# Single-Phase Flow Model in Porous Media

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Posgrado del Instituto Mexicano del Petróleo

2 de mayo de 2023

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# Contenido I

# Introduction

The basic **mathematical model** of fluid flow in a porous medium will be developed. Generally, the final goal of this kind of model is to predict the flow of the fluid (that is, the fluid velocity).

A **porous medium**, is a solid material that contains voids in its interior that are interconnected such that the flow of fluid through them is possible. The solid material that forms the porous medium is referred to as the **solid matrix**, the voids are the **pores**, and the volume fraction of the physical space that is occupied by the pores is the **porosity**.

When considering fluid porous systems the solid porous phase is referred to as the solid matrix. When the solid matrix is compressible, its porosity depends on the fluid pressure; in such a case, generally, the porosity is a function of time.

# Introduction

In single-fluid phase systems, it is assumed that the porous medium is saturated by the fluid; by this we mean that the fluid fills the pores completely, and therefore the volume of the pores equals the volume of the fluid that is contained in the porous medium.

The **fluid pressure** is also referred to as the pore pressure, especially in soil mechanics. The pressure in the pores pushes on the grain surfaces, exerting a force that tends to increase the pores'volume. In general, the grains move in response to changes in fluid pressure such that the porosity increases as the fluid pressure increases, and a decrease in pore fluid pressure results in a decrease in pore volume.

# Conceptual Model

The following assumptions are usually adopted for models of fluid flow through porous media:

- The porous medium is saturated by the fluid, this means that the fluid thoroughly fills the pore space of the solid matrix.
- The solid matrix remains at rest throughout the fluid-flow process.
- The solid matrix is elastic. More precisely, the porosity of the matrix is a function of the fluid pressure. Therefore, the pore-level porosity of the matrix may change as time goes by. This, in spite of the fact that the macroscopic velocity of the solid matrix is zero throughout the whole process.

# Conceptual Model

- The mass of the fluid is conserved.
- The fluid is slightly-compressible. Specifically, the density of the fluid satisfies an equation of state in which the density is a function of the pressure, exclusively.
- The single-phase, is made of only one component, the fluid.
- The fluid velocity fulfills Darcy's law. This is an empirical constitutive equation, which relates the fluid particle velocity to the fluid-pressure distribution.
- The fluid is not subjected to diffusion processes because there are no separate species, so that $\underline{\tau}_f(\underline{x}, t) = \underline{0}$.
- No mass of fluid is generated in the system constituted by the porous material and the fluid (that is, $g_f(\underline{x}, t) = 0$).

# Mathematical Model

We recall **the axiomatic method** for deriving the mathematical models of continuous systems. The flow system is a two-phase system since it consists of the solid matrix and the fluid contained in its pores. However, the fact that the motion of the solid phase is known, since it is at rest, permits dealing with the fluid phase exclusively and treating the system as a single-phase system. This single phase, in turn, is made of only one component, the fluid.

# Mathematical Model

Thus, the family of extensive properties consists of only one **extensive property**, namely the fluid mass:

$$E(t) \equiv M_f(t) \tag{1}$$

When the porous medium is saturated by the fluid, the porosity equals the fraction of the physical space occupied by the fluid, then the **intensive property** associated with the extensive property is

$$\psi(\underline{x}, t) \equiv \rho(\underline{x}, t)\phi(\underline{x}, t) \tag{2}$$

where $\rho(\underline{x}, t)$ is the fluid density and $\phi(\underline{x}, t)$ is the porosity.

# Mathematical Model

So that the mass of the fluid contained in a domain $\Omega(t)$ occupied by the fluid-porous system is given by

$$M_f(t) = \int_{\Omega(t)} \rho(\underline{x}, t)\phi(\underline{x}, t) \quad d\underline{x} \tag{3}$$

Then $\rho(\underline{x}, t)\phi(\underline{x}, t)$ is the mass of fluid per unit volume of the physical space occupied by a fluid porous-medium system.

We are interested in following the motion of fluid bodies, so $\Omega(t)$ will move with the fluid velocity.

# Mathematical Model

**Exercise:**

To write the "global balance equation" (1 minute).

# Mathematical Model

## Global Balance Equation

Since the fluid is not subjected to diffusion because there are no separate species ($\underline{\tau}_f(\underline{x}, t) = \underline{0}$) and assuming that no mass of fluid is generated in the system constituted by the porous material and the fluid (that is, $g_f(\underline{x}, t) = 0$) then the mass of the fluid is conserved.

$$\frac{d}{dt} M_f(t) = \int_{\Omega(t)} 0 \;\; d\underline{x} + \int_{\Omega(t)} \nabla \cdot \underline{0} \;\; d\underline{x} = 0 \qquad (4)$$

# Mathematical Model

**Exercise:**

To write the "local balance equation" (1 minute).

# Mathematical Model

**Local Balance Equation**

Since...

- The intensive property is $\psi(\underline{x}, t) = \rho(\underline{x}, t)\phi(\underline{x}, t)$.
- No mass is created $g_f(\underline{x}, t) = 0$.
- No mass-diffusive processes occur $\underline{\tau}_f(\underline{x}, t) = \underline{0}$

$$\frac{\partial}{\partial t}(\rho\phi) + \nabla \cdot (\rho\phi\underline{v}) = 0 + \nabla \cdot \underline{0} = 0, \qquad \forall \underline{x} \in \Omega \tag{5}$$

It must be pointed out that in regional groundwater studies extraction, or injection of water by wells is frequently incorporated as a distributed external supply, in which case $g_f(\underline{x}, t) = q(\underline{x}, t) \neq 0$

$$\frac{\partial}{\partial t}(\rho\phi) + \nabla \cdot (\rho\phi\underline{v}) = q + \nabla \cdot \underline{0} = q, \qquad \forall \underline{x} \in \Omega \tag{6}$$

# Mathematical Model

**The fluid compressibility** $c_f$ is defined by

$$c_f = \frac{1}{\rho}\frac{\partial \rho}{\partial p} \tag{7}$$

**The rock compressibility** $c_R$ is defined by

$$c_R = \frac{1}{\phi}\frac{\partial \phi}{\partial p} \tag{8}$$

**The total compressibility** $c_t$ is defined by

$$c_t = c_f + c_R = \frac{1}{\rho}\frac{\partial \rho}{\partial p} + \frac{1}{\phi}\frac{\partial \phi}{\partial p} \tag{9}$$

# Mathematical Model

Since...

$$\frac{\partial \rho}{\partial t} = \frac{\partial \rho}{\partial p}\frac{\partial p}{\partial t} \tag{10}$$

by definition of fluid compressibility (7), we have

$$\frac{\partial \rho}{\partial t} = c_f \rho \frac{\partial p}{\partial t} \tag{11}$$

In the other hand

$$\frac{\partial \phi}{\partial t} = \frac{\partial \phi}{\partial p}\frac{\partial p}{\partial t} \tag{12}$$

by definition of rock compressibility (8), we have

$$\frac{\partial \phi}{\partial t} = c_R \phi \frac{\partial p}{\partial t} \tag{13}$$

# Mathematical Model

**Exercise:** Show that

$$\frac{\partial \rho\phi}{\partial t} = \phi\rho c_t \frac{\partial p}{\partial t} \tag{14}$$

in 1 minute.

Hint! You can apply the derivative-of-a-product formula and equations (11), (13) and (9).

# Mathematical Model

Now we proceed to decompose the time derivative of $\phi\rho$ into two contributions: one due to the fluid compressibility and the other due to the elasticity of the solid matrix. Such decomposition follows when the derivative-of-a-product formula is applied:

$$\frac{\partial}{\partial t}(\phi\rho) = \phi\frac{\partial\rho}{\partial t} + \rho\frac{\partial\phi}{\partial t} \tag{15}$$

substituting (11) and (13) in (15), we have

$$\frac{\partial\rho\phi}{\partial t} = \phi\rho c_f\frac{\partial p}{\partial t} + \rho\phi c_R\frac{\partial p}{\partial t} = \phi\rho\left(c_f + c_R\right)\frac{\partial p}{\partial t} \tag{16}$$

from definition of total compressibility (9), we have

$$\frac{\partial\rho\phi}{\partial t} = \phi\rho c_t\frac{\partial p}{\partial t} \tag{17}$$

# Mathematical Model

## Darcy velocity

The Darcy velocity is the volume rate, per unit area, at which the fluid is flowing through the surface of $\Omega(t)$, and it is defined by

$$\underline{u} = \phi \underline{v} \tag{18}$$

where $\phi$ is the porosity and $\underline{v}$ is the fluid velocity.

Combination of equations (18) and (17) with equation (6) yields

$$\phi \rho c_t \frac{\partial p}{\partial t} + \nabla \cdot (\rho \underline{u}) = q, \qquad \forall \underline{x} \in \Omega \tag{19}$$

# Mathematical Model

## Darcy's Law

This is an empirical constitutive equation, which relates the fluid particle velocity to the fluid-pressure spatial distribution.

$$\underline{u} = -\frac{1}{\mu}\underline{\underline{k}} \cdot (\nabla p + \rho\gamma\nabla z) \tag{20}$$

where

$\mu-$ is the dynamic viscosity of the fluid,
$\underline{\underline{k}}-$ is the intrinsic permeability tensor,
$p-$ is the fluid pressure,
$\rho$ is the fluid pressure,
$\gamma-$ is the magnitude of the gravitational acceleration,
$z-$ is the elevation with respect to a given reference level.

# Mathematical Model

When Darcy's law, as given by eq. (20), is incorporated in eq. (19)

$$(\rho\phi c_t)\frac{\partial p}{\partial t} - \nabla \cdot \left( \frac{\rho}{\mu} \underline{\underline{k}} \cdot (\nabla p - \rho\gamma\nabla z) \right) = q \tag{21}$$

When the fluid is slightly compressible, Darcy's velocity is moderate, and the gravity effect and density change are ignored, (21) reduces to

$$\phi c_t \frac{\partial p}{\partial t} - \nabla \cdot \left( \frac{1}{\mu} \underline{\underline{k}} \cdot \nabla p \right) = q/\rho \tag{22}$$

which is a parabolic partial differential equation, as long as $\phi c_t > 0$, because the hydraulic conductivity tensor, $\underline{\underline{k}}$, is always a positive definite matrix.

# Mathematical Model

The governing differential equations for time-dependent problems, when $\phi c_t > 0$, are parabolic. Then well-posed problems are initial-boundary-value problems, which seek a function $p(\underline{x}, t)$ that satisfies equation (22) in a domain $\Omega$ of the physical space, together with suitable boundary conditions defined on its boundary

$$\partial\Omega \equiv \partial\Omega_D \cup \partial\Omega_N \tag{23}$$

$\partial\Omega_D$ is the Dirichlet boundary and $\partial\Omega_N$ is the Neumann boundary, over a specified time interval, which are discussed next. Furthermore, the function $p(\underline{x}, t)$ is required to satisfy additionally the **initial condition**:

$$p(\underline{x}, 0) = p_0(\underline{x}), \qquad \forall \underline{x} \in \Omega \tag{24}$$

Here $p_0(\underline{x})$ is a prescribed function that is known beforehand.

# Mathematical Model

## Dirichlet Condition

Given a porous medium saturated with a fluid in steady-state conditions, the practical problem consists of predicting the pressure distribution in the interior of the porous medium when the pressure is known only on the outer boundary of the porous medium.

In such cases, it is convenient and appropriate to develop mathematical models based on boundary-value problems in which the known pressure levels on the boundary are prescribed.

$$p(\underline{x}) = p_{\partial}(\underline{x}), \quad \forall \underline{x} \in \partial\Omega_D \ y \ t > t_0 \tag{25}$$

# Mathematical Model

**Neumann Boundary Condition:**

In view of the fact that $\underline{u}$ is the Darcy velocity, the term

$$\underline{n} \cdot \left( \frac{1}{\mu} \underline{\underline{k}} \cdot \nabla p \right) = g_{\partial}, \ \forall \underline{x} \in \partial_N \Omega \ y \ t > t_0 \qquad (26)$$

represents the volumetric flow per unit area that flows out from the domain $\Omega$, through its boundary $\partial \Omega_N$.

In this case, the volumetric flow is prescribed; when it is positive it describes the volumetric outflow and describes the volumetric inflow when it is negative, per unit time per unit area of the domain boundary.

# Mathematical Model

Therefore, the basic differential equation describing the flow of a slightly compressible single-phase fluid through a porous medium is:

$$\phi c_t \frac{\partial p}{\partial t} - \nabla \cdot \left( \frac{1}{\mu} \underline{\underline{k}} \cdot \nabla p \right) = q/\rho \quad , \quad \forall \underline{x} \in \Omega \ y \ t \geqslant t_0 \qquad (27)$$

$$p = p_0 \quad , \quad \forall \underline{x} \in \Omega \ y \ t = t_0 \qquad (28)$$

$$p = p_\partial \quad , \quad \forall \underline{x} \in \partial_D \Omega \ y \ t > t_0 \qquad (29)$$

$$-\underline{n} \cdot \left( -\frac{1}{\mu} \underline{\underline{k}} \cdot \nabla p \right) = g_\partial \quad , \quad \forall \underline{x} \in \partial_N \Omega \ y \ t > t_0 \qquad (30)$$

where $\phi$ is the rock porosity, $c_t$ is the total compressibility, $p$ is the fluid pressure, $\mu$ is the fluid viscosity, $\underline{\underline{k}}$ is the permeability tensor, $q$ is the volumetric rate of extraction of the fluid from the porous medium: that is the volume of fluid withdrawn per unit time, per unit volume of the porous system, and $\rho$ is the fluid density.

# Numerical Model

A straightforward approach to solving time-dependent PDEs by the finite element method is to first discretize the time derivative by a finite difference approximation, which yields a recursive set of stationary problems, and then turn each stationary problem into a variational formulation.

$$(\phi c_t)^n \frac{\partial p^n}{\partial t} - \nabla \cdot \left( \left( \frac{1}{\mu} \underline{\underline{k}} \right)^n \cdot \nabla p^n \right) = q^n / \rho^n \tag{31}$$

Let superscript $n$ denote a quanity at time $t_n$, where $n$ is an integer counting time levels. So, $p^n \equiv p(\underline{x}, t_n)$ means $p$ at time level $n$.

The time-derivative can be approximated by a difference quotient.

# Numerical Model

For simplicity and stability reasons, the time-derivative can be approximated by a simple backward finite difference.

$$\frac{\partial p^n}{\partial t} \approx \frac{p^n - p^{n-1}}{\Delta t_n} \tag{32}$$

where

$\Delta t_n \equiv t_n - t_{n-1}$

with $n = 1, ..., N$ is the time discretization parameter.

# Numerical Model

Inserting (32) in (31) yields

$$(\phi c_t)^n \frac{p^n - p^{n-1}}{\Delta t_n} - \nabla \cdot \left( \left( \frac{1}{\mu} \underline{\underline{k}} \right)^n \cdot \nabla p^n \right) = q^n / \rho^n \tag{33}$$

This is our time-discrete version of the single-phase flow model (27), a so-called backward Euler or implicit Euler discretization.

# Numerical Model

**Exercise:** (1 minute)

To reorder eq. (33) so that the left-hand side contains the terms with the unknown $p^n$ and the right-hand side contains computed terms only.

The result is a sequence of spatial (stationary) problems for $p^n$, assuming $p^{n-1}$ is known from the previous time step.

$$(\phi c_t)^n p^n - \Delta t_n \nabla \cdot \left( \left( \frac{1}{\mu} \underline{\underline{k}} \right)^n \cdot \nabla p^n \right) = (\phi c_t)^n p^{n-1} + \Delta t_n q^n / \rho^n \tag{34}$$

Given $p^0 = p_0$, we can solve for $p^1$, $p^2$, $p^3$, and so on.

# Numerical Model

Then to reformulate (27) as a variational problem, we multiply the equation (34) by the test function $w$ and integrate over $\Omega$:

$$\int_{\Omega} (\phi c_t)^n p^n w d\underline{x} - \Delta t_n \int_{\Omega} \nabla \cdot \left( \left( \frac{1}{\mu} \underline{\underline{k}} \right)^n \cdot \nabla p^n \right) w d\underline{x} = \int_{\Omega} ((\phi c_t)^n p^{n-1} + \Delta t_n q^n / \rho^n) w d\underline{x} \tag{35}$$

where
$p \in W = \{p \in H^1(\Omega) : p|_{\partial_D \Omega} = p_\partial\}$
$w \in \hat{W} = \{w \in H^1(\Omega) : w|_{\partial_D \Omega} = 0\}$

# Numerical Model

**Exercise:**
Applying integration by parts to the integrand with second-order derivatives and after applying the boundary condition 30:

$$\int_{\Omega} (\phi c_t)^n p^n w d\underline{x} + \Delta t_n \int_{\Omega} \nabla w \cdot \left( \left( \frac{1}{\mu} \underline{\underline{k}} \right)^n \cdot \nabla p^n \right) d\underline{x}$$

$$= \int_{\Omega} ((\phi c_t)^n p^{n-1} + \Delta t_n q^n / \rho^n) w d\underline{x} + \Delta t_n \int_{\partial_N \Omega} w g_\partial^n d\underline{x} \quad (36)$$

Remember that $\quad w \in \hat{W} = \{ w \in H^1(\Omega) : w|_{\partial_D \Omega} = 0 \}.$

# Numerical Model

The Dirichlet boundary condition is called an **essential boundary condition** because it appears in the variational space.

The Neumann boundary condition is called a **natural boundary condition** because it does not appear in the variational space but rather is implied in the formulation.

# Numerical Model

We need to transform the continuous variational problem (36) to a discrete variational problem, and to solve the following sequence of variational problems to compute the finite element solution:

Find $p_h^n \in W$ such that $a(p_h^n, w_h) = L_n(w_h), \forall w_h \in \hat{W}_h$, for $n = 1, 2, ..., N$, where

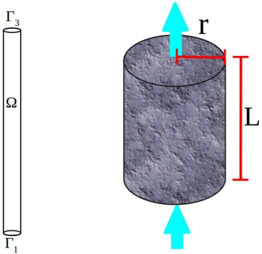
$$a(p_h^n, w_h) = \int_{\Omega} (\phi c_t)^n p_h^n w_h + \Delta t_n \nabla w_h \cdot \left( \left( \frac{1}{\mu} \underline{\underline{k}} \right)^n \cdot \nabla p_h^n \right) d\underline{x} \quad (37)$$

$$L_n(w_h) = \int_{\Omega} ((\phi c_t)^n p_h^{n-1} + \Delta t_n q^n / \rho^n) w_h d\underline{x} + \Delta t_n \int_{\partial_N \Omega} w_h g_{\partial}^n d\underline{x} \quad (38)$$

# Numerical Model

Considering next cilindrical domain and its boundaries, which represents a porous medium at core laboratory scale

$p = p_{out}, \forall \underline{x} \in \Gamma_3$




$\underline{n} \cdot \left( \frac{1}{\mu} \underline{\underline{k}} \cdot \nabla p \right) = u_{in}, \forall \underline{x} \in \Gamma_1$ $\quad$ $\underline{n} \cdot \left( \frac{1}{\mu} \underline{\underline{k}} \cdot \nabla p \right) = 0, \forall \underline{x} \in \Gamma_2$

# Numerical Model

Since

$$\int_{\partial_N \Omega} w g_\partial^n d\underline{x} = \int_{\Gamma_1} w u_{in} d\underline{x} + \int_{\Gamma_2} w \cdot 0 d\underline{x} = \int_{\Gamma_1} w u_{in} d\underline{x} \qquad (39)$$

We need to solve the following sequence of variational problems:

Find $p_h^n \in W$ such that $a(p_h^n, w_h) = L_n(w_h), \ \forall w_h \in \hat{W}_h$, for $n = 1, 2, ..., N$, where

$$a(p_h^n, w_h) = \int_{\Omega} (\phi c_t)^n p_h^n w_h + \Delta t_n \nabla w_h \cdot \left( \left( \frac{1}{\mu} \underline{\underline{k}} \right)^n \cdot \nabla p_h^n \right) d\underline{x} \quad (40)$$

$$L_n(w_h) = \int_{\Omega} ((\phi c_t)^n p_h^{n-1} + \Delta t_n q^n / \rho^n) w_h d\underline{x} + \Delta t_n \int_{\Gamma_1} w_h u_{in}^n d\underline{x} \quad (41)$$

# FEniCS script with Python

```python
from fenics import *
from mshr import *
import  numpy as np
import pandas as pd
```

Listing 1: Python script

**mshr** is the mesh generation component of FEniCS. It is C++ library with Python bindings.

**numpy** is the fundamental package for scientific computing with Python.

**pandas** is an open source, BSD-licensed library providing high-performance, easy-to-use data structures and data analysis tools for the Python programming language.

# FEniCS script with Python

```
import matplotlib.pyplot as plt
```

Listing 2: Python script

**Matplotlib** is a Python plotting library which produces publication quality figures in a variety of hardcopy formats and interactive environments across platforms.

The **pyplot** module provides a MATLAB-like interface.

# FEniCS script with Python

Then we define the geometry and mesh, for example:

```
domain = Cylinder(dolfin.Point(xmin,ymin,zmin),dolfin.Point(
    xmin,ymin, zmax),r,r)
mesh = generate_mesh(domain, 30)
```

Listing 3: Python script

Define function space for pressure and velocity, respectively

```
fs_order = 2 # function space order
V = FunctionSpace(mesh, 'Lagrange', fs_order)
U = VectorFunctionSpace(mesh, 'Lagrange', fs_order)
```

Listing 4: Python script

Define Trial and Test functions

```
p = TrialFunction(V)
w = TestFunction(V)
```

Listing 5: Python script

# FEniCS script with Python

The essential boundary conditions are implemented:

```
boundary_parts = FacetFunction("size_t", mesh)
```

Listing 6: Python script

```
#Boundary condition at the bottom
class BottomBoundary(SubDomain):
    def inside(self, x, on_boundary):
        tol = 1E-14   # tolerance for coordinate comparisons
        return on_boundary and abs(x[2]) < tol

Gamma_1 = BottomBoundary()
Gamma_1.mark(boundary_parts, 1)

#If Dirichlet condition
p_B = Constant(u_in)
bc1=DirichletBC(V, p_B, boundary_parts, 1)
```

Listing 7: Python script

# FEniCS script with Python

```
#Boundary condition at the top
class TopBoundary(SubDomain):
    def inside(self, x, on_boundary):
        tol = 1E-14   # tolerance for coordinate comparisons
        return on_boundary and abs(x[2] - zmax) < tol

Gamma_3 = TopBoundary()
Gamma_3.mark(boundary_parts, 3)

#If Dirichlet condition
p_T = Constant(p_out)
bc3=DirichletBC(V, p_T, boundary_parts, 3)
```

Listing 8: Python script

```
#If any Dirichlet condition
bcs = [bc3]
```

Listing 9: Python script

# FEniCS script with Python

Initial condition

```
p_0 = Expression('p0',p0=p0, degree=fs_order + 1)

p_1 = project(p_0, V)
```

Listing 10: Python script

Variational formulation

```
a = R*p*w*dx + dt*inner(nabla_grad(w), dot(K/mu,nabla_grad(p
    )))*dx

L = (R*p_1 + dt*q/rho)*w*dx + dt*u_in*w*ds(1)
```

Listing 11: Python script

# FEniCS script with Python

Finally, we perform the time-stepping in a loop:

```
A = assemble(a)
b = None

p = Function(V)   # the unknown at a new time level

t = t_0

while t < t_N:
    b = assemble(L, tensor=b)
    p_0.t = t
    for bc in bcs:
        bc.apply(A, b) #if Dirichlet conditions
    solve(A, p.vector(), b, "lu")
    t += dt
    p_1.assign(p)
    ph = project(p)
```

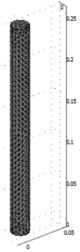
Listing 12: Python script

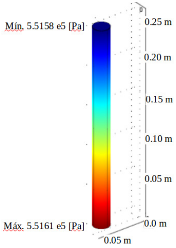
# Case Study

Our program needs to implement the next properties of the core:

| simbol | Description | Value | Unit |
|---|---|---|---|
| $p_o$ | Initial pressure | 80 | $psi$ |
| $r$ | Ratio of the core | 0.02 | $m$ |
| $L$ | Long of the core | 0.25 | $m$ |
| $\mu$ | Oil viscosity | 1.31E-02 | $Pa \cdot s$ |
| $k$ | Permeability | 1.51E-13 | $m^2$ |
| $c_o$ | Oil compresibility ($c_f$) | 0.00001 | $1/psi$ |
| $c_R$ | Rock compresibility | 0.000004 | $1/psi$ |
| $\phi$ | Porosity | 0.1978 | fracción |
| $Q$ | Rate injection | 1.39E-09 | $m^3/s$ |
| $\rho_o$ | Oil density | 8.72E+02 | $Kg/m^3$ |

# Case Study




Mín. 5.5158 e5 [Pa]
Máx. 5.5161 e5 [Pa]
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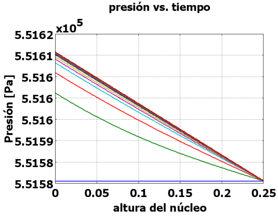
0.05 m

# Case Study


presión vs. tiempo
x10^5
5.5162
5.5161
5.516
5.516
5.516
5.5159
5.5158
5.5158
Presión [Pa]
0
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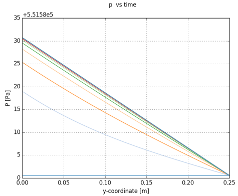
altura del núcleo


p vs time
+5.5158e5
P [Pa]
y-coordinate [m]
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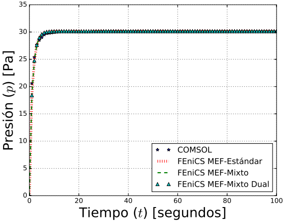
0.05
0.10
0.15
0.20
0.25


Presión (p) [Pa]
Tiempo (t) [segundos]
COMSOL
FEniCS MEF-Estándar
FEniCS MEF-Mixto
FEniCS MEF-Mixto Dual

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# Thank you

# Questions or Comments