Tutorial 1

Question1

Let V be a real vector space and $\mathbf{0}$ be the additive identity of V. Show that for any $\mathbf{x} \in V$,

(a) $0\mathbf{x} = \mathbf{0}$
(b) $(-1)\mathbf{x} = -\mathbf{x}$

Question 2

(a) Show the $W_1 = \{\begin{bmatrix} a_{11} & 0 \\ 0 & a_{22} \end{bmatrix} \mid a_{11}, a_{22} \in \mathbf{R}\}$ is a subspace of $M_{2\times 2}(\mathbf{R})$.

(b) Show that $W_2 = \{\begin{bmatrix} 0 & a_{12} \\ 0 & 0 \end{bmatrix} \mid a_{12} \in \mathbf{R}\}$ is a subspace of $M_{2\times 2}(\mathbf{R})$.

(c) Show that $W_1 \bigcap W_2 = \{\mathbf{0}\}$ where $\mathbf{0}$ is the additive identity of $M_{2\times 2}(\mathbf{R})$.

(d) Show that $W_1 \bigoplus W_2$= Span $\{\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\})$

* Notation: If $W_1 \bigcap W_2 = \{\mathbf{0}\}$, denote $W_1 + W_2$ by $W_1 \bigoplus W_2$.

Question 3

Let V be a real vector space and W_1 and W_2 be subspaces of V.

Show that every vector of $W_1 \bigoplus W_2$ is expressed uniquely. That is, if $\mathbf{x}_1 + \mathbf{y}_1 = \mathbf{x}_2 + \mathbf{y}_2$, $\mathbf{x}_1 = \mathbf{x}_2$ and $\mathbf{y}_1 = \mathbf{y}_2$, where $\mathbf{x}_1, \mathbf{x}_2 \in W_1$ and $\mathbf{y}_1, \mathbf{y}_2 \in W_2$.