Tutorial 2

Question1

Let V be a vector space over $\mathbf{R}$. Show that If $\{\mathbf{v}, \mathbf{u}, \mathbf{w}\} \subset V$ is linearly independent, $\{2\mathbf{v} - 3\mathbf{u}, -4\mathbf{v} + 5\mathbf{w}, \mathbf{2u} - \mathbf{w}\}$ is also linearly independent.

Question 2

Let $V = C^1(\mathbf{R})$ with the standard vector addition and scalar multiplication.

(a) Find a basis for $\mathrm{Span}(\{e^x, e^{2x}, e^{3x}\})$. What is the dimension of the span set?

(b) Let $S = \{f \in \mathrm{Span}(\{e^x, e^{2x}, e^{3x}\} | \, f(0) = f'(0) = 0\}$. Find a basis and the dimension of S.

(c) Let $h(x) = e^x - 2e^{2x} + e^{3x}$.
Define $T : \mathrm{Span}(\{e^x, e^{2x}, e^{3x}\}) \longrightarrow S$ by $T(f(x)) = (c_1 + c_2 + c_3)h(x)$,
where $f(x) = c_1e^x + c_2e^{2x} + c_3e^{3x}$. Is T well defined? Explain it.

(d) If T is well defined, is it a linear transformation?

Question 3

Let V be a real vector space and $\alpha = \{\mathbf{x}_1, \cdots, \mathbf{x}_n\}$ be a basis for V.
Show that the function $[\cdot]_\alpha$:V$\longrightarrow \mathbf{R}^n$ defined by $[\mathbf{x}]_\alpha = (a_1, \cdots, a_n)$, where $\mathbf{x} = a_1\mathbf{x}_1 + \cdots + a_n\mathbf{x}_n$, is a linear transformation. We call $[\mathbf{x}]_\alpha$ the coordinates of the vector $\mathbf{x}$ with respect to the basis α.