

**Tutorial 4**

**Problem 1** Let  $\alpha = \{\mathbf{u}_1, \mathbf{u}_2\}$ ,  $\beta = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ , and  $\gamma = \{\mathbf{w}_1, \mathbf{w}_2\}$  be bases of vector spaces  $U, V$ , and  $W$  respectively. Let  $S : U \longrightarrow V$  be defined by  $S(\mathbf{u}_1) = \mathbf{v}_1 + \mathbf{v}_2 + \mathbf{v}_3$  and  $S(\mathbf{u}_2) = -\mathbf{v}_1 + 2\mathbf{v}_2 - 3\mathbf{v}_3$ . Let  $T : V \longrightarrow W$  be defined by  $T(\mathbf{v}_1) = \mathbf{w}_1 - \mathbf{w}_2$ ,  $T(\mathbf{v}_2) = 2\mathbf{w}_1 + \mathbf{w}_2$ , and  $T(\mathbf{v}_3) = \mathbf{w}_1 - 2\mathbf{w}_2$

- (a) Find  $TS$ .
- (b) Compute  $TS(2\mathbf{u}_1 + 3\mathbf{u}_2)$ .
- (c) Is  $TS$  invertible? Then find  $(TS)^{-1}$ .