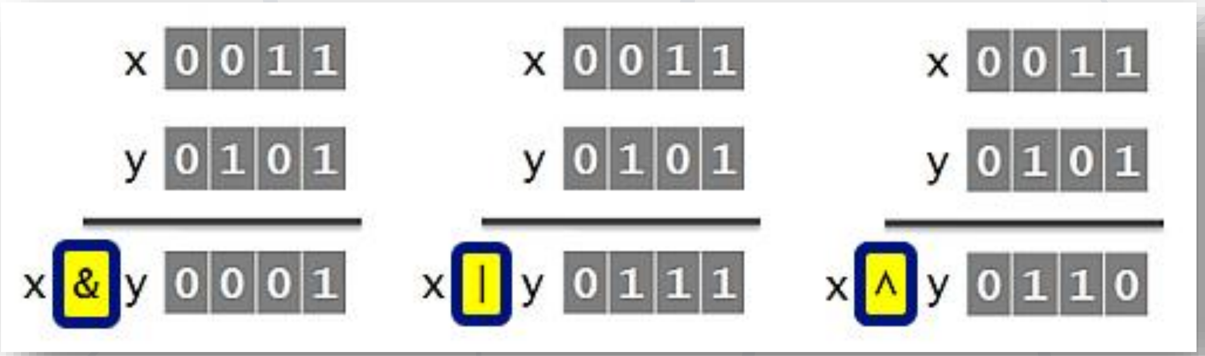
Bits and Bitwise Operations

Bits, Numerals Systems and Bitwise Operations



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Bits

What is a Bit?

Bit

- **Bit** == the smallest **unit of data used in computing**
  - Takes only one of **two values**: either a **0** or **1**
- **1 bit** can store anything with **two separate states**
  - Logical values (true / false)
  - Algebraic signs (+ / -)
  - Activation states (on / off)
- Bits are organized in computer memory in sequences of **8 bits**, called **bytes** (octets)

Bit, Byte, KB, MB, GB, TB, PB

- **Bit** – single **0** or **1**, representing a bit of data
- **Byte** (octet) == **8 bits** == the smallest addressable unit in the computer memory
- **KB** (kilobyte) == **1024 bytes** (sometimes 1000 bytes)
- **MB** (megabyte) == **1024 KB** == **1048576 bytes**
- **GB** (gigabyte) == **1024 MB** == **1073741824 bytes**
- **TB** (terabyte) == **1024 GB** == **1099511627776 bytes**
- **PB** (petabyte) == **1024 TB** == **1125899906842624 bytes**

5

101_b

0x8

Numerals Systems

Decimal, Binary and Hexadecimal

Numeral Systems

- **Numeral system** == system for **representing numbers** in written form using sequence of **digits**
- **Positional numeral systems** == the value of each digit depends on its position
  - These numeral systems have a **base** (e.g., 2, 10, 16)

Decimal (base = 10)	Binary (base = 2)	Hexadecimal (base = 16)
30	111110	1E
45	101101	2D
60	111100	3C

Decimal Numbers

- Decimal numbers (**base 10**)
  - Represented using 10 digits:
    - **0, 1, 2, 3, 4, 5, 6, 7, 8, 9**
  - Each position represents a **power of 10**

```
401 = 4*10² + 0*10¹ + 1*10⁰ =
    = 4*100 + 0*10 + 1*1 =
    = 400 + 0 + 1 = 401
```

- A decimal number $\mathbf{d_{n-1}d_{n-2}...d_1d_0}$ = $\mathbf{d_0}*10^0 + \mathbf{d_1}*10^1 + \mathbf{d_2}*10^2 + ... + \mathbf{d_{n-1}}*10^{n-1}$

Binary Numbers

- The **binary system** is used in computer systems
- Binary numbers (**base 2**)
  - Represented by **sequence of 0** or **1**

```
5 == 101b
```

  - Each position represents a **power of 2**

$101_b = 1 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0 = 4 + 0 + 1 = 5$

$1010_b = 1 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 0 \cdot 2^0 = 8 + 0 + 2 + 0 = 10$

Binary and Decimal Conversion

- **Binary to decimal**
  - Multiply each digit to its magnitude (power of 2)

```
1011b  = 1*2³ + 0*2² + 1*2¹ + 1*2⁰ =
       = 1*8 + 0*4 + 1*2 + 1*1  =
       = 8 + 0 + 2 + 1 =
       = 11
```

- **Decimal to binary**
  - Divide to the base (2) until 0 is reached and take the reminders in reversed order

```
11 / 2 = 5 (1) // last digit
5 / 2 = 2 (1)  // previous digit
2 / 2 = 1 (0)  // previous digit
1 / 2 = 0 (1)  // fist digit
Result: 1011
```

Problem: Binary Digits Count

- You are given a positive integer **n** and a binary digit **b** (0 or 1)
- Write a program that finds the count of **b** digits in the binary representation of **n**

```
20
0
```
→ `3`

```
15
1
```
→ `4`

```
10
0
```
→ `2`

```
23
1
```
→ `4`

Solution: Binary Digits Count

1. **Read the input** from the user: **n** and **b**
2. **Convert the input to binary** system
   (collect the reminders of division by 2)
3. **Count the digits b** in the reminders of **n**
4. Print the **count**

- Another solution is to use **bitwise operations** (think how later)

Hexadecimal Numbers

- Hexadecimal numbers (**base 16**)
  - Represented using **16 literals** (hex digits)
    - **0**, **1**, **2**, **3**, **4**, **5**, **6**, **7**, **8**, **9**, **A**, **B**, **C**, **D**, **E** and **F**
- Usually **prefixed with 0x** in computer science
- Each position represents a **power of 16**

```
0xB7F6 = B*16³ + 7*16² + F*16¹ + 6*16⁰ =
       = 11*4096 + 7*256 + 15*16 + 6*1 =
       = 45056 + 1792 + 240 + 6 = 47094
```

Hex ↔ Decimal Conversions

- **Hexadecimal to decimal**
  - Multiply each digit to its weight (power of 16)

```
0x1F4 = 1*16² + 15*16¹ + 4*16⁰ =
      = 1*256 + 15*16 + 4*1 =
      = 256 + 240 + 4 =
      = 500
```

- **Decimal to hexadecimal**
  - Divide by 16 and take the reminders in reversed order

```
500 / 16 = 31 (4)
31 / 16 = 1 (F)
1 / 16 = 0 (1)
Result: 0x1F4
```

Hex ↔ Binary Conversions

- The conversion from **binary** to **hexadecimal** (and back) is straightforward
  - Each hex digit corresponds to a **sequence of 4 binary digits**

```
A2E3F = 1010 0010 1110 0011 1111
A = 1010
2 = 0010
E = 1110
3 = 0011
F = 1111
```

$1010\ 0010\ 1110\ 0011\ 1111 = A2E3F$

$1010_b = 10_{dec} = A_{hex}$

$0010_b = 2_{dec} = 2_{hex}$

$1110_b = 14_{dec} = E_{hex}$

$0011_b = 3_{dec} = 3_{hex}$

$1111_b = 15_{dec} = F_{hex}$

Representation of Data

Integers, Floating-Point Numbers and Text

Representing Integers in Memory

- **Integer numbers** are sequences of bits
- Can be **signed** (in most cases) or **unsigned**
  - The **sign** == the **Most Significant Bit** (**MSB**)
  - Leading **0** → **positive number**
  - Leading **1** → **negative number**
- Example (8-bit signed integers)

```
0XXXXXXXb > 0  // 00010010b = 18
00000000b = 0
1XXXXXXXb < 0  // 10010010b = -110
```

Representation of Signed Integers

- **Positive 8-bit** numbers have the format **0XXXXXXX**
  - The value is the decimal value of their last **7 bits** (**XXXXXXX**)
- **Negative 8-bit** numbers have the format **1YYYYYYY**
  - The value is **-128** (**-2^7**) **+** the decimal value of **YYYYYYY**

$$
\begin{aligned}
10010010_b &= -2^7 + 0010010_b = \\
&= -128 + 18 = \\
&= -110
\end{aligned}
$$

-2^7

Largest and Smallest Signed Integers

- The **largest signed 8-bit integer** is 2^7-1

```
127 = (2⁷ – 1) = 01111111b
```

- The **smallest negative 8-bit integer** is -2^7

```
-128 = -(2⁷) = 10000000b
```

- The **largest signed 32-bit integer** is 2^{31}-1

```
2147483647 = (2³¹ – 1) = 0111…1111b
```

- The **smallest negative 32-bit integer** is -2^{31}

```
-2147483648 = -(2³¹) = 1000…0000b
```

Integers and Their Ranges in Programming

Bits	Sign	Range	Data Types
8-bit	signed	-128 … 127 (-2^7 … 2^7-1)	**sbyte** in C#, **byte** in Java
8-bit	unsigned	0 … 255 (2^0 … 2^8-1)	**byte** in C#
16-bit	signed	-32768 … 32767 (-2^{15} … 2^{15}-1)	**short** in C#, **short** in Java
32-bit	signed	-2,147,483,648 … 2,147,483,647 (-2^{31} … 2^{31}-1)	**int** in C#, **int** in Java

Representing Real Numbers

- Computers use the **floating-point number** format, defined by the **IEEE 754 technical standard**
- The **IEEE-754** standard defines:
  - Arithmetic and exchange formats – representations of the binary and decimal floating-point data
  - Rounding rules for floating-point numbers
  - Operations – arithmetic and other operations
  - Special numbers – such as **infinity** and **NaN**

Storing Floating-Point Numbers

- Floating-point numbers are stored as sequence of bits: **sign bit**, **exponent** and **mantissa**

Play with the **IEEE-754 converter online**: https://h-schmidt.net/FloatConverter/IEEE754.html

32 Bits

1 10000011 01010010100000000000000 ≈ -21.15625

Sign **Exponent** **Mantissa**

- Note: **errors in calculations** and **precision** may occur
  - Some numbers (e.g., 0.3) cannot be represented in the above format without rounding (as a sum of negative powers of 2)

Representing Text

- Computers represent **text characters** as unsigned integer numbers (i.e. as sequence of bits)
  - Letters, digits, punctuation chars, etc.

A

- The **ASCII** standard represent chars as **8-bit integers**
  - Defines the ASCII code for 127 chars, e.g.

Binary	Dec	Hex	Char
0b01000001	65	0x41	A
0b01000010	66	0x42	B
0b00101011	43	0x2B	+

Representing Unicode Text

- The **Unicode** standard represents 100,000+ text characters as **16-bit integers** (see unicode.org)
  - Supports many alphabets, e.g., Latin, Cyrillic, Arabic

Decimal	Hex	Char	Explanation
65	0x0041	A	Latin "A"
1097	0x0449	щ	Cyrillic letter "Sht"
1576	0x0628	ب	Arabic letter "Beh"
127928	0x1F3B8	🎸	Emoji "Guitar"

  - **UTF-16** uses 2 bytes (16 bits) for each char
  - **UTF-8** uses 1, 2, 3 or 4 bytes for each char

Sequences of Characters

- **Strings** represent **text data** in programming
  - **Strings** are **arrays of characters**, typically represented like this

len	0	1	2	3	4
5	H	e	l	l	o

Takes **14 bytes** in memory
4 bytes (length) + 5 * 2 bytes

  - The string can have its **size as prefix** (used in most languages) or can end with **\0** (null-terminated string – used in C)
  - Characters in the string can be
    - **16-bit** (UTF-16) – default in C#, Java, JS, Python
    - **8-bit** (ASCII / windows-1251) – default in C, C++

Bitwise Operations

Bitwise Operators and Bit Shifts

Bitwise Operators

- **Bitwise operators** works with the binary representations of the numbers, applying **bit by bit** calculations
- The operator `~` turns all `0` to `1` and all `1` to `0` (like `!` for boolean expressions but bit by bit)
- The operators `|`, `&` and `^` behave like `||`, `&&` and `^` for boolean expressions but bit by bit

Operator	|	|	|	|	&	&	&	&	^	^	^	^
Operand1	0	0	1	1	0	0	1	1	0	0	1	1
Operand2	0	1	0	1	0	1	0	1	0	1	0	1
Result	0	1	1	1	0	0	0	1	0	1	1	0

Bitwise Operators – Examples

- Bitwise **NOT (~)**

```
5      // 0101
~5     // 1010
```

- Bitwise **OR (|)**

```
5      // 0101
3      // 0011
5 | 3  // 0111
```

- Bitwise **AND (&)**

```
5      // 0101
3      // 0011
5 & 3  // 0001
```

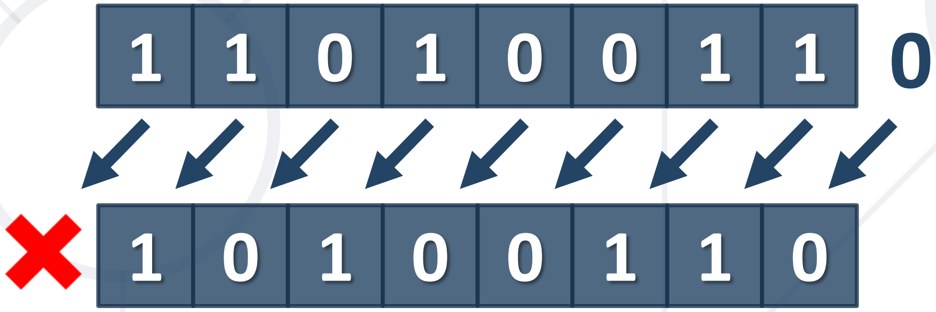
- Bitwise **XOR (^)**

```
5      // 0101
3      // 0011
5 ^ 3  // 0110
```

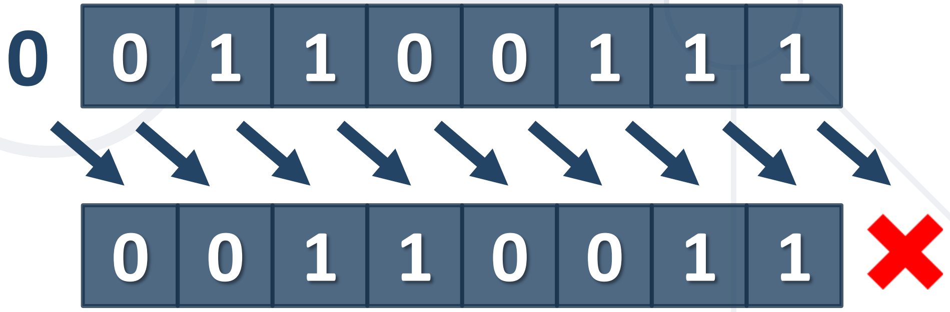
Bit Shifts

- **Bit shifts** are bitwise operations, where
  - Bits are moved (**shifted**) to the **left** or **right**
  - The bits that fall outside the number are **lost** and **replaced by 0**

- **Left** shift (**<<** operator)



- **Right** shift (**>>** operator)



Bitwise Operations: Get the Last Bit

- How to **get the last bit** from a number **n**?
  - The bits are **numbered from 0**, from right to left
  - The position of the last (rightmost) bit is **0**

```
n = 125     //   01111101
mask = 1    // & 00000001
n & mask    //   00000001 = 1
```

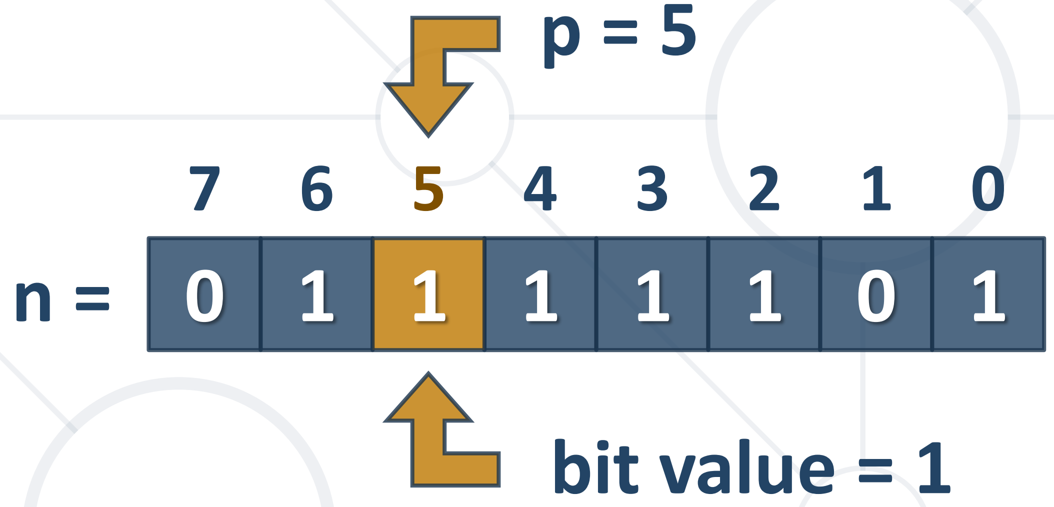
- Last bit – formula:

```
lastBit = n & 1
```

Bitwise Operations: Get Bit at Position

- How to **get the bit at position p** from a number **n**?

```
n = 125     // 01111101
p = 5       // 5th position
125 >> p    // 00000011 = 3
3 & 1       // 00000001 = 1
```



- Bit at position – formula:

```
bit = (n >> p) & 1
```

Bitwise Operations: Set Bit at Position

- How to **set the bit** at given position **p** to **0** or **1**?
  - **Clear** a bit (0) at position **p**

```
p = 5                // 5th position
n = 125              // 01111101
mask = ~(1 << p)     // 11011111
result = n & mask    // 01011101
```

  - **Set** a bit (1) at position **p**

```
p = 5                // 5th position
n = 125              // 01111101
mask = 1 << p        // 00100000
result = n | mask    // 01111101
```

- Assign a bit **b** (0 or 1) at position **p** – formula:

```
n = n & ~(1 << p) | (b << p)
```

Why We Need Bitwise Operations?

- **Networking protocols**
  - Many devices communicate using bit-level protocols
  - e.g., the SYN flag in the **TCP protocol** header is the bit #1 from the 14th byte in the TCP packets
    - Web browsers use bitwise operations to connect to a Web site
- Many **binary file formats** use bits to save space
  - e.g., PNG images use 3 bits to specify the color format used
- **Data compression** replaces byte sequences with bit sequences
  - e.g., the DEFLATE algorithm in **ZIP files**

Problem: Bit #1 (the Bit Before the Last)

- Write a program that prints the **bit** at **position 1** of an **integer**

Input		Output	Binary
51	→	1	**51** == 0 0 1 1 0 0 **1** 1
24	→	0	**24** == 0 0 0 1 1 0 **0** 0
13	→	0	**13** == 0 0 0 0 1 1 **0** 1
2	→	1	**2** == 0 0 0 0 0 0 **1** 0

- Solution:

```
p = 1          // 1st position
n = 51         // 00110011
n = n >> p     // 00011001 = 25
n & 1          // 1
```

Summary

- Computers store data using **bits**
  - Signed **integers** (leftmost bit == sign)
  - **IEEE-754** – floating point numbers
  - **Text** is stored using ASCII / Unicode / other
- **Binary** and **hexadecimal numeral systems** play a key role in computing
- Developers manipulate **bits** in integers using **bitwise operators** and **bit masks**