Tutorial 7

Problem 1 Let $\{1, \sin x, \cos x\})$ be a subset of the vector space $C^\infty(\mathbf{R})$ and let $T : \text{Span}(\{1, \sin x, \cos x\}) \longrightarrow \text{Span}(\{1, \sin x, \cos x\})$ be a linear transformation defined by $T(1) = 1 + 2\sin x + 3\cos x$, $T(\sin x) = 2\sin x + 3\cos x$, and $T(\cos x) = 2\cos x$.

(a) Find the eigenvalues and eigenvectors of T

(b) Determine whether the given linear mapping is diagonalizable. If it is diagonalizable, find a basis of $\text{Span}(\{1, \sin x, \cos x\})$ consisting of eigenvectors.

Problem 2 Let $V = \text{span}\{(1, 1, -1, 1), (1, 0, 0, -1)\}$.

(a) Find the orthogonal complement, $V^\perp$.
(b) Find an orthogonal basis for $V^\perp$.
(c) Find an orthonormal basis for $V^\perp$.