Example 03
Portugal, a shallow Mw 4 earthquake (real data).

Before this, see the introductory hints in isola_EXAMPLE_01.

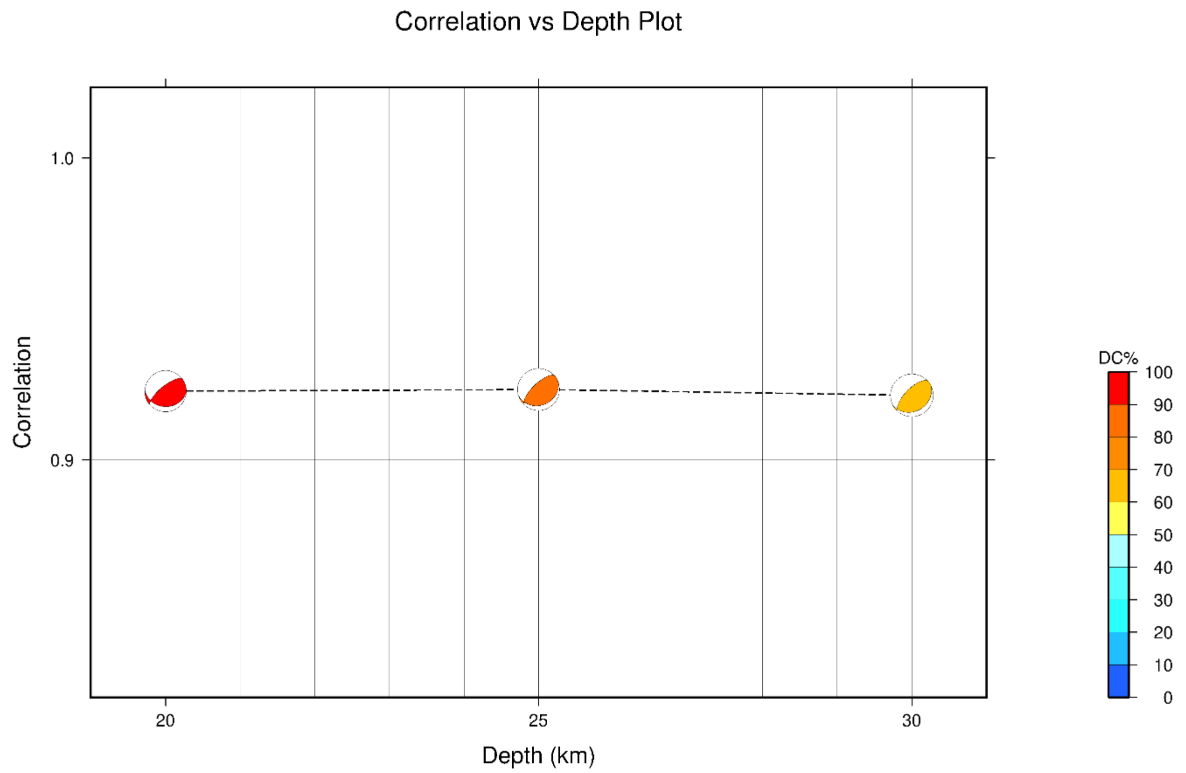
Preparation
Event Info. We study a real Mw 4 event of 20230107, Lat 37.891, Lon -9.534. We choose TWL= 307.2 s, implying waveform sampling with dt= TWL/1024= 0.3 s.
Station Selection. Taken from the file iberia_4stations.stn located in the root folder.
Define Crustal Model. Load the model from file crustal.cru from the root.
Seismic Source Definition. Trial source positions, starting at depth 20 km, depth step 5 km, 3 sources (= 20, 25, 30 km).
Green function calculation. GF is calculated up to the Nyquist frequency 1.67 Hz, assuming delta-function moment rate (default values); 3 trial sources, 4 stations.
Data Preparation. No need to do anything. Real waveforms have been instrumentally corrected and resampled with dt=0.3 s before this test and appear in folder invert as *raw.dat for four stations *. They are also stored (backup) in folder data_IsolaRAW_REALdata.

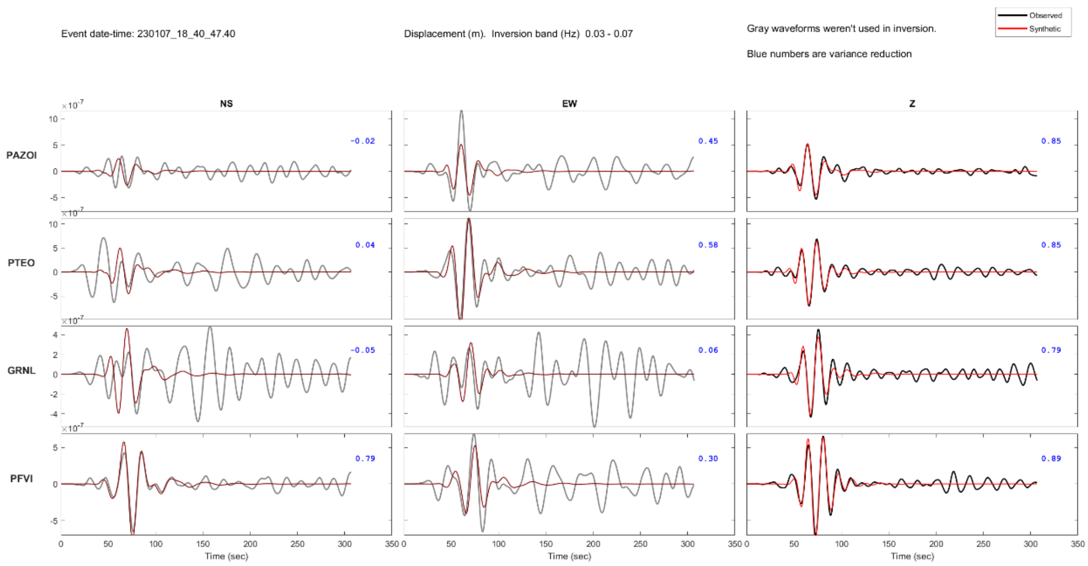
Test 1, frequency range 0.03-0.07 Hz, only Z component
This example mimics the manual approach of a skilled operator. After a few tests, a (more or less) acceptable frequency range is found at 0.03-0.07 Hz, having not much noise and, at the same time, containing relatively low frequencies for which we assume the velocity model to be adequate. As seen in waveforms (plotted below), we find greater noise in the N and E components, so we perform inversion just with the low-noise Z component.

Waveform Inversion: Setting Filter 0.03-0.07, common to all stations, then SelectStations/FrequencyBand, and choosing only Z. Results appear in invert folder and are saved in EXAMPLE_03\invert\test1. From inv1.dat we obtain the centroid and moment tensor (MT) as shown here:

```
Source position 2 … (depth 25 km)
moment (Nm):  9.9327762E+14
   moment magnitude:   4.0
           VOL % :  -8.6
            DC % :  81.0
          CLVD % : -10.4
 strike,dip,rake:         70          24         112
 strike,dip,rake:        226          67          80
 varred=  0.8526452    (= variance reduction, VR)
```

In *inv1.png we see that depth is almost unresolved but a reverse focal mechanism is determined robustly. Its DC% is not very stable, it varies with the trial source position. The Z-component waveform fit obs/syn is very good.



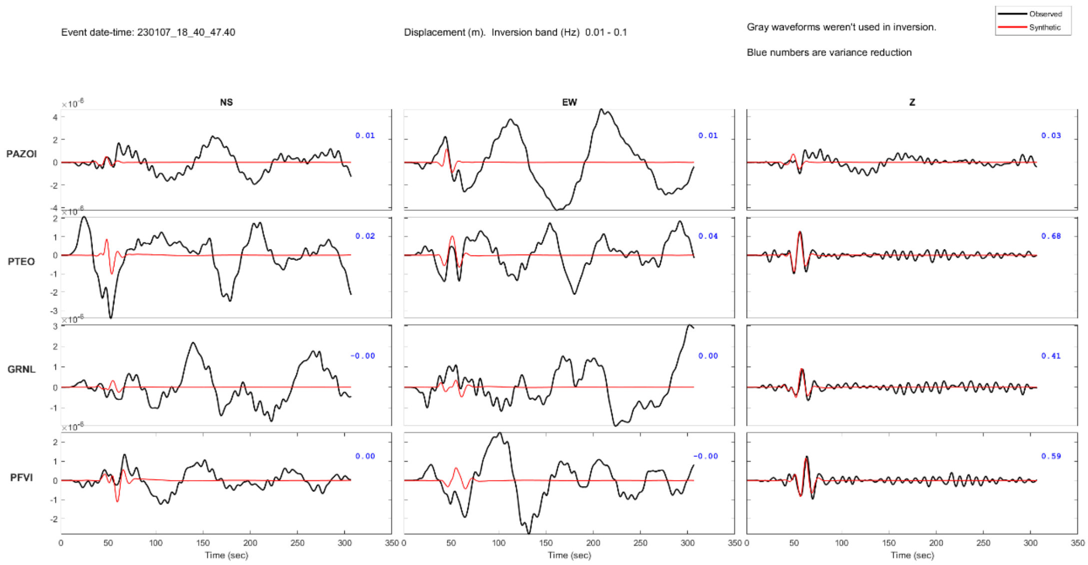


Test 2 – "modern" approach

To avoid the manual search for a suitable low-frequency limit of the inversion, we adopt a relatively broad range of 0.01-0.1 Hz. As a zeroth approximation, we run normal inversion (without covariance matrices, 'no cova'). This time, we use all three components. The results can be seen in your folder invert, and saved in a subfolder invert\test2.

Source position 1
moment (Nm): 8.6587742E+14
moment magnitude: 3.9
VOL % : 38.2
DC % : 55.5
CLVD % :   6.4
strike,dip,rake: 106   36   147
strike,dip,rake: 223   71   58
varred= 1.6747534E-02

Variance reduction (varred=0.0167) is very low, there is almost no waveform fit in N and E component.



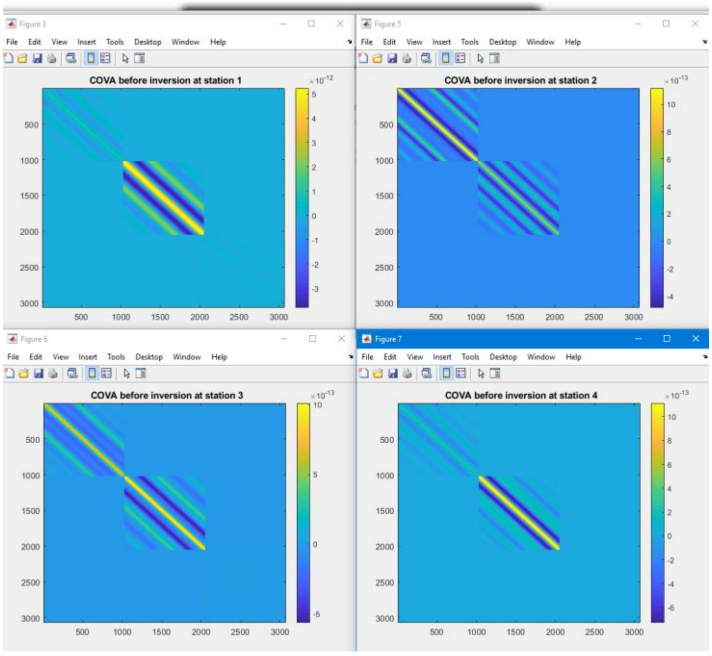
Code automatically created residual waveforms, res = obs minus syn. We assume that the “res” represents a data error. Thus, as a next step, we run the inversion with covariance matrix derived from res, Toeplitz type (Cd formed by autocovariance functions of N, E and Z).

Source position 1
moment (Nm): 8.1958579E+14
moment magnitude: 3.9
VOL % : 29.9
DC % : 65.2
CLVD % :   4.8
strike,dip,rake: 98   29   138
strike,dip,rake: 226   70   66
varred= 1.6151130E-02

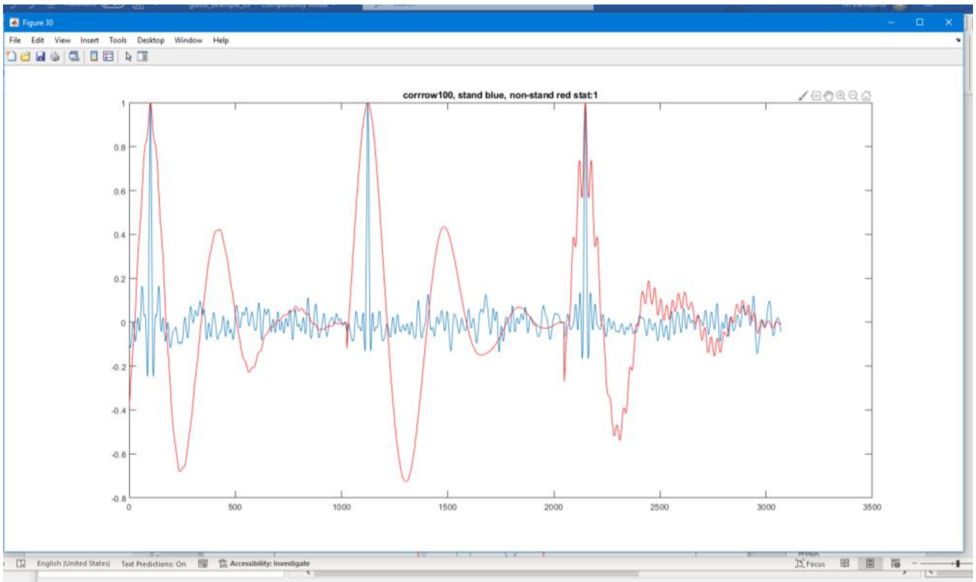
The varred 0.01615 is even (slightly) lower than before. Nevertheless, we can show that the obtained solution may be trustworthy.

In Matlab command window we read varred_classical = 0.01615 and varred_standardized = 0.53809. The latter, considerably larger, is a measure of the fit between real and synthetic data in the so-called standardized domain [Vackar et al., GJI 2017].

Graphical output from Matlab run provides many plots, some of which demonstrate the covariance matrices derived from the residuals (res), shown below. Significant non-zero elements outside the diagonal indicate highly correlated errors. From the to-left to bottom right we have N, E, and Z.

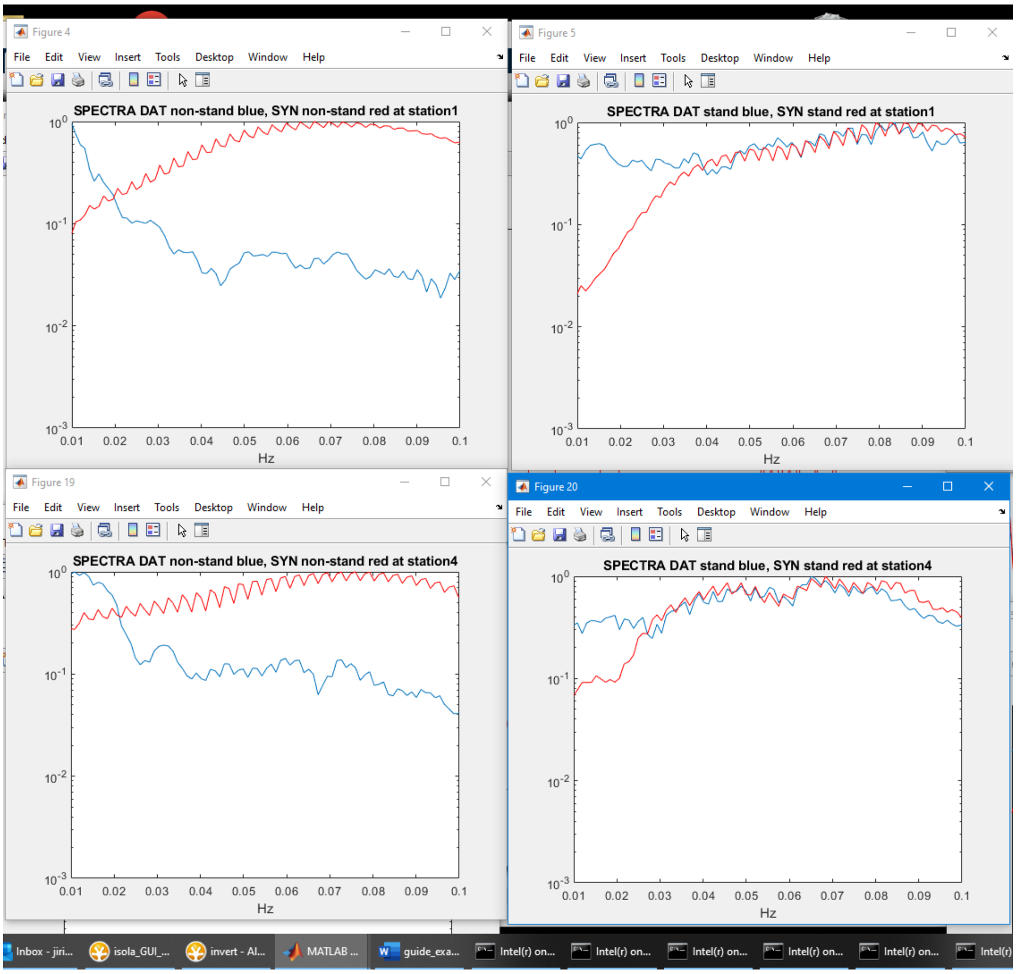


One row of the autocovariance matrix of N, E, and Z component of data residual (from the left to the right), is displayed below, for station 1, red color. Each component is normalized to a maximum value of 1. Shown by the blue color in the same figure is the autocovariance of the standardized residual, which is significantly narrower, implying that the application of the cova approach suppressed the error correlation length, which is equivalent to making the error closer to white noise, as desired in the least-squares MT inversion.

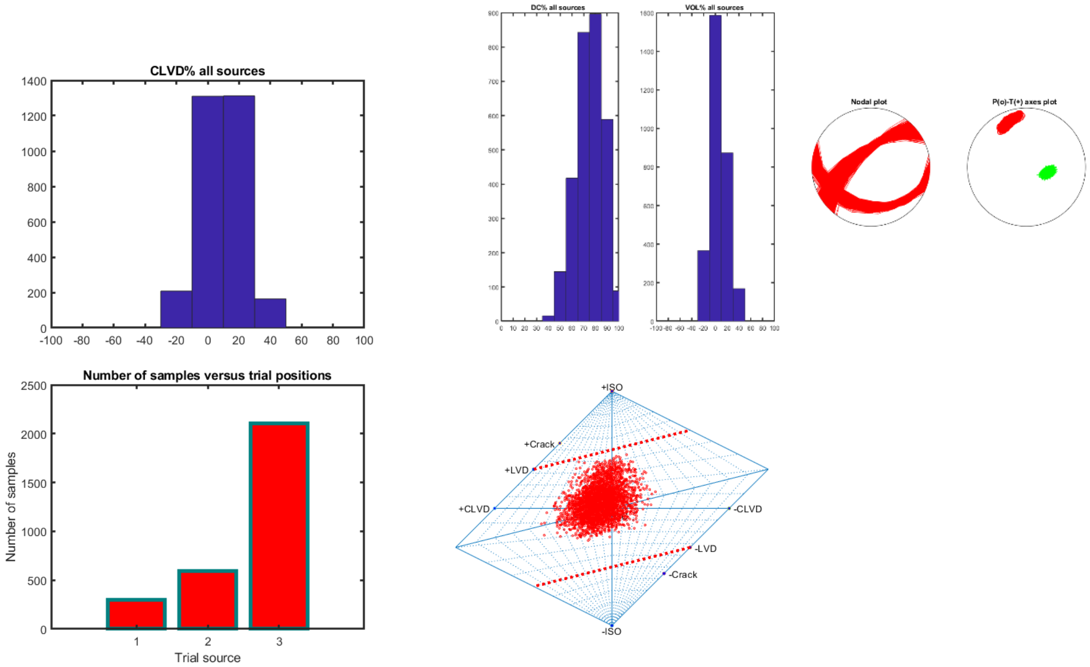


It explains why in the standard obs/syn comparison we see a poor fit (low VR), while the standardized seismograms are fitted much better, see the next two figures.

The same effect in the spectral domain is shown below for stations 1 and 4. Left panels (before standardization) – no fit, right panels (after standardization) – good fit in the range 0.03-0.08 Hz.
[It means we could use Isola without cova in the range 0.03-0.08 and obtain the same result. This is what we did in Test 1.]



Continuing with our Test2, applying Uncertainty Estimation tool, we get the following:

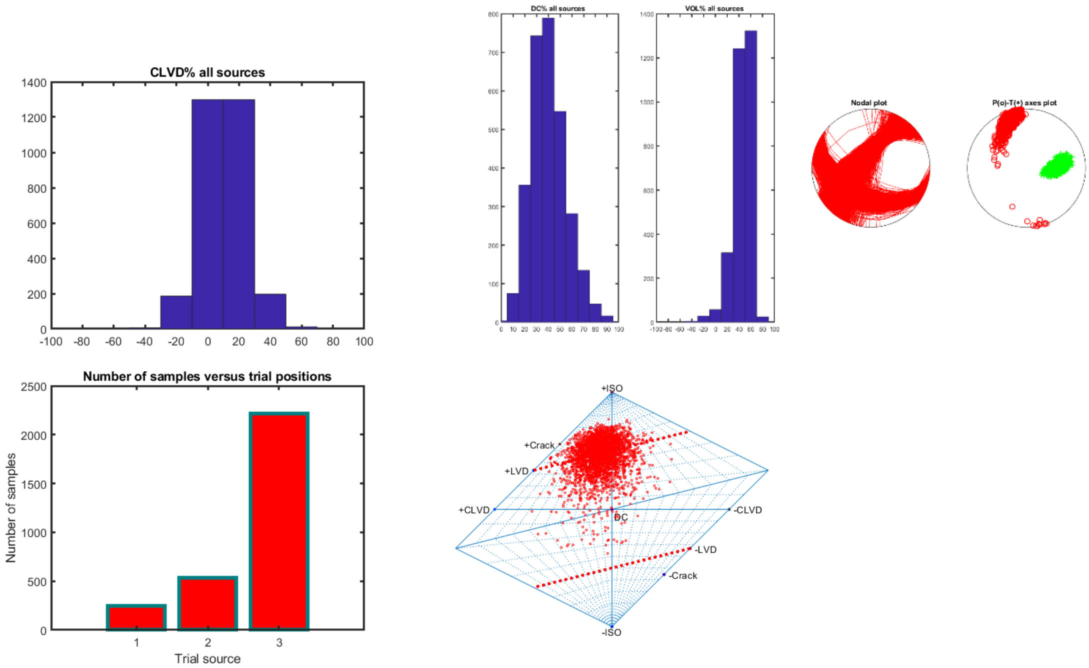


The best-fit solution (above) included VOL % : 29.9, DC % : 65.2, CLVD % : 4.8. Here we see that histograms of VOL and CLVD include value 0, thus the interpretation of uncertainty can be that the pure DC source cannot be ruled out, or that the source has no significant deviation from 100% DC. The same is confirmed by observing that the solution scatter (‘red cloud’ of random moment-tensor samples) in the source-type plot does include the central (DC) point.

A small interpretation problem appears with the source position. This calculation is based on eqs. 5-7 of Vackar et al. (2017) where uncertainty of moment tensor and source position are jointly evaluated. The (formally) best waveform fit is at trial position 1. The most likely depth is the one with the largest number of random samples (here position 3). The number of samples is determined by waveform fit and the determinant of the covariance matrix Cm of model parameters. While misfit is almost constant at the three trial source positions (as also shown above by the plot of correlation versus source position in Test 1), or just weakly prefers position 1, the determinant of Cm differs strongly between the trial positions (see column 3 of newunc_log.dat, varying by two orders of magnitude) and strongly prefers position 3. Nevertheless, the situation is better than in synthetic Example 02 where the trial positions resulted in different source types; here all three are near DC.

Test 3. Is the cova stuff so powerful as to resolve MT from noisy horizontals only?
Here we repeat Test 2, but inverted only N and E components (the option Select Stations under the part Filter). The solution is below, and *assuming* that previous Test 2 and Test 1 provided trustworthy solutions of the real data, we can make a comparison with Test 2. It shows that the answer to the question in the title will be ‘yes and not‘.

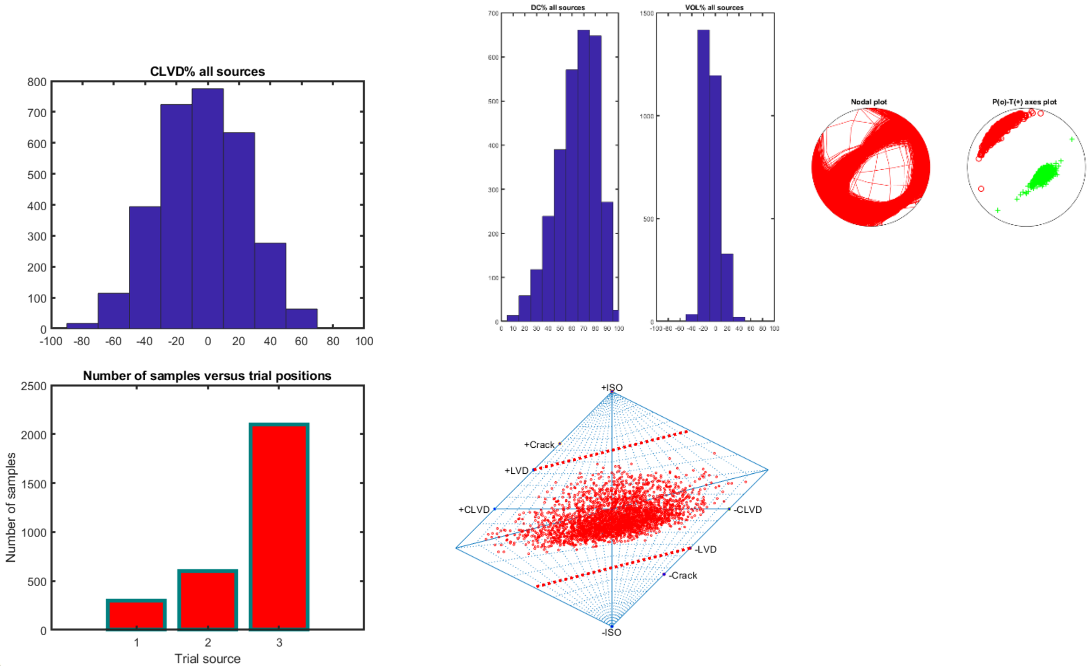
Source position 3
moment (Nm): 1.4400134E+15
  moment magnitude: 4.1
  VOL % : 48.0
  DC % : 44.9
  CLVD % :   7.1
strike,dip,rake: 112   46   153
strike,dip,rake: 222   70   46
varred= 1.0302246E-02



What does it mean ‘yes and not’? We observe similar (just less-well resolved) nodal planes as before, this is the “yes” but, importantly, a significant deviation from 100% DC is now signalized by the source-type plot; that is the “not”.
The example is a good warning that although cova stuff can suppress some noise effects, it is not magic, an “always working” tool, here failing because we used only the noisy N and E components.

Test 4. Is cova so powerful as to resolve MT from low-noise Z only?
Again frequency range 0.01-1.0 Hz, all procedures as in Test 2 (two-step approach, first a no-cova run, followed by a cova run).

Source position 2
moment (Nm): 9.2237751E+14
  moment magnitude: 3.9
  VOL % :   0.6
  DC % : 86.7
  CLVD % : -12.8
strike,dip,rake: 68   25   111
strike,dip,rake: 225   66   80
varred= 0.3051185

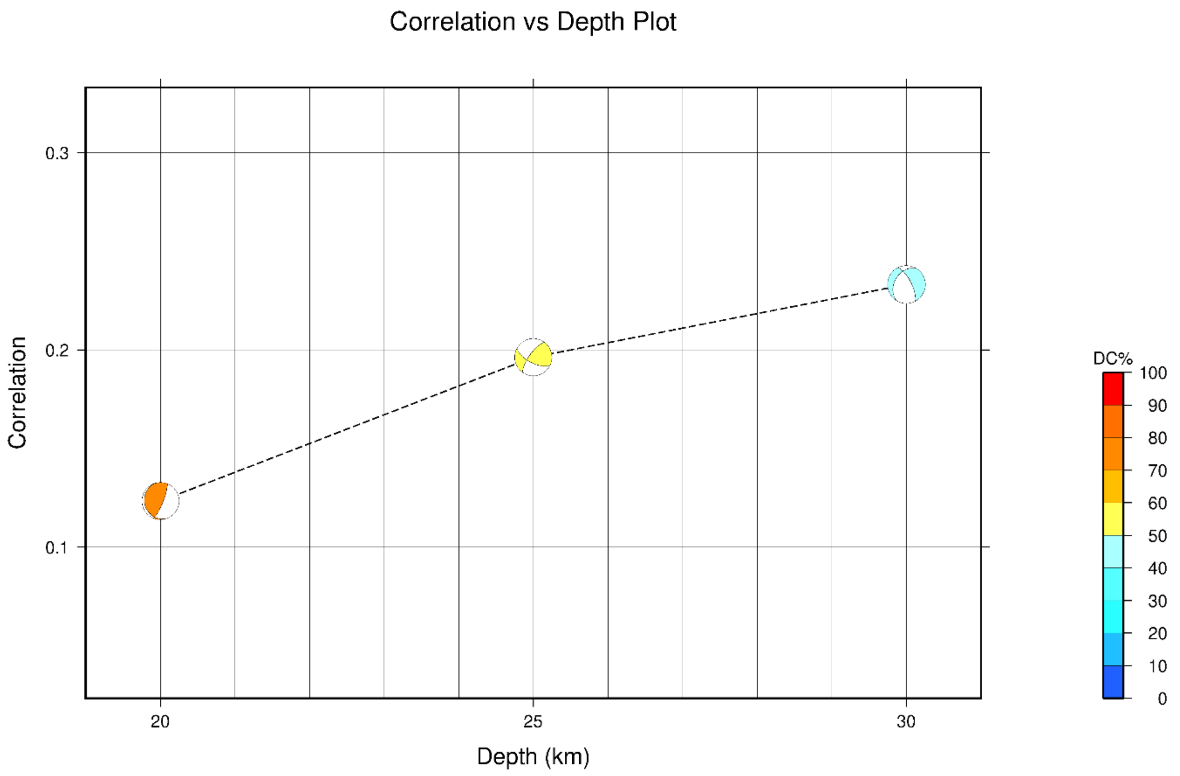


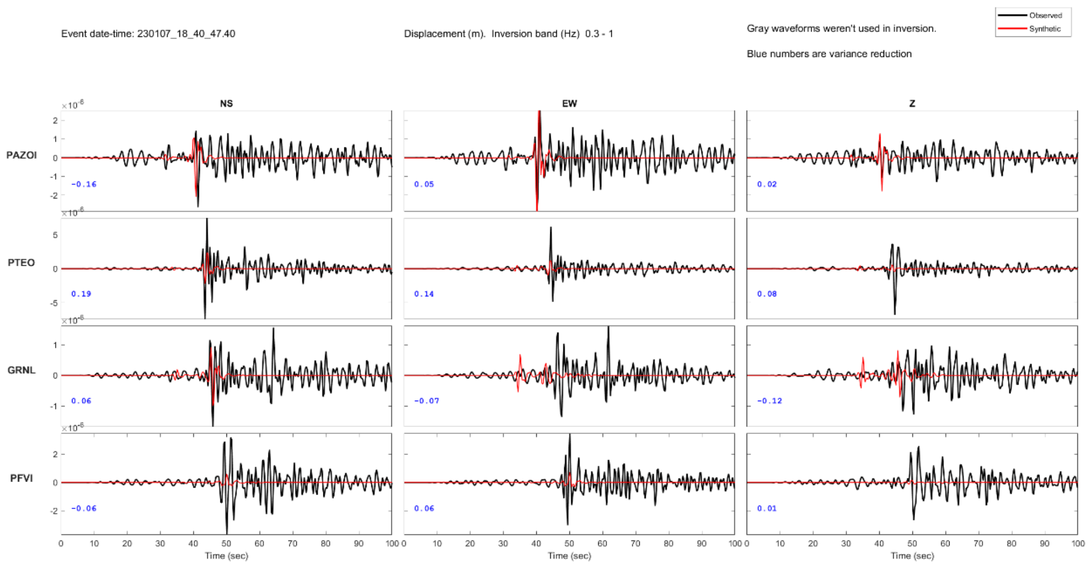
Using non-noisy Z components only, applying the "modern" (two-step) approach is probably a method most acceptable to practical users. We see acceptable VR =0.3, and good waveform fit in Z components (as in Test 2), we appreciate the help of cova in the uncertainty assessment, i.e. we understand that due to reduced data (only Z) the determination of nodal planes is less unique, and we need not to speculate about significant deviation from 100% DC because both histograms of CLVD and VOL indicate a high probability of their 0 value, although the source-type plot is greatly scattered.

Test 5. Can we rely only on frequencies 0.3-1.0 Hz?

Now we invert N, E, Z in two steps (no-cova followed by cova). Frequencies 0.3-1.0 Hz have a very low noise. However, with frequencies up to 1 Hz, we must expect problems with complex waveforms, including coda scattered waves, that are not involved in our deterministic waveform synthetics. We *assume* a trustworthy solution of real data from Test2, so we can state that the present solution is incorrect. First, without cova:

```
Source position 3
moment (Nm):  2.7658873E+14
  moment magnitude:   3.6
          VOL % :  -3.5
           DC % :  43.4
         CLVD % :  53.1
 strike,dip,rake:        333        69       -121
 strike,dip,rake:        213        37        -35
varred=  5.4396152E-02
```

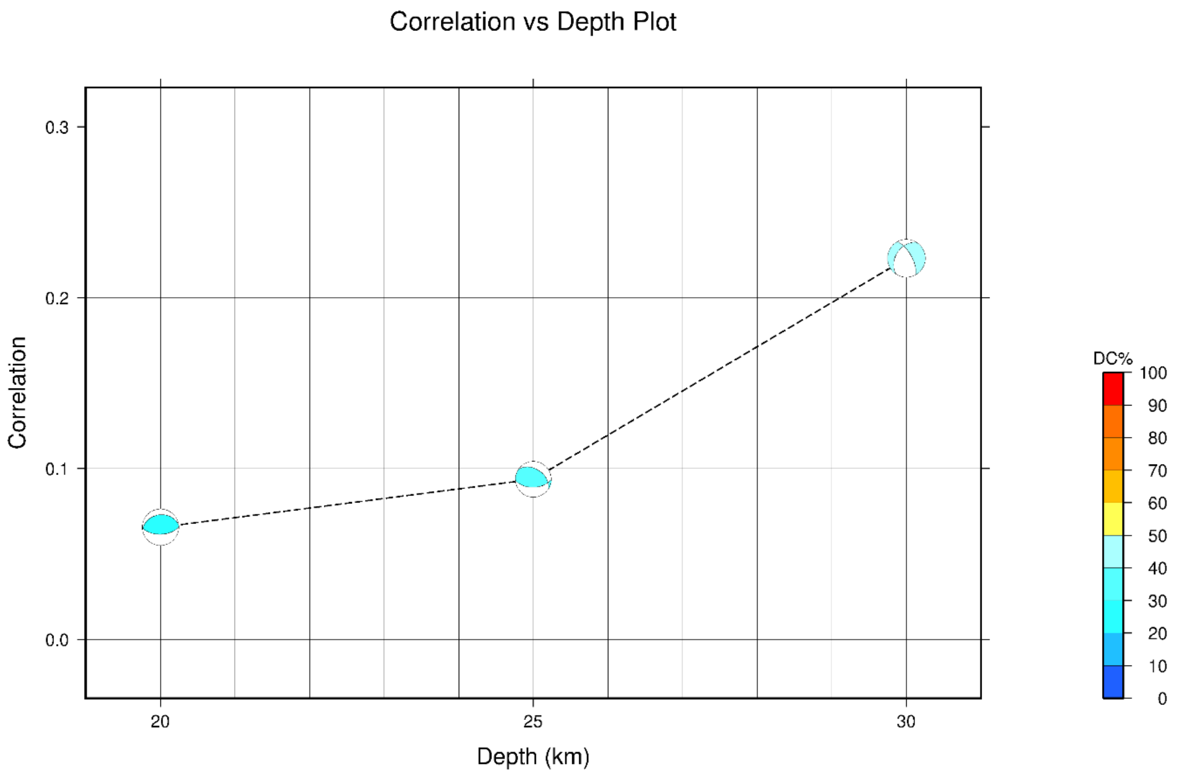




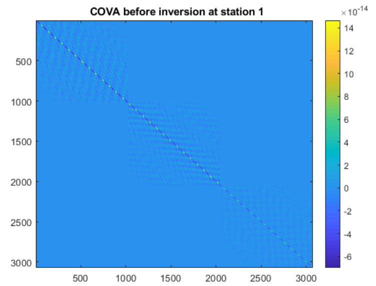
Then with cova:
Source position 3
moment (Nm): 2.5137424E+14
moment magnitude: 3.6
VOL % : 10.5
DC % : 41.5
CLVD % : 48.1

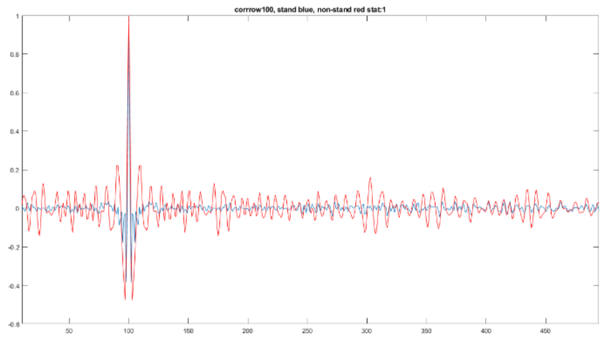
strike,dip,rake:	331	67	-127
strike,dip,rake:	214	42	-34

varred= 5.3533196E-02

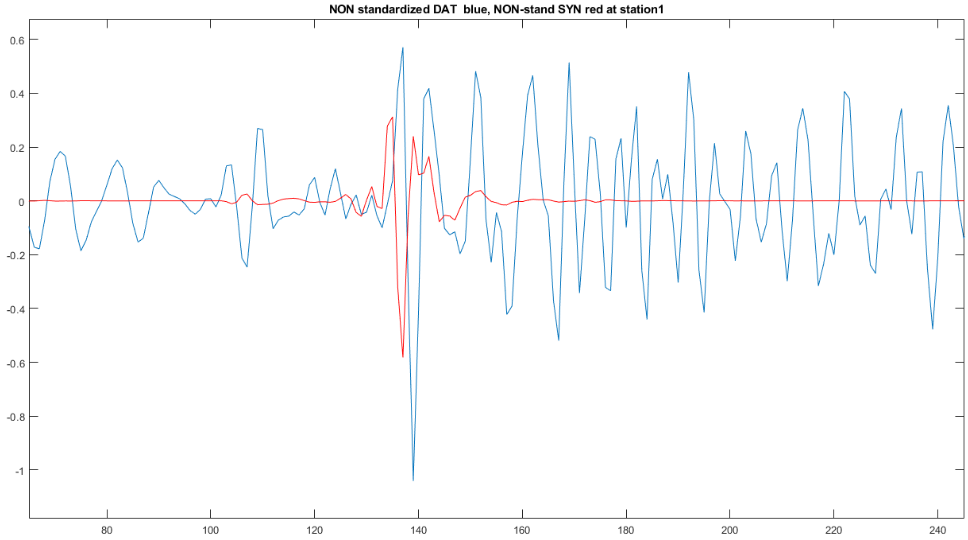


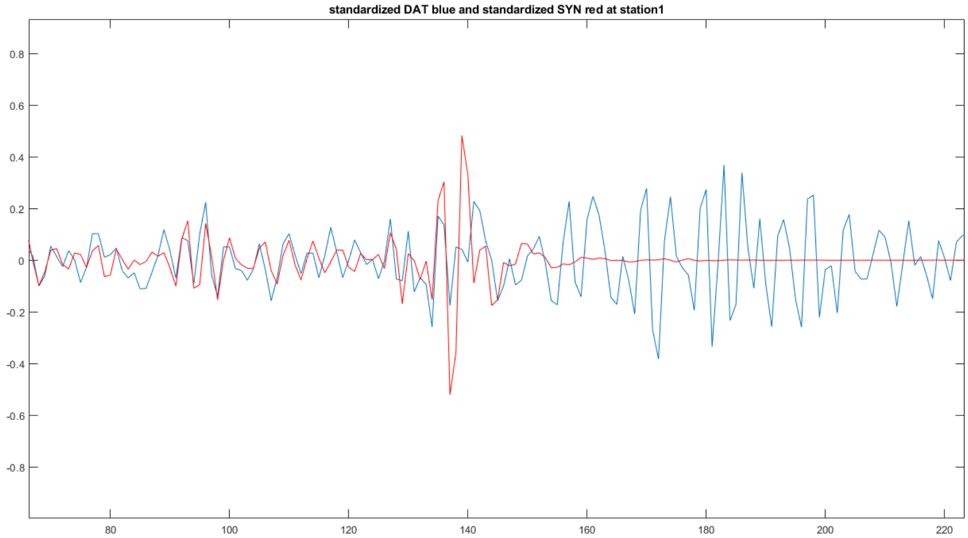
Covariance matrix is diagonal dominant. Non-diagonal elements indicate a short correlation length of errors (red line), standardization further shrinks (narrows) the correlation a bit.



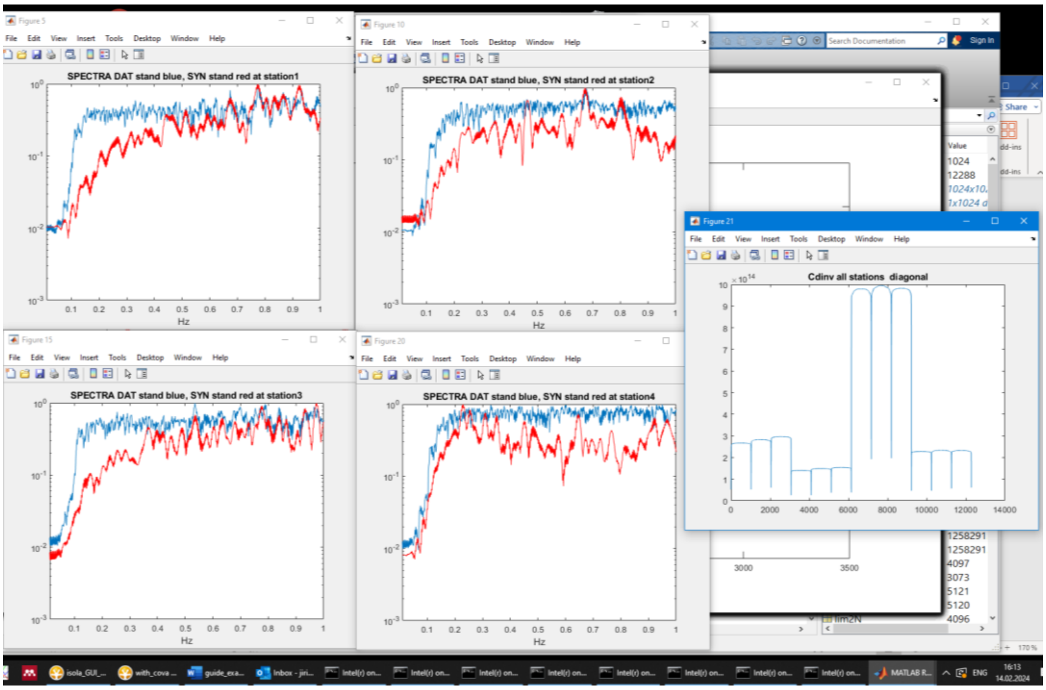


A formal improvement of VR in a standardized sense is found (Matlab command window output): varred_classical = 0.053533, varred_standardized = 0.12795. The poor waveform fit before and the somewhat better fit after standardization can be seen below (station 1, N component) – this is a manual zoom of one of the output plots from Matlab run.

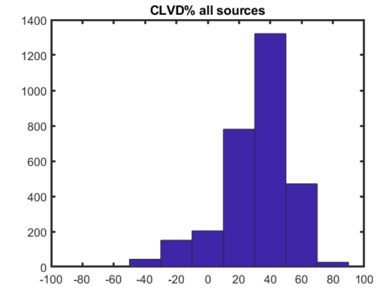


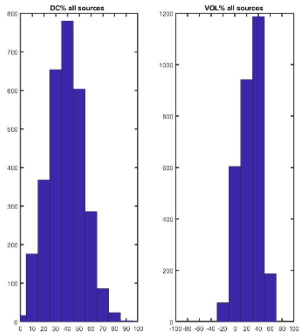


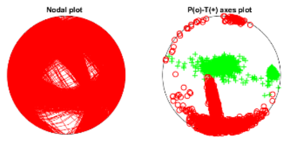
In the spectral domain, checking the comparison of spectra of standardized data and synthetics at stations 1-4, we may see a high-frequency fit at ~0.7-1.0 Hz in station 1 and 3. Station 3 features the largest value of the inverted covariance matrix (see the plot at the right below). These are indications that the cova stuff was helpful.

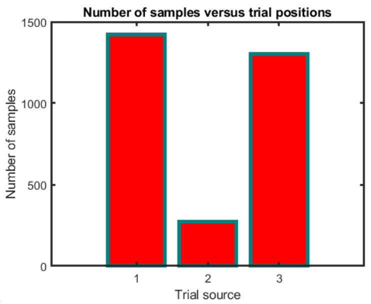


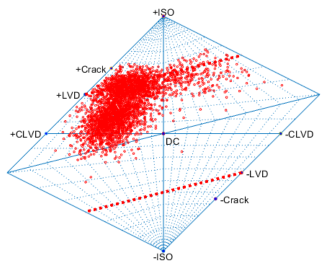
The uncertainty indicates poor resolution of nodal planes, concurrent positions 1 and 3, and strongly suggests a significant non-DC components. We know from previous tests that this result is most likely incorrect.











We conclude with a **warning**. Although above we got indications that cova stuff was helpful, in fact, it was not much helpful in this particular test. The reason is that although formally we see a shortened correlation length of the data error, the covariance matrix (derived from obs-syn residuals) does not correctly reflect the so-called theory error, i.e. absence of coda waves in high-frequency synthetics.