Orientation Tracking

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I. Introduction

Orientation Tgracking

II. Pre-processing

The raw IMU data we get is 10-bit ADC values, which technically ranges from 0 to 1023. Since it's difficult to use those values directly in our filter, we have to convert them so that they have real-world physical units and meanings. Our goal here is to convert the readings from the micro-controller to acceleration with a unit of m/s^2 and angular velocity with a unit of rad/s. The equations used for converting raw values to real values are shown in Eq.1 and Eq.2.

$$\vec{a} = (\vec{a}_{raw} - \vec{a}_{bias}) * s_a \tag{1}$$
$$\vec{\omega} = (\vec{\omega}_{raw} - \vec{\omega}_{bias}) * s_\omega \tag{2}$$

where

$$\vec{a}_{bias} = \begin{bmatrix} 511.5 & 511.5 & 511.5 \end{bmatrix} \tag{3}$$

and $\vec{\omega}_{bias}$ is determined empirically by averaging the first 100 readings of one of the IMU data

$$\vec{\omega}_{bias} = \begin{bmatrix} 373.63 & 375.20 & 369.66 \end{bmatrix} \tag{4}$$

s_a and s_ω are the scale factors and their values are 0.0106 and 0.0171.

III. Methods

For the main unscented kalman filter algorithm, I basically followed the steps outlined in Kraft's paper [1]. For calculating quaternion mean, I used the method proposed by Cheon [2] instead of gradient descent. Another difference is that Kraft uses only $2n$ Sigma points uniform weights, while Cheon uses $2n+1$ Sigma points with weights determined by parameters such as α, β and κ. Here I will describe the algorithm in detail. The following content is either from Kraft or from Cheon, and it focuses more on the steps and implementations than the underlying theorems and principles.

I. Process Model

Process model in kalman filter takes the from

$$x_{k+1} = A(x_k, w_k) \tag{5}$$

where x is the state vector and w is the process noise. The first 3 components of w_k affect the orientation and the last three components of w_k affects angular velocity.

$$w_k = \begin{pmatrix} \vec{w}_q \\ \vec{w}_\omega \end{pmatrix} \tag{6}$$

The final process model is

$$x_{k+1} = A(x_k, w_k) = \begin{pmatrix} q_k q_w q_\Delta \\ \vec{\omega}_k + \vec{w}_\omega \end{pmatrix} \tag{7}$$

where q_k is the quaternion of the previous time step and q_w is the quaternion representing process noise

$$q_w = \begin{bmatrix} \cos\left(\frac{\alpha_w}{2}\right) & \vec{e}_w \sin\left(\frac{\alpha_w}{2}\right) \end{bmatrix} \tag{8}$$
$$\alpha_w = \|w_q\|, \qquad \vec{e}_w = \frac{w_q}{\|w_q\|} \tag{9}$$

*A thank you or further information

and q_Δ is the quaternion representing the rotation during the interval of sensor update

$$q_\Delta = \left[\cos\left(\frac{\alpha_\Delta}{2}\right) \quad \vec{e}_\Delta \sin\left(\frac{\alpha_\Delta}{2}\right)\right] \tag{10}$$

$$\alpha_\Delta = \|\vec{\omega}_k\| \cdot \Delta t, \quad \vec{e}_\Delta = \frac{\vec{\omega}_k}{\|\vec{\omega}_k|} \tag{11}$$

II. Measurement Model

Measurement model in kalman filter takes the from

$$z_k = H(x_k, v_k) \tag{12}$$

Since we have gyro and accelerometer, we have two different type of measurement. The two measurement models are

$$H_1: \quad \vec{z}_{rot} = \vec{\omega}_k + \vec{v}_{rot} \tag{13}$$

$$H_2: \quad \vec{z}_{acc} = \vec{g}' + \vec{v}_{acc} \tag{14}$$

where

$$g' = q_k g q_k^{-1} \tag{15}$$

$$g = (0,\ [g_x, g_y, g_z]) \tag{16}$$

and $[g_x, g_y, g_z]$ is the acceleration readings from the IMU.

III. Initialization

IV. Prediction

V. Correction

IV. Discussion

References

[1] Edgar Kraft, *A quaternion-based unscented kalman filter for orientation tracking*. Information Fusion, 2003. Proceedings of the Sixth International Conference on Information Fusion, Vol. 1 (2003), pp. 47-54.

[2] Y.-J. Cheon and J.-H. Kim, *Unscented filtering in a unit quaternion space for spacecraft attitude estimation*. IEEE International Symposium on Industrial Electronics (ISIE 2007) (2007), pp. 66-71.