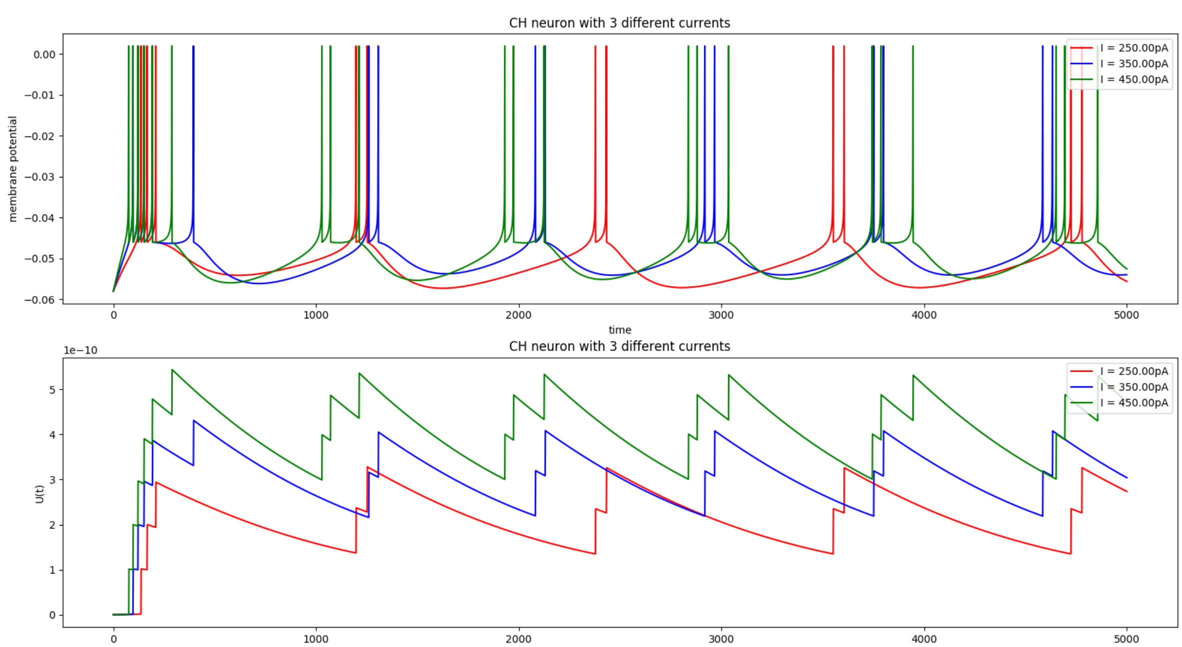


Figure 8: The membrane potential for CH neuron for $t = 0$ to $500ms$ and $I_{app} = 250, 350, 450\ pA$



Problem 4: Spike energy based on Hodgkin-Huxley neuron model

(a) The dynamics of the membrane potential $V(t)$, in the Hodgkin-Huxley neuron model is given by the equations

$$C\frac{dV(t)}{dt} = -i_{Na}(t) - i_K(t) - i_l(t) + I_{ext}(t) \tag{5}$$

where

$$\begin{aligned} i_{Na}(t) &= g_{Na}m^3h(V(t) - E_{Na}) \\ i_K(t) &= g_K n^4(V(t) - E_K) \\ i_l(t) &= g_l(V(t) - E_l) \end{aligned}$$

The variables n, m and h lie in the interval [0, 1] and obey the equation

$$\frac{dx}{dt} = \alpha_x(t)(1-t) - \beta_x(t)x$$

The difference equation for membrane potential and current is:

$$\begin{aligned} V(n+1) &= V(n) + hf(V(n), t_n) \\ i_x(n+1) &= i_x(n) + h\frac{di_x}{dt} \\ t_{n+1} &= t_n + h \end{aligned}$$

In order to solve for steady state, taking $\frac{dV}{dt} = 0$ and $\frac{di}{dt} = 0$, we get

$$V = \frac{g_{Na}E_{Na}\left(\frac{\alpha_m}{\alpha_m+\beta_m}\right)^3\left(\frac{\alpha_n}{\alpha_n+\beta_n}\right) + g_K E_K\left(\frac{\alpha_n}{\alpha_n+\beta_n}\right)^4 + g_l E_l}{g_{Na}\left(\frac{\alpha_m}{\alpha_m+\beta_m}\right)^3\left(\frac{\alpha_n}{\alpha_n+\beta_n}\right) + g_K\left(\frac{\alpha_n}{\alpha_n+\beta_n}\right)^4 + g_l}$$

We have solved this using iterative methods to obtain steady state value of membrane potential

$$\boxed{V = -65.15\ m\text{V}}$$

and corresponding values of

$$m = 0.0520,\ h = 0.6016,\ n = 0.3153.$$

(b) The required plots are listed below.

Figure 9: The membrane potential applied current and parameters of neuron for $t = 0$ to $500ms$

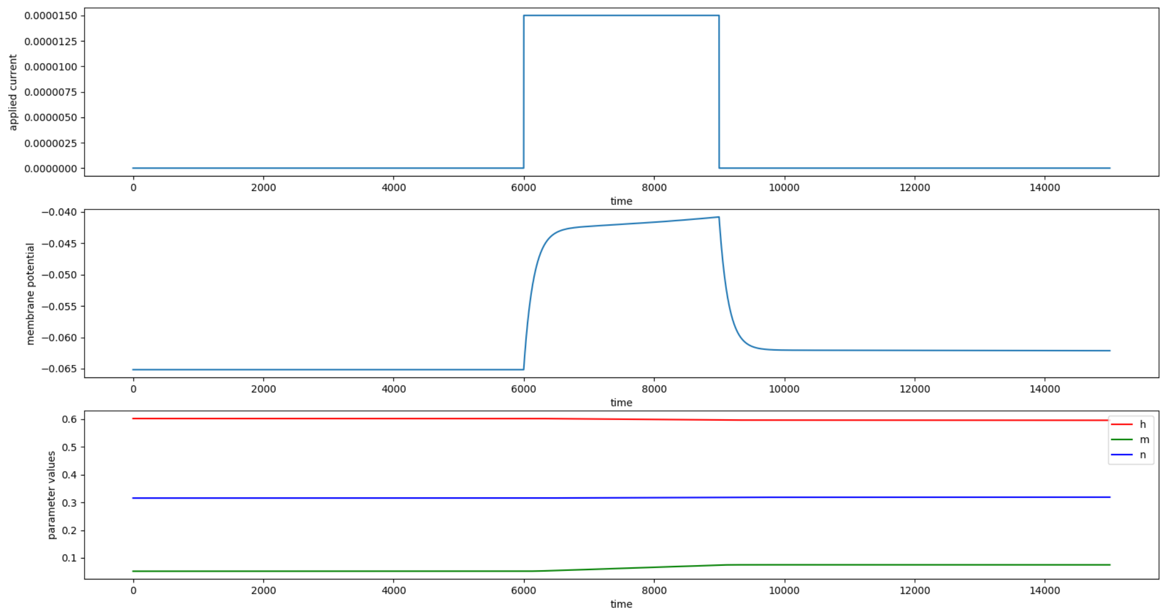
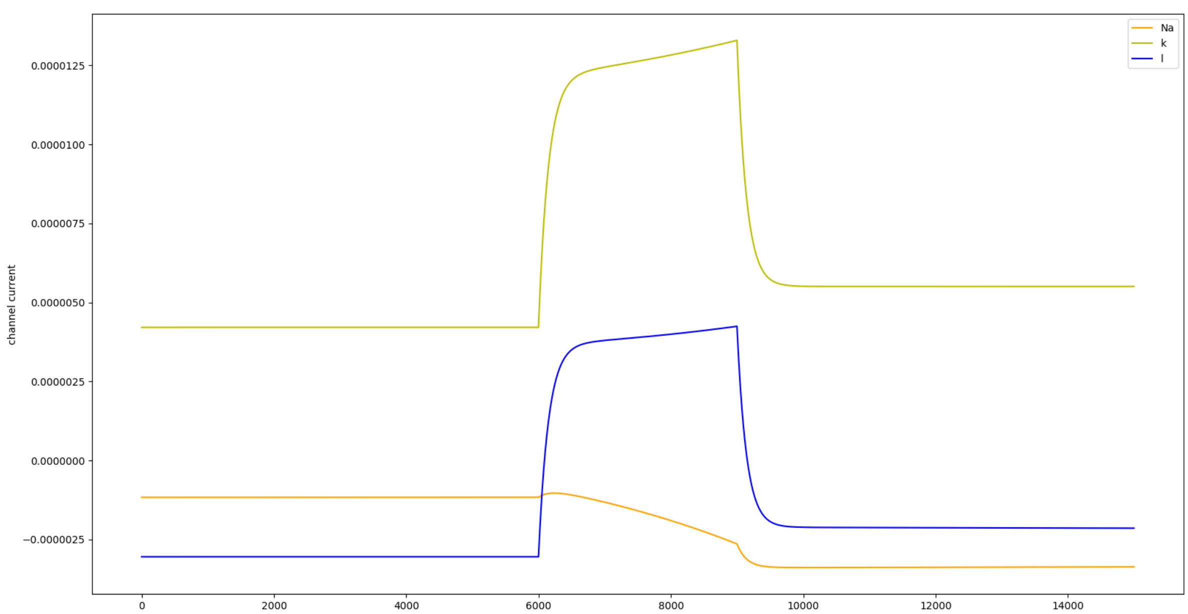
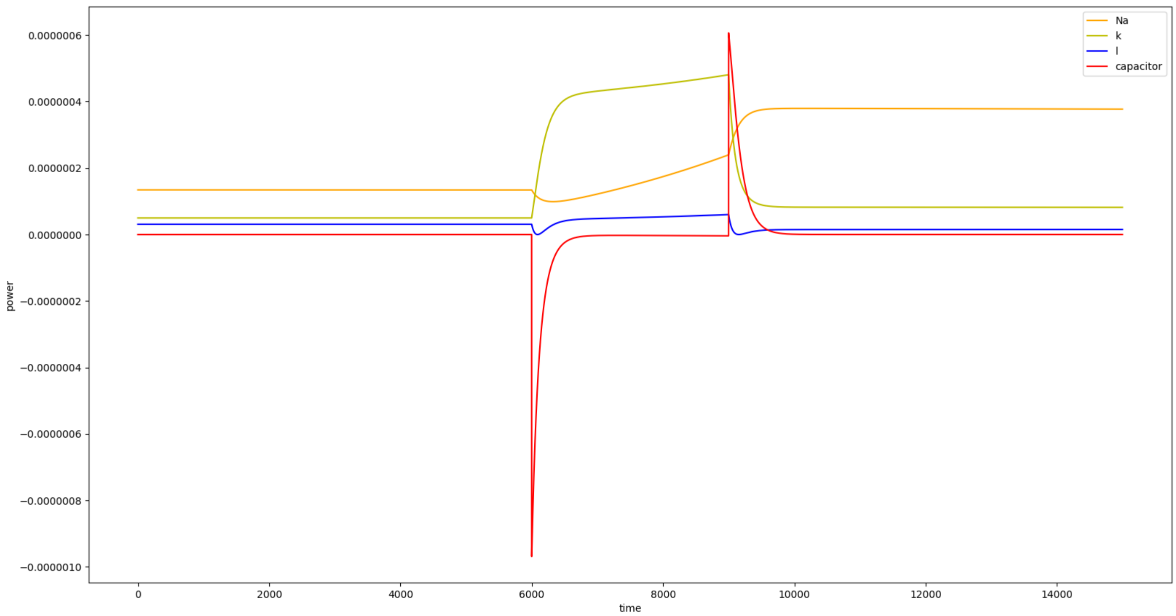


Figure 10: Channel currents $t = 0$ to $500ms$



(c) The power in (b) to determine the total energy dissipated in one cycle of the action potential for a patch of the cell membrane with area of 1 μm^2 is listed.

Figure 11: Power of currents $t = 0$ to $500ms$



Channel	Energy (J)
Na	3.5183×10^{-16}
K	2.1012×10^{-16}
l	4.1322×10^{-16}
C	-1.9012×10^{-16}