Thesis Template for LaTeX

Author:
Jabir Ali Ouassou

Supervisor:
Anonymous

NTNU

Abstract

This LaTeX template was put together by the student Jabir Ali Ouassou ⟨jabirali@switzerlandmail.ch⟩ while working on a Master's project in 2014. I'm sharing this template because a few people have asked me about tips regarding writing a thesis in LaTeX, and I hope that my work can be of some use to others as well. If you find the template useful, then I would really appreciate it if you gave me some credit for my work, such as a short mention in the acknowledgements of your document. But this is of course a request, not a requirement; you're free to share the template with whomever you want, and do whatever you want with it, without my permission.

If you're using a Linux system, then you should start by installing a full LaTeX distribution such as `texlive-full`. After that, you can use the accompanying `Makefile` to compile the LaTeX document; this means that you just have to open a terminal, use `cd /wherever/the/template/is/`, and then run `make` to compile the document. On all the common desktop platforms, you should also be able to install an integrated development environment for LaTeX such as http://texstudio.sourceforge.net, which allows you to compile the document straight from the graphical interface. However, be aware that some development environments may require additional configuration to work well with `bibtex`, `biblatex`, and `makeidx`.

Contents

Preface

You will probably want to replace this text with the preface of your document. In the mean time, the coming section provides a summary of how this LaTeX template is structured.

master.tex: This is the master document which is compiled by LaTeX. This is where you refer to all the other LaTeX files that are a part of the, and also where you define the metadata such as the author, supervisor, and title of the document.

master.ist: This file contains the style for the index that will appear at the back of the document. You probably won't need to touch this file. You can add entries to the index by using the macro `\index` in your regular `.tex` files.

library.bib: This is a BibTeX file that contains a database over all the books and papers that you intend to cite. Only references that are actually used will appear in the bibliography; so remember that anything you add to this file will be invisible in your document until you `\cite` it. This bibliography database can be written and maintained manually (check the included example, or google bibtex to find a guide), but I highly recommend that you instead use some automated tool like Mendeley to generate it for you.

preamble/include.tex: This is where you should include new LaTeX packages using the `\usepackage` macro, or perhaps tweak the arguments of existing packages.

preamble/input.tex: This is where you declare unicode symbols that you wish to use in LaTeX.

preamble/style.tex: This is where you declare stylistic properties of the document, such as margins, headers, footers, and so on.

preamble/macro.tex: This is where you should define your custom LaTeX macros.

preliminaries/*.tex This is the location of the titlepage, abstract and preface of your document.

chapters/*.tex This folder should contain all the chapters of your document as separate .tex files. Remember that for every .tex file you add here, you should also add a corresponding statement `\includemainmatter/filename.tex` in `master.tex` for the contents to appear in the document.

appendices/*.tex This folder should contain your appendices as separate .tex files. Remember to add a corresponding `\includebackmatter/filename.tex` in `master.tex`.

1 Introduction

When an observer of an object moves relative to the object, there is an apparent relative motion in the image plane of the observer. The problem of determining this relative motion from a sequence of images is called the Optical Flow problem. The analysis is not so much dependent on prior knowledge of the scene, but on the image sequence itself. This independency makes it applicable in many different fields. More concisely, one wants to find flow vector components $u, v \in \mathbb{R}$ by looking at the change in brightness $f(x,t)$ at a specific pixel from one frame to another for $x \in \Omega$, where Ω is considered to be a rectangular domain. This is problematic because we can not independently determine a vector of 2 components using one constraint coming from the change in image brightness at a point $x \in \Omega$. Thus we need to impose other constraints to make the problem solvable.

Is there always a relationship between the change in the brightness and the movement of objects in the image? It is not hard to see that the answer is no. For instance, imagine rotating a uniform sphere exhibiting a nonuniform brightness pattern over its surface. This rotation is not observable in the image plane, and would result in zero optical flow. Also, if the illumination of the image scene changes rapidly, brightness changes in the image plane may not be due to moving objects. These examples illustrate that optical flow does not always correspond to the relative movement of an object. Nonetheless, in the following model for optical flow the image scene is assumed to be simple so that brightness changes can be directly related to object motion.

2 The Brightness Constancy Assumption

The starting point for the variational approach to optical flow is the so called brightness constancy assumption of Horn and Schunck[1]. Let $f(x,t)$ be the grayscale value of some image sequence. To constrain the problem one makes an assumption regarding invariance in the brightness: a point moving with velocity $\frac{dx}{dt} = u(x,t)$ along the trajectory $x(t)$ over time t does not change its appearance. This assumption is called the brightness constancy assumption, and it means that if the scene has the same lighting, then movement of an object along a trajectory does not change its brightness.

2.1 The Data Term

In mathematical notation this is (under perfect conditions) equivalent to the following:

$$\frac{d}{dt} f(x(t),t) = 0.$$

By using the chain rule for differentiation, and defining $\frac{dx}{dt} = u$ and $\frac{dy}{dt} = v$, one gets

$$\frac{\partial f}{\partial x} u + \frac{\partial f}{\partial y} v + \frac{\partial f}{\partial t} = 0,$$

or equivalently, by defining $\mathbf{w} = (u,v)^T$, $\nabla f^T \mathbf{w} + f_t = 0$. Following the notation and terminology of Zimmer et al.[2], the constraint coming from the brightness constancy assumption is called the data term $M(u,v)$.

2.2 Penalizing the Data Term

To minimize the data term above, Horn and Schunck used a quadratic penalizer, which is equivalent to least-squares minimization. Least-squares minimization is very sensitive to outliers. In the case of the data term, these outliers would be noise in the image (a pixel that jumps from low intensity to high intensity without corresponding to the motion of an object). Due to the poor robustness to outliers of least-squares minimization other penalization functions have been suggested. Black and Anandan[3] proposed several subquadratic penalizer functions, arguing that a

[1] Horn and Schunck 1980.
[2] Zimmer, Bruhn, and Weickert 2011.
[3] Black and Anandan 1996.

subquadratic penalizer would improve the robustness in the presence of outliers. Letting

$$\rho(\mathbf{w}) = \nabla f \cdot \mathbf{w} + f_t$$

and assuming the data term has some quadratic dependence on ρ, one can define

$$M(u,v) = \Psi_M(\rho^2), \tag{2.1}$$

where Ψ_M is some penalizing function aiming to minimize ρ. Since the flow consists of 2 components, the brightness constancy assumption alone is not enough to determine the flow, but only the components of the flow in the direction of the gradient, or what is known as the normal flow. This is called the Aperture problem, and to be able to compute the components of the flow vector one needs another constraint. This other constraint is known as the smoothness constraint.

3 The Smoothness Constraint

The smoothness constraint, also called the spatial coherence assumption[1], says that points can not move independently in the brightness pattern. There has to be some smoothness in the flow vector for points belonging to the same object. In other words, points on the same object moves with the same velocity. A natural way of obtaining a smoother solution would be to minimize some term depending on the sizes of the gradients in some direction. The smoothness terms considered here will be quadratic with respect to the gradient in each direction, thus it is convenient to write the smoothness term in the form

$$V(\nabla u, \nabla v) = \nabla u^T \Theta_u \nabla u + \nabla v^T \Theta_v \nabla v. \tag{3.1}$$

The interpretation of the matrices Θ_u and Θ_v will become clear in the following section.

[1] Black and Anandan 1996.

4 Variational Formulation

To combine the two constraints into one term we form a global energy function consisting of the data term and the smoothness term:

$$E(u,v) = \frac{1}{2}\int_{\Omega} (M(u,v) + \frac{1}{\sigma^2} V(\nabla u, \nabla v))\,dx\,dy, \tag{4.1}$$

where $\sigma > 0$ is a regularization parameter. The problem is now to find the minimum of the energy functional $E(u,v)$. From calculus of variations we have that if $\mathbf{w}$ minimizes a functional

$$J(\mathbf{w}) = \iint\limits_{\Omega} F(x,y,\mathbf{w},\mathbf{w}_x,\mathbf{w}_y)\,dx\,dy,$$

then the first variation must be zero,

$$\delta J(\mathbf{w}) = \frac{d}{d\epsilon}\left[J(\mathbf{w}+\epsilon\eta)\right] = 0,$$

for any arbitrary function $\eta(x,y)$. We get

$$\begin{aligned}
\delta J(\mathbf{w}) &= \iint\limits_{\Omega} \frac{d}{d\epsilon} F(x,y,\mathbf{w}+\epsilon\eta,\mathbf{w}_x+\epsilon\eta_x,\mathbf{w}_y+\epsilon\eta_y)\,dx\,dy \\
&= \iint\limits_{\Omega} \eta F_{\mathbf{w}} + \eta_x F_{\mathbf{w}_x} + \eta_y F_{\mathbf{w}_y}\,dx\,dy \\
&= \iint\limits_{\Omega} \eta F_{\mathbf{w}} + \frac{d}{dx}(\eta F_{\mathbf{w}_x}) + \frac{d}{dy}(\eta F_{\mathbf{w}_y}) - \eta\left(\frac{d}{dx}F_{\mathbf{w}_x} + \frac{d}{dy}F_{\mathbf{w}_y}\right)dx\,dy
\end{aligned}$$

Now let Γ_E, Γ_W, Γ_N and Γ_S be the east, west, north and south boundary of our domain respectively. Then using Gauss' Theorem gives

$$\begin{aligned}
&\iint\limits_{\Omega} \frac{d}{dx}(\eta F_{\mathbf{w}_x}) + \frac{d}{dy}(\eta F_{\mathbf{w}_y})\,dx\,dy \\
&= \int_{\Gamma_e} \eta F_{\mathbf{w}_x}\,dx - \int_{\Gamma_w} \eta F_{\mathbf{w}_x}\,dx + \int_{\Gamma_n} \eta F_{\mathbf{w}_y}\,dy - \int_{\Gamma_s} \eta F_{\mathbf{w}_y}\,dy
\end{aligned}$$

Using this result, we get

$$\begin{aligned}
\delta J(\mathbf{w}) &= \iint\limits_{\Omega} \eta\left(F_{\mathbf{w}} - \frac{d}{dx}F_{\mathbf{w}_x} - \frac{d}{dy}F_{\mathbf{w}_y}\right)dx\,dy \\
&+ \left(\int_{\Gamma_E} \eta F_{\mathbf{w}_x}\,dx - \int_{\Gamma_W} \eta F_{\mathbf{w}_x}\,dx + \int_{\Gamma_N} \eta F_{\mathbf{w}_y}\,dy - \int_{\Gamma_S} \eta F_{\mathbf{w}_y}\,dy\right) = 0.
\end{aligned}$$

Since this must hold for any arbitrary function $\eta(x,y)$ it follows that

$$\begin{aligned} F_{\mathbf{w}} - \frac{d}{dx}F_{\mathbf{w}_x} - \frac{d}{dy}F_{\mathbf{w}_y} &= 0 \quad \text{in } \Omega \\ F_{\mathbf{w}_x} &= 0 \quad \text{on } \Gamma_e \text{ and } \Gamma_w \\ F_{\mathbf{w}_y} &= 0 \quad \text{on } \Gamma_n \text{ and } \Gamma_s \end{aligned}$$

This is called the Euler-Lagrange equation of variational calculus. From this result it is easy to see that the following must hold for our functional:

$$\begin{aligned} \partial_{\mathbf{w}} M - \frac{1}{\sigma^2}\left(\frac{d}{dx}\partial_{\mathbf{w}_x} V + \frac{d}{dy}\partial_{\mathbf{w}_y} V\right) &= 0 \quad \text{in } \Omega \\ \partial_{\mathbf{w}_x} V &= 0 \quad \text{on } \Gamma_E \text{ and } \Gamma_W \\ \partial_{\mathbf{w}_y} V &= 0 \quad \text{on } \Gamma_N \text{ and } \Gamma_S \end{aligned} \tag{4.2}$$

As previously noted, we are dealing with quadratic smoothness terms. Using the notation of (3.1), the first equation in the Euler-Lagrange system can be written as

$$\partial_q M - \frac{1}{\sigma^2}\operatorname{div}\left(\Theta_q \nabla q\right) = 0 \tag{4.3}$$

for $q \in u, v$. The matrix Θ_q is a diffusion matrix steering the direction of diffusion for each flow component. Its eigenvectors and corresponding eigenvalues gives the direction and magnitude of smoothing respectively. The theoretical framework presented up to this point is the same for all the methods considered here. The main distinction for each method will be across which boundaries the flow field is smoothed, that is, the choice of diffusion matrix. We start with the simplest choice; the uniform smoothness approach by Horn and Schunck.

5 The Approach of Horn and Schunck

The research of Horn and Shunck has formed the basis of further research in the field of optical flow. They proposed the following quadratic penalized data term

$$M(\mathbf{w}) = (\nabla f^T \mathbf{w} + f_t)^2, \tag{5.1}$$

which is equivalent to choosing

$$\Psi_M(\rho^2) = \rho^2$$

in the framework of (2.1). The contribution to (4.2) from the model term is thus

$$\partial_{\mathbf{w}} M = 2\nabla f(\nabla f^T \mathbf{w} + f_t) \tag{5.2}$$

The smoothness term used by Horn and Schunck is

$$V(\nabla u, \nabla v) = |\nabla u|^2 + |\nabla v|^2.$$

This is a homogeneous regularizer which means that it applies an equal amount of diffusion in all directions. In the framework of (4.3), this is equivalent to the diffusion matrices Θ_u and Θ_v being the identity matrix. Using this function as a flow regularizer gives

$$\partial_{\mathbf{w}_{x_i}} V = 2\mathbf{w}_{x_i},$$

for $x_i = x, y$. Dividing by 2 in all terms results in (4.2) taking the form

$$\begin{aligned}
(f_x u + f_y v + f_t) f_x - \frac{1}{\sigma^2}\left(\frac{d}{dx} u_x + \frac{d}{dy} u_y\right) = 0 &\quad \text{in } \Omega, \\
(f_x u + f_y v + f_t) f_y - \frac{1}{\sigma^2}\left(\frac{d}{dx} v_x + \frac{d}{dy} v_y\right) = 0 &\quad \text{in } \Omega \\
\mathbf{w}_x = 0 &\quad \text{on } \Gamma_e \text{ and } \Gamma_w, \\
\mathbf{w}_y = 0 &\quad \text{on } \Gamma_n \text{ and } \Gamma_s,
\end{aligned} \tag{5.3}$$

which can be seen as a system of coupled Poisson equations with Neumann boundary conditions:

$$\begin{aligned}
-\frac{1}{\sigma^2}\Delta u + f_x^2 u &= -(F(v) + f_t f_x) \\
-\frac{1}{\sigma^2}\Delta v + f_y^2 v &= -(F(u) + f_t f_y),
\end{aligned}$$

where $F(q) = f_x f_y q$.

5.1 Discretizing the Horn and Schunck method

Let now our image be of size m-by-n, and let f^1 and f^2 be the image at $t = 1$ an $t = 2$ respectively. Also, we flatten the regular 2-dimensional grid in Ω and consider now $(\xi^i)_{i\in[mn]}$ so that $(\xi^i) = (x, y) = (\lfloor i/m \rfloor, i - \lfloor i/m \rfloor)$, when assuming the distance between vertical and horizontal grid points in Ω are 1. The corresponding vector representation of the image f is denoted as $\mathbf{f}(\xi^i) \in \mathbb{R}^{mn}$. Continuing this notation, the discrete flow values $\mathbf{w}(\xi^i)$ is represented as the following vector in $\mathbb{R}^{2mn}$:

$$\mathbf{w}(\xi^i) = \begin{bmatrix} u(\xi^i)_{i\in[mn]} \\ v(\xi^i)_{i\in[mn]} \end{bmatrix}.$$

For the discretization of the image gradients in (5.3), the forward difference was used on f^1, producing the two vectors $\mathbf{d}_x(\xi^i)$ and $\mathbf{d}_y(\xi^i)$ in $\mathbb{R}^{mn)}$, where the gradients on the boundary are assumed to be zero. The time derivative f_t is discretized using forward difference with time step $\Delta t = t_2 - t_1 = 1$ as shown below:

$$\mathbf{c}(\xi^i) = \mathbf{f}^2(\xi^i) - \mathbf{f}^1(\xi^i),$$

where $\mathbf{c}(\xi^i)$ is a vector in $\mathbb{R}^{mn}$. Normally when choosing derivative approximations one wants as high order as possible so that the truncation error goes to zero as one increases the number of grid points. In this case the number of grid points is fixed, so there is little to gain from choosing higher order approximations; it is best to choose the derivative approximation that results in the simplest discretization. Thus for the flow vector in (5.3), the first derivative was approximated using backward difference, and the second was approximated using forward difference. Let now L_x and L_y be the matrices performing a forward difference on the components of the vector $\mathbf{w}$ in x- and y-direction respectively. The gradient is then represented as

$$L\mathbf{w}(\xi^i) = \begin{bmatrix} \tilde{u}_x(\xi^i)_{i\in[mn]} \\ \tilde{u}_y(\xi^i)_{i\in[mn]} \\ \tilde{v}_x(\xi^i)_{i\in[mn]} \\ \tilde{v}_y(\xi^i)_{i\in[mn]} \end{bmatrix}$$

where $\tilde{u}_x, \tilde{v}_x, \tilde{u}_y, \tilde{v}_y \in \mathbb{R}^{mn}$ are vector approximations to the derivatives. This means that L is the following block matrix:

$$L = \begin{bmatrix} L_x & 0 \\ L_y & 0 \\ 0 & L_x \\ 0 & L_y \end{bmatrix}.$$

Since (5.3) gives one set of equations for the interior nodes and one for the boundary nodes, these have to separated into two coupled systems. The interior system can be written as

$$(D^T D + \frac{1}{\sigma^2} L^T L)\mathbf{w}(\xi^i) = -D^T \mathbf{c}(\xi^i), \tag{5.4}$$

for $(\xi^i) \in \Omega$ where D is the block matrix

$$D = \left[\begin{array}{c|c} D_x & D_y \end{array}\right].$$

D_x and D_y are diagonal matrices with the elements of $\mathbf{d}_x(\xi^i)$ and $\mathbf{d}_y(\xi^i)$ for $(\xi^i) \in \Omega$ along its diagonals respectively. When using a forward difference approximation of the derivative, the derivative approximations in the points next to Γ_E and Γ_S on the grid will be dependent on points on these boundaries respectively, and the derivative on these boundaries are set to zero. Let $\alpha(\xi^i) = (\mathbf{d}_x(\xi^i)u(\xi^i) + \mathbf{d}_y(\xi^i)v(\xi^i) + \mathbf{c}(\xi^i))$. For $(\xi^{i+m}) \in \Gamma_E$ one gets

$$\begin{aligned}
\alpha(\xi^i)\mathbf{d}_x(\xi^i) - \frac{1}{\sigma^2}\left[L_x^T\left(u(\xi^i) - u(\xi^{(i+m)})\right) + L_y^T\left(u(\xi^i) - u(\xi^{(i+1)})\right)\right] &= 0\\
\alpha(\xi^i)\mathbf{d}_y(\xi^i) - \frac{1}{\sigma^2}\left[L_x^T\left(v(\xi^i) - v(\xi^{(i+m)})\right) + L_y^T\left(v(\xi^i) - v(\xi^{(i+1)})\right)\right] &= 0,
\end{aligned}$$

but since the derivative on the boundary is zero, one must enforce that $-L_x^T q(\xi^{i+m}) = 0$ for $q = u, v$. This leads to

$$\begin{aligned}
u(\xi^i) &= u(\xi^{(i+m)})\\
v(\xi^i) &= v(\xi^{(i+m)})
\end{aligned}$$

so

$$\begin{aligned}
\alpha(\xi^i)\mathbf{d}_x(\xi^i) - \frac{1}{\sigma^2}L_y^T L_y u(\xi^i) &= 0\\
\alpha(\xi^i)\mathbf{d}_y(\xi^i) - \frac{1}{\sigma^2}L_y^T L_y v(\xi^i) &= 0.
\end{aligned}$$

Equivalently when $(\xi^{i+1}) \in \Gamma_S$,

$$\begin{aligned}
u(\xi^i) &= u(\xi^{(i+1)})\\
v(\xi^i) &= v(\xi^{(i+1)})
\end{aligned}$$

which results in the following equations:

$$\begin{aligned}
\alpha(\xi^i)\mathbf{d}_x(\xi^i) - \frac{1}{\sigma^2}L_x^T L_x u(\xi^i) &= 0\\
\alpha(\xi^i)\mathbf{d}_y(\xi^i) - \frac{1}{\sigma^2}L_x^T L_x v(\xi^i) &= 0.
\end{aligned}$$

On the two other boundaries Γ_W and Γ_N, enforcing the forward differences to be zero at $(\xi^i) \in \Gamma_W$ leads to

$$\begin{aligned}
u(\xi^i) &= u(\xi^{(i+m)})\\
v(\xi^i) &= v(\xi^{(i+m)}),
\end{aligned}$$

and likewise for $(\xi^i) \in \Gamma_N$,

$$\begin{aligned}
u(\xi^i) &= u(\xi^{(i+1)})\\
v(\xi^i) &= v(\xi^{(i+1)}),.
\end{aligned}$$

Figure 5.1: First image in the Hamburg taxi sequence.

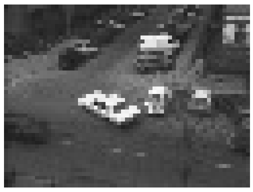

Figure 5.2: Second image in the Hamburg taxi sequence.

5.2 Results for the Horn and Schunck method

The results when running the Horn and Schunck algorithm on our test images shown in Figures 5.1 and 5.2 for different regularization parameters are shown in Figure **??**. The choice of regularization parameter depends on the application, but one often want to get rid of most of the internal structure in each object, since these points will be moving with the same velocity (when assuming no deformation of the object). One also want to have a good enough balance between segmentation and smoothing. It is seen that choosing $\sigma = 0.003$ gives a fairly good segmentation of the objects that are moving, and with almost no internal structure. The flow field with this choice of regularization parameter is shown in Figure **??** on its own. The Horn and Schunck smoothes the flow in all directions, and the flow boundaries are therefore a bit smudged.

6 Anisotropic Image Driven Regularization

The homogeneous regularizer of Horn and Schunck smoothes the flow field in all directions. Since we are looking for a flow field describing relative movement of objects, the boundary between objects will not have a smooth flow field unless they are moving at the same relative velocity. Methods that reduce the smoothing across image edges are called image-driven regularization methods. The anisotropic reguralizer of Nagel and Enkelmann (1986) performs smoothing along the image gradients and prevents smoothing across image edges. This is done by introducing a regularised projection matrix P, defined as

$$P(\nabla f) = \frac{1}{|\nabla f|^2 + 2\kappa^2}(\nabla^{\perp} f (\nabla^{\perp})^T + \kappa^2 I),$$

where $\nabla^{\perp} f = \left[-\frac{\partial f}{\partial y}, \frac{\partial f}{\partial x}\right]^T$, and $\kappa > 0$ is a regularization parameter. The smoothness term of Nagel and Enkelmann can now be written as the following

$$V_{AI}(\nabla u, \nabla v) = \nabla^T u P(\nabla f)\nabla u + \nabla^T v P(\nabla f)\nabla v,$$

or written out

$$V_{AI}(\nabla u, \nabla v) = \frac{\kappa^2}{|\nabla f|^2 + 2\kappa^2}\left(u_{\mathbf{s}_1}^2 + v_{\mathbf{s}_1}^2\right) + \frac{|\nabla f|^2 + \kappa^2}{|\nabla f|^2 + 2\kappa^2}\left(u_{\mathbf{s}_2}^2 + v_{\mathbf{s}_2}^2\right),$$

where $\mathbf{s}_1 = \frac{\nabla f}{|\nabla f|}$, $\mathbf{s}_2 = \frac{\nabla^{\perp} f}{|\nabla f|}$ and $q_{\mathbf{s}_i} = \mathbf{s}_i^T \nabla u$ for $q = u, v$. That is, $q_{\mathbf{s}_i}$ is the directional derivative of q in the direction of the image gradient ($i = 1$) or the orthogonal direction ($i = 2$). This means that setting $D = P$ in (4.3) steers the diffusion so that flow vectors are smoothed in the direction of the image gradients.

6.0.1 Discretizing the Nagel and Enkelmann smoothness term

Setting $\Theta = P$ in (4.3) leads to the following Euler-Lagrange system:

$$\frac{\partial M}{\partial u} - \frac{1}{\sigma^2}\operatorname{div}(P\nabla u) = 0$$
$$\frac{\partial M}{\partial v} - \frac{1}{\sigma^2}\operatorname{div}(P\nabla v) = 0.$$

Using the same discretizations for the derivatives as in 5.1 we get the following system:

$$(D^T D + \frac{1}{\sigma^2} L^T P L)\mathbf{w} = -D^T \mathbf{c}.$$

6.0.2 Results for the anisotropic image-driven regularization

Experiments were run to find the best regularization parameter in the Nagel and Enkelmann smoothness term. The regularization parameter σ found for the Horn and Schunck method is used to regularize the whole smoothness term. The resulting flow field for different choices of the regularization parameter κ, while keeping σ constant, is shown in Figure (**??**). It is seen that choosing $\kappa = 0.8$ gives a fairly good segmentation of the object. Since the values for the gradient from the sobel derivative are relatively high, it is expected to also having to choose a relatively large value for κ for sufficient regularization. Figure (**??**) compares the anisotropic smoothness term of Nagel and Enkelmann with $\kappa = 0.8$ with the homogeneous smoothness term of Horn and Schunck, both with using regularization parameter $\sigma = 0.003$.

7 Testing content

7.1 Test section

This section contains a few examples to demonstrate the most important macros. Let's start with a quick list: $\|A\|$ is the norm, $|A|$ is the absolute value, A^* is the complex conjugate, $A^\dagger$ is the hermitian conjugate, A^T is the matrix transpose, $\langle\phi|$ is a bra vector, $|\psi\rangle$ is a ket vector, $\langle A\rangle = \langle\phi|A|\psi\rangle$ is a quantum expectation value, ${R_\mu}^\nu{}_\rho{}^\sigma$ is a tensor, $[\mathbf{A},\mathbf{B}]_- \equiv \mathbf{AB}-\mathbf{BA}$ is a commutator, and $[\mathbf{A},\mathbf{B}]_+ \equiv \mathbf{AB}+\mathbf{BA}$ is an anticommutator. If you need to typeset for example radioactive isotopes, just use the notationWe can also write out a simple three-dimensional integral:

$$\int_{\mathbb{R}^3} \mathrm{d}^3\mathbf{r}\, f(\mathbf{r}) = \int_0^{2\pi} \mathrm{d}\theta \int_0^{\pi} \mathrm{d}\phi \sin\phi \int_0^{\infty} r^2 f(r,\theta,\phi) \tag{7.1}$$

And how about a few differential equations:

$$\alpha\frac{\mathrm{d}^2 f}{\mathrm{d}x^2} + \beta\frac{\mathrm{d}f}{\mathrm{d}x} + \gamma f(x) = 0 \qquad \nabla^2\hat{\boldsymbol{A}}(\mathbf{x}) = \sum_{ij} \frac{\partial\boldsymbol{\alpha}}{\partial x_i}\cdot\frac{\partial\boldsymbol{\beta}}{\partial x_j} \tag{7.2}$$

And maybe a few matrices and a determinant as well:

$$\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \qquad \mathbf{B} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \qquad \mathbf{C} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} \tag{7.3}$$

The rest of this chapter consists of the traditional *Lorem impsum* example text, with occasional examples of figures, tables, and code listings.

Lorem ipsum dolor sit amet, consectetur adipiscing elit. Ut faucibus, tortor porta imperdiet vestibulum, tortor lorem congue odio, a auctor velit dui a velit. Integer interdum metus ut neque semper, in aliquet nunc tristique. Phasellus ut mi elementum, semper felis vitae, varius urna. Nulla eget malesuada lorem. Nullam pharetra, elit in porta elementum, risus mauris ultrices massa, eget cursus lacus lectus molestie tortor. Vivamus consequat tempor mi vitae mattis. Cras vitae porttitor tellus. Duis tristique, diam nec hendrerit pulvinar, nisl ante consectetur diam, ut sagittis felis purus ut elit. Proin non tellus volutpat, consectetur est ut, mollis augue. Vivamus sollicitudin, nisl vel egestas bibendum, purus tortor rutrum enim, nec porttitor quam turpis eget velit. Mauris lacinia quis mi ut luctus. Sed ut eros metus.

Aenean vitae bibendum ligula, non cursus enim. Ut varius arcu nec nunc pulvinar, posuere mollis sem venenatis. Cras ullamcorper sollicitudin purus eu fermentum. Integer quis nulla eget leo viverra feugiat a eu turpis. Nam imperdiet nisl quis condimentum fermentum. Sed euismod diam egestas, varius magna et, aliquet metus. Nunc quis turpis mi. Maecenas feugiat facilisis ante vel porttitor. Aliquam eget metus sed lacus interdum vehicula at in nunc. Proin sit amet bibendum lorem. Nullam imperdiet at erat ultricies placerat. Sed ac dolor mollis, faucibus augue sed, lobortis tortor.

Figure 7.1: This is a test figure.

Sed id volutpat diam, at vestibulum dui. Mauris convallis justo nec neque luctus tincidunt. Donec vel ullamcorper nisi. Ut pulvinar quam non commodo lacinia. Nunc sollicitudin nibh eu malesuada egestas. Aenean euismod, sapien non molestie tincidunt, libero odio tempor massa, ac accumsan diam sapien sagittis magna. Donec non lectus turpis. Integer in consectetur ante. Nullam mollis suscipit ultricies. Ut auctor risus quis luctus euismod. Sed magna metus, mollis eget felis pretium, bibendum tempor sapien. Suspendisse sodales est sed tellus aliquam, quis bibendum orci laoreet. Sed consectetur justo metus, vitae tristique urna suscipit et. Curabitur id malesuada dui.

Table 7.1: This is a table caption.

Something	Something else	Something different
One	4.5	π
Two	4.7	e
Three	5.5	γ

Duis suscipit congue dolor, a tempus purus consequat sit amet. Sed a pulvinar eros. Phasellus ornare quam pulvinar, vestibulum justo vel, fringilla ipsum. Ut sagittis vehicula nunc, eu feugiat est vulputate vel. In vitae aliquam eros, et luctus dolor. Praesent varius ligula sit amet purus eleifend facilisis. Donec convallis, nisl vitae dignissim interdum, justo arcu convallis tortor, sit amet luctus ligula mi sit amet purus. Nullam congue neque in libero ultrices, a viverra turpis venenatis. Nullam at blandit ligula. Nunc non sodales nibh. Aliquam tempus arcu quis scelerisque convallis.

Listing 7.1: Here is the code caption...

```
#include <iostream>
using namespace std;

cout << "Hello world!" << endl;
```

Vestibulum rutrum placerat dapibus. Donec vitae leo mollis, convallis lorem vitae, tincidunt eros. Sed eleifend non sem a euismod. Fusce vel tincidunt diam. Praesent nec tristique lectus. Suspendisse a quam bibendum, mattis magna sed, feugiat tortor. Aenean ipsum est, fermentum a dui euismod, aliquam tristique nisl. Integer a faucibus elit. Donec pharetra justo ut lorem convallis, ut dictum ligula accumsan. Suspendisse vitae nisl vel massa placerat fringilla at a metus. Integer sed volutpat urna, in rutrum erat. Fusce eget arcu pulvinar dui ullamcorper tincidunt in ut augue. Aliquam erat volutpat. Aliquam tempus arcu non sapien tincidunt, ac ornare justo pharetra. Donec dignissim porttitor ornare. Integer eget sodales diam.

8 References and citations

8.1 Test section

We will now briefly demonstrate how to produce references and citations. The easiest way to consistently refer to equations and floats, is to use the macro `\cref`. For example we can refer to one equation, such as eq. (7.1); we can refer to multiple equations, such as eqs. (7.2) and (7.3); or we can even refer to different kinds of objects at the same time, such as fig. 7.1, table 7.1, and listing 7.1.

When we want to cite an academic paper or book, the standard option is to use the macro `\cite`.[1] By default, this produces a footnote citation, since these are unintrusive yet informative. However, if you prefer any other way of formatting the citations, then you just have to comment out the redefinitions of `\cite` and `\cites` from `preamble/macro.tex`, and then tweak the arguments to BibLaTeX in the file `preamble/include.tex` until you're satisfied. It is also possible to give optional arguments to the citations.[2] This includes the possibility of adding text to the footnote citations.[3] Furthermore, using commands like `\textcite`, it is also possible to cite Feynman, Leighton, and Sands (1963, pp. 10–15) inline. Finally, we can also cite multiple sources at once using the `\cites` macro.[4,5]

Note that all the papers and books that we cited above, have to be declared in the BibTeX bibliography file `library.bib`. The format of the bibliography entries should hopefully be clear from the included examples; if not, then as usual, Google is your friend.

[1] Touboul 2014.
[2] Johnson 2013, pp. 1–5.
[3] As an example, we will now refer to Feynman, Leighton, and Sands 1963, pp. 10–15.
[4] Touboul 2014, pp. 1–5; Johnson 2013, pp. 2,4; Lipovaca 2011, p. 25.
[5] This one is just a regular footnote, not a citation.

A Testing appendices

This is an appendix!

Bibliography

Black, M. J. and P. Anandan (1996). “The Robust Estimation of Multiple Motions: Parametric and Piecewise-Smooth Flow Fields”. In: *Computer Vision and Image Understanding* 63.1, pp. 75 –104. issn: 1077-3142. doi: http://dx.doi.org/10.1006/cviu.1996.0006 (cit. on pp. 3, 5).

Feynman, R., R. Leighton, and M. Sands (1963). *The Feynman Lectures on Physics*. 2nd Edition. Boston: Addison-Wesley. isbn: 9780201021165 (cit. on p. 19).

Horn, B. K. and B. G. Schunck (1980). *Determining Optical Flow*. Tech. rep. Cambridge, MA, USA (cit. on p. 3).

Johnson, V. E. (2013). “Revised standards for statistical evidence”. In: *Proceedings of the National Academy of Sciences of the United States of America* 110.48. issn: 1091-6490. doi: 10.1073/pnas.1313476110 (cit. on p. 19).

Lipovaca, M. (2011). *Learn You a Haskell for Great Good!* 1st ed. San Francisco, CA, USA: No Starch Press. isbn: 9781593272838 (cit. on p. 19).

Touboul, J. (2014). “The hipster effect: When anticonformists all look the same”. In: arXiv: 1410.8001 (cit. on p. 19).

Zimmer, H., A. Bruhn, and J. Weickert (2011). “Optic flow in harmony”. In: *International Journal of Computer Vision* 93.3, pp. 368–388 (cit. on p. 3).