EE 746 : Neuromorphic Engineering

Assignment 1: Discerning timing dependent signals

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Note: The codes for all the Neuron models are available *here*.

Problem 1: AEF neuron driven by a synapse receiving Poisson stimulus

(a) Poisson stimulus with $T = 500\ ms,\ \Delta t = 0.1\ ms$ and $\lambda = 10\ /s$ is generated and shown in figure 1

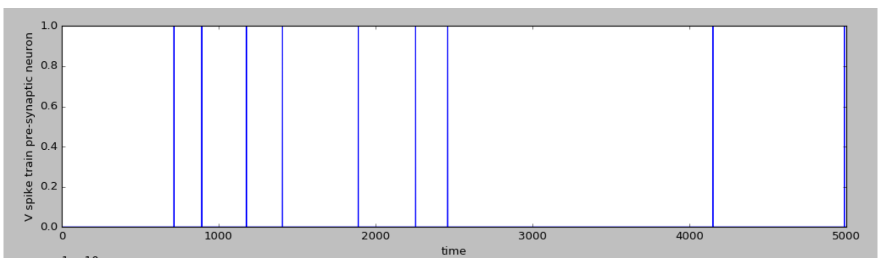

Figure 1: Poisson spike train as a function of t

The time instances (in ms) at which spikes were generated are

$$t_k = [\ 71.6,\ 89.3,\ 117.8,\ 140.6,\ 189.0,\ 225.5,\ 245.9,\ 246.0,\ 415.0,\ 498.8]$$

(b) This stimulus arrives at an AEF RS neuron through a synapse. Then, the total current flowing into the neuron through the synapse at any time t will depend on the stimulus arrival times prior to time t (Say $t_1,\ t_2,\ t_3,\ ...\ t_n,$ with $t_n < t$). Modelling the current at time t according to the expression,

$$I_{app}(t) = I_0 w_e \sum_{m=1}^{n} [e^{-(t-t_m)/\tau} - e^{-(t-t_m)/\tau_s}] \tag{1}$$

Assuming $I_0 = 1p\text{A},\ w_e = 500,\ \tau = 15ms,\ \tau_s = \tau/4$, the response of the neuron along with the input current and stimulus is shown in 2 and 3

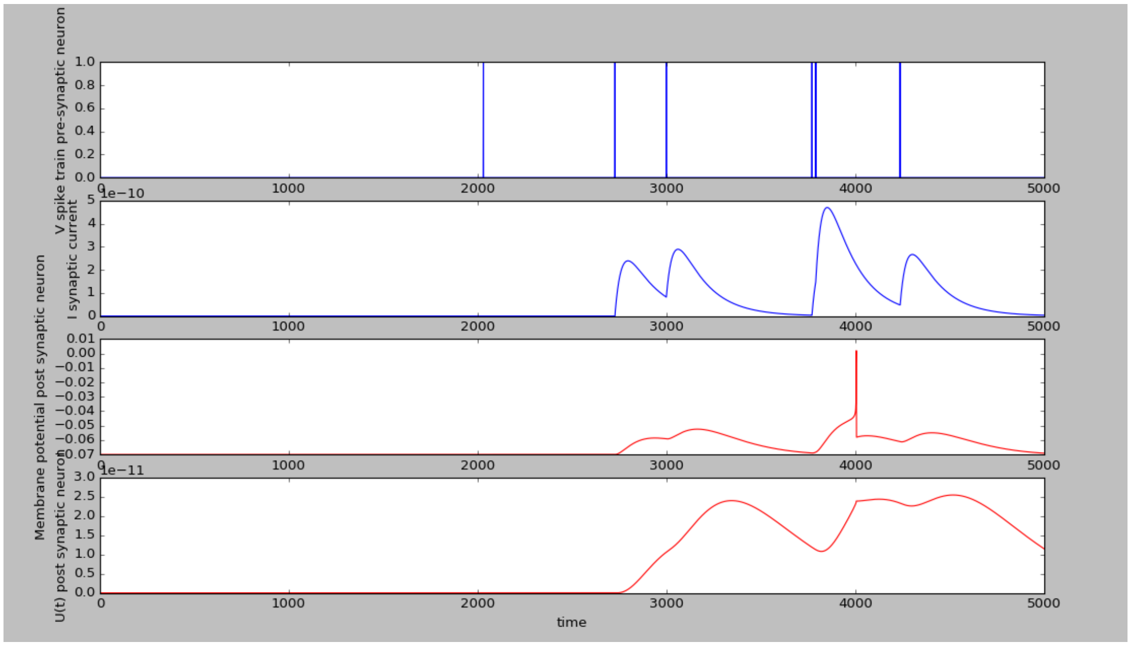

Figure 2: The neuron emitting a spike as closely spaced stimulus are present

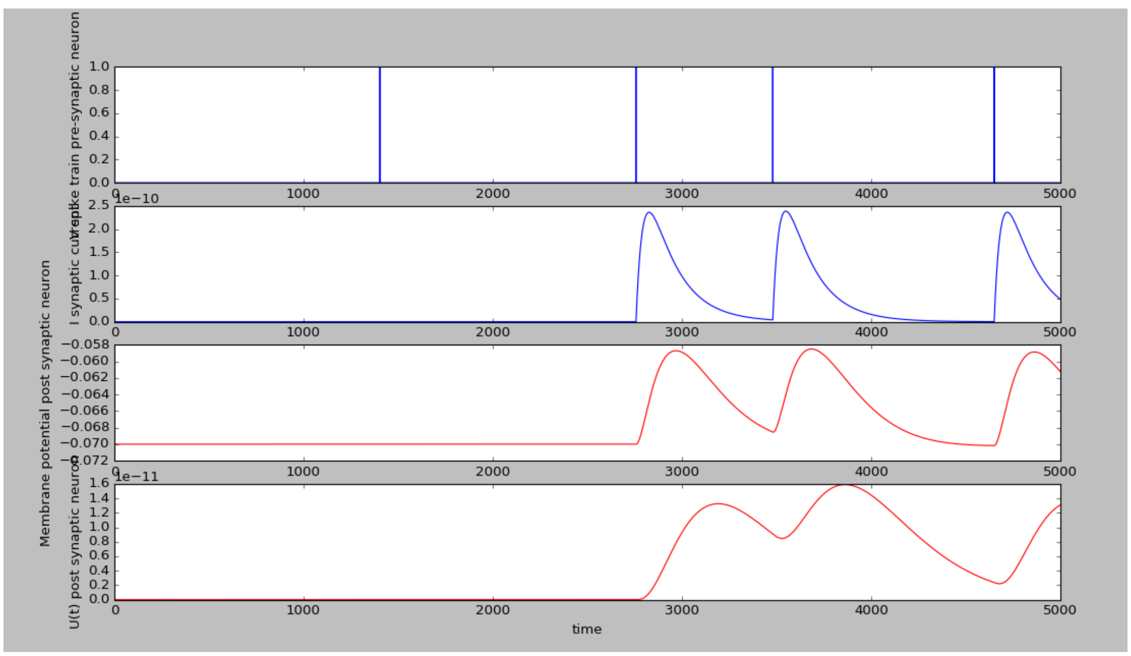

Figure 3: The neuron fails to emit spikes as closely spaced stimuli are absent

Problem 2: AEF neuron driven by multiple synapses

(a) We have a total of $N_s = 100$ synapses driving the neuron, whose connection strengths are Gaussian distributed, with a mean strength of w_0 and standard deviation of σ_w. For $w_0 = 50$ and $\sigma_w = 5$, and with Poisson stimulus (as in Problem 1) to each synapse. The response of the neuron is shown in 4. The total number of spikes generated were **121**.

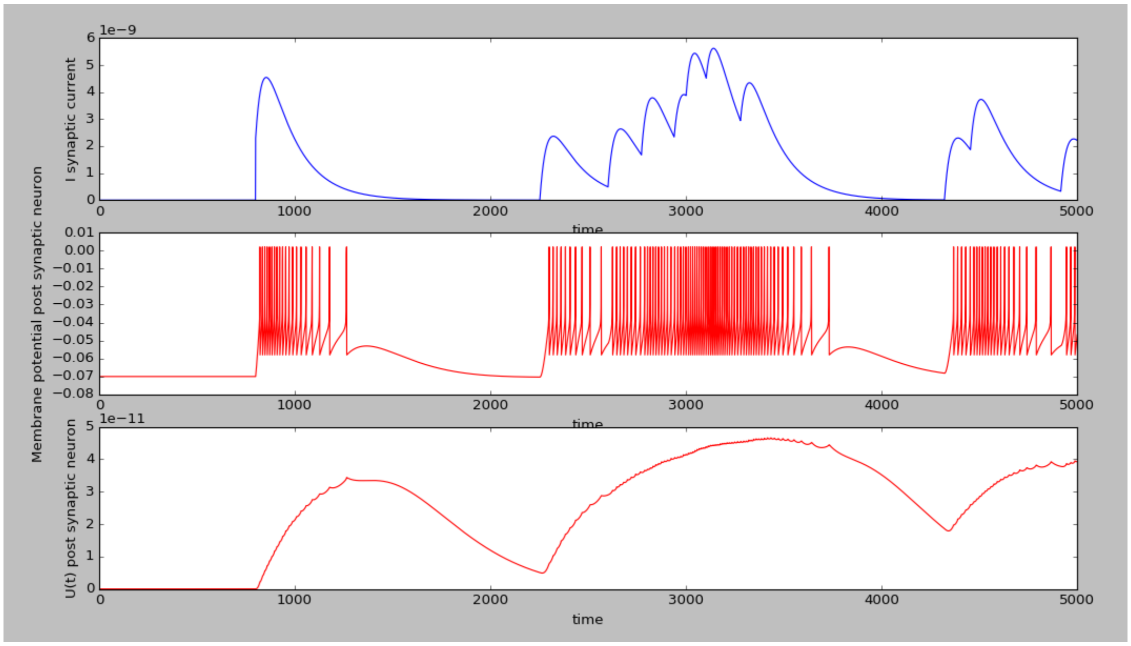

Figure 4: The total current flowing into the neuron based on equation (1) and plot the response of the neuron, for $w_0 = 50,\ \sigma_w = 5$. Total number of spikes generated are 121.

(b) For the same stimulus in (a), the total current and the response of the neuron for the new configuration of synaptic strengths defined by a guassian distribution whose mean is $w_0 = 250,\ \text{and}\sigma_w = 25$ is shown in figure 5. Note here that, the number of spikes are **392**, which is more three times more as compared to the previous case.

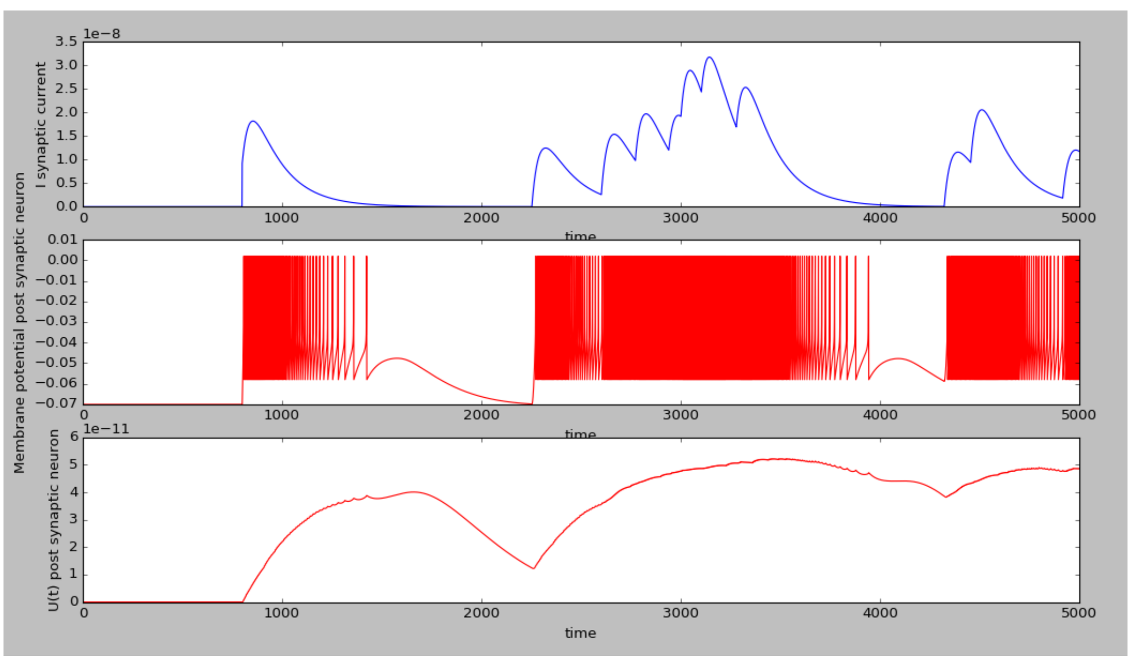

Figure 5: The total current flowing into the neuron based on equation (1) and plot the response of the neuron, for $w_0 = 250,\ \sigma_w = 25$. Total number of spikes generated are 392.

Problem 3: Adjusting the weights to elicit a spike response

(a) For the response of the neuron we have determined the time instant, t_{max} where the neuron membrane potential was the maximum in the interval $[0, T]$. t_k is the time instant for which there was a stimulus just prior to t_{max} for a particular neuron.

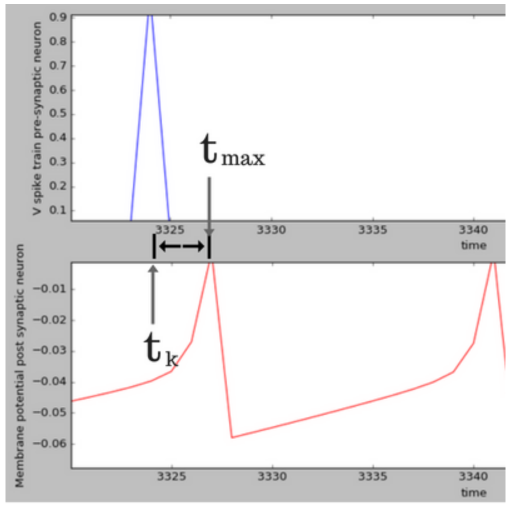

Figure 6: t_k and t_{max} shown for a particular pre-synaptic neuron

The minimum of the difference between t_{max} and t_k gives us the neuron which *most probably* caused the spiking in post synamptic membrane potential. Thus the synaptic strengths can be changed in order to elicit a spike by modifying the strength of each synapse according to (3)

$$\Delta w_k = +w_k \gamma (e^{(-\Delta t_k/\tau)} - e^{(-\Delta t_k/\tau_s)}) h(\Delta t_k) \tag{2}$$

.where γ is a parameter that controls the rate of learning and $h(x)$ is the heaviside step function. With this rule, synapses that have the potential to contribute the most to a neuron spike are selectively increased.

The number of iterations that are required to cause the neuron to create at least one spike for $\gamma = 1$ are **7 (This number varies depending upon the input train which is random, but usually 15)**. The set of weights after training are:

Synapse Id	weight
0	1.101567120741183
1	0.9079021105445441
2	0.982629777387221
3	0.9340789613462565
4	1.0380487749337854
5	1.0056810217595777
6	1.0414063070488255
7	0.9476605843310189
8	0.972161463023082
9	1.049810526278132
10	0.9659322886758613
11	1.0631540928006367
12	1.055840474457723

13	0.9008884661800293
14	0.9987174529197511
15	1.0645874967289328
16	0.9520651628699761
17	0.9586393539232061
18	1.0725026063857612
19	0.9090918562363787
20	1.0175681463413502
21	0.9726308465811287
22	0.944188613964285
23	1.0714766044397188
24	0.9888432992583449
25	0.9581684507371133
26	1.031278881703862
27	1.0246407931623736
28	1.056985950521146
29	0.9849986180128654
30	1.0140136311186052
31	0.9360106757380522
32	1.0588399238628252
33	1.0139792046451261
34	1.0369712171772225
35	0.9901791120233224
36	1.0339356928590362
37	0.9983302110712534
38	1.0058370153373286
39	1.067508080254972
40	1.0202098503216028
41	0.994553849665956
42	0.9518158677816628
43	0.9819054187373224
44	1.0185328680648156
45	1.0464407382305154
46	0.979620135339494
47	1.0231498086168234
48	1.0191972439184793
49	0.9403273724414839
50	1.012952175870823
51	0.9864838188738483
52	1.0017616482972402
53	1.084676080836976
54	0.9949793622934008
55	0.976358751957712
56	1.0555870512805126
57	1.0461790655922787
58	1.0159140759923229
59	1.06462824182256
60	1.0612919601514228
61	0.9529990796238043
62	1.0371745927402924
63	0.9925765866385583
64	1.0327990934086508
65	1.0472893858209538
66	1.0180688053574023

67	0.968385224632883
68	0.9247357691074088
69	1.0434959988153638
70	0.9660568615554284
71	1.009632927893424
72	0.983717320601381
73	1.0491022554697056
74	1.0876322710567177
75	1.032814973052905
76	8.321300640106934
77	1.0030320045594237
78	0.9914818114360234
79	1.0201762123873974
80	0.9705852469818824
81	1.0404989654725034
82	1.0465091633667254
83	0.991901535377666
84	1.020534699744071
85	0.9554462569035407
86	1.0593804707323713
87	1.0496445093753648
88	0.9586343269746105
89	1.0379347053394887
90	1.1111049445667647
91	0.9583774076595636
92	1.0566357846305963
93	0.9866329746757445
94	0.9915430682078544
95	0.9646096386681636
96	1.0101067382819375
97	0.9458302617694241
98	0.943090464905202
99	0.9806124226591645

(b) The required plot is shown in figure 7.

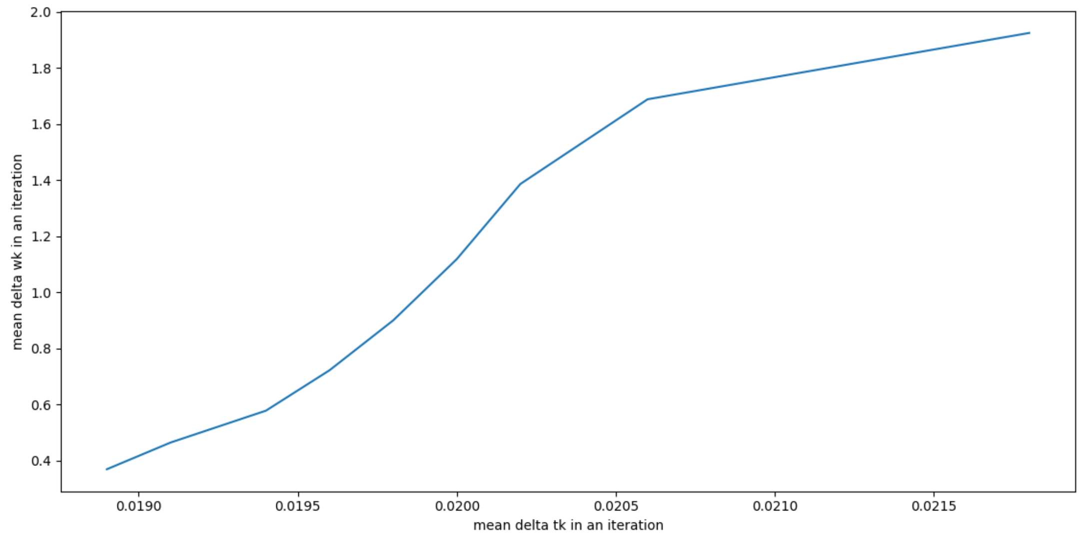

Figure 7: The value of Δw_k vs Δt_k for every synapse and every training iteration

Problem 4: Adjusting the weights to remove all spike responses

(a) Similar to previous question, to implement the scheme of changing the synaptic strengths in order to remove all spike responses is to modify the strength of each synapse depending on the value of Δt_k, the following rule is used:

$$\Delta w_k = -w_k \gamma (e^{(-\Delta t_k/\tau)} - e^{(-\Delta t_k/\tau_s)}) h(\Delta t_k) \tag{3}$$

The number of iterations that are required to cause the neuron to suppress [1] all spikes varies a lot and for some spike trains could not be reached in maximum number of iterations we checked for (see footnote), but for the once where it could be reached for $\gamma = 1$ is **1 (usually 5)**. The set of weights after training are:

Synapse Id	weight
0	25.508825027276256
1	6.395583499929999
2	22.770704833636266
3	12.964009427213169
4	17.25799985902397
5	26.193481340234488
6	23.89607046189178
7	17.700635758802687
8	18.40885809514051
9	10.948340239836824
10	4.719854055055702
11	20.789197120457732
12	19.916489736529527
13	19.39730708135929
14	23.02263203215594
15	20.59888035476015

[1] Note here that after reaching to a particular number of spikes, the spikes don't decrease further since the neurons which were identified as having the significant effect on the spikes in the response, their synaptic weight is clipped to *10*. Thereafter, they keep getting the same input currents but the response doesn't change since the weights are the same.

16	11.16277620773892
17	28.640431082915857
18	21.056637226702538
19	26.662390307515775
20	23.89711702107843
21	23.93048695119431
22	20.041643767794397
23	4.843532469340911
24	18.597987029382946
25	26.113524374733295
26	22.3303646149524
27	16.300975531991448
28	18.825682524001255
29	20.3974930786247
30	21.79690750900051
31	21.37629291824962
32	23.46023082113198
33	22.74544643383698
34	13.762451964307052
35	25.54465294972326
36	2.1633627313309733
37	3.0296724124687606
38	21.867259533165395
39	1
40	15.023531254481838
41	19.732220488432556
42	24.91395823202315
43	19.43236518573193
44	16.515827367371255
45	26.583957489363826
46	18.35685307774998
47	10.418541260286338
48	14.863064638907549
49	28.128244510463944
50	19.704365970884645
51	25.417049761526357
52	20.82907074170286
53	17.805385182672904
54	21.31464573408605
55	0.9987813835813086
56	21.45198655640875
57	21.00320764847007
58	1
59	17.959510206303126
60	13.509592887931152
61	14.788824095417054
62	1
63	1.3451251139203486
64	17.818556082726094
65	1
66	19.661171126617344
67	25.715112865392705
68	20.219207146293932
69	25.91449646627087

70	24.659384849344512
71	27.882731475190365
72	23.86506030091371
73	10.578903311064034
74	23.36183684234902
75	28.98412153351609
76	19.661874406509686
77	13.400242974243469
78	17.157619073992766
79	22.382661000863976
80	15.496572185906368
81	15.79336835146255
82	17.358336262973783
83	26.18233677034802
84	24.988191758434606
85	3.370458283693515
86	12.122446711936554
87	13.796540801859427
88	1
89	23.64311074872076
90	22.32694465847281
91	25.5815071991528
92	18.137610831929695
93	21.136486659903074
94	23.93448427728398
95	1
96	18.199547152491974
97	18.601523039139547
98	13.606921394272472
99	24.75404585689782

(b) The required plot is shown in figure 8.

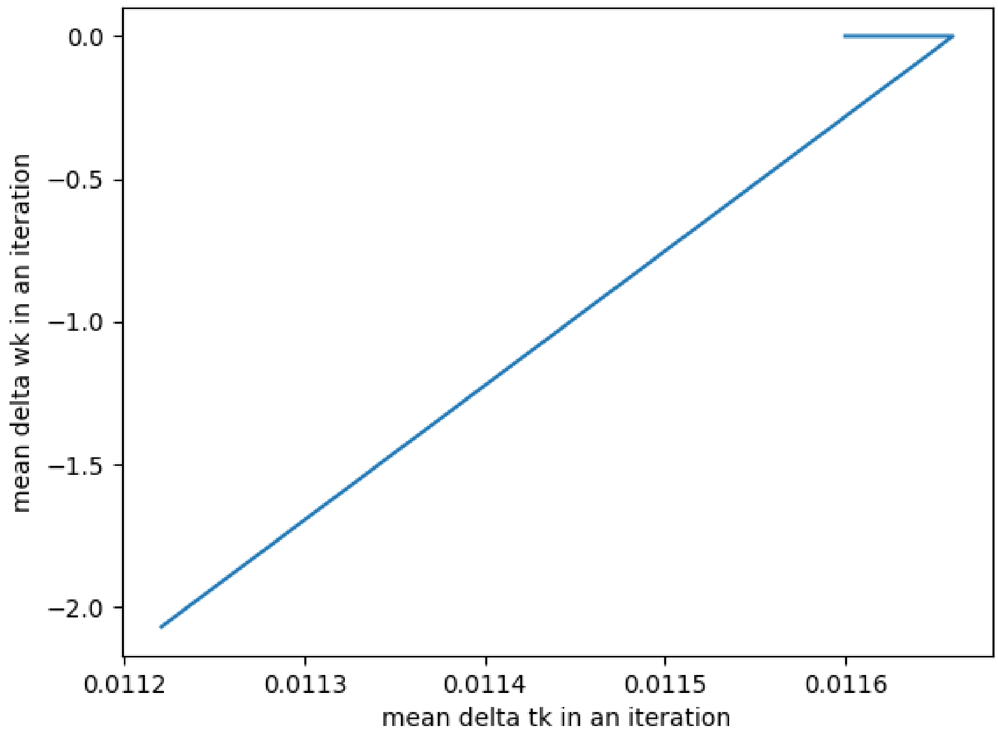

Figure 8: The value of Δw_k vs Δt_k for every synapse and every training iteration

Problem 5: Discriminating stimuli with similar statistical characteristics

(a) Initially, the synaptic population has a $w_0 = 200$ and $\sigma_w = 20$. Creating two stimulus patterns S_1 and S_2 with $T = 500\ ms,\ \Delta t = 0.1\ ms$ and $\lambda = 10\ /s$.
The response of the neuron for stimulus $S_1 and S_2$ for the same starting synaptic strengths is shown in figure 9 and 10.

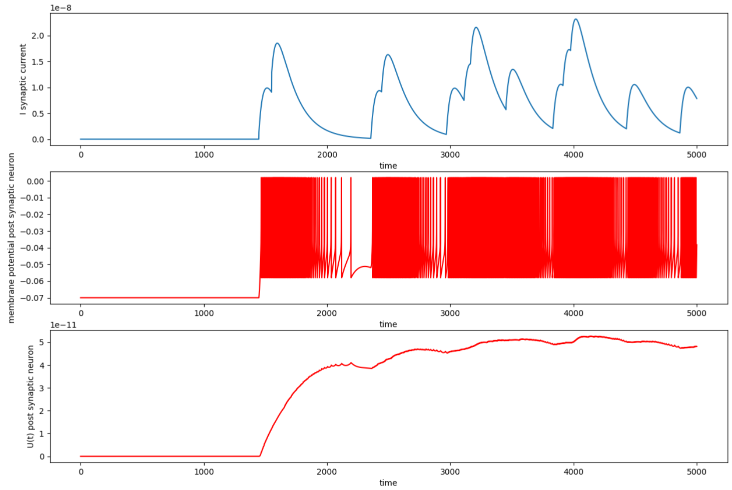

Figure 9: response due to S1 with initial synapse weights

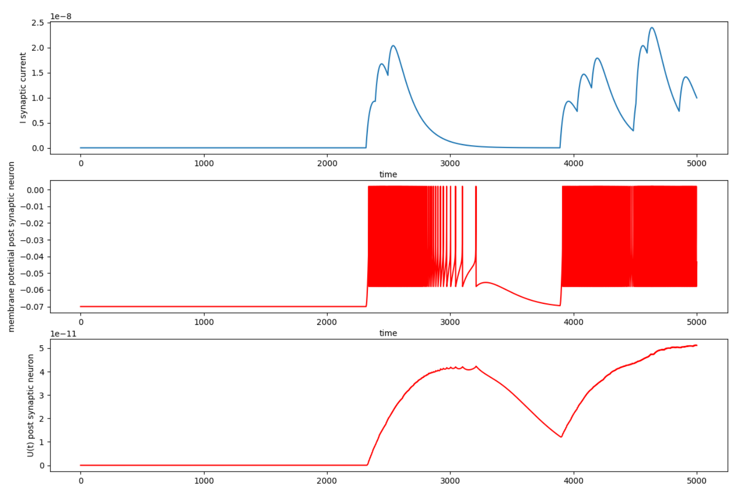

Figure 10: response due to S2 with initial synapse weights

(b) **(Results combined for b and c)** Removing all the spikes with stimulus S_1 while having at-least one spike with stimulus S_2, starting with w_0 , results into a trained network whose response is shown in 11 and 12

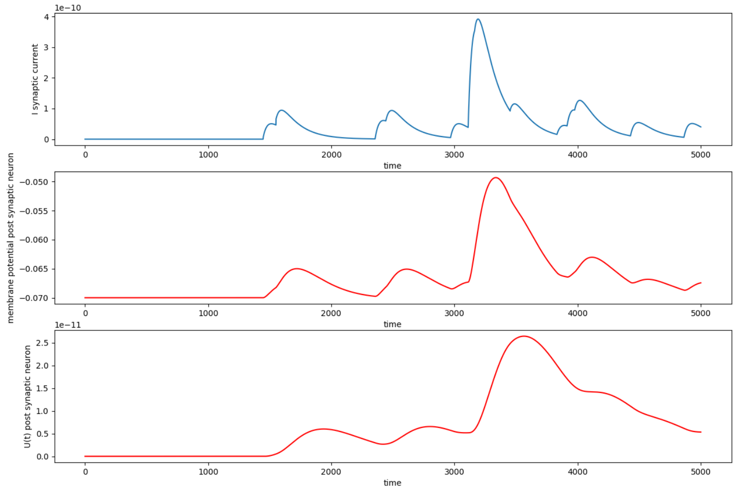

Figure 11: response due to S1 with tuned weight to eliminate spikes

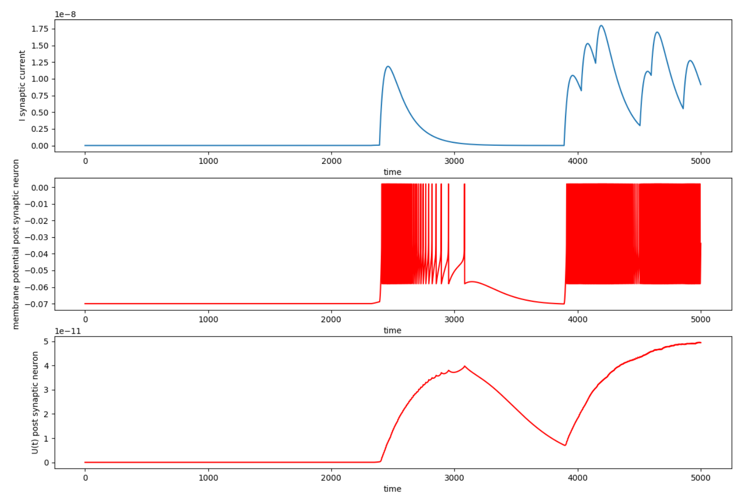

Figure 12: response due to S2 with tuned weight to allow spikes

The above results are obtained using tuned weights:

Synapse Id	weight
0:	1.330547016821936
1:	193.6115413854239
2:	210.95513380591305
3:	1.2802604957888393
4:	221.28432025115563
5:	207.72787180955896
6:	188.54441396682378

7:	1
8:	221.85863297651125
9:	1
10:	0.7354542063302701
11:	182.68187497119018
12:	1
13:	197.0254049251621
14:	189.80864979486614
15:	202.38157308075787
16:	168.83356065520894
17:	207.13917042970021
18:	178.0424830115955
19:	1
20:	1
21:	209.30688392164353
22:	192.28922801467397
23:	185.89759281088394
24:	234.23772572141965
25:	188.58012144660862
26:	185.36742022671197
27:	202.88944935647564
28:	200.54329497464624
29:	195.82787764334014
30:	184.6478087507225
31:	186.81711725293454
32:	212.98147445797338
33:	222.15369829112237
34:	212.7571577821592
35:	190.04981569340978
36:	229.1784414583166
37:	211.9558849155673
38:	164.9798714270231
39:	183.2894106135809
40:	182.2774231499361
41:	201.88016933150965
42:	206.18105744181912
43:	209.40042108461785
44:	177.27994388976094
45:	186.78492779379312
46:	193.1934850720556
47:	232.6575528598328
48:	213.11272118580976
49:	185.32332439241927
50:	7.0760616185023775
51:	189.91271816123324
52:	211.36735024612176
53:	214.4113311622469
54:	190.91165816881818
55:	202.522317692394
56:	201.61348419546937
57:	1
58:	173.55315245965176
59:	154.65313002055285
60:	204.8107274260385

61:	176.35064992666375
62:	175.46427952305416
63:	185.41478134742536
64:	214.29523603597713
65:	194.0090889849494
66:	196.8137736427202
67:	176.14435767278437
68:	169.9326435199673
69:	1
70:	181.2600614356438
71:	232.43627469693058
72:	180.61166548866683
73:	191.58884095484675
74:	205.4155683465986
75:	172.7244148433746
76:	249.96969074645244
77:	219.11063575280366
78:	221.94275252862494
79:	189.05285385675907
80:	187.64736491214376
81:	213.20749687126022
82:	200.52188867301885
83:	1
84:	187.0220626552711
85:	222.34675292868857
86:	190.70199437781875
87:	216.357086584494
88:	211.41719189201694
89:	200.33028391369257
90:	196.60313925527925
91:	205.647434687375
92:	192.69970818985732
93:	202.86619093288198
94:	1
95:	218.30644718592976
96:	145.32283225331764
97:	212.57680222948017
98:	209.80364811040891
99:	196.43278439350775

(c) **(Results for d)** Following a similar procedure necessary to cause a spike response for presentation of S_2 and no spike for presentation of S_1, the results obtained are shown in 13 and 14

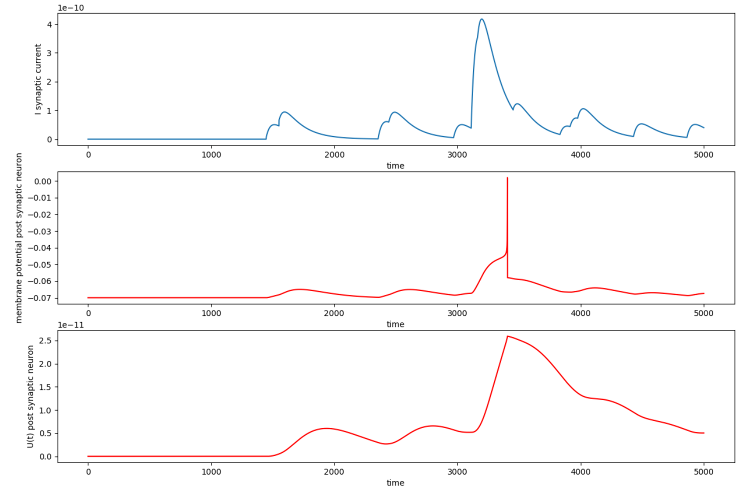

Figure 13: response due to S1 with tuned weight to allow for spikes

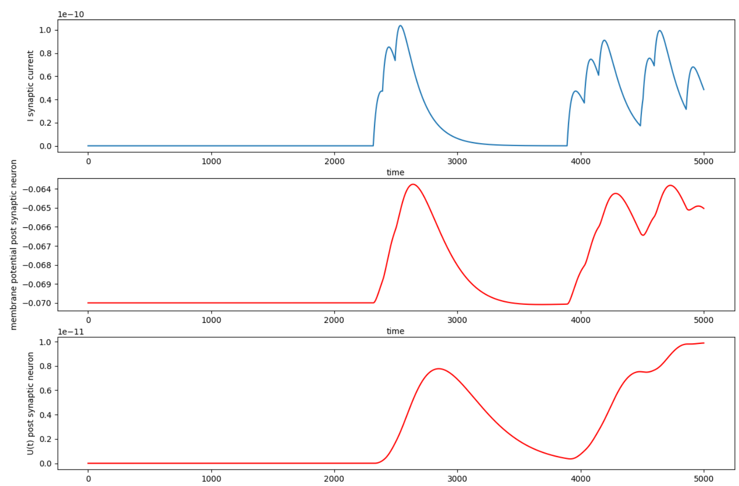

Figure 14: response due to S2 with tuned weight to eliminate spikes

The above results are obtained using weights:

Synapse Id	weight
0:	0.8478917863962703
1:	0.5458464008283266
2:	210.95513380591305
3:	1.2802604957888393
4:	221.28432025115563
5:	207.72787180955896
6:	188.54441396682378

7:	1
8:	221.85863297651125
9:	1
10:	0.7354542063302701
11:	182.68187497119018
12:	1
13:	197.0254049251621
14:	189.80864979486614
15:	202.38157308075787
16:	168.83356065520894
17:	207.13917042970021
18:	178.0424830115955
19:	1
20:	1
21:	209.30688392164353
22:	1
23:	185.89759281088394
24:	234.23772572141965
25:	188.58012144660862
26:	185.36742022671197
27:	202.88944935647564
28:	200.54329497464624
29:	195.82787764334014
30:	184.6478087507225
31:	186.81711725293454
32:	212.98147445797338
33:	1
34:	212.7571577821592
35:	1
36:	229.1784414583166
37:	211.9558849155673
38:	164.9798714270231
39:	183.2894106135809
40:	182.2774231499361
41:	201.88016933150965
42:	206.18105744181912
43:	209.40042108461785
44:	177.27994388976094
45:	186.78492779379312
46:	193.1934850720556
47:	232.6575528598328
48:	213.11272118580976
49:	185.32332439241927
50:	7.0760616185023775
51:	189.91271816123324
52:	211.36735024612176
53:	214.4113311622469
54:	190.91165816881818
55:	202.522317692394
56:	201.61348419546937
57:	1
58:	173.55315245965176
59:	154.65313002055285
60:	204.8107274260385

61:	176.35064992666375
62:	175.46427952305416
63:	185.41478134742536
64:	214.29523603597713
65:	1
66:	196.8137736427202
67:	176.14435767278437
68:	169.9326435199673
69:	1.6720769029608693
70:	181.2600614356438
71:	232.43627469693058
72:	180.61166548866683
73:	191.58884095484675
74:	205.4155683465986
75:	172.7244148433746
76:	1
77:	219.11063575280366
78:	221.94275252862494
79:	189.05285385675907
80:	187.64736491214376
81:	213.20749687126022
82:	200.52188867301885
83:	1
84:	187.0220626552711
85:	222.34675292868857
86:	190.70199437781875
87:	216.357086584494
88:	211.41719189201694
89:	200.33028391369257
90:	196.60313925527925
91:	205.647434687375
92:	192.69970818985732
93:	202.86619093288198
94:	1
95:	218.30644718592976
96:	145.32283225331764
97:	212.57680222948017
98:	209.80364811040891
99:	196.43278439350775