

Tutorial 3

Problem 1 Let $F : V \longrightarrow W$ be a bijective (injective and surjective) linear mapping and $V = U_1 \oplus U_2$. Show that $W = F(U_1) \oplus F(U_2)$.

Problem 2 (MAT223 question) With respect to the standard bases, let $T : \mathbf{R}^3 \longrightarrow \mathbf{R}^2$ be given by

$$\begin{bmatrix} 2 & 1 & 3 \\ -4 & -2 & -6 \end{bmatrix}$$

Determine bases $\alpha = \{\mathbf{u}, \mathbf{v}_1, \mathbf{v}_2\}$ and $\beta = \{\mathbf{w}_1, \mathbf{w}_2\}$ in such a way that

- $\text{Ker}(T) = \text{Span}\{\mathbf{v}_1, \mathbf{v}_2\}$
- $\text{Im}(T) = \text{Span}\{\mathbf{w}_1\}$
- $T(\mathbf{u}) = \mathbf{w}_1$